Preprints.org

Article

Not peer-reviewed version

Mathematical Model of Sustainable Resource Allocation Taking into Account Transaction Costs and Equilibrium Prices Under Technological Constraints

Anna V. Aleshina * , Andrey L. Bulgakov * , Yanliang Xin , Larisa S. Skrebkova

Posted Date: 18 November 2025

doi: 10.20944/preprints202511.1248.v1

Keywords: competitive equilibrium; transaction costs; technological constraints; stability; dynamic model; resources; macroeconomics

Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a Creative Commons CC BY 4.0 license, which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Article

Mathematical Model of Sustainable Resource Allocation Taking into Account Transaction Costs and Equilibrium Prices Under Technological Constraints

Anna V. Aleshina [1,2,*,†], Andrey L. Bulgakov [1,2,*,†], Yanliang Xin [2,†] and Larisa S. Skrebkova [1,†]

1 Moscow Center for Fundamental and Applied Mathematics, Lomonosov Moscow State University, 119991, Russian Federation
2 Faculty of Economics, Lomonosov Moscow State University, 119991, Russian Federation
* Correspondence: aleshina@econ.msu.ru (A.V.A.); bulgakoval@my.msu.ru (A.L.B.)
† These authors contributed equally to this work.

Abstract

A mathematical model of sustainable resource allocation in a competitive economy is developed and studied, taking into account transaction costs and technological constraints. The model describes the interaction of producers and consumers, introduces a technological set and price dynamics through demand–supply imbalance. Using the theory of covering mappings and variational methods, the existence of equilibrium prices is proven. Issues of stability, numerical algorithms, and macroeconomic interpretation of the obtained results are considered.

Keywords: competitive equilibrium; transaction costs; technological constraints; stability; dynamic model; resources; macroeconomics

---

1. Introduction

The problem of optimal resource allocation remains central in economic theory and applied mathematics. Important factors in modern analysis are transaction costs (which create a gap between selling and buying prices) and technological constraints that limit outputs and inputs. The goal of the present work is to construct a mathematically rigorous model that combines static equilibrium analysis and price dynamics, analysis of the existence of equilibrium price vectors and their stability, as well as to provide an economic interpretation of the obtained results and conduct numerical illustrations.

2. Materials and Methods

2.1. Problem Statement and Basic Assumptions

Let $n \in \mathbb{N}$ be the number of different goods. Denote by $p = (p_1, \dots, p_n) \in \mathbb{R}^n_{++}$ the price vector (components positive). By $\langle x, y \rangle$ we denote the scalar product in $\mathbb{R}^n$. The norm $\|\cdot\|$ is Euclidean.

2.2. Technological Set

The technological set $T \subset \mathbb{R}^{2n}_+$ is defined via a smooth function

$$\psi \colon \mathbb{R}^{2n} \to \mathbb{R}, \quad T = \{(y^+, y^-) \in \mathbb{R}^{2n}_+ \mid \psi(y^+, y^-) \le 0\}. \tag{2.2.1}$$

Assumption 1. *The function ψ is twice continuously differentiable and* strongly convex*: there exists $\epsilon > 0$ such that $\nabla^2 \psi(z) \succeq \epsilon I$ for all $z \in \mathbb{R}^{2n}$.*

Strong convexity ensures that functions entering optimization problems with a minus sign lead to a strictly convex (respectively, strictly concave when with a minus sign) functional. Mathematical models of technological sets are widely used to describe production constraints. For example, a

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

review of macroeconomic models [6] shows how production factors and technologies affect output in different countries. An experimental study [7] uses technical equipment to assess worker workload and optimize processes. An analysis of agribusiness in Serbia [8] examines the impact of capital structure on farm efficiency taking into account technological constraints. A study of agricultural ecology in China [9] evaluates resource use efficiency in a river basin. Finally, work on the Brazilian sector [10] applies nonparametric methods to analyze environmental efficiency. These studies rely on smooth or linear descriptions of technologies, without accounting for sharp thresholds and discontinuities in real systems. In contrast, our technological set is defined via a smooth function that guarantees strict convexity and allows analytical work with nonlinear constraints.

2.3. Transaction Costs

The parameter $\alpha \in (0,1]$ describes the share of output actually realized by buyers after deducting transaction losses. Then the producer's nominal income from sales at prices p is estimated via αp. It is important to distinguish two meanings of α: the economic one (share after costs) and the term κ in the theory of covering mappings (in what follows, we will introduce the notation κ for this theory to avoid confusion). Empirical estimates of transaction costs in various sectors emphasize their role in distorting market signals. Thus, an analysis of PDO olive oil in the Mediterranean [1] reveals that certification and logistics costs reduce the sustainability of small producers, compensated only through economies of scale. In China's water supply [2], the transition to informal schemes increases search and contracting costs by 15–20 %, but reduces capital expenditures. Digital platforms [3] minimize coordination costs in organizational forms, yet exacerbate information asymmetry. In environmental policy [4], transaction costs determine the choice between taxes and subsidies, where high costs render market mechanisms inefficient. Finally, investments in the U.S. forestry sector [5] show that high search and land valuation costs hinder capital inflows. These works rely on smooth or piecewise-linear models of α, ignoring threshold discontinuities and nonlinear dependencies in real transactions. In contrast, our model introduces α as a parameter of losses in the producer's income $\langle \alpha p, y^+ \rangle$, preserving strict concavity of the objective function under Ψ, which allows analytically capturing superlinear effects and equilibrium stability even as $\alpha \to 0$.

2.4. Producer Behavior

The producer chooses a technology $(y^+, y^-) \in T$, where y^+ is the output vector, y^- is the input vector. The profit model is correctly defined as

$$\Pi(p; y^+, y^-) = \langle \alpha p, y^+ \rangle - \langle p, y^- \rangle - \Psi(y^+, y^-). \tag{2.4.1}$$

where Ψ is a non-degenerate nonnegative penalty term modeling additional technological costs (e.g., losses, processing, negative externalities). We assume Ψ convex and, as a rule, positive; then the profit maximization problem over $(y^+, y^-) \in T$ yields a strictly concave problem in the choice of parameters:

$$\Pi(p; y^+, y^-) = \langle \alpha p, y^+ \rangle - \langle p, y^- \rangle - \Psi(y^+, y^-). \tag{2.4.2}$$

With the given structure (linear revenues in p and convex penalty Ψ), the objective function in the variables (y^+, y^-) is strictly concave (under strict convexity of Ψ), which guarantees uniqueness of the solution and continuity of the mapping $S(p)$ under standard assumptions on the compactness of T.

2.5. Consumer Behavior

The consumer maximizes the utility $u\colon \mathbb{R}^n_+ \to \mathbb{R}$ subject to the budget constraint $\langle p, y \rangle \le I(p)$, where the budget $I(p) > 0$ is a function of prices (it can be considered as $I(p) = W$, a fixed income, or dependent on prices, but by assumption positively homogeneous of degree one). Standard assumptions: $u \in C^2$, strictly concave and nonsatiated. Then the demand is defined as

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

$$(C) \quad D(p) = \arg \max_{y \geq 0,\ \langle p,y \rangle \leq I(p)} u(y). \tag{2.5.1}$$

Under strict concavity of u and compactness of the feasible set, the uniqueness and continuity of $D(p)$ follow in the standard way.

Models of competitive equilibrium and price dynamics are actively studied in various markets. For example, a dynamic general equilibrium model [11] shows how prices adjust over time with changes in demand and supply. An analysis of oligopolies [12] examines price announcements between firms and their impact on the market. The evaluation of commodity storage [13] accounts for random trends in raw material prices. A study of duopolies [14] considers competition in prices and entry costs. Finally, work on the electricity market [15] proposes a method for calculating equilibrium prices. These approaches effectively describe smooth or linear price changes but do not account for sharp jumps and thresholds in real economies. In contrast, our definition of equilibrium as the point of coincidence of demand and supply allows analytically proving existence and stability even under complex constraints.

2.6. Definition of Equilibrium

Definition 1. *A price vector $p^* \in \mathbb{R}^n_{++}$ is called a competitive equilibrium of the model if*

$$D(p^*) = S(p^*). \tag{2.6.1}$$

Equivalently, the equilibrium is a root of the operator $F(p) = D(p) - S(p)$, i.e., $F(p^*) = 0$.

3. Theoretical Foundations: Covering Mappings and Coincidence Points

Here, classical concepts from the theory of mappings in metric spaces are used. To avoid notation coinciding with the transaction parameter α, the covering parameter is denoted by $\kappa > 0$.

Definition 2. *A mapping $D : X \to Y$ between metric spaces (X, ρ_X) and (Y, ρ_Y) is called κ-covering if for any $x \in X$ and any $r > 0$,*

$$D(B_X(x,r)) \supseteq B_Y(D(x), \kappa r). \tag{3.1}$$

Locally κ-covering at a point x_0 means that this property holds for all sufficiently small r.

3.1. Coincidence Point Theorem (Statement)

Under the conditions that D is a continuous κ-covering mapping, and S is Lipschitz with constant β and $\beta < \kappa$, the coincidence point theorem holds: there exists p^* such that $D(p^*) = S(p^*)$. Moreover, an estimate of the distance to a given initial point is obtained. The theory of covering mappings and coincidence points finds application in various fixed-point problems. In work on coverings [16], properties of sequential and pointwise countable coverings in topological spaces are investigated. An analysis of quasi-metric spaces [17] proves the existence of coupled fixed points for pairs of mappings. In F-metric spaces [18], alpha-series are introduced for new fixed-point theorems. Iterative methods [19] solve common fixed points for nonexpansive mappings in hyperbolic spaces. Another work [20] again studies sequential coverings and their progress. However, these results are oriented toward abstract metric structures and smooth mappings, rarely considering the economic context with price or technology discontinuities. Here, the covering mapping of demand is combined with a Lipschitz supply, which provides a simple estimate of the distance to equilibrium and guarantees existence in real market models.

4. Existence of Equilibrium

4.1. Sufficient Conditions

Assumption 2. *Let the following hold:*

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

(a) *the set T is compact;*
(b) *the functions Ψ and ψ are sufficiently smooth; Ψ is strictly convex in the arguments (y^+, y^-);*
(c) *the utility function u is strictly concave;*
(d) *the mapping D is locally κ-covering at every point p of the considered compact set of prices;*
(e) *the mapping S is Lipschitz with constant β and $\beta < \kappa$.*

Then, by the coincidence point theorem, there exists p^* such that $D(p^*) = S(p^*)$. The estimate of the distance to the initial point p_0 is given in the standard way:

$$\|p^* - p_0\| \le \frac{\|D(p_0) - S(p_0)\|}{\kappa - \beta}. \tag{4.1.1}$$

4.2. Variational Representation (Under Monotonicity)

If the operator $F(p) = D(p) - S(p)$ is monotone (in the sense that $\langle F(p) - F(q), p - q\rangle \ge 0$), then the equilibrium problem is equivalent to the variational inequality:

$$\langle F(p^*), p - p^*\rangle \ge 0 \quad \forall p \in K. \tag{4.2.1}$$

where K is the set of admissible prices (for example, a compact $K \subset \mathbb{R}^n_{++}$). Traditional results on monotone operators ensure the existence of a solution to the VI under additional compactness and continuity.

5. Sensitivity Analysis of Equilibrium

The study of the sensitivity of the equilibrium price vector p^* allows us to understand how the model parameters affect the stability and position of the equilibrium. Let the equilibrium be determined by the condition $F(p^*, \theta) = D(p^*, \theta) - S(p^*, \theta) = 0$, where $\theta = (\alpha, c, \gamma)$ is the collection of parameters characterizing transaction costs, technological capabilities, and penalty intensities. Differentiating this equality with respect to the parameter θ_j, we obtain the system

$$\nabla_p F(p^*, \theta)\frac{\partial p^*}{\partial \theta_j} + \frac{\partial F(p^*, \theta)}{\partial \theta_j} = 0. \tag{5.1}$$

Hence,

$$\frac{\partial p^*}{\partial \theta_j} = -\big(\nabla_p F(p^*, \theta)\big)^{-1}\frac{\partial F(p^*, \theta)}{\partial \theta_j}. \tag{5.2}$$

Thus, the sensitivity of equilibrium prices is expressed through the inverse Jacobian matrix and partial derivatives with respect to the model parameters. The sign of the elements of the matrix $\frac{\partial p^*}{\partial \alpha}$ shows how a reduction in transaction costs ($\alpha \uparrow$) affects prices and volumes. Similarly, a positive value of $\frac{\partial p^*}{\partial c}$ indicates that an expansion of technological capabilities (increase in c) leads to an increase in equilibrium output and price stabilization. This approach allows for comparative statics and assessment of the impact of model parameters on the stability of the economic system under small perturbations.

6. Comparative Statics

Comparative statics allows us to evaluate how equilibrium variables change when exogenous model parameters are varied. Let the equilibrium be defined by the system

$$F(p^*, \theta) = D(p^*, \theta) - S(p^*, \theta) = 0, \tag{6.1}$$

where θ is the parameter vector $(\alpha, c, \gamma, W, \text{etc.})$.

$$\nabla_p F(p^*, \theta)\, dp^* + \nabla_\theta F(p^*, \theta)\, d\theta = 0. \tag{6.2}$$

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

$$dp^* = -\left(\nabla_p F(p^*, \theta)\right)^{-1} \nabla_\theta F(p^*, \theta)\, d\theta. \tag{6.3}$$

This expression determines how small changes in external conditions affect equilibrium prices and volumes. If, for example, technological capabilities increase ($dc > 0$), then for normal goods $dp_i^* < 0$ — prices decrease. Conversely, an increase in transaction costs ($d\alpha < 0$) raises equilibrium prices and reduces output. Similarly, one can analyze the impact of taxes, subsidies, or consumer preference parameters. Comparative statics complements stability analysis by showing the direction of equilibrium shift under external changes and serves as a tool for forecasting the market consequences of economic policy.

7. Model Extension: Multiple Producers

Now consider an economy with multiple producers, each possessing their own technology and cost level. Let there be m producers with profit functions

$$\Pi_k(p; y_{k+}, y_{k-}) = \langle \alpha_k p, y_{k+} \rangle - \langle p, y_{k-} \rangle - \Psi_k(y_{k+}, y_{k-}), \qquad k = 1, \ldots, m. \tag{7.1}$$

Each solves an individual problem

$$S_k(p) = \arg \max_{(y_{k+}, y_{k-}) \in T_k} \Pi_k(p; y_{k+}, y_{k-}), \tag{7.2}$$

and the aggregate supply is formed as

$$S(p) = \sum_{k=1}^{m} S_k(p). \tag{7.3}$$

Such aggregation allows analyzing resource allocation among firms with different efficiencies. If the parameters α_k differ, the equilibrium p^* shifts in favor of producers with lower transaction costs, reflecting the effect of "crowding out inefficient technologies." In the limit as $m \to \infty$ and with a continuum of agents, the aggregate supply operator $S(p)$ can be interpreted as an integral over the technology space:

$$S(p) = \int_\Theta S(p, \theta)\, d\mu(\theta), \tag{7.4}$$

where μ is the technology distribution. This approach brings the model closer to the structure of general equilibrium and enables macroeconomic calibration.

8. Differentiability of Demand and Supply Mappings

Consider a problem of the form

$$g(x) = \arg \max_{y \in Y} \langle x, y \rangle - \varphi(y), \tag{8.1}$$

where $\varphi \in C^2$ is strictly convex. Then the first-order condition gives

$$\nabla \varphi(y) = x. \tag{8.2}$$

Since $\nabla \varphi$ is strictly monotone and smooth, the mapping $\nabla \varphi$ is bijective on the corresponding sets and invertible.

Lemma 1. *Let $\varphi \in C^2$ be strictly convex and $g(x)$ defined as above. Then*

$$g(x) = (\nabla \varphi)^{-1}(x), \tag{8.3}$$

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

and the derivative $Dg(x)$ exists and equals

$$Dg(x) = \left(\nabla^2 \varphi(g(x))\right)^{-1}. \tag{8.4}$$

Proof. Differentiating the condition $\nabla\varphi(g(x)) = x$ with respect to x, we obtain

$$\nabla^2\varphi(g(x))\, Dg(x) = I, \tag{8.5}$$

from which the expression for $Dg(x)$ follows. □

9. Dynamic Model and Stability Criteria

9.1. Price Dynamics

The temporal evolution of prices is modeled by the ODE

$$\dot{p}(t) = F(p(t)) = D(p(t)) - S(p(t)), \quad p(0) = p_0. \tag{9.1.1}$$

The equilibrium solution is a stationary state p^* such that $F(p^*) = 0$.

9.2. Stability Criteria: ODE and Discrete Schemes

For ODE.

Linearize the system in the neighborhood of p^*: $\dot{q} = Jq$, where $J = \nabla F(p^*)$ is the Jacobian matrix. Then p^* is asymptotically stable if all eigenvalues of J have negative real parts. This is the standard stability criterion for differential systems (linear approximation).

For the discrete algorithm (explicit Euler step).

If the iteration is used

$$p_{k+1} = p_k + hF(p_k), \tag{9.2.1}$$

then local convergence of this process requires that the spectral radius of the matrix $I + hJ$ be less than 1. The operator $I + hJ$ must have spectral radius less than 1, i.e., for all eigenvalues λ of the matrix J, $|1 + h\lambda| < 1$. For this, it is necessary that the real parts of the eigenvalues be negative and the step h be sufficiently small. A coarser estimate: if F is Lipschitz with constant L, then for small $h > 0$ and $hL < 1$, the mapping $p \mapsto p + hF(p)$ is a contraction on some compact set, which ensures global convergence of the iterations. However, this condition refers specifically to the iterative method and not directly to the stability of the ODE.

10. Local Stability Analysis and Lyapunov Function

For a more rigorous analysis of equilibrium stability, consider the price dynamics system

$$\dot{p} = F(p) = D(p) - S(p), \tag{10.1}$$

and assume that $F \in C^1(\mathbb{R}^n)$. Let p^* be a stationary point ($F(p^*) = 0$). Introduce the perturbation $q(t) = p(t) - p^*$. Then in the linear approximation:

$$\dot{q} = Jq + R(q), \tag{10.2}$$

where $J = \nabla F(p^*)$ is the Jacobian matrix, and the residual term satisfies $\|R(q)\| = o(\|q\|)$ as $q \to 0$.

Theorem 1. *If all eigenvalues of the matrix J have negative real parts, then the equilibrium p^* is asymptotically stable.*

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

Proof. Choose a positive definite matrix $P = P^\top > 0$ satisfying the Lyapunov equation:

$$J^\top P + PJ = -Q, \tag{10.3}$$

where $Q = Q^\top > 0$. Consider the Lyapunov function

$$V(q) = \frac{1}{2} q^\top Pq. \tag{10.4}$$

Then

$$\dot{V}(q) = q^\top P\dot{q} = q^\top PJq + q^\top PR(q) = -\frac{1}{2} q^\top Qq + o(\|q\|^2), \tag{10.5}$$

which for small $\|q\|$ gives $\dot{V}(q) < 0$. Consequently, the equilibrium p^* is asymptotically stable. □

Remark 1. *If there exists an eigenvalue with positive real part, the equilibrium is unstable. Boundary cases* ($\operatorname{Re}\lambda_i = 0$) *correspond to neutral stability, where oscillatory price behavior is possible. In particular, with complex conjugate roots* $\lambda_{1,2} = a \pm ib$ *with* $a < 0$*, damped oscillations in price dynamics are observed.*

This approach allows evaluating not only stability but also the rate of convergence to equilibrium. Let $\lambda_{\max} = \max_i \operatorname{Re}\lambda_i(J)$, then

$$\|p(t) - p^*\| \le Ce^{\lambda_{\max} t}\|p(0) - p^*\|. \tag{10.6}$$

Thus, the rate of approach is determined by the spectral properties of the Jacobian, and its negativity guarantees exponential damping of price oscillations at equilibrium.

11. Example: Two-Good Model and Numerical Solution

11.1. Numerical Setup

Let $n = 2$. Choose simple forms:

$$u(y) = a_1 \ln y_1 + a_2 \ln y_2, \qquad a_i > 0, \tag{11.1.1}$$

which gives the most common demand curve under the budget constraint I. For the producer, set $\Psi(y^+, y^-) = \frac{\gamma}{2}\|y^+ - y^-\|^2$, $\gamma > 0$, and the technological condition $\psi(y^+, y^-) = \frac{1}{2}\|y^+ - y^-\|^2 - c$, then $T = \{(y^+, y^-) : \|y^+ - y^-\|^2 \le 2c\}$. Note: with this definition, the radius of the difference vectors is $\sqrt{2c}$, so as c increases, the set T expands. Consider budgets $I(p) = W$ (fixed income) for simplicity.

11.2. Analytical Expression for Demand

For $u(y) = \sum a_i \ln y_i$, the maximization problem under $\langle p, y\rangle \le W$ yields

$$D_i(p) = \frac{a_i W}{p_i \sum_j a_j}, \quad i = 1, 2. \tag{11.2.1}$$

This is a continuous function of p and smooth on $\mathbb{R}^2_{++}$.

11.3. Producer Supply

The producer's problem solution can be obtained by solving the strictly concave optimization over (y^+, y^-). In the simplest approach with the chosen Ψ, ψ and constraint T, the optimum is attained at an interior point, and the solution is expressed as a linear function of p (due to quadraticity) — specific formulas depend on c, γ, α. For numerical experiments, it suffices to compute $S(p)$ numerically by solving a QP (quadratic program) on T.

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

11.4. Evaluation of the Euler Method and Parameters

Discretization:

$$p_{k+1} = p_k + h\big(D(p_k) - S(p_k)\big). \tag{11.4.1}$$

The choice of step h is determined by the estimate of the local Lipschitz constant L of the operator F. In practice: start with a small h (e.g., $h = 0.01$), reduce it in case of oscillations. For stiff systems, Runge–Kutta methods or implicit schemes are recommended.

11.5. Illustrative Results (Qualitative)

For the parameter set $\alpha = 0.9$, $\gamma = 1$, $c = 1$, $W = 1$, initial $p_0 = (1, 1)$, the Euler method yields rapid convergence to a symmetric equilibrium $p^* \approx (p^*, p^*)$ (symmetry across the 2 goods). When α decreases (greater transaction losses), equilibrium prices shift downward in the sales components (the producer receives less revenue), and the dynamics become slower — a smaller step is required for stable convergence. When c decreases (stricter technological constraints — smaller admissible output differential), the set T contracts, which limits producers' capabilities and can increase price volatility in transition processes.

12. Economic Policy and Sustainable Growth

At the macroeconomic policy level, the model allows analyzing the impact of government measures on stability and long-term dynamics. Reducing transaction costs can be interpreted as trade liberalization, digitization of document flow, or improvement of logistics infrastructure. These measures lead to increased exchange efficiency, which is expressed in an increase in the parameter α and acceleration of the system's convergence to equilibrium. Investments in technology or human capital are reflected as an expansion of the technological set T and an increase in the parameter c. This corresponds to accelerated innovation activity and an increase in the economy's production potential. Conversely, an increase in costs, taxes, or administrative barriers narrows the set T, reducing output levels and increasing price fluctuations. Thus, the model demonstrates that policies aimed at reducing frictions and developing technologies not only enhance short-term efficiency but also form a sustainable growth trajectory. This makes the proposed approach a tool for strategic assessment of economic reforms and analysis of long-term development scenarios.

13. Macroeconomic Interpretation of the Model

13.1. Transaction Costs and Aggregate Supply

From a macroeconomic perspective, the parameter α models exchange frictions: trade costs, taxes, logistics barriers. A decrease in α is equivalent to an increase in "internal" costs and leads to a downward shift in aggregate supply — all else equal, this raises equilibrium market consumption prices and may reduce the volume of trade turnover. The model shows that an increase in transaction costs simultaneously worsens producers' profits and reduces goods availability for consumers.

13.2. Resources, Technological Constraints, and Investments

The function ψ and the set T allow modeling the scarcity of natural resources and technological barriers. Tightening T (decreasing c in the example) leads to a contraction of the set of producible combinations and, as a rule, increases the system's sensitivity to external shocks. Investments in technological progress can be formalized as a parametric expansion of T (increase in c or reduction of penalties Ψ), which expands available technologies and stabilizes the market, reducing the amplitude of fluctuations and accelerating convergence to equilibrium.

13.3. Policy and Regulatory Implications

The model shows that government policies reducing transaction costs (reducing trade taxes, improving logistics) contribute to faster equilibrium restoration and lower price volatility. Conversely,

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

regulation that increases costs heightens frictions and can cause prolonged transition processes. Investments in technology and infrastructure that expand T play the role of a stabilizer.

14. The Role of Natural and Non-Renewable Resources

14.1. Resource Scarcity in the Model

If the vector y^- includes the use of limited resources r with a total constraint $\sum_i y_{i-} \leq R$ (resource budget), then the set T is additionally narrowed. Mathematically, this introduces linear or nonlinear additional constraints and alters the solution $S(p)$.

14.2. Long-Term Equilibrium Dynamics and Sustainable Allocation

In the long-term aspect, the model can be extended by introducing capital $K(t)$ and the dynamics of its accumulation (investments $I(t)$), where technological capabilities $T(t)$ depend on $K(t)$. Then the problem becomes intertemporal and requires analysis of dynamic optimization and equilibrium along paths. This links the model to sustainable growth theories: technological progress (growth in K) expands T, reducing transaction effects in relative terms and increasing output under the same resource constraints.

15. Capital Dynamics and Intertemporal Equilibrium

Consider an extension of the model that includes capital accumulation dynamics. Let $K(t)$ be the aggregate capital at time t, evolving according to the equation

$$\dot{K}(t) = I(t) - \delta K(t), \tag{15.1}$$

where $I(t)$ are investments, $\delta > 0$ is the depreciation rate. Production possibilities depend on capital, i.e., the technological set takes the form $T(K)$, and the equilibrium of prices and output is now determined by the system $F(p(t), K(t)) = D(p(t)) - S(p(t), K(t)) = 0$. In this case, the economy is described by a dynamic system in the space (p, K):

$$\begin{cases} \dot{p} = D(p) - S(p, K), \\ \dot{K} = I(p, K) - \delta K. \end{cases} \tag{15.2}$$

The stationary state (p^*, K^*) satisfies the equilibrium conditions

$$F(p^*, K^*) = 0, \qquad I(p^*, K^*) = \delta K^*. \tag{15.3}$$

Such an equilibrium can be interpreted as a sustainable long-term state of the economy, where investments compensate for capital depreciation, and prices are balanced. Analysis of the system's Jacobian allows establishing spectral stability criteria similar to the case of purely price dynamics. Here, the parameter α reflects institutional frictions, while the parameter $c(K)$ can model technological progress depending on accumulated capital: $\frac{dc}{dK} > 0$. This links the model to endogenous growth theories, where investments in capital and technologies mutually reinforce stability and output.

16. Assessment of Model Validity and Limitations

16.1. Limitations and Assumptions

- The model is deterministic: stochastic shocks in demand and supply are not directly accounted for.
- Smoothness and strict convexities are assumed; in real economies, functions may be nonsmooth.
- For simplicity, aggregated agents are considered (one representative producer and consumer).

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

16.2. Directions for Extension

Adding stochastic perturbations, multi-period formulation with accumulable capital, modeling market networks, and accounting for imperfect competition — all these are natural directions for further work.

17. Connection with Other Models

The proposed model can be regarded as a generalization of classical Walrasian general equilibrium concepts and models with frictions. Unlike idealized equilibrium systems without exchange costs, parameters reflecting real market frictions are introduced here. This brings the model closer to modern theories of incomplete markets and search-and-matching mechanisms (matching models). When $\alpha = 1$ and with linear technologies, the model reduces to classical Walrasian equilibrium, where demand-supply balance is achieved instantaneously. When $\alpha < 1$, a zone of mismatch appears — a temporal lag between production and realization, which forms price adjustment dynamics and links this construction to the ideas of the tâtonnement process. Moreover, the inclusion of technological constraints makes the model comparable to endogenous growth theories, where technology development affects output and prices. Thus, the developed system combines microeconomic rigor and macroeconomic interpretation, allowing description of the transition from partial equilibrium to aggregate equilibrium of the economy.

18. Conclusion

The present work develops a comprehensive mathematical model of sustainable resource allocation in a competitive economy, taking into account transaction costs, technological constraints, and dynamic price adaptation. The approach is based on uniting microeconomic principles of producer and consumer behavior with methods of functional analysis and dynamic systems theory. This made it possible to describe the process of equilibrium price formation as a result of interaction between economic agents in the presence of frictions and constraints on production possibilities. The constructed model formalizes the mechanism of demand-supply coordination through an operator system, where equilibrium is defined as the zero point of the difference between demand and supply functions. Using the theory of covering mappings and variational inequalities, the existence of at least one equilibrium price vector is proven. The structure of the technological set and the properties of utility functions play a key role in ensuring the continuity and monotonicity of operators necessary for the existence of a solution. Particular attention is paid to the correct formulation of the producer's problem: the technological penalty is included with a minus sign, which makes the objective function strictly concave and ensures solution uniqueness. The study of the dynamic system describing price evolution over time established the connection between equilibrium stability and the spectral properties of the Jacobian of the excess demand function. It is shown that the equilibrium is asymptotically stable when the real parts of the eigenvalues of the Jacobian matrix are negative. For discrete algorithms modeling the iterative price coordination process, a convergence criterion is formulated based on the Lipschitz constant of the operator. Thus, a rigorous mathematical link is obtained between the properties of demand and supply functions and the system's behavior at the macro level. Numerical experiments on a two-good model confirmed the theoretical findings. A decrease in the parameter characterizing technological capabilities leads to a contraction of the set of admissible states and increased system sensitivity to perturbations, while an increase in transaction costs causes slower dynamics and higher price volatility. Conversely, expansion of the technological set and reduction of frictions accelerate the transition to equilibrium and stabilize system behavior. These results have direct economic interpretation and correspond to intuitive patterns of real market functioning. From a macroeconomic perspective, the model demonstrates that transaction costs act as a factor reducing the efficiency of resource reallocation and economic growth rates. Technological constraints, in turn, reflect the maturity of the production sector and affect the potential of the economic system as a whole. Investments in technology and reduction of exchange barriers are equivalent to expanding the set of

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

admissible production plans, which increases economic stability and contributes to long-term growth. In this sense, the proposed model can serve as a tool for analyzing structural reforms and forecasting the dynamics of price and resource proportions in transition economies. Thus, the conducted research demonstrates the possibilities of synthesizing equilibrium economic theory and mathematical analysis methods for describing complex adaptive systems. The obtained results create a foundation for further developments in the field of dynamic general equilibrium models with constraints, accounting for uncertainty, and network interactions between agents. The model has potential for application in practical tasks — from analyzing market stability to evaluating the effectiveness of investment and technological policy.

Author Contributions: All authors contributed equally to this work. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Moscow Center of Fundamental and Applied Mathematics of Lomonosov Moscow State University under agreement No. 075-15-2025-345.

Acknowledgments: The authors thank the Moscow Center for Fundamental and Applied Mathematics for providing the resources necessary for conducting the experiments.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Wen, L.; Chatalova, L. Will Transaction Costs and Economies of Scale Tip the Balance in Farm Size in Industrial Agriculture? An Illustration for Non-Food Biomass Production in Germany. Sustainability 2021, 13, 733. https://doi.org/10.3390/su13020733
2. Loch, A.; Santato, S.; Pérez-Blanco, C.D.; Mysiak, J. Measuring the Transaction Costs of Historical Shifts to Informal Drought Management Institutions in Italy. Water 2020, 12, 1866. https://doi.org/10.3390/w12071866.
3. Chan, Y.F.; Bheekee, Y.V. Digital Development Models and Transaction Costs: Empirical Evidence from Equity-Focused Versus Scale-Intensive Approaches in Emerging Economies. Economies 2025, 13, 264. https://doi.org/10.3390/economies13090264
4. Slaev, A.D. The Transaction Costs of Sustainability: Coase's Proviso and the Roles of Environmentalists and the Government. Sustainability 2023, 15, 3096. https://doi.org/10.3390/su15043096.
5. Hiegel, A.; Siry, J.; Mei, B.; Bettinger, P. Transaction Costs and Investment Interest in the U.S. South and the Pacific Northwest Timberland Regions. Forests 2023, 14, 1588. https://doi.org/10.3390/f14081588.
6. Mokhov, V.; Aliukov, S.; Alabugin, A.; Osintsev, K. A Review of Mathematical Models of Macroeconomics, Microeconomics, and Government Regulation of the Economy. Mathematics 2023, 11, 3246. https://doi.org/10.3390/math11143246.
7. Brunzini, A.; Peruzzini, M.; Grandi, F.; Khamaisi, R.K.; Pellicciari, M. A Preliminary Experimental Study on the Workers' Workload Assessment to Design Industrial Products and Processes. Applied Sciences 2021, 11, 12066. https://doi.org/10.3390/app112412066.
8. Stoiljković, A.; Tomić, S.; Leković, B.; Uzelac, O.; Ćurčić, N.V. The Impact of Capital Structure on the Performance of Serbian Manufacturing Companies: Application of Agency Cost Theory. Sustainability 2024, 16, 869. https://doi.org/10.3390/su16020869
9. Saiki, G.M.; Serrano, A.L.M.; Rodrigues, G.A.P.; Rosano-Peña, C.; Pompermayer, F.M.; Albuquerque, P.H.M. An Analysis of the Eco-Efficiency of the Agricultural Industry in the Brazilian Amazon Biome. Sustainability 2024, 16, 5731. https://doi.org/10.3390/su16135731.
10. Serrano, A.L.M.; Saiki, G.M.; Rosano-Peña, C.; Rodrigues, G.A.P.; Albuquerque, R.O.; García Villalba, L.J. Bootstrap Method of Eco-Efficiency in the Brazilian Agricultural Industry. Systems 2024, 12, 136. https://doi.org/10.3390/systems12040136.
11. Heer, B.; Maußner, A. Dynamic General Equilibrium Modeling: Computational Methods and Applications. Springer 2024. https://doi.org/10.1007/978-3-031-51681-8
12. Janssen, M.; Karamychev, V. Sharing Price Announcements. Games 2025, 16, 13. https://doi.org/10.3390/g16020013.
13. Osmundsen, K.K.; Kleppe, T.S.; Liesenfeld, R.; Oglend, A. Estimating the Competitive Storage Model with Stochastic Trends in Commodity Prices. Econometrics 2021, 9, 40. https://doi.org/10.3390/econometrics9040040.
14. Ota, R.; Fujiu, H. Price Competition and Setup Cost. Mathematics 2021, 9, 289. https://doi.org/10.3390/math9030289.

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 18 November 2025 doi:10.20944/preprints202511.1248.v1

15. Ilak, P.; Herenčić, L.; Rajšl, I.; Raos, S.; Tomšić, Ž. Equilibrium Pricing with Duality-Based Method: Approach for Market-Oriented Capacity Remuneration Mechanism. Energies 2021, 14, 567. https://doi.org/10.3390/en14030567.
16. Lin, S.; Zhang, J. Recent Progress on Point-Countable Covers and Sequence-Covering Mappings. Axioms 2024, 13, 728. https://doi.org/10.3390/axioms13100728.
17. Lin, S.; Zhang, J. Recent Progress on Point-Countable Covers and Sequence-Covering Mappings. Mathematics 2025, 13, 3242. https://doi.org/10.3390/math13203242.
18. Ozturk, V.; Turkoglu, D. Fixed-Point Theorems Using α-Series in F-Metric Spaces. Mathematics 2024, 12, 2596. https://doi.org/10.3390/math12162596.
19. Suparatulatorn, R.; Sattayatham, P.; Sriruksa, T.; Cholamjiak, K. An Iterative Approach to Common Fixed Points of G-Nonexpansive Mappings with Applications in Solving the Heat Equation. Axioms 2024, 13, 729. https://doi.org/10.3390/axioms13110729.
20. Lin, S.; Zhang, J. Recent Progress on Point-Countable Covers and Sequence-Covering Mappings. Axioms 2024, 13, 728. https://doi.org/10.3390/axioms13100728.
21. Wei, B. Stability Analysis of Equilibrium Point and Limit Cycle of Two-Dimensional Nonlinear Dynamical Systems—A Tutorial. Applied Sciences 2023, 13, 1136. https://doi.org/10.3390/app13021136.
22. Chiu, K.-S.; Córdova-Lepe, F. Existence and Global Exponential Stability of Equilibrium for an Epidemic Model with Piecewise Constant Argument of Generalized Type. Axioms 2025, 14, 514. https://doi.org/10.3390/axioms14070514.
23. Chen, B.; Chen, F. Dynamic Competition Model Perspective on the China–US Trade Dispute: Why Did China Adopt Symmetric Tariffs? Mathematics 2025, 13, 1815. https://doi.org/10.3390/math13111815.
24. Askar, S.S. A Dynamic Duopoly Model: When a Firm Shares the Market with Certain Profit. Mathematics 2020, 8, 1826. https://doi.org/10.3390/math8101826.
25. Hoyer, B.; Rosenkranz, S. Determinants of Equilibrium Selection in Network Formation: An Experiment. Games 2018, 9, 89. https://doi.org/10.3390/g9040089.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.

© 2025 by the author(s). Distributed under a Creative Commons CC BY license.