Short Note

Optimization of Denominations in Poland

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Optimization of Denominations in Poland

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Abstract: The purpose of this work is to investigate whether the optimization of currency denominations in Poland is possible. Specific denomination values have been indicated that would allow to reduce the average amount of cash needed to represent any amount, while maintaining the property of divisibility. Moreover, short analysis reveals this approach outperforms the solutions identified by Tesler and Wynne in terms of the mergeability property of money.

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1. Introduction

The costs of keeping cash in circulation are significant, including the costs of producing, transporting banknotes and security costs. This work aims to investigate whether it is possible to reduce the amount of cash in circulation by changing the value of denominations, while maintaining cash's value. The objective was to minimize the average number of coins and banknotes needed to represent any amount. Such a solution will increase the mergeability of money, while maintaining the divisibility property.

For decades people have been examining how money behaves in the market. The most important properties of money, discovered through years of research, include divisibility, ease of use, and mergeability. "Mergeability is always achievable by collecting multiple units of the same denomination. However, storing more units may require more resources. With respect to physical cash, mergeability can be measured as the minimum number of units required to sum to an arbitrary number, x." (Hull & Sattath, 2021).

Shy (2020) points out that consumers prefer cash payments at certain multiples of denominations, suggesting that changing these denominations may disrupt their established payment habits; Raghubir and Srivastava (2009) prove that denomination affects the perceived value of money. Studies show the denomination effect can also be translated into everyday situations, such as tipping: the shame associated with tipping smaller denominations makes consumers prefer larger denominations (Zenkić, Lei, Millet, & Rotman, 2023). Thus, this solution might raise questions about money's ease of use.

Denomination optimization research has been conducted by Prescott and Shy (2023), demonstrating that eliminating the penny coin would reduce cash exchange costs without significant inflationary effects. Another research on denomination optimization done by Tesler (1995) suggests the most optimal solution would be denominations with a value of powers of 3.

In this work a local minimum was found that would decrease the average number of coins and banknotes needed to represent any amount by 8.7175%, however, it does not take inflation, ease of use, and other significant factors into account. A detailed analysis of the denomination structure in Poland is a topic for broader research. For example, an extensive paper done by Manikowski (2017) proved the introduction of a higher denomination, e.g. PLN 500, should not affect the inflation in Poland, and additionally, coins 1 grosz and 2 grosze should be demonetized, due to their unprofitability.

2. Method and Results

The mergeability property of money can be strengthened by reducing the average number of coins and banknotes needed to represent any amount. A simple computer program using a greedy

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algorithm will be used to calculate the averages. The same algorithm is used for both old and new denominations, so while the greedy algorithm may not result in the most optimal solution, it does allow for a simple and uniform comparison of both sets of denominations. Any amount equal to or above 500 can be represented with the use of multiples of a PLN 500 banknote, thus the average is calculated using every amount from 0 to 499.99.

The following set of numbers represents current denominations in Poland.

$$X = \{0.01, 0.02, 0.05, 0.10, 0.20, 0.50, 1.00, 2.00, 5.00, \\ 10.00, 20.00, 50.00, 100.00, 200.00, 500.00\}$$

The current monetary system in Poland allows for an average of 8 ± 0.008716, with the standard deviation $\sigma \approx 1.94903$.

$$X' = \{0.01, 0.02, 0.05, 0.13, 0.21, 0.50, 1.07, 2.38, 5.05, \\ 10.92, 23.70, 51.13, 109.98, 234.05, 500.00\}$$

However, changing the denominations to those in the above set of numbers will minimize this average to 7.3026 ± 0.007382, with the standard deviation $\sigma' \approx 1.65067$.

Using Student's two-tailed t-test, the t-value $t \approx 61.06$, $|t| >\sim 1.96$, thus the difference is statistically significant.

3. System Comparison

Wynne (1997) contradicts Tesler's (1995) findings: "A simple numerical comparison of the binary system $1, 2, 4, 8, 16, \ldots, 2^k$ with the ternary system $1, 3, 9, 27, \ldots, 3^k$ for transactions between 0.01 and 100.00 shows that in terms of minimizing the number of coins or notes that must change hands if transactions are conducted efficiently, the binary system is indeed superior". By defining given sets of denominations and using previous methodologies, we can calculate the average number of coins and banknotes needed to represent any amount.

$$X_3 = \{0.01, 0.03, 0.09, 0.27, 0.81, 2.43, 7.29, 21.87, 65.61, 196.83, 590.49, \\ 1771.47, 5314.41, 15943.23, 47829.69\}$$
$$X_2 = \{0.01, 0.02, 0.04, 0.08, 0.16, 0.32, 0.64, 1.28, 2.56, 5.12, 10.24, \\ 20.48, 40.96, 81.92, 163.84\}$$

The ternary system allows for an average of 9.6314 ± 0.011004, with the standard deviation $\sigma_3 \approx 2.46064$. The binary system allows an average of 7.99488 ± 0.009091, with the standard deviation $\sigma_2 \approx 2.03292$.

Using Student's two-tailed t-test to compare my solution with both systems, the t-value $t_3 \approx 175.75$, $|t_3| >\sim 1.96$ and $t_2 \approx 59.11$, $|t_2| >\sim 1.96$. Thus, the result confirms the superiority and significance of my solution in the context of mergeability, being more suitable and optimal compared to Wynne's and Tesler's solutions.

4. Conclusions

This solution allows to represent any amount, thus it maintains the divisibility property and improves the mergeability property, however, questions the ease of use. The mergeability property can be further improved by finding a more optimal set of denominations. Alternative methods, such as dynamic programming or metaheuristic algorithms, probably could better explore the space of possible solutions.

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Another limitation is the lack of consideration of the costs associated with introducing a new system. This includes the production of new coins and banknotes, potential raise in inflation and the adaptation of society to the new values.

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