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Article

# Quark Confinement as Charged Micro-Universes

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**Abstract:** This study explores the significant influence of charge  $Q$  on the discrete energy spectrum and interactions of particles within a confined system. We analyze how variations in  $Q$ , particularly as it transitions from zero to non-zero values, reveal critical insights into particle behavior at the quantum level. Central to our findings is the strong coupling factor  $\kappa$ , which underscores the intricate relationship between intrinsic system energies and those generated through particle interactions. Our results demonstrate the inverse proportionality of  $\kappa$  to energy density and its direct relationship with the confinement radius, reflecting the principles of confinement theory in hadrons. Additionally, we elucidate the connection between the energy of a free particle and the energy resulting from strong interactions, as governed by  $\kappa$ . This framework enhances our understanding of the energetic dynamics in particle physics, highlighting the complex interplay between charge and confinement.

**Keywords:** discrete energy; strong coupling; quark; confinement; Stress energy momentum tensor

## Introduction

A point-like concentration of mass is associated with two fundamental intrinsic length scales, the Compton wavelength and the Schwarzschild radius. The length scale at which quantum field theory is required is determined by the Compton wavelength [1,2]. Relativistic effects (e.g. pair creation, relativistic kinematics, etc.) are connected to interactions that probe such distances since they need energies similar to the particle's rest mass. Conversely, the Schwarzschild radius establishes the length scale at which gravitational effects (such as the presence of an event horizon) and space-time curvature become significant. It is important to remember that the Schwarzschild radius and the Compton wavelength have opposite relationships with the particle's mass. An electron's Schwarzschild radius is around  $10^{-57}$  meters, while its Compton wavelength is roughly  $10^{-12}$  m. The electron dynamics are thus entirely controlled by QFT effects at distances within the experimentally accessible range [3,4].

Compare this to a stellar mass black hole, which has a mass of about  $10^{30}$  kg. The Schwarzschild radius is roughly 1.5 km, while the Compton wavelength is  $10^{-72}$  m. GR effects totally overpower QFT effects, unlike the electron example, at least until one looks (ludicrously) far inside the event horizon.

The Planck mass scale, or mass range  $m \sim 10 \mu\text{m} \sim 10 \mu\text{g}$ , is situated in the middle of these two extremes. The Compton wavelength and the Schwarzschild radius are on the same order of magnitude in this mass regime. Space-time curvature effects and relativistic quantum effects are equally significant for such objects. This is the scale at which a cogent theory of quantum gravity would be required because the dynamics of such a particle cannot be sufficiently described by one without the other [5–8].

The stress-energy momentum tensor plays a vital role in understanding the dynamics of a perfect fluid, especially in contexts where negative pressure is present. Negative pressure fluids, represented mathematically as  $p = -\rho c^2$ , are essential to theoretical frameworks concerning cosmic fluids and dark energy [9–11].

The standard Einstein field equation, represented as

$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu}$  is revisited here with an innovative modification [12], that substitutes the Schwarzschild radius  $R_s = \frac{2GM}{c^2}$  with the Compton length  $\lambda_c (R_s \rightarrow \alpha^2\lambda_c)$  yielding  $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{4\pi\alpha^2\hbar}{mc}\widetilde{T}_{\mu\nu}$ .

## Stress Energy Momentum Tensor

Stress energy momentum tensor of a perfect fluid, in Background cosmic fluid with negative pressure  $p = -\rho c^2$  is given as  $T_{\mu\nu} = g_{\mu\sigma}g_{\nu\zeta}(\rho + \frac{p}{c^2})c^2 - pg_{\mu\nu} \equiv \rho g_{\mu\nu}$ , where Einstein equation is  $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu}$ .

Put R.H.S. in terms of the rational stress energy momentum tensor and Schwarzschild radius as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -4\pi R_s \widetilde{T}_{\mu\nu} \quad (1)$$

where  $R_s = \frac{2GM}{c^2}$  and  $\widetilde{T}_{\mu\nu} = \frac{T_{\mu\nu}}{Mc^2}$

## Modified Einstein Field Equation

When the Compton length  $\lambda_c$  is substituted for the Schwarzschild radius,  $R_s \rightarrow \alpha^2\lambda_c$ , Einstein field Equation (1) becomes:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{4\pi\alpha^2\hbar}{mc}\widetilde{T}_{\mu\nu} \quad (2)$$

where M represents the macroscopic mass and m represents the microscopic mass.

The Einstein equation in the background of a cosmic fluid with negative pressure can now be rewritten as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{4\pi\alpha^2\hbar}{mc}\tilde{\rho}g_{\mu\nu} \quad (3)$$

where  $\tilde{\rho} = \rho/Mc^2 \propto a^{-3}$ ,  $a$  is the scale factor of the system.

Solving Einstein field equation in conformal flat space time with metric tensor,  $g_{\mu\nu} = e^\psi\eta_{\mu\nu}$ ,  $\psi$  is coordinate function. We recourse the result in [13], to write

$$R_{\mu\nu} = \frac{\partial^2\psi}{\partial x^\mu\partial x^\nu} - \frac{1}{2}\frac{\partial\psi}{\partial x^\mu}\frac{\partial\psi}{\partial x^\nu} + \frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}\left(\frac{\partial^2\psi}{\partial x^\alpha\partial x^\beta} + \frac{\partial\psi}{\partial x^\alpha}\frac{\partial\psi}{\partial x^\beta}\right) \quad (4)$$

$$R_{\mu}{}^\nu = \frac{\partial^2\psi}{\partial x^\mu\partial x^\nu} + \frac{1}{2}\delta_\mu^\nu\frac{\partial^2\psi}{\partial x^\alpha\partial x^\alpha}, \quad R = 3\Box\psi, \quad \Box\psi \equiv \frac{\partial^2\psi}{\partial x^\alpha\partial x^\alpha},$$

Einstein equation becomes

$$\frac{\partial^2\psi}{\partial x^\mu\partial x^\nu} + \frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}\frac{\partial^2\psi}{\partial x^\alpha\partial x^\beta} - \frac{3}{2}g_{\mu\nu}\Box\psi = -\frac{4\pi\alpha^2\hbar}{mc}\tilde{\rho}g_{\mu\nu} \quad (5)$$

$$\frac{\partial^2\psi}{\partial x^\mu\partial x^\nu} + \frac{1}{2}\delta_\nu^\mu\frac{\partial^2\psi}{\partial x^\alpha\partial x^\alpha} - \frac{3}{2}\delta_\nu^\mu\psi = -\frac{4\pi\alpha^2\hbar}{mc}\tilde{\rho}(1+2\psi)\delta_\nu^\mu \quad (6)$$

Equation (6) implies, that

$$\Box\psi - \frac{32\pi\alpha^2\hbar}{3mc}\tilde{\rho}\psi = \frac{16\pi\alpha^2\hbar}{3mc}\tilde{\rho} \quad (7)$$

In static case

$$\nabla^2\psi - \frac{32\pi\alpha^2\hbar}{3mc}\tilde{\rho}\psi = \frac{16\pi\alpha^2\hbar}{3mc}\tilde{\rho} \quad (8)$$

## Mass and Energy in Confining System

Compare Equation (8) with Klein – Gordon equation

$$\left(\nabla^2 - m_\phi^2 c^2 / \hbar^2\right) \phi = \frac{1}{c^2} \left(\frac{\partial \phi}{\partial t}\right)^2, \quad m_\phi \text{ is the Klein - Gordon mass.}$$

We find equivalent mass as:

$$m_\phi = \sqrt{\frac{16\pi\alpha^2\hbar^3}{3mc^3}} \tilde{\rho} \quad (9)$$

And average energy as:

$$E = \sqrt{\frac{16\pi\alpha^2\hbar^3 c}{3m}} \tilde{\rho} \quad (10)$$

To calculate the equivalent mass and energy, we assume the system is sphere of radius  $r$ . Thus, we find,

$$m_\phi = \sqrt{\frac{4\alpha^2\hbar^3}{mc^3}} r^{-3} \quad (11)$$

$$E = \sqrt{\frac{4\alpha^2\hbar^3 c}{m}} r^{-3} \quad (12)$$

If the radius of interaction of a particle ( $m$ ) is proportion to  $\lambda_c$  ( $\lambda_c$  is Compton length),  $r = \frac{\beta^2 \hbar}{mc}$ , where  $2\alpha \geq \beta \geq \alpha$ . One finds:

$$m_\phi = 2 \frac{\alpha}{\beta} m \quad (13)$$

$$E = 2 \frac{\alpha}{\beta} mc^2 \quad (14)$$

The interaction range of virtual particles, estimated as  $2c\Delta t > \hbar/mc$ , further explains the principles underlying quantum behavior. Equations (9-14) substantiate the tenets of the uncertainty principle by demonstrating that as the radius of confinement diminishes, both the mass and energy of the interacting particles correspondingly escalate. This phenomenon offers a compelling explanation for the disparate masses observed in mesons such as the rho meson ( $\rho$ ) and the pion ( $\pi$ ). While both particles are indeed mesons, their considerable mass differences do not stem solely from their compositions but also from their intrinsic quantum properties, namely spin and quark content.

The confining radius of a meson is the effective size of the region within which the quark and antiquark are bound together by the strong force. This radius is determined by the balance between the attractive strong force (mediated by gluons) and the kinetic energy of the quarks[14], [15].

The pion is the lightest meson, with a mass of about 140 MeV/c<sup>2</sup> for the charged pions ( $\pi^+$ ,  $\pi^-$ ) and 135 MeV/c<sup>2</sup> for the neutral pion ( $\pi^0$ ). Because the pion is so light, the quark and antiquark are bound relatively loosely, and the confining radius is larger compared to heavier mesons. The confining radius of the pion is estimated to be around femtometers. The large radius is also due to the fact that the pion is a pseudoscalar meson (spin-0), where the quark and antiquark have antiparallel spins, resulting in a weaker binding energy.

The rho meson is much heavier, with a mass of about 770 MeV/c<sup>2</sup>. The higher mass of the rho meson indicates that the quark and antiquark are bound more tightly, resulting in a smaller confining radius compared to the pion.

The confining radius of the rho meson is estimated to be around 0.5–1 fm.

The smaller radius is due to the rho meson being a vector meson (spin-1), where the quark and antiquark have parallel spins, leading to a stronger binding energy and a more compact structure.

The rho meson's higher mass means that the quark and antiquark are more tightly bound, reducing the confining radius. In contrast, the pion's lower mass corresponds to a looser binding and a larger radius.

The spin-1 configuration of the rho meson (parallel spins) results in a stronger spin-spin interaction, which increases the binding energy and reduces the size of the meson. The spin-0 configuration of the pion (antiparallel spins) leads to a weaker interaction and a larger size.

The quarks in the rho meson move more rapidly due to the stronger binding, which also contributes to the smaller confining radius.

The sizes of mesons are not directly measurable but are inferred from theoretical models (e.g., quark models, lattice QCD) and experimental data (e.g., scattering experiments and decay rates)[16–18].

Lattice QCD calculations, which simulate the strong force on a discrete space-time lattice, provide estimates for the sizes of mesons, confirming that the rho meson is smaller than the pion [19,20].

### Confinement of Charged Micro-Universes

On the other hand, Reissner-Nordstrom metric in spherical coordinate system is given by:

$$ds^2_{NR} = \left(1 - \frac{2\mu}{r} + \frac{r_Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2\mu}{r} + \frac{r_Q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

The spherically symmetric Schwarzschild-de sitter metric is

$$ds^2_{SD} = \left(1 - \frac{2\mu}{r} + \frac{r^2}{R^2}\right) dt^2 - \left(1 - \frac{2\mu}{r} + \frac{r^2}{R^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

For the sake of simplicity, we consider the case where  $\mu = 0$ . In the microscopic realm, the relevant scale is the Compton wavelength, leading us to assume that  $r = \frac{2\hbar}{m_\phi c}$ . This assumption allows us to derive the relationship  $r_Q^2 = \frac{\hbar Q^2}{4\pi\epsilon_0 m_Q^2 c^3}$  with  $m_Q = \pm \sqrt{\frac{\hbar}{4\pi\epsilon_0 c^3} \frac{Q m_\phi c}{\hbar}}$  [12].

The Reissner-Nordstrom metric can then be expressed as:

$$ds^2_{NR} = \left(1 + \frac{Q^2 m_\phi^4 c}{64\pi\epsilon_0 m_Q^2 \hbar^3} r^2\right) dt^2 - \left(1 + \frac{Q^2 m_\phi^4 c}{64\pi\epsilon_0 m_Q^2 \hbar^3} r^2\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (15)$$

By comparing this metric with the Schwarzschild-de Sitter metric, we find the relationship

$$\frac{1}{R^2} = \frac{Q^2 m_\phi^4 c}{64\pi\epsilon_0 m_Q^2 \hbar^3} \quad (16)$$

From this, we can derive

$$\frac{Q^2}{4\pi\epsilon_0 r_Q} = \frac{\hbar c}{R} \quad (17)$$

Abdus Salam et al, Confinement through Tensor Gauge Fields, explores quark confinement using tensor gauge fields. They utilize the O(3,2)-symmetric de Sitter solution of Einstein's equations to describe a strongly interacting tensor field. They proposed that hadronic bags, which confine quarks, can be modeled as de Sitter "micro-universes," with specific radii determined by the strong-coupling and cosmological constants in the Einstein equation [21].

Interestingly, the energy spectrum for two-body hadronic states in this model resembles that of a harmonic oscillator potential, although the wave functions diverge. They posit that these tensor fields can be extended to include color, offering a novel perspective on quark confinement mechanisms.

The confinement mechanism in particle physics, particularly within Quantum Chromodynamics (QCD), is a complex phenomenon. Color confinement refers to the principle that color-charged particles, such as quarks and gluons, cannot be isolated and observed individually under normal conditions. Instead, they exist in groups that form color-neutral particles, such as protons, neutrons, and mesons.

Gluons, the force carriers of the strong nuclear force, bind quarks together. Unlike photons in electromagnetism, gluons carry color charge, resulting in the formation of a narrow flux tube (or string) between quarks. As quarks separate, the energy within the gluon field increases, making it energetically favorable to create new quark-antiquark pairs rather than extending the flux tube further.

A key feature of QCD is asymptotic freedom, which indicates that the force between quarks diminishes as they come closer together. Conversely, as quarks move apart, the force intensifies, leading to confinement. This behavior starkly contrasts with other fundamental forces, such as electromagnetism, which weaken with distance.

In high-energy collisions, such as those in particle accelerators, quarks cannot exist freely. Instead, they quickly combine with other quarks to form hadrons, a process known as hadronization or fragmentation, ensuring that only color-neutral particles are detected.

The confining phase in QCD can be analyzed using the Wilson loop, a path in space-time traced by a quark-antiquark pair. In a confining theory, the action of the Wilson loop is proportional to its area, indicating the suppression of free quarks. This mathematical framework aids in comprehending the confinement mechanism.

Understanding confinement has profound implications for our grasp of the universe at its most fundamental level. It elucidates why quarks and gluons are never observed in isolation and provides insights into the structure and behavior of hadrons, which is crucial for exploring the fundamental properties of matter.

This modification underscores that the energy spectrum  $\omega_{nl}$  depends not only on the quantum numbers  $n$  and  $l$  but also intricately on the charge  $Q$ . Additionally, through further refinements, we express  $\omega_{nl}$  in terms of  $Q$  as follows:

$$\omega_{nl} = \frac{\hbar c}{R} \left( 2n + l + \frac{3}{2} + \sqrt{\frac{9}{4} + k^2 R^2} \right) \quad (18)$$

with

$$k = \frac{m_\phi c}{\hbar} \quad (19)$$

Substituting (17) in equation (18) we find

$$\omega_{nl} = \frac{Q^2}{4\pi\epsilon_0 r_Q} \left( 2n + l + \frac{3}{2} + \sqrt{\frac{9}{4} + \bar{k}^2 r_Q^2} \right) \quad (20)$$

The parameter  $\bar{k}$  is parameterized as follows

$$\bar{k} = \frac{4\pi\epsilon_0 m c^2}{Q^2} \quad (21)$$

Further refinement leads us to a new expression for  $\omega_{nl}$  as

$$\omega_{nl} = \sqrt{\frac{Q^2 m_\phi^4 c^3}{64\pi\epsilon_0 m_Q^2 \hbar}} \left( 2n + l + \frac{3}{2} \right) + \sqrt{\frac{9}{64} \frac{Q^2 m_\phi^4 c^3}{4\pi\epsilon_0 m_Q^2 \hbar} + m_\phi^2 c^4} \quad (22)$$

In this context, where  $n = 0, 1, 2, \dots$ , and when  $Q = 0$ , we find that  $\omega = m_\phi c^2$  illustrating the intrinsic link between energy discreteness and charge. Therefore, we can express equation (10) in a new form:

$$\frac{\rho}{\zeta} = E_{k-G}^2 \quad (23)$$

The introduction of the parameter  $\zeta$  and its relationship with the energy states yields another dimension to our understanding.

Where  $\zeta$  is defined as

$$\zeta = \frac{3m_\phi^2 c}{64\pi\hbar^3} \quad (24)$$

This equation facilitates a connection between particle energy, charge, and mass, portraying how these fundamental properties interact within a defined system characterized by the radius of confinement.

We assign  $\alpha = 2$  and define  $\varrho$  as:

$$\varrho = \left( 2n + l + \frac{3}{2} + \sqrt{\frac{9}{4} + \frac{64\pi\epsilon_0\hbar m_Q^2 c}{m_\phi^2 Q^2}} \right) \hbar c \quad (25)$$

The relationship between  $\omega_{nl}$  and other parameters is given by:

$$\omega_{nl}^2 = \frac{Q^2 m_\phi^2}{3\epsilon_0 m_Q^2} \varrho^2 \zeta \quad (26)$$

Furthermore, we observe the equivalence:

$$\zeta^2 E_{k-G}^2 = \zeta \rho \quad (27)$$

This leads to the expression:

$$\omega_{nl} = \kappa E_{k-G} \quad (28)$$

The ensuing equations, particularly the derived relationship of  $\omega_{nl}$  and  $E_{k-G}$  with the discrete energy  $\omega_{nl}$ , discernibly assert that the coupling constant  $\kappa$ , defined as:

$$\kappa = \frac{\zeta Q \varrho m_\phi}{m_Q \sqrt{3\epsilon_0 \rho}} \quad (29)$$

is critical for comprehending how energetic interactions manifest under different charges. When  $Q$  assumes a non-zero value, the interplay between confinement and energetic states elucidates how charge influences particle dynamics, especially in a system confined to a specific radius.

In the context of particle physics,  $\kappa$  denotes a dimensionless factor that signifies the strength of the coupling in a given system. As we recognize from equation (29), the inverse proportionality between  $\kappa$  and the energy density. This leads the strong coupling  $\kappa$  to direct proportion to the radius of confinement of the system as it is in confinement theory of Hadrons. Equation (28) elucidates the relationship between the energy of a free particle, denoted as  $E_{k-G}$ , and the energy resulting from strong interactions, represented by  $\omega$ . This relationship is established within a confined system characterized by a radius of  $2\lambda_c$ , and it is mediated by the strong coupling factor  $\kappa$ . Through this framework, one can understand how confinement influences energetic interactions in particle dynamics.

When  $Q = 0$  ;  $\kappa = \sqrt{\zeta/\rho} m_\phi c^2$  .

When  $Q = 0, \kappa = 1$  we find  $\rho = \zeta m_\phi^2 c^4$ , if the radius  $r = \frac{2\hbar}{m_\phi c}$  , the energy becomes  $E = \frac{1}{2} m_\phi c^2$

When  $Q \neq 0, \kappa = 1$  we find  $\rho = \frac{(\zeta Q \varrho m_\phi)^2}{3\epsilon_0 m_Q^2}$ .

Eventually, we find that, the charge  $Q$  is pivotal in modifying the energy spectrum, shaping the interactions and energy states of particles within a defined system. The strong coupling factor  $\kappa$  that emerges reflects the interplay between the inherent energies of the system and those arising from interactions, revealing the rich tapestry of particle physics where confinement and charge are inexorably linked. As the charge  $Q$  transitions from zero to non-zero values, the transformations in

energy reveal fundamental truths about the behaviors and nature of particles at a quantum level. The relationship between the modified energy spectrum  $\omega_{nl}$  and its governing parameters can be succinctly expressed through equation (26).

## Conclusions

In summary, our findings underscore the essential role of charge  $Q$  in shaping the energy spectrum and influencing the interactions and energy states of particles within a given system. The strong coupling  $\kappa$  highlights the intricate interplay between the system's intrinsic energies and those arising from particle interactions. This relationship illustrates the fundamental connections in particle physics, wherein confinement and charge are deeply intertwined.

As  $Q$  transitions from zero to non-zero values, the resultant alterations in energy provide critical insights into the behavior and nature of particles at the quantum level. The link between the modified energy spectrum  $\omega_{nl}$  and its associated parameters is effectively captured in equation (26). In the realm of particle physics,  $\kappa$  serves as a dimensionless factor, reflecting the strength of coupling within the system.

As demonstrated in equation (29),  $\kappa$  exhibits an inverse proportionality to the energy density, indicating that stronger coupling corresponds to the system's confinement radius, aligning with confinement theory in hadrons. Furthermore, equation (28) clarifies the relationship between the energy of a free particle, denoted as  $E_{k-G}$ , and the energy from strong interactions. This interaction is contextualized within a confined system with a radius of  $2\lambda_c$  and is mediated by the strong coupling factor. Through this framework, we gain a deeper understanding of how confinement impacts energetic interactions in particle dynamics.

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