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# Recovering Absolute Time in the Lorentz Transformation -The Apparent Nature of Relative Simultaneity

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**Abstract**

Relative simultaneity remains a highly debated issue. It is presented as a necessary physical consequence of the Lorentz transformation (LT). However, we demonstrate that the phenomenon is an artefact of 'mixed-coordinate' algebraic representation, which does not guarantee that the stationary system S 4-vector transformed to the moving system S′ are both covariant. A covariant representation of a transformed 4-vector which components are explicit functions of time, requires them to be expressed as functions of the local time in S and in the moving frame S′. While the one step multiplication of LT matrix by the 4-vector in S, yields correct algebraic expressions, the 'raw' resulting 4-vector retains the variable *t* throughout all components. This 'mixed-coordinate' representation is incomplete; it is not in a form covariant with the vector in S because its components are not functions of *t′*. The variable *t* must be replaced by equating the first component of the transformed vector to *ct′* and substituting the resulting expression into all instances of *t* in the 'raw' 4-vector. After this procedure applied to two simultaneous events in S, the apparent time difference between the events in S′ becomes $\Delta t'=0$. The effect of relative simultaneity, which appears in the 'mixed-coordinate' representation, is absent. This highlights the role of emergent 'absolute-like time' hidden within the structure of LT equations affecting temporal relations, suggests that the "Relative Now" discussed by Eddington in 1927 resulting from widely known conclusion $\Delta t'\neq 0$ is a mathematical artefact of coordinates convention rather than the physical reality.

**Keywords:** coordinate homogeneity; Lorentz invariance; relative simultaneity; equations of motion; temporal relations

## 1. Introduction

Relative simultaneity has been a hallmark of Special relativity since 1905 when Einstein released of his seminal paper [1] (first widely available 1923 translation in English). The overwhelming interpretation was that every observer has their own timeline in which they judge events happening elsewhere such that they may disagree on whether events that really happened in another system are simultaneous or not. A summary of this peculiar and counterintuitive phenomenon has been captured by Eddington [2] (p. 49) who famously declared: "There is no absolute Now, but only the various relative Nows differing according to the reckoning of different observers...". We found it illuminating to analyse this situation using the LT matrix approach to transform 4-vectors from a stationary system S vector to its image in the moving system S′. We shall demonstrate how the matrix approach may clarify potential misunderstandings regarding relative simultaneity.

The following LT matrix representation in standard axes configuration will be used in subsequent considerations:

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$$\Lambda_v = \begin{bmatrix} \gamma_v & -v\dfrac{\gamma_v}{c} & 0 & 0 \\ -v\dfrac{\gamma_v}{c} & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$\gamma_v = 1/\sqrt{1 - v^2/c^2}$$

## 2. Methods

*2.1. Lorentz Transformation Procedure*

There are two approaches to Lorentz Transformation. The traditional one, in the simplest case uses explicit algebraic expressions of LT as shown in Equation 2.

$$\begin{cases} t' = \gamma_v(t - vx/c^2) \\ x' = \gamma_v(x - vt) \end{cases} \tag{2}$$

The other approach uses the LT matrix from Equation (1) applied to the 4-vectors. This approach was not in common use in 1905; thus, the change of coordinates was accomplished by substituting time and space coordinates expressions or constants in the set of algebraic Equations (2). This was not like first demonstrated by Einstein, proving that the speed of light is invariant in all inertial frames [1] (p.46) because he had to use the inverse transformation of the Equation (2). The earlier mentioned method continued to be used and still is today in many publications and textbooks for example Tolman [3] (p.51), Feynman [4] (p. 15-13), Ugarov [5] (p. 73) and University of California lecture notes[1] to prove relative simultaneity. The LT matrix approach is elegant, more robust and less error-prone hence eliminating hand-picked substitutions in a series of algebraic equations. Educational approach to the matrix method can be found in the book of Steane [6].

The typical use of the LT matrix in physics is as follows: $\Lambda_v \boldsymbol{K} = \boldsymbol{K}'$ where $\boldsymbol{K}, \boldsymbol{K}'$ are wave 4-vectors before and after the transformation respectively. For many physical problems this is a one step complete matrix multiplication. The problem becomes more complicated when the 4-Vector is an explicit function of time. The matrix multiplication result contains the stationary system time $t$, which makes it a non-homogenous or 'mixed-representation'. It is not immediately clear how to obtain the homogenous representation of the original vector in the general case. In classical mechanics using Galilean matrix the problem does not exist because $t = t'$. We propose a logical approach where the second step needs to be added after the matrix multiplication, where the first component is used to convert time $t$ to $t'$ by equating the temporal component as: $ct'$=$(\boldsymbol{X}'_c)_1$ as in Equation (5). This procedure immediately recovers the covariant form of the equations of motion (EOM) for all explicit EOMs in the 4-vector form. However, this becomes debatable because the relativity of simultaneity no longer appears. We formulate a hypothesis and attempt the proof in the following section.

## 3. The Hypothesis of Coordinate Homogeneity

The Lorentz Transformation process is typically treated as a sequence of coordinate substitutions or as a single-step LT matrix multiplication by a 4-vector. While the latter is sufficient for vectors not containing an explicit time variable, the resulting vector has a non-homogeneous representation—containing the variable $t$ remaining present where $t'$ could be reasonably expected—causing significant interpretation problems. To address this issue, we put forward the following hypothesis.

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1 Keith Fratus , The Lorentz Transformation - Lecture Notes, University of California, Santa Barbara (updated in 2025) Page3 within

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**Hypothesis:**

For an inertial system S′ to be functionally equivalent to S, its physical state must be expressed as a homogeneous 4-vector, reflecting the original representation in system S. Furthermore, there is only one mathematically consistent form possible for each vector to maintain this functional equivalence.

Problem: Standard 'raw' result of LT matrix multiplication yields a mixed-coordinate representation containing the stationary frame variable $t$

Necessity: Because observers in S′ have no direct awareness of time variable the other system, $t$ must be eliminated.

Objective: Achieving true covariance requires that all components of the transformed vector are explicit functions of local time $t'$.

The most reliable EOM of light emitted from the origin in S is represented by the following 4-vector:

$$\boldsymbol{X_C} = \begin{bmatrix} ct \\ ct \\ 0 \\ 0 \end{bmatrix}. \tag{3}$$

The transformation of the EOM 4-vector $\boldsymbol{X_C}$ using the LT matrix yields:

$$\boldsymbol{X'_c} \equiv \Lambda_v \boldsymbol{X_c} = \Lambda_v \begin{bmatrix} ct \\ ct \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma_v ct - \gamma_v vt \\ \gamma_v ct - \gamma_v vt \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma_v (c - v)t \\ \gamma_v (c - v)t \\ 0 \\ 0 \end{bmatrix}. \tag{4}$$

The result does not yet constitute a functional EOM for a moving observer. There is no explicit time coordinate, and the spatial component $\gamma_v(c-v)t$ does not immediately imply the invariant speed of light. It is still a representation of the light propagation 4-vector in S′; however, it is not in the covariant form directly comparable to $\boldsymbol{X_C}$. The presence of the other system's time variable $t$, of which S′ has no direct awareness, makes it a non-homogeneous representation. Traditionally, it would require some narrative and algebraic manipulation to demonstrate that the speed of light is invariant. A simple two step conversion makes this straightforward. Solving the equation $(\boldsymbol{X'_c})_{\boldsymbol{1}} = ct'$ for the variable $t$ from the first temporal element then substituting the equivalent equation in terms of $t'$ to any occurrence of $t$ as follows:

$$(\boldsymbol{X'_c})_{\boldsymbol{1}} = ct' \Rightarrow t = \frac{ct'}{\gamma_v (c - v)}. \tag{5}$$

Substituting $t$ using the formula from Equation (5) is the only possible way to make the resulting 4-vector EOM covariant; therefore it is unique as implied by the hypothesis. The truth of the hypothesis is demonstrated, however; this is only one specific case, and this is not a formal proof, this is physically valid in general case. After substituting $t$ to equation (4) the covariant form of the light vector in S′ is:

$$\boldsymbol{X'_C}(t') = \begin{bmatrix} ct' \\ ct' \\ 0 \\ 0 \end{bmatrix}. \tag{6}$$

This confirms the postulates of special relativity. We can now apply the same methodology to events. Because the interpretation of the intermediate form of the EOM in Equation (4) was initially problematic, we can anticipate difficulties in interpretation after the transformation of event. coordinates.

Coordinates of events in the stationary frame S and locations $x_1$ and $x_2$ are:

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$$\boldsymbol{E_1} \equiv \begin{bmatrix} ct \\ x_1 \\ 0 \\ 0 \end{bmatrix}, \tag{7}$$

$$\boldsymbol{E_2} \equiv \begin{bmatrix} ct \\ x_2 \\ 0 \\ 0 \end{bmatrix}. \tag{8}$$

For the event $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ LT matrix transformation yields:

$$\boldsymbol{E'_1} \equiv \Lambda_v \boldsymbol{E_1} = \Lambda_v \begin{bmatrix} ct \\ x_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma_v ct - \gamma_v v x_1/c \\ -\gamma_v vt + \gamma_v x_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma_v (ct - \frac{v x_1}{c}) \\ \gamma_v (-vt + x_1) \\ 0 \\ 0 \end{bmatrix}. \tag{9}$$

$$(\boldsymbol{E'_1})_{\boldsymbol{1}} = ct't \Rightarrow t = \frac{t'c^2 + \gamma_v x_1}{\gamma_v c^2}, \tag{10}$$

$$E'_2 \equiv \Lambda_v E_2 = \Lambda_v \begin{bmatrix} ct \\ x_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma_v ct - \gamma_v v x_2/c \\ -\gamma_v vt + \gamma_v x_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma_v (ct - \frac{v x_2}{c}) \\ \gamma_v (-vt + x_2) \\ 0 \\ 0 \end{bmatrix}, \tag{11}$$

$$(\boldsymbol{E'_2})_{\boldsymbol{1}} = ct't \Rightarrow t = \frac{t'c^2 + \gamma_v x_2}{\gamma_v c^2}. \tag{12}$$

After substituting the time expression from Equation (10), the covariant form of the first event vector in S′ is:

$$\boldsymbol{E'_1}(t') = \begin{bmatrix} ct' \\ -vt' + \gamma_v x_1 - \frac{\gamma_v x_1 v^2}{c^2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ct' \\ -vt' + x_1/\gamma_v \\ 0 \\ 0 \end{bmatrix}. \tag{13}$$

For the event $\boldsymbol{E_2}$ by the same process we get:

$$\boldsymbol{E'_2}(t') = \begin{bmatrix} ct' \\ -vt' + \gamma_v x_2 - \gamma_v x_2 v^2/c^2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ct' \\ -vt' + x_2/\gamma_v \\ 0 \\ 0 \end{bmatrix}. \tag{14}$$

Note that the conventional 'mixed-coordinate' subtraction of the raw transformed time components (Equations 9 and 11) immediately yields the familiar non-zero result $\Delta t' = -\gamma_v (v x_2/c^2 - v x_1/c^2)$. However, once both events are expressed in the fully covariant form using only the local time $t'$ of frame S′ (Equations 13 and 14), the time difference vanishes ($\Delta t' = t' - t' = 0$). This shows that the textbook claim of relative simultaneity may be an artefact of reading an incomplete, mixed-coordinate representation rather than using the completed covariant 4-vector required for coordinate homogeneity in the target frame. That does not necessarily mean that this is an error but rather one aspect of the more nuanced problem of conventional simultaneity. The term 'conventional` does not carry a negative connotation because it was a deliberate stipulation which Einstein admitted deciding on the definition of simultaneity of his own free will [7] (p.28). Strohm [8] (p.109) confirms the convention as the correct classification, because there are other valid alternatives including the Tangherlni transformation (TT) operating in absolute time, also known as the absolute transformations.

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The $\Delta \boldsymbol{t}' = \mathbf{0}$ conclusion appears to be at variance with the standard interpretation of simultaneity first presented by Einstein [1] (p.42). Our approach may be questioned because, the deliberate choice of the assigning different events the same value of $ct'$. However, assigning $x_1$ to the first spatial component in the event coordinate 4-vector is not different than having an equation of motion but with zero velocity, which is perceived like any other relative motion in S′. The fact that without the discussed substitution there would be no valid equation of motion, is compelling. We now attempt to falsify the assertion about absolute simultaneity of distant events. It is natural to assume that in system S two concurrent light beams emitted from the origin at a delay $\delta_1$ and $\delta_2$ can generate events simultaneously expressed as:

$$\boldsymbol{X_{C1}} = \begin{bmatrix} ct \\ c(t-\delta_1) \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{X_{C2}} = \begin{bmatrix} ct \\ c(t-\delta_2) \\ 0 \\ 0 \end{bmatrix}. \qquad (15)$$

The LT applied to those EOMs using the previously described procedure yields:

$$\boldsymbol{X'_{C1}}(t') = \begin{bmatrix} ct' \\ -\gamma_v(v+c)\delta_1 + ct' \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{X'_{C2}}(t') = \begin{bmatrix} ct' \\ -\gamma_v(v+c)\delta_2 + ct' \\ 0 \\ 0 \end{bmatrix}. \qquad (16)$$

From the above we can draw a simple conclusion, that for any pair od $\delta_1$ and $\delta_2$, there can be two simultaneous events at $x_1(t) = c(t-\delta_1)$ and $x_2(t) = c(t-\delta_2)$and two simultaneous events at time $t'$ at corresponding locations $x_1'(t') - \gamma_v(v+c)\delta_1$ and $x_2'(t') - \gamma_v(v+c)\delta_2$ which confirms our previous conclusion that the time difference vanishes ($\Delta t' = t' - t' = 0$) for any two simultaneous events in S.

## 4. Discussion

There seems to be no doubt that the systems S and S′ are properly configured covariant systems. In this representation the temporal component is always $ct$ or $ct'$, where $t$ and $t'$ represent the local master origin clock proper time initially projected to all clocks, which remain synchronised in the idealised scenario. Even in conventional synchronisation every clock state is the same as the master clock. All with an implicit assumption, both local system clocks ensembles along the axes are synchronised by light signals enforcing Einstein's synchronisation. Only then, does the application of the LT matrix yield a physically valid transformed event. The idealised master clocks in uniform motion instantaneously synchronised to 0 when coinciding, subsequently carry their own proper time from the synchronised value onwards, together with their later synchronised local clock ensembles. The clocks on $x$ and x′ axes can be synchronised at the speed of light immediately after $t = 0$, well before an experiment is conducted. Although in relative motion and different rates, they remain in continuous temporal relation predictable by the theory of relativity. For any location there can always be two independent clocks running at different rates in the respective systems. Any event can be simultaneously recorded by them, providing a lasting record of what and when it happened, in which system.

The apparent absence of relative simultaneity requires further discussions to gain a wider consensus or falsification. We use traditional LT equations allowing more than one choice. One is the most common in literature [3] (p. 51), [5] (p. 73) and lecture notes[1] (p.2), using the original Einstein derived formulation, the other by using their inverse variant:

$$\begin{cases} t' = \gamma_v(t - vx/c^2) \\ x' = \gamma_v(x - vt) \end{cases} \Rightarrow \begin{cases} t = \gamma_v(t' + vx'/c^2) \\ x = \gamma_v(vt' + x') \end{cases} \qquad (17)$$

To convert the vector $\boldsymbol{E_1}$ component manually, we need to equate the x component content to the right hand side of the $x$ conversion: $(\boldsymbol{E_1})_2 = x_1 \Rightarrow x = x1$, then $x_1 = \gamma_v(vt' + x')$ and solve this for

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$x'$ and obtain $x' = -vt' + x_1/\gamma_v$. This is exactly what was obtained in the second step after which, $(\boldsymbol{E'}_1)_2 = -vt' + x_1/\gamma_v$. Using the manual method, if there is any time variable left, we need to substitute $t$ with the remaining transformation equation introducing $x'$ in the middle of the transformed equation and then it must be solved for $x'$ again. The equivalent of the matrix transformation second step is 'automatically' completed. Now, in the temporal component $ct$ we can substitute the formula for $t$ from the Equation 17. However, it becomes a mixed representation and it is not possible to represent the 4-vector in the covariant format. Therefore, the only choice is to put $ct'$ where it belongs, thus securing a covariant representation. We have just demonstrated that the second step concludes the unfinished Lorentz transformation.

The disappearance of relative simultaneity is the result of demanding covariant representations in the equivalent systems, and the 'mixed representation' contains all necessary information to accomplish this process. So, the moving observer's time found its way out of the shell of synchronisation convention which allowed to master relativistic physics without instantaneous signals. We can see it as an emergence of the 'absolute-like' time came naturally. The nature of this time is yet to be revealed. Absolute time is not fiction. It is explicit in independently developed Tangherlini's theory [8] from Einstein's field equations of General Relativity (GR) which preserves all relativistic effects as Einstein's theory. Both theories are equivalent by the virtue of GR coordinate independent formulation; it is not then a surprise that 'absolute-like' time had to emerge somewhere in special relativity as demonstrated. For more than one hundred years physicists derive relativistic equations of motion this way or another, not being aware of what kind of time they are dealing with. It is also not surprising that relative simultaneity has no physical effect because in Tangherlini's theory it is inherently absent. Until this moment it was unthinkable these two theories could ever agree on relative simultaneity, yet they did eventually. Furthermore, the TT (or absolute transformation) can be derived using Reinchenbach synchronisation approach as demonstrated by Strohm [7] which is the second non-arbitrary alternative. The emergent 'absolute-like' time is the mathematical time like Newton said, but not flowing equably without relation to anything external, but running at different relative rates depending on relative velocity of the observers. There are possible clues how to describe the time just emerged. One method is to simulate light propagation in the system S and determine in any location what Einstein's time is there. If the light is emitted at $t = 0$ from the origin of S it automatically synchronises the ensemble of clock on x' axis as t'=x'/c =$\gamma_v x/c$ while the proper time according to Lorentz transformation is $\gamma_v t = \gamma_v x/c$=t' which is exactly the same as always has been but ignoring coordinates convention because GR is formulated as coordinate independent.

Concluding Summary

The investigation of the Lorentz transformation (LT) reveals that the long-standing debate over relative simultaneity is based on an intermediate algebraic stage rather than a completed covariant physical representation. By changing the way LT is applied in the traditional and matrix approach the rationale of coordinate homogeneity, establishes a unified procedure applicable to both continuous EOMs and discrete, separated events.

The transition from a 'mixed-coordinate' form to a fully covariant representation is not merely an optional interpretation but a mathematical necessity to maintain the functional equivalence of inertial frames. Because the local time $t$ cannot be explained in terms of the moving spatial coordinate $x$ without circularity, it becomes evident that 'absolute-like' time—represented by the is naturally encoded within the structure of special relativity being still the same relativistic time as ever before and independent of coordinates convention. Special relativity is not affected but becomes stronger.

Ultimately, the apparent interval Δt≠0 is identified as a coordinate artefact of the manual method not elevated to achieve full covariance of the two systems. When the transformation is fully resolved, the interval returns to ***Δt=0***, suggesting that while the "Relative Now" has served as a prominent academic narrative, it does not reflect the underlying physical reality of a covariant universe.

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## 5. Conclusions

1. The covariant Lorentz transformation of two simultaneous events gives the interval $\boldsymbol{\Delta t} = \mathbf{0}$ in the moving frame. The phenomenon of relative simultaneity is found to be an artefact of the intermediate mixed-coordinate representation and vanishes in the complete covariant form.
2. The apparent relative simultaneity $\Delta t' \neq 0$ of the standard interpretation arises from reading the temporal components of the incomplete, 'mixed-coordinate' intermediate result, which retains the stationary frame variable $t$ in the 4-vector components, as if it represented a physical measurement. However, In a system-specific observation only the local homogeneous variable $t'$ possesses physical measurable significance.
3. This result may need to revive a debate on Eddington's abolition of absolute "Now" and is offered for broader discussion by the community.
4. The Hypothesis of Coordinate Homogeneity has been proven within the limited scope of special cases under analysis, but requires general validation and mathematical rigour.
5. The findings presented here do not suggest an error in the empirical successes of relativistic physics, as incomplete transformations and possible side-effects are unlikely to survive the rigorours experimental verification or practical application. Rather, this correction applies the academic and philosophical narratives that have grown around the 'incomplete' representation of the Lorentz transformation. While these narratives—such as the abolition of an absolute "Now'—are prominent in theoretical debate, they appear to be inconsequential artefacts of a specific mathematical representation, with no impact on the underlying physical reality.

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## Abbreviations

The following abbreviations are used in this manuscript:

| | |
|---|---|
| EOM | Equation of Motion |
| GR | General Relativity |
| LT | Lorentz Transformation |

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