Article

Study of Contact Pressures in Total Hip Replacement

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Featured Application: The results of the analytical tests concern the interaction of the friction pair, which is the joint of the hip joint endoprosthesis. The joint consists of two diametrically different materials with significant differences in Young's modulus: e.g. PE-UHMW polymer with E=0.625 GPa and titanium NitridedGRADE 2 with E=112 GPa. Knowledge of these interactions, and especially the maximum contact pressures, will allow you to design a personalized endoprosthesis that will meet your functional expectations.

Abstract: Total hip arthroplasty is a complex procedure. The achievements of implantology enabled the development of a faithful representation of hip joint physiology as well as the production of materials that can successfully replace damaged natural tissues. A very important issue is the correct selection of the geometry of the endoprosthesis adequate to the load of the joint. Materials used for endoprosthesis are a metal head and a polymer cup (e.g. PE-UHMW). The main interactions in the endoprosthesis are friction and surface pressure, which must be limited, exceeding them causes the destruction of the biomechanical system - plastic deformation of the polymer and the formation of too large and unacceptable radial clearances. Based on the author's developed calculation method of hip joint endoprosthesis contact parameters, the impact on maximum contact pressure and the angle of contact of the joint load was estimated depending on the diameter of the endoprosthesis and radial clearance. The correctness of changing the values of maximum contact pressure from the mentioned parameters was determined. Correspondingly: an increase in joint load causes a linear increase in the maximum contact pressure; increasing the diameter of the endoprosthesis head - their non-linear decrease, and increasing radial clearance - their increase.

1. Introduction

Hip endoprosthesis are considered the greatest stride forward in orthopedic surgery in the last 100 years. Over 1 000 000 surgeries are conducted every year, over 300000- only in the USA.

Hip arthroplasty is an operation involving the replacement of a diseased hip joint with an artificial one. During the operation, the damaged femoral head and the inside of the acetabulum are removed and replaced with artificial elements. Each endoprosthesis consists of an acetabulum, acetabular insert, stem and metal head. The artificial cup is mounted in place of the natural acetabulum. Most often, it is made of titanium, and inside it, there is an insert made of polyethylene or ceramics. The endoprosthesis mandrel is fixed in the femur. It is also made of titanium and a metal or ceramic head is placed on it. In this way, the metal head and acetabular insert form a new joint, capable of making movements.

A very important issue when designing, implanting and using them is the assessment of contact pressure depending on the load, diameter of the endoprosthesis and the size of the gap (radial clearance) in the joint. Unfortunately, the literature lacks justified assessments using appropriate calculation methods based on classic methods of contact mechanics. Solutions based on FEM [1-8 et

al] and approximate calculation method [9] are known. Because "... the accuracy of FEM predictions depends on the input from laboratory experiments [5]" then appropriate justified mathematical modeling can be used to estimate contact parameters in endoprosthesis.

Estimating the amount of surface pressure is a very important issue from the point of view of materials used today to build hip joint prostheses. Biocompatible polymers are commonly used, which, after exceeding their limit pressures, undergo plastic deformation. These in turn cause an increase in radial clearances and, consequently, loosening of the hip prosthesis.

Loosening of the hip is one of the most serious complications of joint arthroplasty. The result of this phenomenon is that a new artificial hip joint ceases to perform its function, since too large a radial clearance in extreme cases, causes loss of the head of the prosthesis in the acetabulum of the polymer and loss of support of the human body while disconnecting a pair of head-cup prosthesis and the immobilization of man.

Another negative consequence of the loosening of the hip endoprosthesis because of exceeding the surface pressure is the formation of inflammation, which causes further degradation of human bone material as well as accelerated wear of the tribological pair that is hip endoprosthesis.

Hip endoprosthesis is a hip ball joint of a limited deviation angle in a sliding movement at a single-track working load. In this dissertation, the results of numerical analysis of hip endoprosthesis using the original method are presented. Such methods as the solution method of contact in theory of resilience was used in order to assess the parameters of contact between the circular bodies of similar diameters occurring in an internal contact point [10-13].

2. The issue of contact pressure

In order to use the indicated method of testing flat contact problems, this type of endoprosthesis (3D system) was modeled (Fig. 1b) with the diameter of the head with a cylindrical joint with a fixed socket (3D system) (Fig. 1b) by introducing a model radius (contractual).

This method was previously used to test the load capacity of resting joints, as well as initial contact pressures in the moving systems: slide bearings, reciprocating cylindrical guides and pendulum joints [5, 6, 7].

The tested 3D system was transformed into a 2D flat system reducing the total compressive load N (Fig. 1b) per head of the prosthesis to one unit of its length (head diameter), i.e. $N' = \frac{N}{D_2}$. It is assumed that $R_1 \approx R_2 = R$, where, respectively R_1, R_2- the radius of the bushing (bearing shells) 1 and the radius of the disk 2. In the hip is a small radial clearance (aperture) $\varepsilon = R_1 - R_2 \geq 0 \ll R$. Under the influence of load, there is contact of the system elements in the zone defined by the angle $2\alpha_0$ and arises contact pressure p_α. The elastic characteristics of the bodies are not the same. In the state of unidirectional pendulum motion (partial rotation of the endoprosthesis head towards its cup during gait in the loading phase), we will model this system with a plain bearing (Fig. 1b).
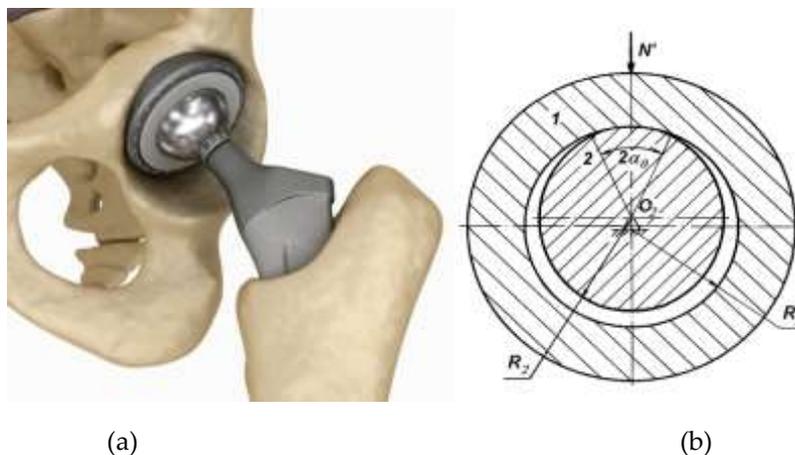

(a)   (b)

Figure 1. Calculation schemes a) hip arthroplasty [20], b) joint scheme

3. Solution method

The solution method consists of determining the maximum contact pressure, the contact angle and distribution of pressures in the contact sphere. The equation of contact pressures p_α in the case of symmetric contact against the external loading of the components in the layout shall be in the form [11-13]:

$$c_1 \int_{-\alpha_0}^{\alpha_0} \cot \frac{(\alpha - \theta)}{2} p'_\theta d\theta = c_2 p_\alpha + c_3 \int_{-\alpha_0}^{\alpha_0} p_\alpha \, d\alpha + c_4 cos\alpha \int_{-\alpha_0}^{\alpha_0} p_\alpha cos\alpha \, d\alpha + \frac{\varepsilon}{R^2} \, , \tag{1}$$

where $p'_\theta = \frac{dp}{d\theta}$; α– polar angle; $0 \leq \alpha \leq \theta$; $0 \leq \theta \leq \alpha_0$;
$c_1 = \frac{1}{8\pi}(\frac{1+\kappa_1}{G_1 R_1} + \frac{1+\kappa_2}{G_2 R_2})$; $c_2 = \frac{1}{4}(\frac{1-\kappa_1}{G_1 R_1} - \frac{1-\kappa_2}{G_2 R_2})$;
$c_3 = \frac{1+\kappa_1}{8\pi G_1 R_1}$; $c_4 = \frac{1}{2\pi}(\frac{\kappa_1}{G_1 R_1} + \frac{1}{G_2 R_2})$;

$\quad G_1, G_2$- the modulus of material elasticity of all the components in the layout
$\quad \nu_1, \nu_2$–Poisson's ratio; $\kappa = 3 - 4\nu$ – the state of flat deformation.

The approximate solution of the equation (1) is achieved by the collocation method. The function of contact pressures p_α is presented in the form of [12-16]:

$$p_\alpha \approx E_0 \varepsilon \sqrt{tan^2 \frac{\alpha_0}{2} - tan^2 \frac{\alpha}{2}} \tag{2}$$

where $E_0 = (\frac{e_4}{R_2}) cos^2 (\frac{\alpha_0}{4})$ – simplified version,

$E_0 = \frac{e_4}{R_2} \left[cos^{-2}(\frac{\alpha_0}{4}) - e_1 \sqrt{tan^2(\frac{\alpha_0}{2}) - tan^2(\frac{\alpha_0}{4})} - 0.5 sin^2 \frac{\alpha_0}{4} \left(e_2 cos^{-1}(\frac{\alpha_0}{2}) + 2e_3 cos(\frac{\alpha_0}{2}) \right) \right]^{-1}$ – detailed version,

$e_1 = \frac{2}{Z}[(1 - \kappa_1)(1 + \mu_1)E_2 - (1 - \kappa_2)(1 + \mu_2)E_1]$; $e_2 = \frac{2}{Z}(1 + \kappa_1)(1 + \mu_1)E_2$;
$e_3 = \frac{4}{Z}[\kappa_1(1 + \mu_1)E_2 + (1 + \mu_2)E_1]$; $e_4 = \frac{4E_1 E_2}{Z}$; $Z = (1 + \kappa_1)(1 + \nu_1)E_2 + (1 + \kappa_2)(1 + \nu_2)E_1$;
$E = \frac{2G}{(1+\nu)}$ – Young's modulus.

Maximum contact pressures p_0 are obtained when $\alpha = 0$. Then

$$p_0 \approx E_0 \varepsilon tan \frac{\alpha_0}{2} \tag{3}$$

The balanced forces exerted on the second target determine an unknown half-angle α_0.

$$N' = R \int_{-\alpha_0}^{\alpha_0} p_\alpha \cos\alpha d\alpha = 4\pi R E_0 \varepsilon sin^2 \frac{\alpha_0}{4} \tag{4}$$

4. Loading conditions in the hip joint

The total force N on the femoral head is determined based on various literature data [17]. It is a geometric sum of the two forces - body weight K as well as muscular strength M (Fig. 2).
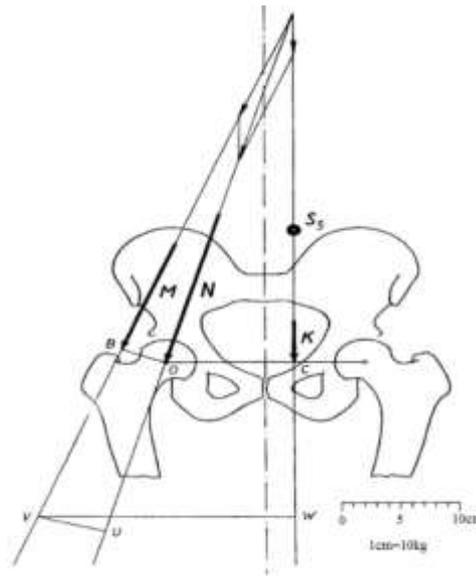
Figure 2. Quasi-static estimation of hip joint loading [8]

In the cycle of movement, it is a substantial change – $1.45K \leq N \geq 4.4K$ (Fig. 3). Therefore, in normal conditions of walking in two peaks, pressure of 3.0 and 4.4 K occurs. In extreme cases, it reaches up to 9 K.
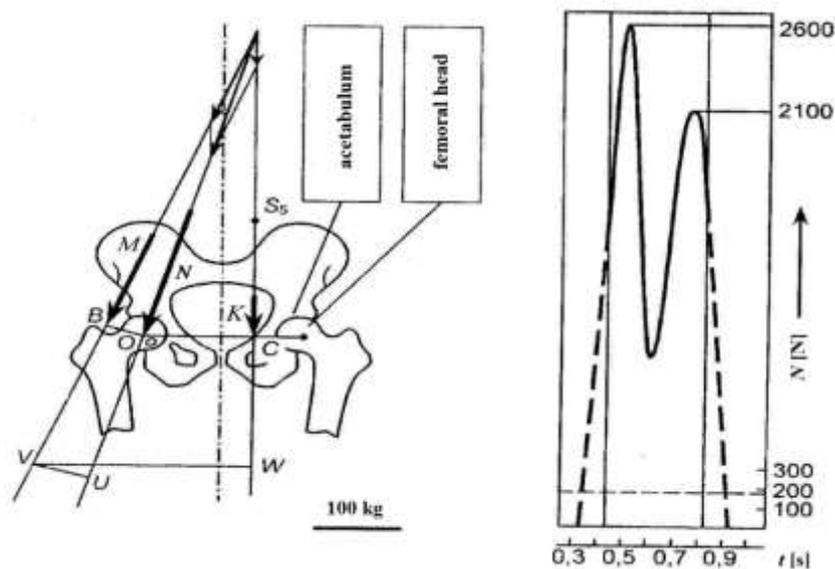

Figure 3. Diagram of the change in pressure on the head of the hip joint during support with one foot during the physiological gait phase [17, 18]

Therefore, taking all the data into consideration, the average value of the compression force N_{av} registers about 1900 N (if K = 700 N).

5. Numerical solution of the problem

Numerical solution of the problem was conducted for the examined layout when $\varepsilon > 0$. To calculate the parameters of the contact in hip endoprosthesis the following data were selected: N_{max} = 2900 N, N_{av} = 1900 N, N_{min} = 1000 N; $N' = \dfrac{N}{D_2}$; D_2 = 28, 48 and 58 mm; ε=0.02 ÷ 0.2 mm; for calculations were adopted accordingly (Tab. 1):

Table 1. Input data for calculations

D_2 = 28 mm	D_2 = 48 mm	D_2 = 58 mm
N'_{max}=103.6 N	N'_{max}=60.4 N	N'_{max}=50 N
N'_{av}= 68 N	N'_{av}= 39.6 N	N'_{av}= 32.8 N
N'_{min}= 35.7 N	N'_{min}= 20.8 N	N'_{min}= 17.2 N

A simplified version was applied in case of E_0.

In the calculation of the model cylindrical joint, a conventional radius $R' = 0.5\sqrt{R_1 R_2} = 0.5\sqrt{(R_2 + \varepsilon)R_2}$ was introduced. As a result, received a replacement plane system for the prosthesis as the spatial system with R_1 and R_2 rays interacting elements.

Materials used for the endoprosthesis: head 2 – NitridedGRADE 2 (TDN) [7], for which E_2 = 112 GPa, ν_2 = 0.32 (GRADE 2 - titanium); acetabulum 1 – polyethylene PE-UHMW, for which E_1 = 0.625 GPa (37^{0}C), ν_1 = 0.46.

The results of the solution are given in figures 4 and 5 as well as in Tab. 2. In Fig. 4 the ratio correlation of maximum contact pressures p_0 of radial clearance ε is presented with the head diameter D_2 of endoprosthesis and the reduced compression force N′.
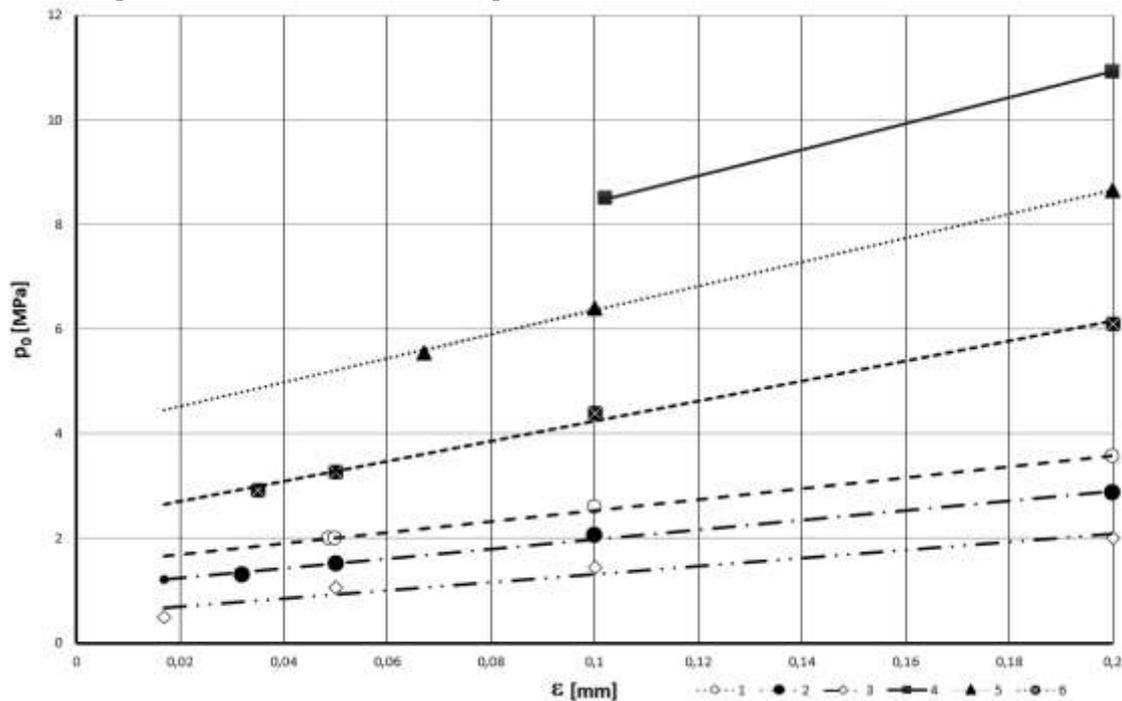

Figure 4. The impact of radial clearance on maximum contact pressures. D_2=58mm: 1 – N′$_{max}$=50N/mm, 2 – N′$_{av}$=32.8N/mm, 3 - N′$_{min}$=17.2N/mm; D_2=28mm: 4 - N′$_{max}$=103.6N/mm,5 - N′$_{av}$=68N/mm, N′$_{min}$=35.7N/mm

Analysis of the results allows us to draw conclusions from linear relationship of increasing p_0 from ε within the scope of 0.05≤ ε ≤0.2 mm. In case of ε ≤0.05 mm the above-mentioned correlation, become non-linear. The intensity of growth p_0 depends on the size of head diameter D_2. In Tab. 2 more specific parameters of p_0 and $2\alpha_0$ are presented.

Table 2. Input data for calculations

Data Param.	D_2 = 58 mm N_{max} = 2900N N'_{max} = 50N/mm				D_2 = 58 mm N_{av} = 1900N N'_{av} = 32.8N/mm				D_2 = 58 mm N_{min} = 1000N N'_{min} = 17.2N/mm			
ε [mm]	0.049	0.05	0.1	0.2	0.032	0.05	0.1	0.2	0.017	0.05	0.1	0.2
p_0 [MPa]	1.978	1.98	2.6	3.56	1.30	1.52	2.06	2.86	0.48	1.06	1.44	2.0
$2\alpha_0$ [°]	158.8	157.6	106.6	74	148.8	126.6	85.8	60	158.8	85.8	60	42.2

Data \ Param.	$D_2 = 28$ mm $N_{max} = 2900N$ $N'_{max} = 103.6N/mm$		$D_2 = 28$ mm $N_{av} = 1900N$ $N'_{av} = 68N/mm$				$D_2 = 28$ mm $N_{min} = 1000N$ $N'_{min} = 35.7N/mm$			
ε [mm]	0.1023	0.2	0.0671	0.1	0.2	0.035	0.05	0.1		0.2
p_0 [MPa]	8.48	10.92	5.56	6.41	8.64	2.92	3.26	4.4		6.1
$2\alpha_0$ [°]	158.8	108.4	158.8	128.2	86.8	158.8	128.2	88.2		61.6

For the maximum and average value of compression force and the examined radial clearances, the relationship between maximum contact pressures p_0 with the head diameter of endoprosthesis D_2 was presented in fig. 5.
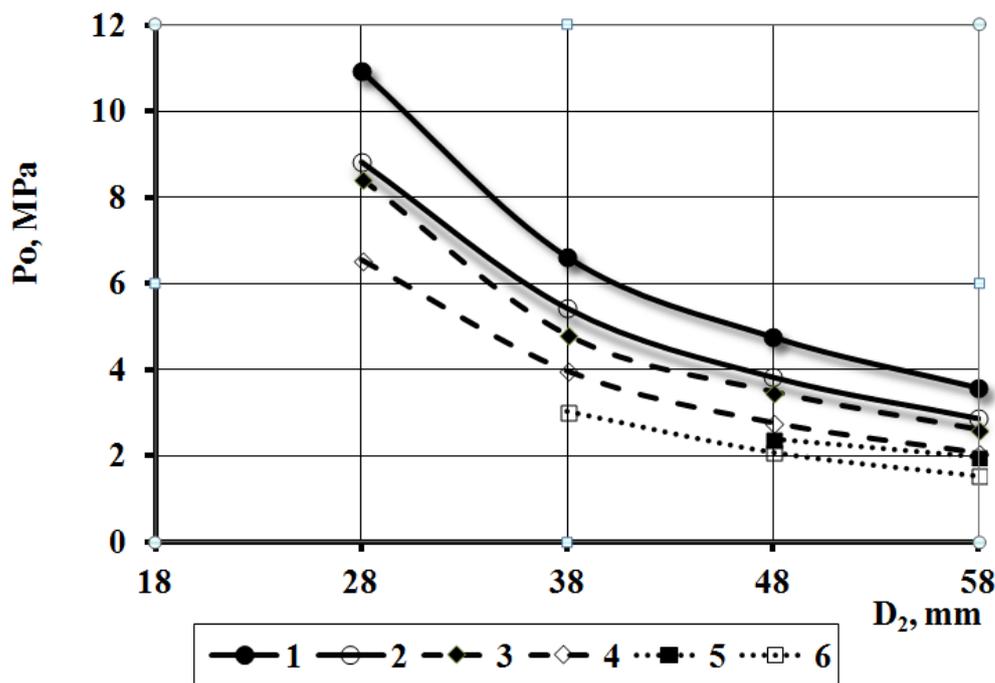

Figure 5. The impact of the head diameter of endoprosthesis on maximum contact pressures: 1– $N_{max}=$ 2900N, ε=0.2 mm; 2 – N_{av} = 1900N, ε=0.2mm; 3 – N_{max}=2900N ,ε=0.1mm;   4 – N_{av}=1900N, ε=0.1mm;5 – N_{max}=2900N , ε=0.05mm; 6 – N_{av}= 1900N, ε=0.05mm

The results of calculation indicate that an increase of D_2 2.07 times causes the reduction of p_0 2.96 ÷4.24 times depending on the value of N as well as ε.

According to the study [7]

$$p_0 = a_1 + a_2 N^{a_3} + a_4 \varepsilon^{a_5} + a_6 N^{a_3} \varepsilon^{a_5} \qquad (5)$$

If $\varepsilon \geq 0$; a_1, a_2, ….. factors of approximation.

And according to [9]

$$p_0 = c_0 \frac{\varepsilon E_1}{R}, \qquad (6)$$

Where $E_2 = \infty$, $E_1 = 0.625$ GPa, c_0 – collocation rate depends of α_0.

In our case $E_2/E_1 = 112/0.625 = 179.2$ times (GRADE2/PE-UHMW). Formally, it may be stated that the head of endoprosthesis is very rigid in relation to the bearing and deformation does not occur due to the contact force.

In terms of the flat contact formula the maximum contact pressure p_0 were also estimated according to the Hertz equation for the contact disc with aperture:

$$p_0 = 0.564 \sqrt{\frac{N'}{\beta} \frac{R_1 - R_2}{R_1 R_2}} \qquad (7)$$

where $\beta = \frac{(1-v_1^2)}{E_1} + \frac{(1-v_2^2)}{E_2}$ – shear modulus, $R_1 - R_2 = \varepsilon$, $R_1 = R_2 + \varepsilon$.

Hertz formula for the internal ball connection of slightly different radii is not possible to use within the examined issue of flat contact strength of theory of elasticity due to the two-dimensional (circle shape) area of contact.

In Tab. 3 and in pictures 6 and 7 the results of the calculations p_0 according the certain methods and their relative change for the layout where radial clearance $\varepsilon=0.1$ and $\varepsilon=0.2$ mm occurs.

Table 3. Results of the calculations

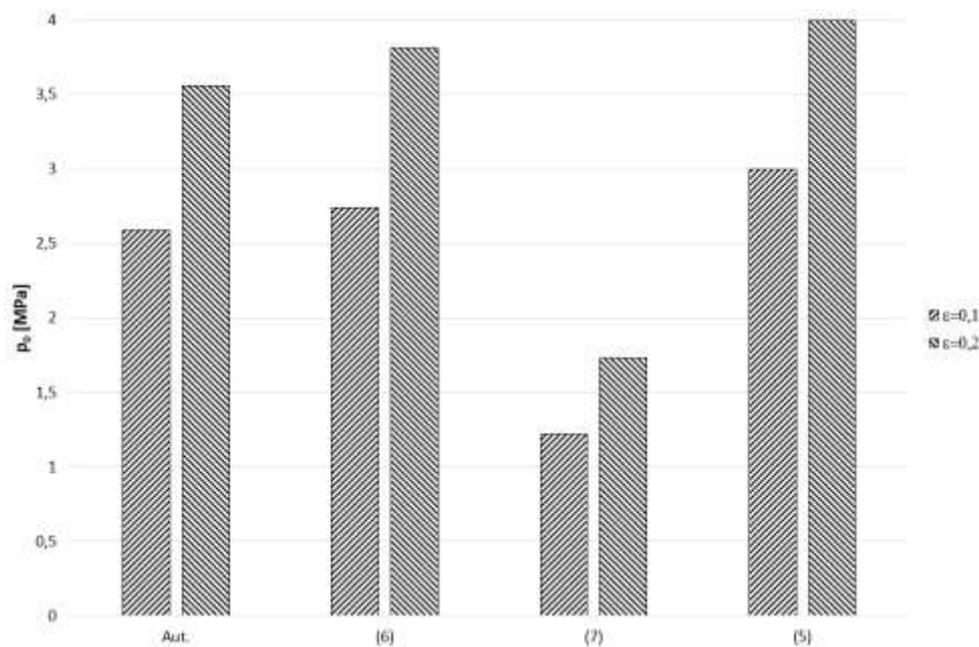| Formula \ Param.	D$_2$ = 58 mm N = 2900N N′ = 50N/mm		(6)		(7)		(5)	
ε [mm]	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2
p_0 [MPa]	2.59	3.56	2.74	3.81	1.22	1.73	3.0	4.0
$2\alpha_0$ [°]	106.6	74.0	106.6	74.0	-	-	-	-
p_{0i}/p_0	1.0	1.0	1.06	1.07	0.47	0.485	1.16	1.12

Figure 6. Alteration of maximum contact pressures for radial clearance $\varepsilon=0.1$mm and $\varepsilon=0.2$mm
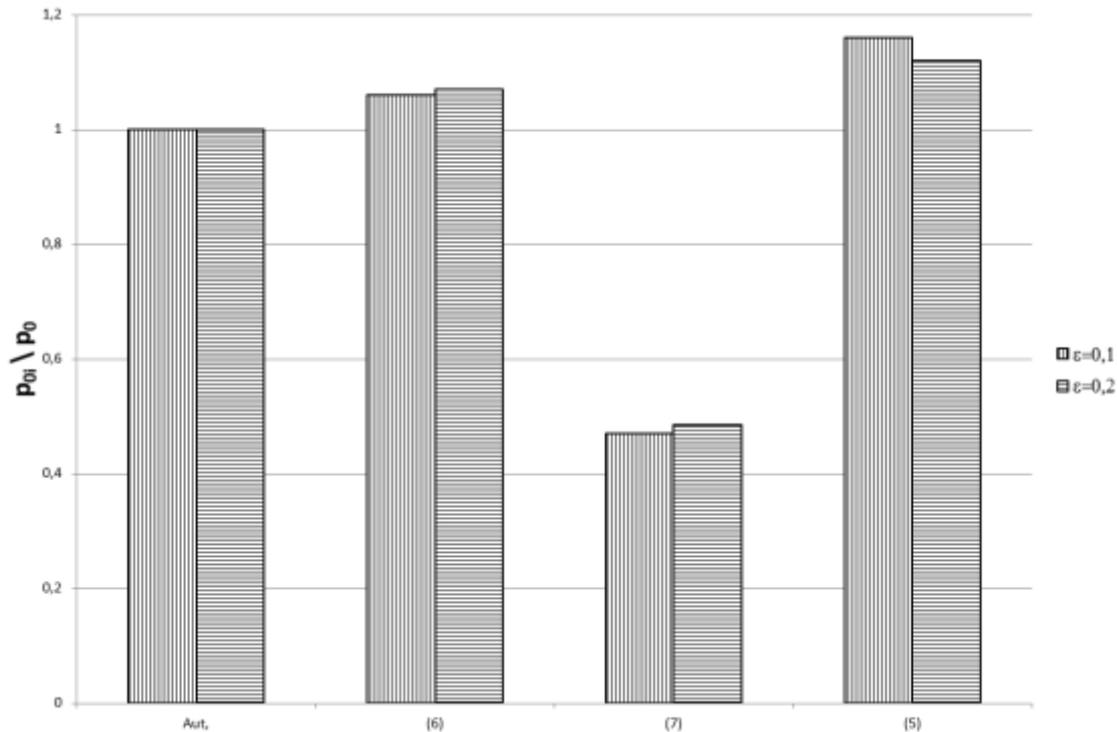
Figure 7. Relative change of maximum contact pressure for radial clearance ε=0.1mm and ε=0.2mm

Due to the method [9] certain pressures p_0 of slightly higher value (1.07) were determined and according to the Hertz formula the result is inappropriate and according to the method [7] – higher than the original method up to 1.16 times.

According to the data established in this paper [4] the solution for the endoprosthesis problem was conducted CoCr – CoCr, when N=3200 N, D_2 = 58.6 mm, ε = 0.05 mm. It was established that the maximum contact pressures reached 24.05 MPa by the original method and 22.0 MPa according to FEM [4] that means they are 1.093 times higher.

However, according to the scholarly work [8]and the original authors method for the endoprosthesis CoCrMo –PE-UHMW, if N = 2500 N, D_2 = 32 mm, ε = 0.098 mm the pressures are identical – 10.2 MPa.

Producers of hip endoprosthesis anticipate the initial radial clearance ε = 0.05 ÷ 0.25 mm [7]. As a practical matter, the assessment of maximum contact pressures outside the scope is not targeted. Nonetheless, reduction ε of the above-mentioned $ε_{min}$, even more hypothetically factored to zero does not provide any substantial benefits in an increase of the contact surface $2α_0$ of all the components of endoprosthesis and the reduction of level p_0.

The results of the calculation (tab. 3) show that there are certain values of clearance ε in each examined case where the angle of the contact $2α_0$ reaches the limit value of 160 degrees. It is caused by the increased deformation of the bearing material (PE-UHMW) upon the contact pressures. In Figure 4 those threshold values ε were established for which the contact angle for this method will be verging. Especially in cases D_2= 28 mm and N_{max}= 2900 N, N_{av}= 1900 N appears at ε>0.05 mm, and for D_2= 58 mm, N_{max}= 2900 N, whenε > $ε_{min}$= 0.05 mm.

The reduction of the initial radial clearance in endoprosthesis of the examined type below 0.05 mm aims at the reduction of the initial contact pressures is not justified since those pressures will be reduced while acetabulum wear. It is also known that at acetabulum wear the size of its diameter approaches the size of the diameter of rigid head, which technically is hardwearing. Therefore, the construction clearance declines which results in the reduction of maximum contact pressures.

6. Conclusions

1. Maximum contact pressures p_0 depend on the loading force N against the endoprosthesis head, its diameter D_2 and radial clearance ε in the layout (fig. 4, 5).
2. Their values increase linearly making radial clearance escalate of $0.05 < \varepsilon < 0.2$ mm (fig. 4). At $\varepsilon \leq 0.05$ mm their alteration slightly deviates from the linear one.
3. An escalation of pressures with the reduction of diameter of head endoprosthesis with an increase of radial clearance was observed (fig. 4).
4. Maximum contact pressure p_0 increases linearly with an increase in the force N loading the endoprosthesis head.
5. A comparative analysis of results regarding the assessment of p_0 according to the author's method is in accordance with other methods (fig. 6 and 7).
6. From the practical point of view, the results of the examined issue indicate that the reduction of radial clearance below 0.05 mm does not seem to be deliberate.
7. The developed method enables the effective assessment of maximum contact pressures in endoprosthesis in case of the occurrence of radial clearance.

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