Dimension and Coloring alongside Domination in Neutrosophic Hypergraphs

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Abstract

New setting is introduced to study resolving number and chromatic number alongside dominating number. Different types of procedures including set, optimal set, and optimal number alongside study on the family of neutrosophic hypergraphs are proposed in this way, some results are obtained. General classes of neutrosophic hypergraphs are used to obtain these numbers and the representatives of the colors, dominating sets and resolving sets. Using colors to assign to the vertices of neutrosophic hypergraphs and characterizing resolving sets and dominating sets are applied. Some questions and problems are posed concerning ways to do further studies on this topic. Using different ways of study on neutrosophic hypergraphs to get new results about numbers and sets in the way that some numbers get understandable perspective. Family of neutrosophic hypergraphs are studied to investigate about the notions, dimension and coloring alongside domination in neutrosophic hypergraphs. In this way, sets of representatives of colors, resolving sets and dominating sets have key role. Optimal sets and optimal numbers have key points to get new results but in some cases, there are usages of sets and numbers instead of optimal ones. Simultaneously, three notions are applied into neutrosophic hypergraphs to get sensible results about their structures. Basic familiarities with neutrosophic hypergraphs theory and hypergraph theory are proposed for this article.

Keywords: Dimension, Coloring, Domination **AMS Subject Classification:** 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [15], neutrosophic set in **Ref.** [2], related definitions of other sets in **Refs.** [2,13,14], graphs and new notions on them in **Refs.** [5–11], neutrosophic graphs in **Ref.** [3], studies on neutrosophic graphs in **Ref.** [1], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [12], related definitions of other graphs based on neutrosophic graphs in **Ref.** [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. Is it possible to use mixed versions of ideas concerning "neutrosophic domination", "neutrosophic dimension" and "neutrosophic coloring" to define some notions which are applied to neutrosophic hypergraphs?

It's motivation to find notions to use in any classes of neutrosophic hypergraphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors, dominating and domination. Thus they're used to define new ideas which conclude to the structure of coloring, dominating and domination. The concept of having general neutrosophic hyperedge inspires me to study the behavior of general neutrosophic hyperedge in the way that, three types of coloring numbers, dominating number and resolving set are the cases of study in individuals and families.

The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In section "New Ideas For Neutrosophic Hypergraphs", new notions of coloring, dominating and domination are applied to neutrosophic vertices of neutrosophic graphs as individuals. In section "Optimal Numbers For Neutrosophic Hypergraphs", specific numbers have the key role in this way. Classes of neutrosophic graphs are studied in the terms of different numbers in section "Optimal Numbers For Neutrosophic Hypergraphs" as individuals. In the section "Optimal Sets For Neutrosophic Hypergraphs", usages of general neutrosophic sets and special neutrosophic sets have key role in this study as individuals. In section "Optimal Sets and Numbers For Family of Neutrosophic Hypergraphs", both sets and numbers have applied into the family of neutrosophic hypergraphs. In section "Applications in Time Table and Scheduling", one application is posed for neutrosophic hypergraphs concerning time table and scheduling when the suspicions are about choosing some subjects. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

1.2 Preliminaries

Definition 1.2. (Graph).

G = (V, E) is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Definition 1.3. (Hypergraph).

H = (V, E) is called a **hypergraph** if V is a set of objects and for every nonnegative integer $t \leq n$, E is a set of t-subsets of V where V is called **vertex set** and E is called **hyperedge set**.

Definition 1.4. (Neutrosophic Hypergraph).

 $NHG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic hypergraph** if it's hypergraph, $\sigma_i : V \to [0, 1], \ \mu_i : E \to [0, 1],$ and for every $v_1v_2 \cdots v_t \in E$,

$$\mu(v_1v_2\cdots v_t) \leq \sigma(v_1) \wedge \sigma(v_2) \wedge \cdots \sigma(v_t).$$

- (i): σ is called **neutrosophic vertex set**.
- (ii): μ is called **neutrosophic hyperedge set**.

- (iii): |V| is called **order** of NHG and it's denoted by $\mathcal{O}(NHG)$.
- $(iv): \Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NHG and it's denoted by $\mathcal{O}_n(NHG)$.
- (vi): |E| is called **size** of NHG and it's denoted by $\mathcal{S}(NHG)$.
- $(vii): \Sigma_{e \in E} \mu(e)$ is called **neutrosophic size** of NHG and it's denoted by $S_n(NHG)$.

Example 1.5. Assume Figure (5).

- (i): Neutrosophic hyperedge $n_1n_2n_3$ has three neutrosophic vertices.
- (ii): Neutrosophic hyperedge $n_3n_4n_5n_6$ has four neutrosophic vertices.
- (iii): Neutrosophic hyperedge $n_1n_7n_8n_9n_5n_6$ has six neutrosophic vertices.
- $\begin{array}{l} (iv):\ \sigma=\{(n_1,(0.99,0.98,0.55)),(n_2,(0.74,0.64,0.46)),(n_3,(0.99,0.98,0.55)),\\ (n_4,(0.54,0.24,0.16)),(n_5,(0.99,0.98,0.55)),(n_6,(0.99,0.98,0.55)),\\ (n_7,(0.99,0.98,0.55)),(n_8,(0.99,0.98,0.55)),(n_9,(0.99,0.98,0.55))\})\ \text{is}\\ \text{neutrosophic vertex set.} \end{array}$
- (v): $\mu = \{(e_1, (0.01, 0.01, 0.01)), (e_2, (0.01, 0.01, 0.01)), (e_3, (0.01, 0.01, 0.01))\})$ is neutrosophic hyperedge set.
- $(vi): \mathcal{O}(NHG) = 9.$
- $(vii): \mathcal{O}_n(NHG) = (8.21, 7.74, 4.47).$
- $(viii): \mathcal{S}(NHG) = 3.$
- $(ix): S_n(NHG) = (0.03, 0.03, 0.03).$

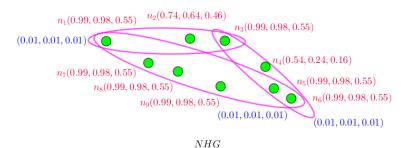


Figure 1. There are three neutrosophic hyperedges and two neutrosophic vertices.

Definition 1.6. (Neutrosophic Edge t-Regular Hypergraph).

A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic edge** t-**regular hypergraph** if every neutrosophic hyperedge has only t neutrosophic vertices.

Question 1.7. What-if all neutrosophic hypergraphs are either edge t-regular or not?

In the following, there are some directions which clarify the existence of some neutrosophic hypergraphs which are either edge t-regular or not.

Example 1.8. Two neutrosophic hypergraphs are presented such that one of them is edge t-regular and another isn't.

(i): Assume Figure (5). It isn't neutrosophic edge t-regular hypergraph.

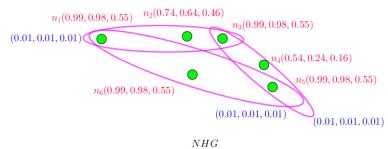


Figure 2. $NHG = (V, E, \sigma, \mu)$ is neutrosophic edge 3-regular hypergraph

(ii): Suppose Figure (2). It's neutrosophic edge 3-regular hypergraph.

Definition 1.9. (Neutrosophic vertex t-Regular Hypergraph).

A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic vertex** t-**regular hypergraph** if every neutrosophic vertex is incident to only t neutrosophic hyperedges.

Example 1.10. Three neutrosophic hypergraphs are presented such that one of them is vertex t-regular and anothers aren't.

- (i): Consider Figure (5). It isn't neutrosophic edge t-regular hypergraph.
- (ii): Suppose Figure (2). It's neutrosophic edge 3—regular hypergraph but It isn't neutrosophic vertex 3—regular hypergraph.
- (iii): Assume Figure (3). It's neutrosophic vertex 2-regular hypergraph but It isn't neutrosophic edge t-regular hypergraph.

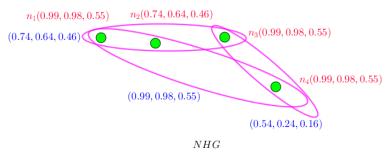


Figure 3. $NHG = (V, E, \sigma, \mu)$ is neutrosophic strong hypergraph.

Definition 1.11. (Neutrosophic Strong Hypergraph).

A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic strong** hypergraph if it's hypergraph and for every $v_1v_2 \cdots v_t \in E$,

$$\mu(v_1v_2\cdots v_t)=\sigma(v_1)\wedge\sigma(v_2)\wedge\cdots\sigma(v_t).$$

Example 1.12. Three neutrosophic hypergraphs are presented such that one of them is neutrosophic strong hypergraph and others aren't.

- (i): Consider Figure (5). It isn't neutrosophic strong hypergraph.
- (ii): Assume Figure (2). It isn't neutrosophic strong hypergraph.

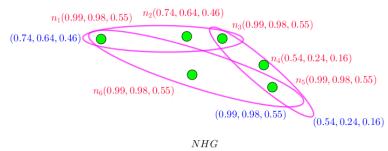


Figure 4. $NHG = (V, E, \sigma, \mu)$ is neutrosophic strong hypergraph.

- (iii): Suppose Figure (3). It isn't neutrosophic strong hypergraph.
- (iv): Assume Figure (4). It's neutrosophic strong hypergraph. It's also neutrosophic edge 3—regular hypergraph but it isn't neutrosophic vertex t—regular hypergraph.

Definition 1.13. (Neutrosophic Strong Hypergraph).

Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ A neutrosophic hyperedge $v_1v_2\cdots v_t \in E$ is called a **neutrosophic strong hyperedge** if

$$\mu(v_1v_2\cdots v_t)=\sigma(v_1)\wedge\sigma(v_2)\wedge\cdots\sigma(v_t).$$

Proposition 1.14. Assume neutrosophic strong hypergraph $NHG = (V, E, \sigma, \mu)$ Then all neutrosophic hyperedges are neutrosophic strong.

Definition 1.15. (Neutrosophic Hyperpath).

A path $v_0, E_0, v_1, v_1, E_1, v_2, \dots, v_{t-1}, E_{t-1}, v_t$, is called **neutrosophic hyperpath** such that v_{i-1} and v_i have incident to E_{i-1} for all nonnegative integers $0 \le i \le t$. In this case, t-1 is called **length** of neutrosophic hyperpath. Also, if x and y are two neutrosophic vertices, then maximum length of neutrosophic hyperpaths from x to y, is called **neutrosophic hyperdistance** and it's denoted by d(x, y). If $v_0 = v_t$, then it's called **neutrosophic hypercycle**.

Example 1.16. Assume Figure (5).

- $(i): n_1, E_1, n_3, E_2, n_6, E_3, n_1$ is a neutrosophic hypercycle.
- (ii): $n_1, E_1, n_n, E_2, n_6, E_3, n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.
- (iii): $n_1E_1n_3E_2n_6E_3n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.
- $(iv): n_1, n_3, n_6, n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.
- (v): $n_1E_1, n_3, E_2, n_6, E_3, n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.
- $(vi): n_1, E_1, n_3, E_2, n_6, E_3, n_7$ is a neutrosophic hyperpath.
- (vii): Neutrosophic hyperdistance amid n_1 and n_4 is two.
- (viii): Neutrosophic hyperdistance amid n_1 and n_7 is one.
- (ix): Neutrosophic hyperdistance amid n_1 and n_2 is one.
- (x): Neutrosophic hyperdistance amid two given neutrosophic vertices is either one or two.

2 New Ideas For Neutrosophic Hypergraphs

Definition 2.1. (Dominating, Resolving and Coloring). Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

- (a): Neutrosophic-dominating set and number are defined as follows.
 - (i): A neutrosophic vertex x neutrosophic-dominates a vertex y if there's at least one neutrosophic strong hyperedge which have them.
 - (ii): A set S is called **neutrosophic-dominating set** if for every $y \in V \setminus S$, there's at least one vertex x which neutrosophic-dominates vertex y.
 - (iii): If S is set of all neutrosophic-dominating sets, then

$$\Sigma_{x \in X} \sigma(x) = \min_{S \in \mathcal{S}} \Sigma_{x \in S} \sigma(x)$$

is called **optimal-neutrosophic-dominating number** and X is called **optimal-neutrosophic-dominating set**.

- (b): Neutrosophic-resolving set and number are defined as follows.
 - (i): A neutrosophic vertex x neutrosophic-resolves vertices y, w if

$$d(x,y) \neq d(x,w)$$
.

(ii): A set S is called **neutrosophic-resolving set** if for every $y \in V \setminus S$, there's at least one vertex x which neutrosophic-resolves vertices y, w.

(iii): If S is set of all neutrosophic-resolving sets, then

$$\Sigma_{x \in X} \sigma(x) = \min_{S \in S} \Sigma_{x \in S} \sigma(x)$$

is called **optimal-neutrosophic-resolving number** and X is called **optimal-neutrosophic-resolving set**.

- (c): Neutrosophic-coloring set and number are defined as follows.
 - (i): A neutrosophic vertex x neutrosophic-colors a vertex y differently with itself if there's at least one neutrosophic strong hyperedge which have them.
 - (ii): A set S is called **neutrosophic-coloring set** if for every $y \in V \setminus S$, there's at least one vertex x which neutrosophic-colors vertex y.
 - (iii): If \mathcal{S} is set of all neutrosophic-coloring sets, then

$$\Sigma_{x \in X} \sigma(x) = \min_{S \in \mathcal{S}} \Sigma_{x \in S} \sigma(x)$$

is called **optimal-neutrosophic-coloring number** and X is called **optimal-neutrosophic-coloring set.**

Example 2.2. Consider Figure (5) where the improvements on its hyperedges to have neutrosophic strong hypergraph.

- (a): The notions of dominating are clarified.
 - (i): n_1 neutrosophic-dominates every vertex from the set of vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 neutrosophic-dominates every vertex from the set of vertices $\{n_6, n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

- (ii): $\{n_1, n_3\}$ is neutrosophic-coloring set but $\{n_1, n_4\}$ is optimal-neutrosophic-dominating set.
- (iii): (1.53, 1.22, 0.71) is optimal-neutrosophic-dominating number.
- (b): The notions of resolving are clarified.
 - (i): n_1 neutrosophic-resolves two vertices n_4 and n_6 .
 - (ii): $V \setminus \{n_1, n_4\}$ is neutrosophic-resolves set but $V \setminus \{n_2, n_4, n_9\}$ is optimal-neutrosophic-resolving set.
 - (iii): (5,94,6.36,3.3) is optimal-neutrosophic-resolving number.
- (c): The notions of coloring are clarified.
 - (i): n_1 neutrosophic-colors every vertex from the set of vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 neutrosophic-colors every vertex from the set of vertices $\{n_6, n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_7, n_8, n_9\}$.
 - (ii): $\{n_1, n_5, n_7, n_8, n_9, n_6, n_4\}$ is neutrosophic-coloring set but $\{n_1, n_5, n_7, n_8, n_2, n_4\}$ is optimal-neutrosophic-coloring set.
 - (iii): (5.24, 4.8, 2.82) is optimal-neutrosophic-coloring number.

Example 2.3. Consider Figure (3).

- (a): The notions of dominating are clarified.
 - (i): n_1 neutrosophic-dominates every vertex from the set of vertices $\{n_5, n_6, n_2, n_3\}$. n_4 neutrosophic-dominates every vertex from the set of vertices $\{n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_6\}$.
 - (ii): $\{n_1, n_3\}$ is neutrosophic-dominating set but $\{n_1, n_4\}$ is optimal-neutrosophic-dominating set.
 - (iii): (1.53, 1.22, 0.71) is optimal-neutrosophic-dominating number.
- (b): The notions of resolving are clarified.
 - (i): n_1 neutrosophic-resolves two vertices n_4 and n_6 .
 - (ii): $V \setminus \{n_1, n_4\}$ is neutrosophic-resolves set but $V \setminus \{n_2, n_4, n_6\}$ is optimal-neutrosophic-resolving set.
 - (iii): (5,94,6.36,3.3) is optimal-neutrosophic-resolving number.
- (c): The notions of coloring are clarified.
 - (i): n_1 neutrosophic-colors every vertex from the set of vertices $\{n_5, n_6, n_2, n_3\}$. n_4 neutrosophic-colors every vertex from the set of vertices $\{n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_6\}$.
 - (ii): $\{n_1, n_5, n_6\}$ is neutrosophic-coloring set but $\{n_5, n_2, n_4\}$ is optimal-neutrosophic-coloring set.
 - (iii): (2.27, 1.86, 1.17) is optimal-neutrosophic-coloring number.

3 Optimal Numbers For Neutrosophic Hypergraphs

Proposition 3.1. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. S is maximum set of vertices which form a hyperedge. Then optimal-neutrosophic-coloring set has as cardinality as S has.

Proof. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Every neutrosophic hyperedge has neutrosophic vertices which have common neutrosophic hyperedge. Thus every neutrosophic vertex has different color with other neutrosophic vertices which are incident with a neutrosophic hyperedge. It induces a neutrosophic hyperedge with the most number of neutrosophic vertices determines optimal-neutrosophic-coloring set. S is maximum set of vertices which form a hyperedge. Thus optimal-neutrosophic-coloring set has as cardinality as S has.

Proposition 3.2. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. S is maximum set of vertices which form a hyperedge. Then optimal-neutrosophic-coloring number is

$$\Sigma_{s \in S} \sigma(s)$$
.

Proof. Assume neutrosophic hypergraph $NHG=(V,E,\sigma,\mu)$. Every neutrosophic hyperedge has neutrosophic vertices which have common neutrosophic hyperedge. Thus every neutrosophic vertex has different color with other neutrosophic vertices which are incident with a neutrosophic hyperedge. It induces a neutrosophic hyperedge with the most number of neutrosophic vertices determines optimal-neutrosophic-coloring set. S is maximum set of vertices which form a hyperedge. Thus optimal-neutrosophic-coloring number is

$$\Sigma_{s \in S} \sigma(s)$$
.

Proposition 3.3. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-coloring number is

$$\Sigma_{v \in V} \sigma(v),$$

then there's at least one hyperedge which contains n vertices where n is the cardinality of the set V.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider optimal-neutrosophic-coloring number is

$$\sum_{v \in V} \sigma(v)$$
.

It implies there's one neutrosophic hyperedge which has all neutrosophic vertices. Since if all neutrosophic vertices are incident to a neutrosophic hyperedge, then all have different colors. \Box

Proposition 3.4. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If there's at least one hyperedge which contains n vertices where n is the cardinality of the set V, then optimal-neutrosophic-coloring number is

$$\sum_{v \in V} \sigma(v)$$
.

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Suppose there's at least one hyperedge which contains n vertices where n is the cardinality of the set V. It implies there's one neutrosophic hyperedge which has all neutrosophic vertices. If all neutrosophic vertices are incident to a neutrosophic hyperedge, then all have different

colors. So V is optimal-neutrosophic-coloring set. It induces optimal-neutrosophic-coloring number is

$$\Sigma_{v \in V} \sigma(v)$$
.

Proposition 3.5. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-dominating number is

$$\sum_{v \in V} \sigma(v)$$
,

then there's at least one neutrosophic vertex which doesn't have incident to any neutrosophic hyperedge.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider optimal-neutrosophic-dominating number is

$$\sum_{v \in V} \sigma(v)$$
.

If for all given neutrosophic vertex, there's at least one neutrosophic hyperedge which the neutrosophic vertex has incident to it, then there's a neutrosophic vertex x such that optimal-neutrosophic-dominating number is

$$\sum_{v \in V - \{x\}} \sigma(v)$$
.

It induces contradiction with hypothesis. It implies there's at least one neutrosophic vertex which doesn't have incident to any neutrosophic hyperedge. \Box

Proposition 3.6. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then optimal-neutrosophic-dominating number is <

$$\sum_{v \in V} \sigma(v)$$
.

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Thus $V - \{x\}$ is a neutrosophic-dominating set. Since if not, x isn't incident to any given neutrosophic hyperedge. This is contradiction with supposition. It induces that x belongs to a neutrosophic hyperedge which has another vertex s. It implies s neutrosophic-dominates s. Thus $V - \{x\}$ is a neutrosophic-dominating set. It induces optimal-neutrosophic-dominating number is s

$$\sum_{v \in V} \sigma(v)$$
.

Proposition 3.7. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-resolving number is

$$\Sigma_{v \in V} \sigma(v),$$

then every given vertex doesn't have incident to any hyperedge.

Proof. Consider neutrosophic hypergraph $NHG=(V,E,\sigma,\mu)$. Let optimal-neutrosophic-resolving number be

$$\Sigma_{v \in V} \sigma(v)$$
.

It implies every neutrosophic vertex isn't neutrosophic-resolved by a neutrosophic vertex. It's contradiction with hypothesis. So every given vertex doesn't have incident to any hyperedge. \Box

Proposition 3.8. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then optimal-neutrosophic-resolving number is <

$$\Sigma_{v \in V} \sigma(v)$$
.

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-resolving number is

$$\Sigma_{v \in V} \sigma(v)$$
,

then there's a contradiction to hypothesis. Since the set $V \setminus \{x\}$ is neutrosophic-resolving set. It implies optimal-neutrosophic-resolving number is <

$$\Sigma_{v \in V} \sigma(v)$$
.

Proposition 3.9. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-coloring number is

$$\Sigma_{v \in V} \sigma(v)$$
,

then all neutrosophic verties which have incident to at least one neutrosophic hyperedge.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider optimal-neutrosophic-coloring number is

$$\Sigma_{v \in V} \sigma(v)$$
.

If for all given neutrosophic vertices, there's no neutrosophic hyperedge which the neutrosophic vertices have incident to it, then there's neutrosophic vertex x such that optimal-neutrosophic-coloring number is

$$\Sigma_{v \in V - \{x\}} \sigma(v)$$
.

It induces contradiction with hypothesis. It implies all neutrosophic vertices have incident to at least one neutrosophic hyperedge.

Proposition 3.10. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then optimal-neutrosophic-coloring number isn't <

$$\sum_{v \in V} \sigma(v)$$
.

Proof. Consider neutrosophic hypergraph $NHG=(V,E,\sigma,\mu)$. Thus $V-\{x\}$ isn't a neutrosophic-coloring set. Since if not, x isn't incident to any given neutrosophic hyperedge. This is contradiction with supposition. It induces that x belongs to a neutrosophic hyperedge which has another vertex s. It implies s neutrosophic-colors s. Thus $V-\{x\}$ isn't a neutrosophic-coloring set. It induces optimal-neutrosophic-coloring number isn't s

$$\Sigma_{v \in V} \sigma(v)$$
.

Proposition 3.11. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then optimal-neutrosophic-dominating set has cardinality which is greater than n-1 where n is the cardinality of the set V.

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. The set V is neutrosophic-dominating set. So optimal-neutrosophic-dominating set has cardinality which is greater than n where n is is the cardinality of the set V. But the set $V \setminus \{x\}$, for every given neutrosophic vertex is optimal-neutrosophic-dominating set has cardinality which is greater than n-1 where n is is the cardinality of the set V. The result is obtained.

Proposition 3.12. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. S is maximum set of vertices which form a hyperedge. Then S is optimal-neutrosophic-coloring set and

$$\Sigma_{s \in S} \sigma(S)$$

 $is\ optimal-neutrosophic-coloring\ number.$

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider S is maximum set of vertices which form a hyperedge. Thus all vertices of S have incident to hyperedge. It implies the number of different colors equals to cardinality of S. Therefore, optimal-neutrosophic-coloring number \geq

$$\Sigma_{s \in S} \sigma(S)$$
.

In other hand, S is maximum set of vertices which form a hyperedge. It induces optimal-neutrosophic-coloring number \leq

$$\Sigma_{s\in S}\sigma(S)$$
.

So S is neutrosophic-coloring set. Hence S is optimal-neutrosophic-coloring set and

$$\Sigma_{s \in S} \sigma(S)$$

is optimal-neutrosophic-coloring number.

4 Optimal Sets For Neutrosophic Hypergraphs

Proposition 4.1. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If S is neutrosophic-dominating set, then D contains S is neutrosophic-dominating set.

Proof. Consider neutrosophic hypergraph $NHG=(V,E,\sigma,\mu)$. Suppose S is neutrosophic-dominating set. Then all neutrosophic vertices are neutrosophic-dominated. Thus D contains S is neutrosophic-dominating set.

Proposition 4.2. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If S is neutrosophic-resolving set, then D contains S is neutrosophic-resolving set.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider S is neutrosophic-resolving set. Hence All two given neutrosophic vertices are neutrosophic-resolved by at least one neutrosophic vertex of S. It induces D contains S is neutrosophic-resolving set.

Proposition 4.3. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If S is neutrosophic-coloring set, then D contains S is neutrosophic-coloring set.

Proof. Suppose neutrosophic hypergraph $NHG=(V,E,\sigma,\mu)$. Consider S is neutrosophic-coloring set. So all neutrosophic vertices which have a common neutrosophic hyperedge have different colors. Thus every neutrosophic vertex neutrosophic-colored by a neutrosophic vertex of S. It induces every neutrosophic vertex which has a common neutrosophic hyperedge has different colors with other neutrosophic vertices belong to that neutrosophic hyperedge. then D contains S is neutrosophic-coloring set.

Proposition 4.4. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then V is neutrosophic-dominating set.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Since $V \setminus \{x\}$ is neutrosophic-dominating set. Then V contains $V \setminus \{x\}$ is neutrosophic-dominating set.

Proposition 4.5. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then V is neutrosophic-resolving set.

Proof. Suppose neutrosophic hypergraph $NHG=(V,E,\sigma,\mu)$. If there's no neutrosophic vertex, then all neutrosophic vertices are neutrosophic-resolved. Hence if I choose V, then there's no neutrosophic vertex such that neutrosophic vertex is neutrosophic-resolved. It implies V is neutrosophic-resolving set but V isn't optimal-neutrosophic-resolving set. Since if I construct one set from V such that only one neutrosophic vertex is out of S, then S is neutrosophic-resolving set. It implies V isn't optimal-neutrosophic-resolving set. Thus V is neutrosophic-resolving set.

Proposition 4.6. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then V is neutrosophic-coloring set.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. All neutrosophic vertices belong to a neutrosophic hyperedge have to color differently. If V is chosen, then all neutrosophic vertices have different colors. It induces that n colors are used where n is the number of neutrosophic vertices. Every neutrosophic vertex has unique color. Thus V is neutrosophic-coloring set.

5 Optimal Sets and Numbers For Family of Neutrosophic Hypergraphs

Proposition 5.1. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then V is neutrosophic-dominating set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. Thus V is neutrosophic-dominating set for every given neutrosophic hypergraph of \mathcal{G} . It implies V is neutrosophic-dominating set for all members of \mathcal{G} , simultaneously.

Proposition 5.2. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then V is neutrosophic-resolving set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. Thus V is neutrosophic-resolving set for every given neutrosophic hypergraph of \mathcal{G} . It implies V is neutrosophic-resolving set for all members of \mathcal{G} , simultaneously.

Proposition 5.3. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then V is neutrosophic-coloring set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. Thus V is neutrosophic-coloring set for every given neutrosophic hypergraph of \mathcal{G} . It implies V is neutrosophic-coloring set for all members of \mathcal{G} , simultaneously.

Proposition 5.4. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then $V \setminus \{x\}$ is neutrosophic-dominating set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. Thus $V \setminus \{x\}$ is neutrosophic-dominating set for every given neutrosophic hypergraph of \mathcal{G} . One neutrosophic vertex is out of $V \setminus \{x\}$. It's neutrosophic-dominated from any neutrosophic vertex in $V \setminus \{x\}$. Hence every given two neutrosophic vertices are neutrosophic-dominated from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ is neutrosophic-dominating set for all members of \mathcal{G} , simultaneously.

Proposition 5.5. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then $V \setminus \{x\}$ is neutrosophic-resolving set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. Thus $V \setminus \{x\}$ is neutrosophic-resolving set for every given neutrosophic hypergraph of \mathcal{G} . One neutrosophic vertex is out of $V \setminus \{x\}$. It's neutrosophic-resolved from any neutrosophic vertex in $V \setminus \{x\}$. Hence every given two neutrosophic vertices are neutrosophic-resolved from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ is neutrosophic-resolving set for all members of \mathcal{G} , simultaneously.

Proposition 5.6. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then $V \setminus \{x\}$ isn't neutrosophic-coloring set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. Thus $V \setminus \{x\}$ isn't neutrosophic-coloring set for every given neutrosophic hypergraph of \mathcal{G} . One neutrosophic vertex is out of $V \setminus \{x\}$. It isn't neutrosophic-colored from any neutrosophic vertex in $V \setminus \{x\}$. Hence every given two neutrosophic vertices aren't neutrosophic-colored from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ isn't neutrosophic-coloring set for all members of \mathcal{G} , simultaneously.

Proposition 5.7. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then union of neutrosophic-dominating sets from each member of \mathcal{G} is neutrosophic-dominating set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. For every chosen neutrosophic hypergraph, there's one neutrosophic-dominating set in the union of neutrosophic-dominating sets from each member of \mathcal{G} . Thus union of neutrosophic-dominating sets from each member of \mathcal{G} is neutrosophic-dominating set for every given neutrosophic hypergraph of \mathcal{G} . Even one neutrosophic vertex isn't out of the union. It's neutrosophic-dominated from any neutrosophic vertex in the union. Hence every given two neutrosophic vertices are neutrosophic-dominated from any neutrosophic vertex in union of neutrosophic-coloring sets. It implies union of neutrosophic-coloring sets is neutrosophic-dominating set for all members of \mathcal{G} , simultaneously.

Proposition 5.8. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then union of neutrosophic-resolving sets from each member of \mathcal{G} is neutrosophic-resolving set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. For every chosen neutrosophic hypergraph, there's one neutrosophic-resolving set in the union of neutrosophic-resolving sets from each member of \mathcal{G} . Thus union of neutrosophic-resolving sets from each member of \mathcal{G} is neutrosophic-resolving set for

every given neutrosophic hypergraph of \mathcal{G} . Even one neutrosophic vertex isn't out of the union. It's neutrosophic-resolved from any neutrosophic vertex in the union. Hence every given two neutrosophic vertices are neutrosophic-resolved from any neutrosophic vertex in union of neutrosophic-coloring sets. It implies union of neutrosophic-coloring sets is neutrosophic-resolved set for all members of \mathcal{G} , simultaneously.

Proposition 5.9. Assume \mathcal{G} is a family of neutrosophic hypergraphs. Then union of neutrosophic-coloring sets from each member of \mathcal{G} is neutrosophic-coloring set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. For every chosen neutrosophic hypergraph, there's one neutrosophic-coloring set in the union of neutrosophic-coloring sets from each member of \mathcal{G} . Thus union of neutrosophic-coloring sets from each member of \mathcal{G} is neutrosophic-coloring set for every given neutrosophic hypergraph of \mathcal{G} . Even one neutrosophic vertex isn't out of the union. It's neutrosophic-colored from any neutrosophic vertex in the union. Hence every given two neutrosophic vertices are neutrosophic-colored from any neutrosophic vertex in union of neutrosophic-coloring sets. It implies union of neutrosophic-coloring sets is neutrosophic-colored set for all members of \mathcal{G} , simultaneously.

Proposition 5.10. Assume \mathcal{G} is a family of neutrosophic hypergraphs. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then $V \setminus \{x\}$ is optimal-neutrosophic-dominating set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. For all neutrosophic hypergraphs, there's no neutrosophic-dominating set from any of member of \mathcal{G} . Thus $V \setminus \{x\}$ is neutrosophic-dominating set for every given neutrosophic hypergraph of \mathcal{G} . For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. Only one neutrosophic vertex is out of $V \setminus \{x\}$. It's neutrosophic-dominated from any neutrosophic vertex in the $V \setminus \{x\}$. Hence every given two neutrosophic vertices are neutrosophic-dominated from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ is neutrosophic-dominating set for all members of \mathcal{G} , simultaneously. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then $V \setminus \{x\}$ is optimal-neutrosophic-dominating set for all members of \mathcal{G} , simultaneously.

Proposition 5.11. Assume \mathcal{G} is a family of neutrosophic hypergraphs. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the neutrosophic vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then $V \setminus \{x\}$ is optimal-neutrosophic-resolving set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. For all neutrosophic hypergraphs, there's no neutrosophic-resolving set from any of member of \mathcal{G} . Thus $V \setminus \{x\}$ is neutrosophic-resolving set for every given neutrosophic hypergraph of \mathcal{G} . For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. Only one neutrosophic vertex is out of $V \setminus \{x\}$. It's neutrosophic-resolved from any neutrosophic vertex in the $V \setminus \{x\}$. Hence every given two neutrosophic vertices are neutrosophic-resolving from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ is

neutrosophic-resolved set for all members of \mathcal{G} , simultaneously. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then $V \setminus \{x\}$ is optimal-neutrosophic-resolving set for all members of \mathcal{G} , simultaneously.

Proposition 5.12. Assume \mathcal{G} is a family of neutrosophic hypergraphs. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the neutrosophic vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then V is optimal-neutrosophic-coloring set for all members of \mathcal{G} , simultaneously.

Proof. Suppose \mathcal{G} is a family of neutrosophic hypergraphs. For all neutrosophic hypergraphs, there's no neutrosophic-coloring set from any of member of \mathcal{G} . Thus V is neutrosophic-coloring set for every given neutrosophic hypergraphs of \mathcal{G} . For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. No neutrosophic vertex is out of V. It's neutrosophic-colored from any neutrosophic vertex in the V. Hence every given two neutrosophic vertices are neutrosophic-colored from any neutrosophic vertex in V. It implies V is neutrosophic-coloring set for all members of \mathcal{G} , simultaneously. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then V is optimal-neutrosophic-coloring set for all members of \mathcal{G} , simultaneously.

6 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

- **Step 1. (Definition)** Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.
- Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.
- Step 3. (Model) As Figure (5), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (1), clarifies about the assigned numbers to these situation.

Table 1. Scheduling concerns its Subjects and its Connections as a Neutrosophic Hypergraph in a Model.

Sections of NHG	n_1	$n_2\cdots$	n_9
Values	(0.99, 0.98, 0.55)	$(0.74, 0.64, 0.46)\cdots$	(0.99, 0.98, 0.55)
Connections of NHG	E_1	E_2	E_3
Values	(0.01, 0.01, 0.01)	(0.01, 0.01, 0.01)	(0.01, 0.01, 0.01)

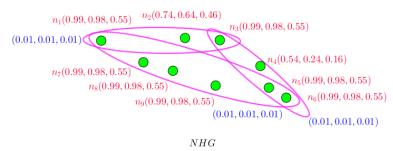


Figure 5. Vertices are suspicions about choosing them.

- **Step 4. (Solution)** As Figure (5) shows, neutrosophic hyper graph as model, proposes to use different types of coloring, resolving and dominating as numbers, sets, optimal numbers, optimal sets and et cetera.
 - (a): The notions of dominating are applied.
 - (i): n_1 neutrosophic-dominates every vertex from the set of vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 neutrosophic-dominates every vertex from the set of vertices $\{n_6, n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_7, n_8, n_9\}$.
 - (ii): $\{n_1, n_3\}$ is neutrosophic-coloring set but $\{n_1, n_4\}$ is optimal-neutrosophic-dominating set.
 - (iii): (1.53, 1.22, 0.71) is optimal-neutrosophic-dominating number.
 - (b): The notions of resolving are applied.
 - (i): n_1 neutrosophic-resolves two vertices n_4 and n_6 .
 - (ii): $V \setminus \{n_1, n_4\}$ is neutrosophic-resolves set but $V \setminus \{n_2, n_4, n_9\}$ is optimal-neutrosophic-resolving set.
 - (iii): (5,94,6.36,3.3) is optimal-neutrosophic-resolving number.
 - (c): The notions of coloring are applied.
 - (i): n_1 neutrosophic-colors every vertex from the set of vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 neutrosophic-colors every vertex from the set of vertices $\{n_6, n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_7, n_8, n_9\}$.
 - (ii): $\{n_1, n_5, n_7, n_8, n_9, n_6, n_4\}$ is neutrosophic-coloring set but $\{n_1, n_5, n_7, n_8, n_2, n_4\}$ is optimal-neutrosophic-coloring set.
 - (iii): (5.24, 4.8, 2.82) is optimal-neutrosophic-coloring number.

7 Open Problems

The three notions of coloring, resolving and dominating are introduced on neutrosophic hypergraphs. Thus,

Question 7.1. Is it possible to use other types neutrosophic hyperedges to define different types of coloring, resolving and dominating on neutrosophic hypergraphs?

Question 7.2. Are existed some connections amid the coloring, resolving and dominating inside this concept and external connections with other types of coloring, resolving and dominating on neutrosophic hypergraphs?

Question 7.3. Is it possible to construct some classes on neutrosophic hypergraphs which have "nice" behavior?

Question 7.4. Which applications do make an independent study to apply these three types coloring, resolving and dominating on neutrosophic hypergraphs?

Problem 7.5. Which parameters are related to this parameter?

Problem 7.6. Which approaches do work to construct applications to create independent study?

Problem 7.7. Which approaches do work to construct definitions which use all three definitions and the relations amid them instead of separate definitions to create independent study?

8 Conclusion and Closing Remarks

This study uses mixed combinations of different types of definitions, including coloring, resolving and dominating to study on neutrosophic hypergraphs. The connections of neutrosophic vertices which are clarified by general hyperedges differ them from each other and and put them in different categories to represent one representative for each color, resolver and dominator. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic hypergraphs theory. One way is finding some relations amid three definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study is pointed out.

Table 2. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations	
1. Defining Dimension	1. General Results	
2. Defining Domination		
3. Defining Coloring	2. Connections Amid New Notions	
4. Applying on Individuals		
5. Applying on Family	3. Connections of Results	

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