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# Relativistic Effects Appearing at Non-Relativistic Speeds

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**Abstract**

We study the center of energy (CE) before and after the separation of superposed wave from a moving medium (MM). It is assumed that two out-of-phase mechanical transverse waves (MTWs) propagating from the opposite directions on a medium moving at non-relativistic speeds are superposed and the superposed portion (SP) is separated from the MM at that moment. We consider the CE of the SP before and after the separation from the MM. The location of CE (LCE) of the SP seems to be at the center of it at the moment of superposition. The SP rotates due to the separation from the MM since the velocity of each portion symmetric with respect to the center of the SP is equal in magnitude and opposite in direction. The magnitudes of velocities of the symmetric portions become different as soon as the SP begins to rotate with the separation from the MM. Then the energies of their symmetric portions are not the same, so the LCE of the SP is not at the center of it. As a result, the LCE of it looks different before and after the separation from the MM. We must find a mechanism to keep the LCE of the SP constant. We propose that the two out-of-phase MTWs propagating from the opposite directions on a MM originally have hidden difference in relativistic energy (HDRE) and it suddenly appears within the range observable in Newtonian mechanics when the SP starts rotating. This means that the concept of HDRE is necessary to keep the LCE constant.

**Keywords:** hidden difference in relativistic energy; center of energy; relativity of simultaneity; time dilation; mechanical transverse wave; superposition of waves; rest mass in wave motion; special relativity

## 1. Introduction

It is regarded as one of the most fundamental principles of physics that the location of center of mass in an isolated frame of reference is always constant. From this principle, Einstein has derived the equivalence of mass and energy in special relativity (SR) [1,2]. According to it, the LCE is always constant in an isolated frame of reference [3–5]. This is called the CE theorem in SR [3–6]. The concept of CE is the relativistic generalization of center of mass because it includes not only rest energy but also all forms of energy [3]. If the CE in a frame of reference is at rest, then its total momentum is zero [3,6,7]. Here the CE theorem is related to the law of conservation of momentum. Furthermore, physical quantity like hidden momentum mainly in the electromagnetic field has been proposed from this theorem [7–12]. The hidden momentum is a notion devised to prevent the change in energy density, in other words, to keep the LCE constant [8,13]. In contrast, the concept like hidden energy has not been proposed.

We study whether the concept of hidden energy is needed to keep the LCE constant. This is irrelevant to the above hidden momentum in the electromagnetic field. We consider the superposition of two out-of-phase MTWs propagating from the opposite directions on a medium moving at non-relativistic speeds and the separation of the SP from the medium. The LCE of the SP seems to be at the center of it at the moment of superposition. The SP starts rotating with the separation from the MM because the velocity of each portion symmetric with respect to the center of it is equal in magnitude and opposite in direction. The magnitudes of velocities of the symmetric

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portions become different as soon as the SP begins to rotate at the same time as the separation from the MM. Hence, the energies of the symmetric portions are not same. As a result, the LCE of the SP is not at the center of it. This means that the LCE of the SP looks different before and after the separation from the MM. We cannot find any previous work examining this. We must discover a mechanism to keep the LCE of the SP constant under non-relativistic speeds at which the medium moves.

To keep the LCE constant, we postulate that the two out-of-phase MTWs propagating from the opposite directions on the MM originally have the HDRE. It suddenly appears as the observable difference in energy even under non-relativistic speeds when the SP begins to rotate. Therefore, we apply SR to physical phenomenon under non-relativistic speeds to analyze this HDRE.

## 2. The LCE of Out-of-phase MTWs on a MM

First of all, we assume two inertial frames of reference, $S'$ and $S$, that are in a state of uniform relative motion. $S'$ moves with constant non-relativistic velocity $V$ in the positive direction of the $x$ axis in $S$. Here the $x$ axis in $S$ and the $x'$ axis in $S'$ are on the same straight line. A box is stationary in $S'$. Moreover, the box is equipped with the string overlapping with the $x'$ axis in $S'$.

We simultaneously move both ends of the string quickly up and down, in other words, along the $y'$ axis perpendicular to the $x'$ axis in $S'$. We thereby generate two MTWs that have $1/2$ wavelength respectively and are symmetric with respect to the $x'$ axis and $y'$ axis in $S'$. The two out-of-phase MTWs propagating in the opposite and advancing direction of the MM are generated above and below the medium that is in equilibrium, respectively. In other words, they are generated in the range of $y' > 0$ and $y' < 0$, respectively. The magnitude of velocity of each portion symmetric with respect to the $x'$ and $y'$ axes is identical, so the symmetric portions have the same energy. Thus, the LCE of the two MTWs is constant and is on the $x'$ axis overlapping with the medium in equilibrium (ME).

Thereafter, the two MTWs propagating from the opposite directions are superposed on the $x'$ axis instantaneously and all of their energies become kinetic energies. At that moment, the magnitude of the transverse velocity (TV) of each portion symmetric with respect to the center of the SP is identical in $S'$. Hence, those portions have the same kinetic energy. As a result, the LCE of the SP is on the $x'$ axis in $S'$, so it remains constant.

This, within the range observable, also seems to hold true for $S$ and hence the LCE of the SP looks located on the $x$ axis.

## 3. The LCE of the SP Separated from the MM

Here we separate the SP from the MM instantaneously. As mentioned above, when the two waves are superposed, the velocity of each portion symmetric with respect to the center of the SP is equal in magnitude and opposite in direction. Hence, the SP starts rotating clockwise due to the separation from the MM because its left and right sides have upward and downward velocities respectively. Suppose that the center of rotation of the SP is at $x' = 0$ and $y' = 0$. Then the SP becomes vertical with respect to the $x'$ and $x$ axes momentarily. In this state, the distribution of the rest mass (RM) of the SP is symmetric with respect to each axis. For simplicity, we consider the SP in this state.

The velocities of two portions above and below the $x'$ axis, in other words, in the range of $y' > 0$ and $y' < 0$ are $v_x{}'$ and $-v_x{}'$ in $S'$ respectively since the SP rotates clockwise. Suppose that each portion with the same mass is symmetric with respect to the $x'$ axis. Then each magnitude of velocity is identical in $S'$. When observing this from $S$, the portion with $v_x{}'$ and that with $-v_x{}'$ are observed to have $V + v_x{}'$ and $V - v_x{}'$ respectively, so the former magnitude of velocity is large compared with the latter one of velocity. Therefore, in $S$, the energy of the portion above the $x$ axis is larger than that of the portion below the $x$ axis. In sum, the distribution of energy of the SP is not symmetric with respect to the $x$ axis in $S$. Let $E_1$ and $E_2$ be the energies at $y_1$ and $y_2$ symmetric with respect

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to the $x$ axis, respectively. Suppose that $y_1$ and $y_2$ are located above and below the $x$ axis, in other words, in the range of $y_1 > 0$ and $y_2 < 0$, respectively. Moreover, let $y_c$ be the center of $y_1$ and $y_2$. It is on the $x$ axis. Then, since $E_1$ at $y_1$ is larger than $E_2$ at $y_2$, we obtain

$$E_1(y_1 - y_c) > E_2(y_c - y_2). \quad (1)$$

This can be applied to any portion symmetric with respect to the $x$ axis. Let $E_i$ and $E_j$ be the energies at arbitrary $y$ coordinates $y_i$ and $y_j$ symmetric with respect to the $x$ axis, respectively. Moreover, suppose that $y_i$ and $y_j$ are located above and below the $x$ axis, in other words, in the range of $y_i > 0$ and $y_j < 0$, respectively. Then, since $E_i$ at $y_i$ is larger than $E_j$ at $y_j$, we have

$$E_i(y_i - y_c) > E_j(y_c - y_j), \quad (2)$$

where $y_c$ is the center of $y_i$ and $y_j$ and moreover it is on the $x$ axis.

From these facts, the LCE of the SP is not on the $x$ axis overlapping with the ME and is located above it. In other words, it is in the range of $y > 0$. Consequently, in $S$, the LCE of the SP seems to be different before and after the separation from the MM.

By contrast, the production of the two out-of-phase MTWs propagating from the opposite directions on the medium results in the counterclockwise rotation of the box itself or other body. Suppose that one portion of the rotating body is above $x'$ axis and the other portion of it is below the $x'$ axis, respectively. The velocities of portions above and below the $x'$ axis, in other words, in the range of $y' > 0$ and $y' < 0$ are $-v_x'$ and $v_x'$ in $S'$ respectively since the body rotates counterclockwise. Again, suppose that each portion with the same mass is symmetric with respect to the $x'$ axis. Then each magnitude of velocity is identical in $S'$. When observing this from $S$, the portion with $-v_x'$ and that with $v_x'$ are observed to have $V - v_x'$ and $V + v_x'$, respectively. Therefore, in $S$, the energy of a portion above the $x$ axis is small compared to that of a portion below the $x$ axis. This can also be applied to any portion symmetric with respect to the $x$ axis. From these, the LCE of the rotating body shifts below the $x$ axis.

In sum, the two shifts in the CE from the $x$ axis cancel each other. Hence, the LCE in $S$ is always on the $x$ axis, so it is constant. Nevertheless, the problem is that, within the range observable, the LCE of the medium does not seem to shift with the generation of the waves and thereafter suddenly seems to shift due to the separation of the SP from the MM.

## 4. Physical Quantities Contributing to Relativistic Energy of the Two Waves

We must find a mechanism to keep the LCE constant. To do so, we consider whether two out-of-phase MTWs propagating from the opposite directions on a MM originally have HDRE.

We start by analyzing the relativistic kinetic energies of the two out-of-phase waves propagating from the opposite directions on a MM. Two physical quantities that contribute to them are the velocity and RM of each portion which is in wave motion (WM).

### *4.1. Velocity Contributing to Relativistic Kinetic Energy (RKE)*

We analyze the velocity $v$ of a portion which is in WM in $S$. Let $v'$ be the velocity of the corresponding portion observed from $S'$. The velocity $v$ is one obtained by converting $v'$ according to the Lorentz transformation.

On the basis of the law of velocity addition in SR, two components of $v$ of the portion, i.e., $v_x$ and $v_y$ are written as:

$$v_x = \frac{V + v_x'}{1 + V v_x'/c^2}, v_y = \frac{v_y'\sqrt{1 - V^2/c^2}}{1 + V v_x'/c^2}. \quad (3)$$

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Here $V$ is the velocity of $S'$ in the positive direction of the $x$ axis in $S$. Moreover, since $v = \sqrt{(v_x^2 + v_y^2)}$, substituting Eq. (3) into it, we have a transformation formula of velocity:

$$v = \sqrt{\left(\frac{V + v_x'}{1 + Vv_x'/c^2}\right)^2 + \left(\frac{v_y'\sqrt{1 - V^2/c^2}}{1 + Vv_x'/c^2}\right)^2}$$
$$= \frac{\sqrt{V^2 + 2Vv_x' + v_x'^2 + v_y'^2(1 - V^2/c^2)}}{1 + Vv_x'/c^2}$$
$$= \frac{\sqrt{V^2 + 2Vv_x' + v_x'^2 + v_y'^2 - V^2 v_y'^2/c^2}}{1 + Vv_x'/c^2}. \quad (4)$$

In the case of the MTWs, $v_x' = 0$. Hence, the longitudinal velocity of each portion in WM observed from $S$ remains $V$. Substituting $v_x' = 0$ for Eq. (4), we obtain

$$v = \sqrt{V^2 + v_y'^2 - V^2 v_y'^2/c^2}. \quad (5)$$

As indicated above, the velocity of each portion of the wave contributing to the RKE obeys the Lorentz transformation as well as that of a particle.

### *4.2. A Relativistic Peculiarity in the Generation of MTWs*

Another physical quantity contributing to the RKE of the wave is the RM in WM. We need to indicate a relativistic peculiarity in the generation of MTWs before considering the RM in WM.

The continuous supply of energy from a wave source (WS) is done on a medium adjacent to it. In contrast, transfer of energy on the medium is performed at distant places from the WS. We assume the medium moving with constant velocity. According to the relativity of simultaneity, the time of supply of energy at the WS generally differs from the time of transfer of energy on the medium.

### *4.3. RM Contributing to RKE*

The RM in WM is generally the amount obtained by multiplying unit length by mass density. Viewed from a different aspect, the RM in WM corresponds to the coordinate interval (CI) of the wave rather than the length of it.

Firstly, we consider the amount of RM of the wave propagating in the opposite direction of the MM in $S$. Let $W_o$ be its wave. Here we need to take the relativity of simultaneity in SR into account. Suppose that clocks are fixed at certain equal intervals along the $x$ and $x'$ axes respectively. Moreover, we assume that they are synchronized in some way on each axis.

According to the relativity of simultaneity, the clocks at the back of the MM always go by fast compared to those at the front of it. This means that, in $S$, an event A in the back of the MM occurs earlier than one B in the front of it even if both of them take place in $S'$ simultaneously. When a wave propagates in the opposite direction of the MM, the $x'$ coordinate corresponding to the leading end of the wave (LEW) is at the back of the MM compared to that corresponding to the WS. Hence, the time on the former $x'$ coordinate goes by faster than that on the latter one. Let $x_0'$ and $-x_1'$ be the positions of the WS and the LEW at a certain time, $t_1'$, in $S'$, respectively. When observing the wave from $S$, if the position $x_0$ and time $t_1$ of the WS coincide with $x_0'$ and $t_1'$ in $S'$ respectively, then the LEW is not at $-x_1'$. The time on $-x_1'$ is not $t_1'$ and $t_1'$ is already past. Therefore, the LEW observed from $S$ has already passed through $-x_1'$ and propagates backward on the MM compared to that observed in $S'$. When observing the LEW from $S$, it is at $-x_2'$ that is positioned farther from $x_0'$ compared with $-x_1'$. Also, at that moment, the clock on $-x_2'$ observed in $S$ shows $t_2'$. Here we can represent the CI by the absolute value. Then the coordinate interval of the portion in WM (CIPWM) observed from $S$ is $|-x_2 - x_0|$, while that in $S'$ is $|-x_1' - x_0'|$. Therefore, we obtain

$$\left|-x_2 - x_0\right| > \left|-x_1{}' - x_0{}'\right|. \ (6)$$

Moreover, we assume that the portion corresponding to $\left|-x_1{}' - x_0{}'\right|$ has a TV $v_y{}'$ in $S'$. Converting $v_y{}'$ in $S'$ into the corresponding TV, $v_y$, in $S$, from Eq. (3), $v_y = v_y{}'\sqrt{1 - V^2/c^2}$ since $v_x{}' = 0$. As indicated above, the LEW in $S$ arrives at $-x_2$ when the TV of the portion becomes $v_y{}'$ in $S'$. Hence, the CIPWM with $v_y{}'\sqrt{1 - V^2/c^2}$ is $\left|-x_2 - x_0\right|$. As a result, we can find that the CIPWM having $v_y$ in $S$, $I_{W_o}$, is larger than one having $v_y{}'$ in $S'$, $I'$, i.e.,

$$I_{W_o} > I'. \ (7)$$

Here the term, interval, does not denote the distance between the coordinates of the portion in WM, in other words, the length of the wave. On the other hand, the CI corresponds to RM. Therefore, we can calculate the former using the latter. The RM of the portion having $v_y$ in $S$, $m_{W_o}$, is defined as:

$$m_{W_o} = I_{W_o}\rho', \ (8)$$

where $\rho'$ is RM per unit CI in $S'$. Here $m_o$ in $S$ is determined using $\rho'$ in $S'$ because the RM corresponding to the CI is depended on the proper $\rho'$ in $S'$ relative to which the ME is at rest. The RM, $m'$, having $v_y{}'$ in $S'$ is equal to a value obtained by multiplying $I'$ by $\rho'$, i.e., $m' = I'\rho'$. Then, since $I_{W_o} > I'$ from inequality (7), we find that $m_{W_o}$ having $v_y$ in $S$ is larger than $m'$ having $v_y{}'$ in $S'$. The difference in the RM between the former and the latter results from that of the CI because $\rho'$ is the same value in each frame of reference. Since its difference is proportional to the ratio of the CI, we get

$$m_{W_o} = m'\frac{I_{W_o}}{I'} > m'. \ (9)$$

The difference in the CI, in other words, that in the RM between $S$ and $S'$, as indicated above, depends upon the time difference at the coordinates on the $x'$ axis observed from $S$. This time difference is determined according to the velocity $V$ of $S'$ relative to $S$.

Secondly, we analyze the amount of RM of the wave propagating in the advancing direction of the MM in $S$. Let $W_a$ be its wave. In contrast to $W_o$, in $W_a$, the $x'$ coordinate corresponding to the LEW is at the front of the MM compared with that corresponding to the WS. Hence, the time on the former $x'$ coordinate goes by slower than that on the latter $x'$ coordinate. Then, if the position and time of the WS in $S$ coincide with them of the WS in $S'$ respectively, then the LEW in $S'$ propagates forward on the MM compared to that observed from $S$. In other words, the latter is at the back of the MM compared with the former. This means that the CIPWM in $S$ is smaller than that in $S'$. Let $-v_y{}'$ and $-v_y$ be the transverse velocities of the portions corresponding to each CIPWM in $S'$ and $S$, respectively. The latter velocity is also one obtained by converting $-v_y{}'$ according to the Lorentz transformation. Then the CIPWM having $-v_y$ in $S$, $I_{W_a}$, is smaller than $I'$ having $-v_y{}'$ in $S'$, i.e.,

$$I_{W_a} < I'. \ (10)$$

The RM of the portion having $-v_y$ in $S$, $m_{W_a}$, is defined as:

$$m_{W_a} = I_{W_a}\rho'. \ (11)$$

Furthermore, since $I_{W_a} < I'$ from inequality (10), we find that $m_{W_a}$ having $-v_y$ in $S$ is smaller than $m'$ having $-v_y{}'$ in $S'$. Again, since its difference is proportional to the ratio of the CI, we obtain

$$m_{W_a} = m' \frac{I_{W_a}}{I'} < m'. \ (12)$$

We assume that the magnitudes of velocities of $v_y{}'$ and $-v_y{}'$ are the same. Thus, those of velocities of $v_y$ and $-v_y$ are also the same. Then combining expressions (7) and (10) yields

$$I_{W_o} > I' > I_{W_a}. \ (13)$$

Furthermore, combining expressions (9) and (12), we obtain

$$m_{Wo} > m' > m_{W_a}. \ (14)$$

Consequently, the RM of a portion of $W_o$ is larger than that of a portion of $W_a$ when each portion has the TV of the same magnitude.

Inequalities (13) and (14) hold true for any CIPWM having the TV of the same magnitude. Therefore, we get

$$TI_{Wo} > TI' > TI_{Wa}. \ (15)$$

where $TI_{Wo}$ and $TI_{Wa}$ are the total CIPWM of $W_o$ and $W_a$ in $S$ respectively and $TI'$ is that of one wave in $S'$. Furthermore, we obtain

$$TM_{Wo} > TM' > TM_{Wa}. \ (16)$$

where $TM_{Wo}$ and $TM_{Wa}$ are the total RM of $W_o$ and $W_a$ in $S$ respectively and $TM'$ is that of one wave in $S'$.

Here we need to bear in mind that we consider and compare only the amount of RM of the portions that are in WM. Total RM including the portions that are not in WM is invariant for the Lorentz transformation.

## 5. HDRE Between the Two Waves

### *5.1. The Amount of RKE*

We firstly compare the RKE of a portion of $W_o$ with that of a portion of $W_a$. The RKE of a portion having RM, $m$, and velocity, $v$, is given by

$$\mathrm{RKE} = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right), (17)$$

where $c$ is the speed of light. Substituting Eq. (5) into Eq. (17), we have

$$\mathrm{RKE} = mc^2\left\{\frac{1}{\sqrt{1 - \left(\sqrt{V^2 + v_y'^2 - V^2 v_y'^2/c^2}\right)^2/c^2}} - 1\right\}. (18)$$

Here let $m_{Wo1}$ and $m_{Wa1}$ be the RM of a portion of $W_o$ and that of a portion of $W_a$, respectively. Moreover, let $v_y$ be the TV of the portion with $m_{Wo1}$. Similarly, let $-v_y$ be the TV of the portion with $m_{Wa1}$. These $v_y$ and $-v_y$ are the velocities obtained by converting $v_y{}'$ and $-v_y{}'$ observed in $S'$ according to the Lorentz transformation respectively. Again, from Eq. (3), $v_y = v_y'\sqrt{1 - V^2/c^2}$ and $-v_y = -v_y'\sqrt{1 - V^2/c^2}$ since $v_x' = 0$. Suppose that the portions with $m_{o1}$ and $m_{a1}$ have the same displacement in the direction of $y$ axis. Hence, each magnitude of TV is also the

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same. Again, each longitudinal velocity is the same $V$. Let $RKEm_{Wo1}$ and $RKEm_{Wa1}$ be the RKE of the portion having $m_{Wo1}$ and that of the portion having $m_{Wa1}$. They are given by the following equations:

$$RKEm_{Wo1} = m_{Wo1}c^2 \left\{ \frac{1}{\sqrt{1-\left(\sqrt{V^2+v_y'^2-V^2v_y'^2/c^2}\right)^2/c^2}} - 1 \right\} \quad (19)$$

and

$$RKEm_{Wa1} = m_{Wa1}c^2 \left\{ \frac{1}{\sqrt{1-\left(\sqrt{V^2+\left(-v_y'\right)^2-V^2\left(-v_y'\right)^2/c^2}\right)^2/c^2}} - 1 \right\}. \quad (20)$$

From inequalities (14), $m_{Wo1} > m_{Wa1}$. In contrast, other physical quantities contributing to the RKE are the same. In Eqs. (19) and (20), even if the signs of $v_y$ and $-v_y$ in two equations are different, their squared values are the same. Therefore, we find

$RKEm_{Wo1} > RKEm_{Wa1}$. (21)

Let $TRKE_{Wo}$ and $TRKE_{Wa}$ be the total RKE of $W_o$ and that of $W_a$, respectively. $TRKE_{Wo}$ is given by

$$TRKE_{Wo} = RKEm_{Wo1} + RKEm_{Wo2} + RKEm_{Wo3} + \cdots = \sum_{i=1}^{n} RKEm_{Woi}. \quad (22)$$

Likewise, $TRKE_{Wa}$ is expressed as

$$TRKE_{Wa} = RKEm_{Wa1} + RKEm_{Wa2} + RKEm_{Wa3} + \cdots = \sum_{i=1}^{n} RKEm_{Wai}. \quad (23)$$

Inequality (21) holds true for any portion having the velocity of the same magnitude. Therefore, the RKE of each term in Eq. (22) is larger than that of the corresponding term in Eq. (23) on the premise that each portion of the corresponding terms has the velocity of the same magnitude. Consequently, we find the following inequality

$$TRKE_{Wo} > TRKE_{Wa}. \quad (24)$$

This difference due to that of the RM in WM does not appear within the range observable during WM. Therefore, $TRKE_{Wo} - TRKE_{Wa}$ is the HDRE.

*5.2. The Amount of Potential Energy (PE)*

Secondly, we analyze the PE of the two waves. A portion in WM with a certain displacement in the direction of the $y'$ axis in $S'$ has not only the TV but also a constant proper PE corresponding to it. The amount of PE stored in the portion depends on the CIPWM because a CI corresponds to the certain displacement. This holds true even though the forms of each portion, in other words, their

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waveforms look different in $S$. When observing the two CIPWMs with the displacement in $S$, as indicated above, each CIPWM included in $W_o$ and $W_a$ is different.

Let $PE_{I_{Wo}}$ and $PE_{I_{Wa}}$ be the PE stored in the portion corresponding to $I_{Wo}$ and that stored in the portion corresponding to $I_{Wa}$, respectively. Suppose that the displacement of each portion is the same. Then the PE per CI of each portion is also the same. Since, from inequalities (13), $I_{Wo} > I_{Wa}$, we obtain

$$PE_{I_{Wo}} > PE_{I_{Wa}}. \quad (25)$$

Furthermore, inequality (25) is true for any CI having an arbitrary same displacement. Let $TPE_{Wo}$ and $TPE_{Wa}$ be each total PE corresponding to $TI_{Wo}$ and $TI_{Wa}$, respectively. Since, from inequalities (15), $TI_{Wo} > TI_{Wa}$, we get

$$TPE_{Wo} > TPE_{Wa}. \quad (26)$$

This difference due to that of the CI in WM does not appear within the range observable during WM. Therefore, $TPE_{Wo} - TPE_{Wa}$ is also the HDRE.

## 6. Appearance of HDRE and Its Mechanism

We compare the total relativistic energy of $W_o$ with that of $W_a$. Let $TRE_{Wo}$ and $TRE_{Wa}$ be each total relativistic energy of $W_o$ and $W_a$. As already indicated, the former and latter waves correspond to those which have been generated above and below the $x$ axis overlapping with the ME, respectively. Combining inequalities (24) and (26) yields

$$TRE_{Wo} = TRKE_{Wo} + TPE_{Wo} > TRE_{Wa} = TRKE_{Wa} + TPE_{Wa}. \quad (27)$$

These expressions are derived from the difference in the CI of the two waves and the corresponding difference in RM.

From expressions (27), the CE of the two waves has been located above $x$ axis since before the separation of the SP from the MM although this has not been within the range observable under non-relativistic speeds. We conclude that the two waves have the HDRE and its difference suddenly appears due to the separation of the SP from the MM. This means that the unobserved difference in energy, $TRE_{Wo} - TRE_{Wa}$, appears as that in observable kinetic energy. In other words, this shows that relativistic effect normally non-observable can be observed within Newtonian mechanics.

The difference in total relativistic energy between two waves before they superpose is due to the amount of RM of each portion with the same velocity in the $y$ axis direction. On the other hand, after two waves superpose and separate, the RM of the SP is evenly distributed in it. When the SP is perpendicular to the $x$ axis, the HDRE appears as the difference in the RKE of each portion located symmetrically to the $x$ axis. What effect does the RM have on the LCE after the separation of the SP? Here we assume that the SP appears to overlap with the $x$ axis. To be precise, due to the relativity of simultaneity, the SP does not completely overlap with the $x$ axis. In this case, the velocity of each portion of the SP is almost the same. For example, when the front end in the direction of travel of the SP overlaps with the $x$ axis, the rear end in the direction of travel of it is already above it due to the counterclockwise rotation because the latter time goes by fast compared to the former one. According to the formula $E = mc^2$, rest energy becomes enormous value. Moreover, this rest energy is equivalent to RM. The position of the RM of each end effects on the LCE of them. Even for extremely small distance that is non-observable in Newtonian mechanics, multiplying such distance by rest energy results in the change in the LCE in the form of kinetic energy observable in its mechanics.

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## 7. Conclusions

We studied a mechanism to keep the LCE constant in the phenomenon that the superposed wave separates from the MM. We proposed that the two out-of-phase MTWs propagating from the opposite directions on a medium moving at non-relativistic speeds have a HDRE. Its difference suddenly appears within the range observable when the SP begins to rotate due to the separation from the MM. The LCE observed from $S$ remains constant because that of the two waves has originally been above the $x$ axis. Consequently, the concept of HDRE is necessary to keep the LCE constant.

We conducted a qualitative research on appearance of HDRE. Quantitative researches in this regard are expected in the future.

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