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Big bang process, magnetic monopoles and super unification theory of all forces using the GEM wave

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Abstract;
Space-time evolution is briefly explained by using the 3-dimensional quantized space model (TQSM) based on the 4-dimensional (4-D) Euclidean space. The energy (mass), charges and absolute time are newly defined based on the 4-D Euclidean space. The photons and gravitons are understood as the 2-D space fluctuations along the space axes and 1-D time fluctuations along the time axis, respectively. This indicates that the electromagnetic (EM) and gravitational (G) waves can be unified into the so called gravityelectromagnetic (GEM) wave which has both space and time fluctuations. The photon with zero force fields is the 2EM wave because an EM wave corresponds to a charged fermion. The dark matter bosons, weak force bosons, strong force bosons are the 1-D time fluctuations which can be expressed by the G waves. This indicates that all five forces are unified by the GEM wave. It is called as the super unification theory in the present work. And the force carrying bosons and mesons are, for the first time, proposed as the possible candidates of the magnetic monopoles like the fermions and baryons are the electric monopoles. The signs of the magnetic charges quantized as $q_m=cq$ are newly defined. The big bang is understood by the space-time evolution of the 4-D Euclidean space but not by the sudden 4-D Minkowski space-time creation. The big bang process created the matter universe with the positive energy and the partner anti-matter universe with the negative energy from the CPT symmetry.

Key words; Big bang, Super unification theory, 4-D Euclidean space, CPT symmetric universe, Origins of the energy (mass), charge and time, Magnetic monopoles

1. Introduction

The space-time geometry research is closely related to the special and general relativities, quantum mechanics, manifold mathematical physics, cosmology, and standard model [1-42]. The particles and their interactions have been studied in terms of the quantum field theory (QFT), too. In the quantum field theory (QFT), the particles except the graviton are defined as the excited states of the corresponding fields [23]. Whether the gravitons have the non-zero rest mass and its corresponding field is not clear. In QFT, the gravitational wave is defined as the warped space-time movement emitted from the source. The string theory has been applied to unify all four interactions in nature [24]. String theory requires the 10 dimensions. Six of the 10 dimensions are microscopically curled up. Therefore, only the three space-dimensions and one time-dimension are observed in the real macroscopic world. It has been reported that most of the results predicted by the string theory cannot be experimentally observed. And the leptoquarks [25] and supersymmetry [26] have been tried without the experimental evidences, too.

What happened before and after the big bang moment, whether all forces can be unified in a consistent way, whether the magnetic monopoles can be discovered [43], why the charge has positive value or negative value unlike the mass with only the positive value, and where the fermions and boson are originated from etc. are discussed in the present work. Please see section

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2 for the unification of all forces (super unification theory (SUT)), section 3 for the space-time evolution and big bang, sections 4 and 5 for the magnetic monopoles and section 5 for the force fields of the photon space and particles.

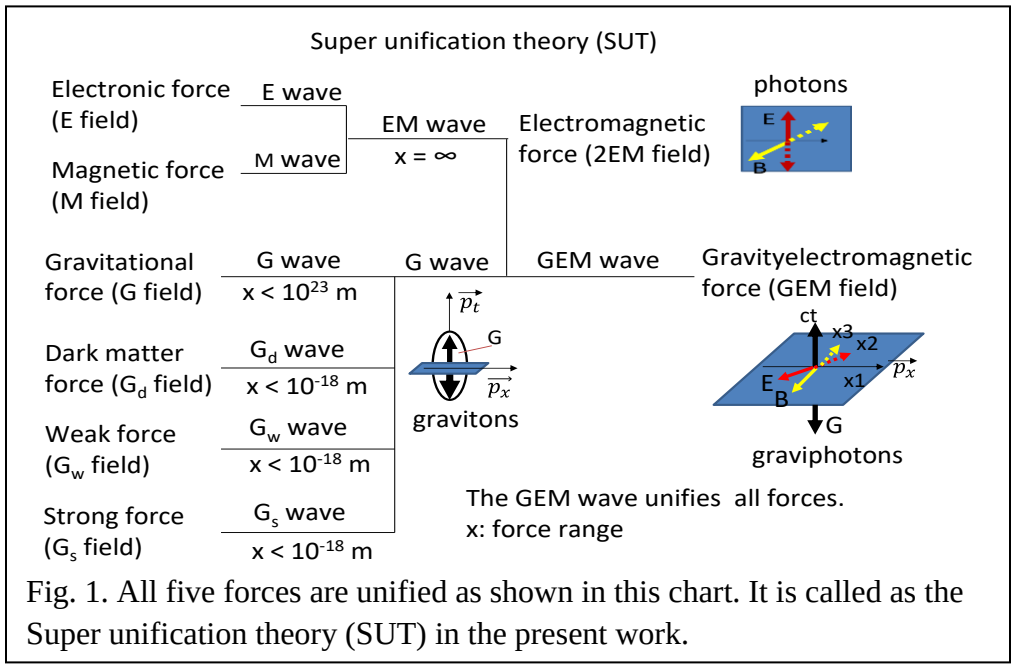

Fig. 1. All five forces are unified as shown in this chart. It is called as the Super unification theory (SUT) in the present work.

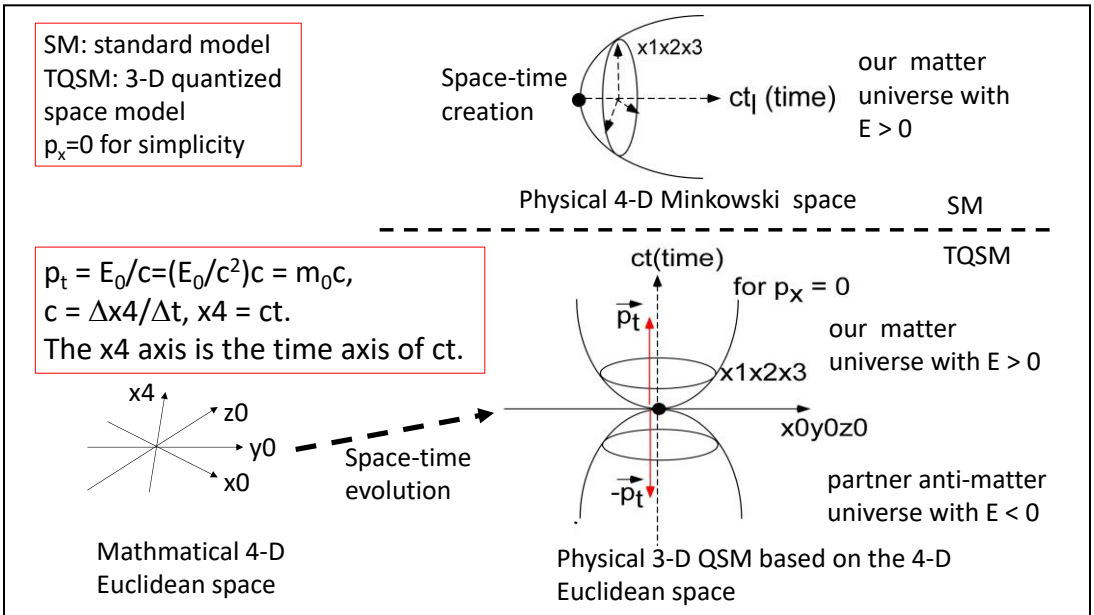

Fig. 2. The big bang process is compared for the space-time evolution in TQSM and space-time creation in SM. The x1x2x3 space coordinates and x0y0z0 space coordinates can be always overlapped with the corresponding time gap of cΔt.

The unification theory has the history extending from the electromagnetic interaction to the electroweak interaction. The grand unification theory (GUT) indicates a unification of the electromagnetic-, weak- and strong interactions by using the unified force coupling (strength) constant without the experimental evidences. In the present work, the physical concept of the

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electromagnetic wave is applied to the possible unification of all forces as shown in Fig. 1. In other words, the magnetic wave and electric wave are combined to form the electromagnetic (EM) wave for the electromagnetic interaction. Therefore, it is great if the gravitational wave, magnetic wave and electric wave are combined to build the so called gravityelectromagnetic (GEM) wave for the so called gravitoelectromagnetic (GEM) interaction as shown in Fig. 1. In this case, the 4-D (four dimension) Euclidean space is needed in order to unify three waves because 3-D (three dimension) Euclidean space has been used in order to unify two waves of the magnetic wave and electric wave.

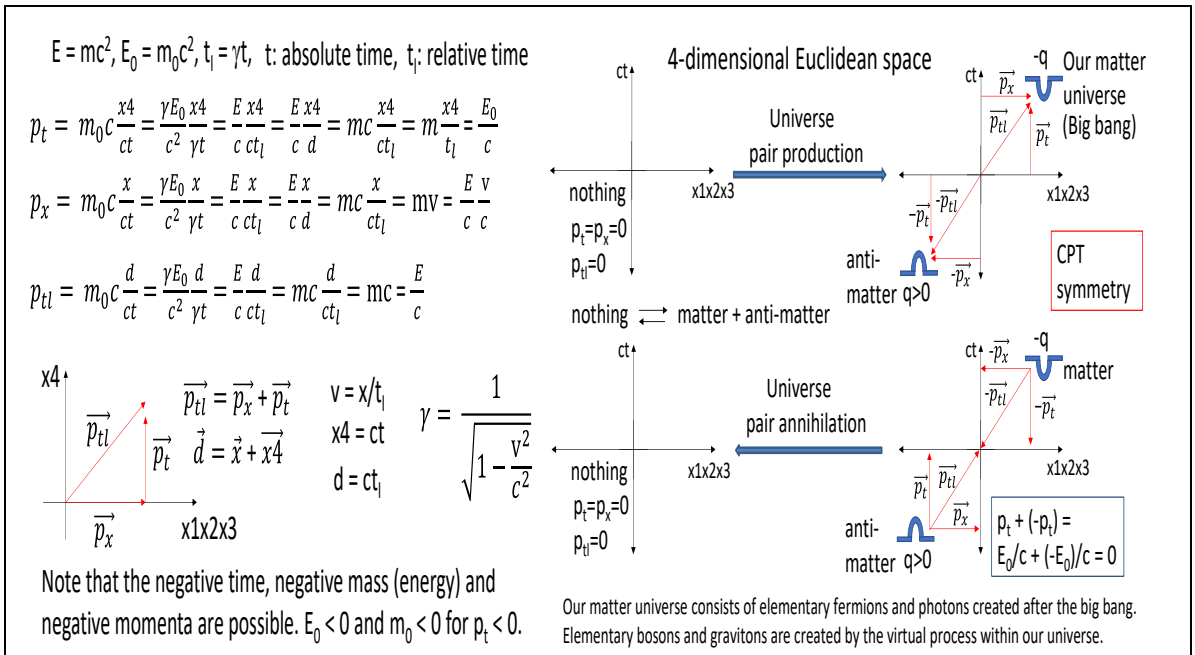

Fig. 3. 4-dimensional momenta are defined for the absolute time (t) along the x4 axis. Here, the big bang is explained as the universe pair production (see Fig. 1). The x1x2x3 space coordinates can be always replaced with the x0y0z0 space coordinates if needed.

Here, the moving direction of the unified wave takes one of three dimension space axes. As well known in the various experiments, the electric (E) wave and magnetic (M) wave are the space fluctuations along other two dimension space axes in the EM wave. Also, the magnetic (B) field and electric (E) field have been separately observed. Our real world is expressed as the 3-D space and 1-D time. If the fourth Euclidean space axis is equal to the 1-D time axis in our world, the problem for the missing third wave can be easily solved. Under this assumption, the third wave should be the time axis fluctuation. Then, the question is what kind of the new interaction is connected to the new third wave. This question can be easily solved if the gravitation interaction is connected to the third wave. In other words, it is reported in the present work that the gravitational wave is the time (axis) fluctuation that completes the GEM wave. Already the G (gravitational) field has been observed. This is the unification of the EM interaction and G interaction. I will start to write the present paper with this assumption. I will go even further to include the dark matter force (interaction), weak force and strong force for the possible unification of all forces by using the GEM wave in section 2 as shown in Fig. 1.

The big bang process is the sudden space-time creation from the nothing in the standard model as shown in Fig. 2. There have been many researches to explain this sudden 4-D space-time creation. The sudden 4-D space and time creation does not look like the natural process in terms of the CPT

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symmetry. It is because the partner universe is not proposed in the well-known big bang theory. The corresponding space-time is based on the 4-D Minkowski space where the observed time of t_l is changing with changing of the particle velocity. In this well-known big bang theory, the observed time and energy (mass) are positive in Fig. 2. This means that the time momentum of $p_{tl}=E/c$ is positive along the time axis of ct_l, and our universe is moving toward the positive (forward) time direction since the birth of the universe. This explains the fact that our universe cannot be moving to the backward time direction and the observed mass (energy) are positive. At

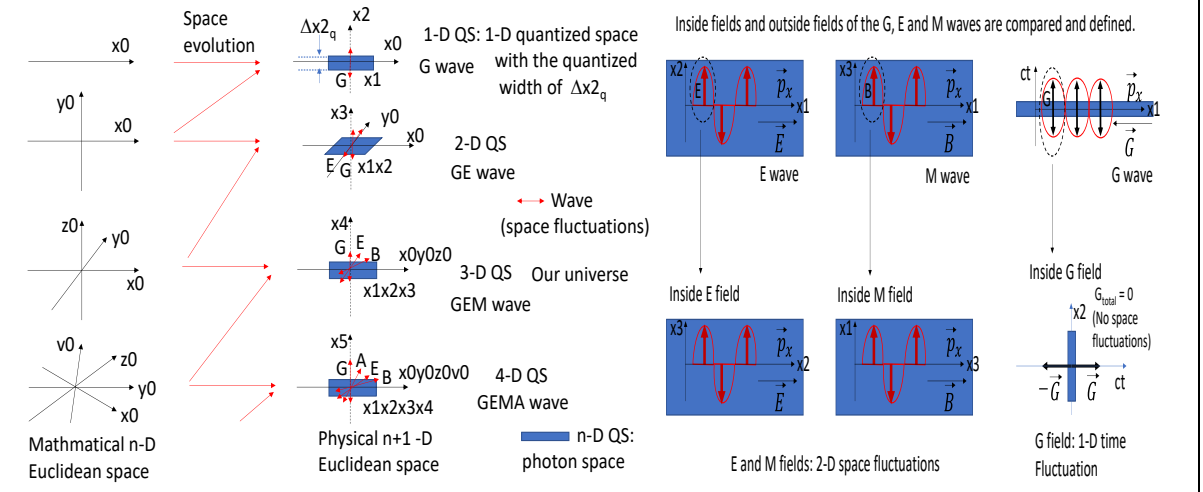

Fig. 4. The mathematical n-D Euclidean spaces are developed to the physical n+1 -D Euclidean spaces with the energy (photon space). The inside fields and outside fields are compared with the corresponding waves as defined in the present work.

least this is true in our universe where we live. The standard model including the big bang process has been successfully established to explain many properties of the elementary particles, for a long time.

In the present work, our space-time matter universe is explained by the extension from the 4-D x0y0z0x4 Euclidean space to the 3-D x1x2x3 Euclidean space on the 4-D x0y0z0x4 Euclidean space in Figs. 2, 3 and 4. This x1x2x3 space is called as the 3-D quantized space with the quantized time width of $c\Delta t_q$ (3-D QS) or the 4-D quantized space and time (4-D QST) in Figs. 2,3 and 4. Then the time axis of ct is the fourth dimension x4 axis. The quantized time width of $c\Delta t_q$ could be the Planck size scale. Our matter universe corresponds to the x1x2x3 quantized space in Figs. 2, 3 and 4 because only the three space dimensions and one time dimension are observed in the real macroscopic world. Also, two E and M space fluctuations in the EM wave indicates that we live in the 3-D quantized x1x2x3 space in Fig. 4. This is called as the space-time evolution which looks like the natural process in terms of the CPT symmetry because our matter x1x2x3 universe and the partner anti-matter x1x2x3 universe are created at the big bang by the CPT symmetry. In the president 3-D quantized space model, the universe can move toward the positive time direction or negative time direction. It is because the time momentum of p_t is positive or negative. Because the time momentum is $p_t = E_0/c$ in Fig. 3, the energy of the universe with the positive time momentum is positive and the energy of the partner universe with the negative time momentum is negative. Of course, the zero time momentum of $p_t = E_0/c = 0$ means the nothing with the zero energy. Therefore, the universe with the positive time momentum and the partner universe with the negative time momentum are created from the nothing with the zero time momentum of $p_t =$

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$E_0/c = 0$ as shown in Figs. 2 and 3. The created universe is called as the x1x2x3 space with the very small time width of $c\Delta t_q$ which could be the Planck time scale. The present big bang theory in terms of the 3-D quantized space model (TQSM) is called as the space-time evolution theory while the well-known big bang theory in terms of the standard model (SM) is called as the space-time creation theory in Fig. 2.

2. Possible unification of all forces (super unification theory (SUT))

The present work is entirely based on the 4 dimension Euclidean space but not on the 4 dimension Minkowski space in Figs. 2, 3, 4, 5 and 6. Fourth dimension axis is the absolute time axis of ct. The ideas proposed by the present model are graphically explained for the readers who want to understand the basic physical concepts. In the present model, the photon is the flat space with the zero rest mass in Figs. 5 and 6. The particles and matters with non-zero rest masses (m_0) including the gravitons are considered as the warped spaces in Figs. 5 and 6. Here, the graviton has the non-

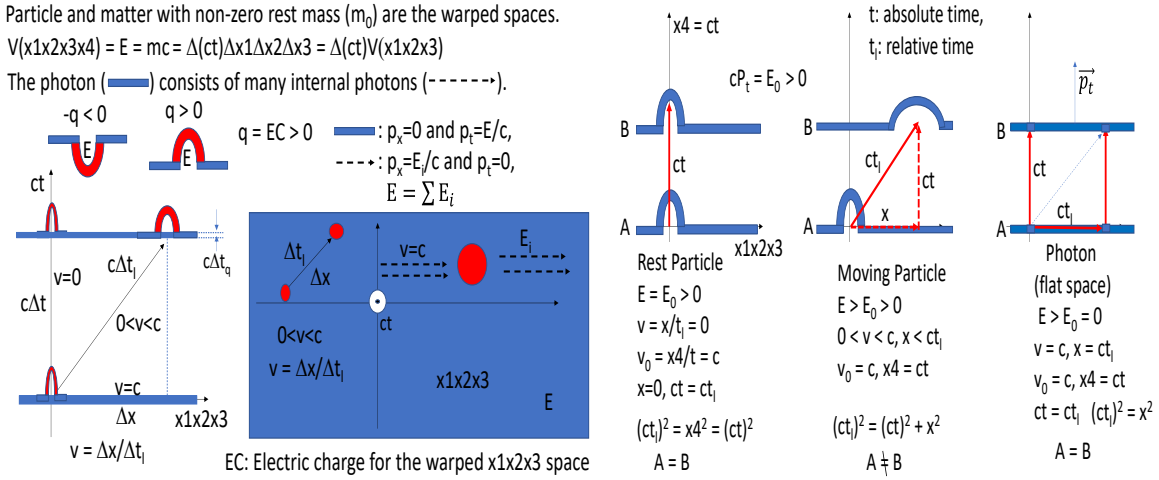

Fig. 5. The 3-dimensional quantized space is the photon space. The photon has the constant velocity based on the time definition of $\Delta t_l = \Delta x/c$. The photon consists of several internal photons. The particle is the warped space with the velocity (v) slower than the photon velocity of c. Then the rest mass energy of the particle is defined as the 4-dimensional volume of the warped space. Under this definition of the particle mass energy, it is proposed that the electric charge is positive for the positive warped space and negative charge for the negative warped space in the x1x2x3 space.

zero rest mass. This tells that all the particles including the gravitons with the non-zero rest masses are the warped spaces which are created from the flat photon space. Only the photon has the zero rest mass which indicates the flat space. The rest mass energy of $E_0 = m_0c^2$ is defined as the four-dimension space volume of $\Delta(ct)\Delta x1\Delta x2\Delta x3$ in Fig. 5. Because the 4-D volume is the only factor to define the different particles in Figs. 5 and 6, it is reasonable to say that the 4-D volume is the rest mass energy of the corresponding particle. This four-dimension space volume of $\Delta(ct)\Delta x1\Delta x2\Delta x3$ is connected to the wave function of the quantum mechanics. This could be the origin of the quantum mechanics. The wave function is formed along the absolute time axis of ct but not along the relative time axis of ct_l. It will be interesting to research the relation between the

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four-dimension space volume of Δ(ct)Δx1Δx2Δx3 and the particle energy in quantum mechanics. The relation between the four-dimension space volume and wave function in the quantum

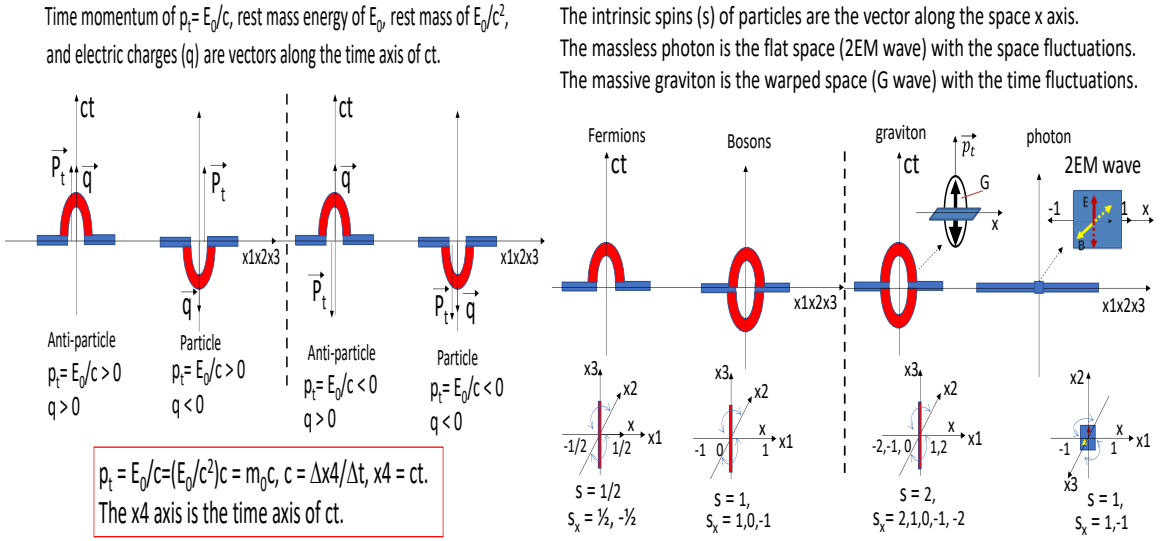

Fig. 6. Time momentum and energy are vectors along the time axis. The particle spin is a vector along the space axis.

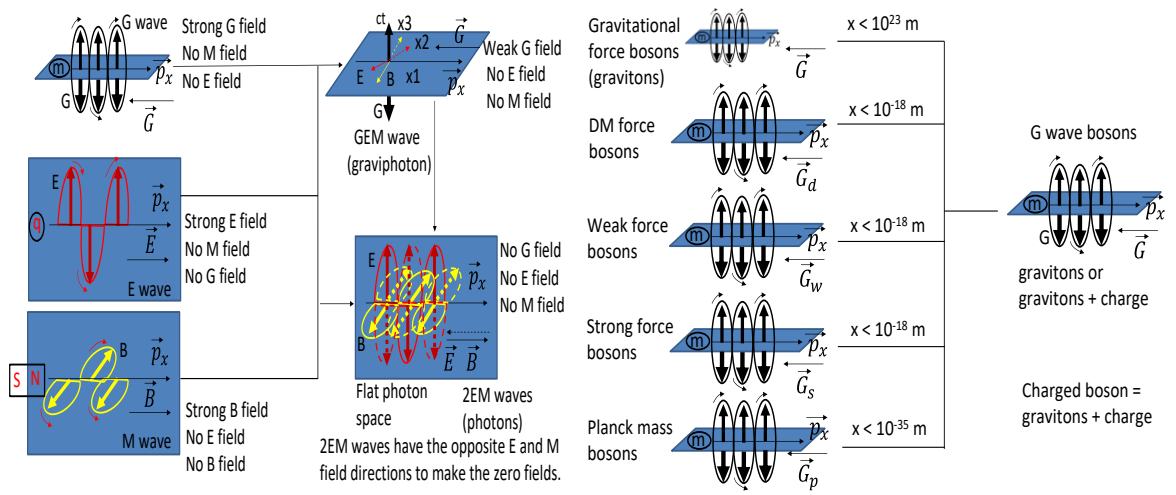

Fig. 7. Photon and graviton are unified as the gravityelectromagnetic (GEM) wave. The γ rays, x rays and flat photon space without the E- and M- fields consist of the 2EM waves (photon waves) without the E- and M- fields. See Figs. 4, 8 and 9.

mechanics will be discussed in the following paper.

The 3-dimensional quantized flat space is the photon space in Fig. 5. The particle is the warped space with the velocity (v) slower than the photon velocity of c in Fig. 5. Under this definition of the particle mass, it is proposed that the electric charge is positive for the positive warped space and negative charge for the negative warped space in the x1x2x3 space in Figs. 5 and 6. Based on these concepts, the 3-dimensional quantized space model is developed for the elementary fermions

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and bosons [11]. It is surprising that several new particles including three fermionic dark matters exist in terms of the 3-dimensional quantized space model [11].

The elementary fermions are originated from the photons, which are the electromagnetic wave (EM wave) with the space fluctuations in Figs. 6 , 7, 8 and 9. And the elementary bosons are originated from the gravitons, which are the gravitational wave (G wave) with the time fluctuations in Figs. 6, 7, 8 and 9. In Figs. 2, 3, 6 and 7, the time momentum and energy are vectors along the time axis. The particle spin is a vector along the space axis in Fig. 6. The fermion with the intrinsic

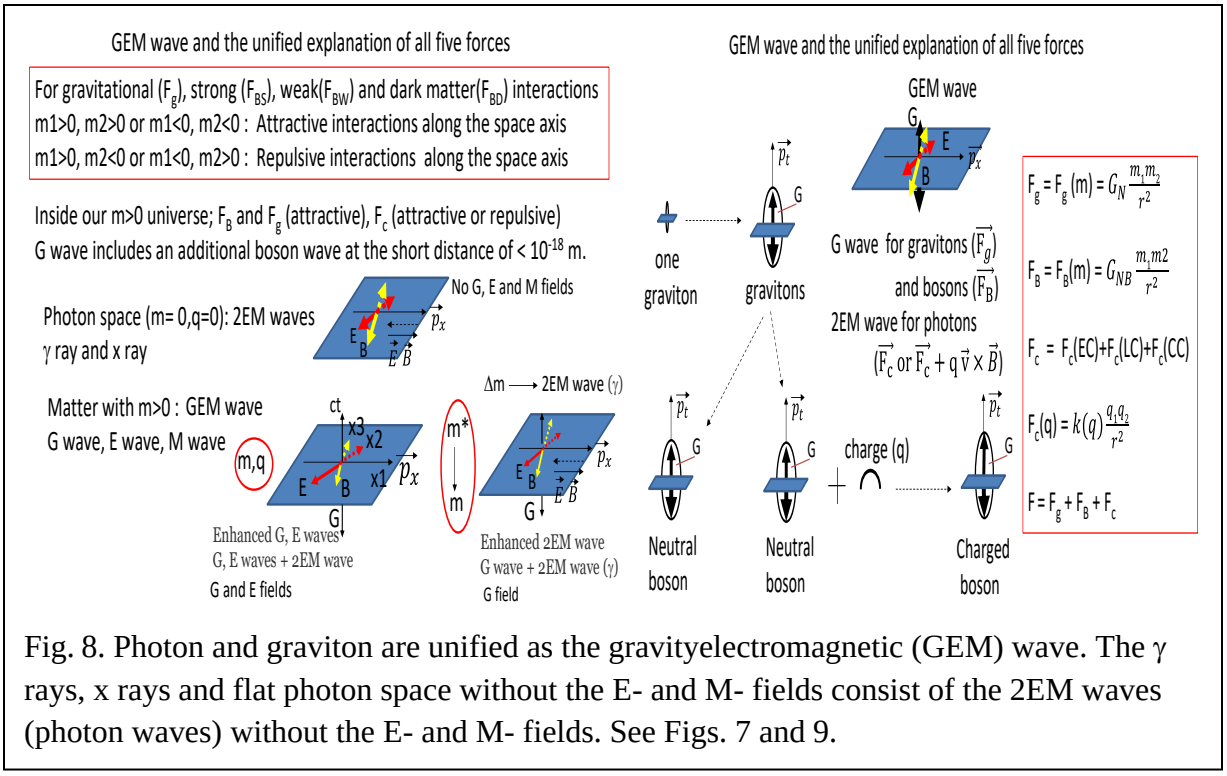

Fig. 8. Photon and graviton are unified as the gravityelectromagnetic (GEM) wave. The γ rays, x rays and flat photon space without the E- and M- fields consist of the 2EM waves (photon waves) without the E- and M- fields. See Figs. 7 and 9.

spin of 1/2 is the open warped photon space along the positive or negative direction of the time axis in Figs. 6, 7 and 9. The boson with the intrinsic spin of 1 is the closed warped photon space along both positive and negative directions of the time axis in Figs. 6 and 9. And the graviton with the intrinsic spin of 2 is the closed warped photon space along both positive and negative directions of the time axis and the photon with the intrinsic spin of 1 is just the flat space. All particles including the fermions are originated from the photon space, and, specially, the bosons are originated from the gravitons, too. As shown in Fig. 8, the massive bosons can be formed by using many massive gravitons. This indicates the possibility that the dark matter force, weak force and strong force could be unified with the gravitational force as shown in Figs. 1, 6, 7, 8 and 9 by using the GEM wave. The EM wave and G wave could be two parts belonging to the gravtyelectromagnetic wave (GEM wave) with both of space fluctuations and time fluctuations. This could be treated as the possible unification of the electromagnetic force and gravitational force as shown in Figs. 1 and 7. The massive force carrying bosons including the gravitons can be expressed as the G waves and the massless force carrying bosons (photons) are expressed as the 2EM waves. Also, the Planck mass boson is additionally shown for comparison in Fig. 7. This means that the GEM waves result from the unification of all five forces like the electromagnetic force, dark matter force, weak force, strong force and gravitational force in Figs. 1, 7 and 8. It is

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called as the super unification theory (SUT) in terms of the 3-dimensional quantized space model (TQSM).

Now I am writing about the photon and graviton. In Fig. 9, the elementary fermions are created from the pair production from the photon and the elementary bosons are created from the pair production from the graviton. In the 3-D Euclidean space, one axis is for the moving direction of the 2EM wave, one axis is for the electric space fluctuation (E) and one axis is for the magnetic space fluctuation (B). This is called as the photon that builds the 2EM wave. Then in the 4-D Euclidean space, the fourth axis is for the x4 axis fluctuation which is called as the time fluctuation (G). This time fluctuation is defined as the graviton that builds the gravitational (G) wave. The photon is on the elementary fermion side and the graviton is on the elementary boson side. In Fig. 9, it is shown how the photon and graviton are related to the elementary fermions and elementary bosons, respectively. These fermions and bosons are created by the pair production of the particle

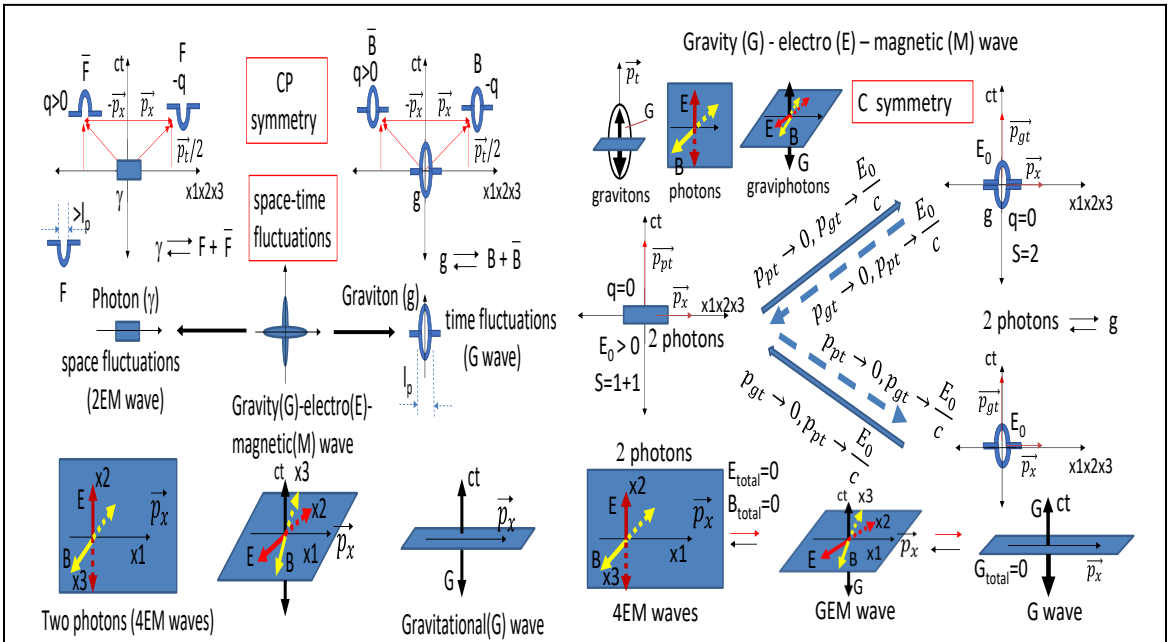

Fig. 9. Photon and graviton are unified as the gravityelectromagnetic (GEM) wave. The γ rays, x rays and flat photon space without the E- and M- fields consist of the 2 EM waves (photon waves) without the E- and M- fields. See Figs. 7 and 19.

and anti-particle from one photon and one graviton, respectively. In Fig. 9, the properties of the photon and graviton are described. The photon is the space fluctuations along the space axes and the graviton is the time fluctuation along the time axis. The space fluctuation is the flat space fluctuation along the space axes which means the zero rest mass. But the time fluctuation is the warped space fluctuation along the time axis of ct which means the non-zero rest mass. Therefore, the graviton has the non-zero rest mass which indicates the limited gravitational force range of 10^{23} m, which could be around the galaxy cluster size. Therefore, there are two kinds of space fluctuations such as the electric (E) field and magnetic (B) field perpendicular to the moving direction of the electromagnetic (EM) wave. In fact, there is another time fluctuation such as the gravitational (G) field perpendicular to the moving direction of the gravitational wave that is based on the gravitons.

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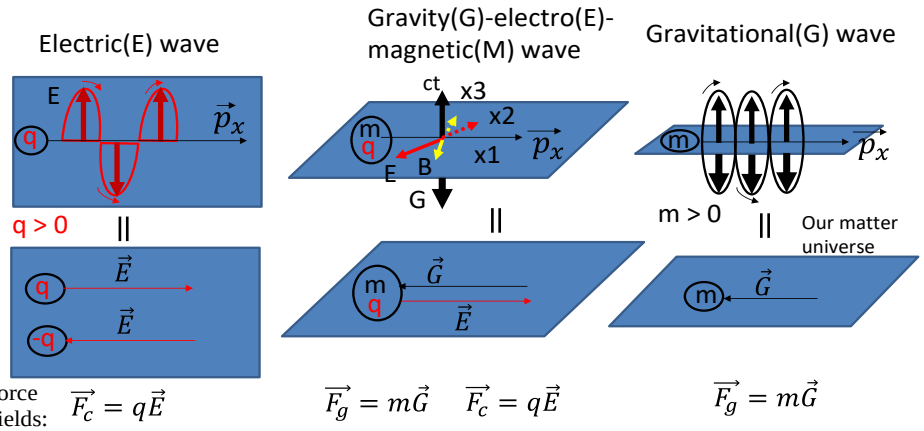
In Fig. 9, the photon and the graviton are compared for the particle pair production by the CP symmetry. The pair of the fermion and anti-fermion with the spin of 1/2 is created from a photon with the spin of 1. And the pair of the boson and anti-boson with the spin of 1 is created from a graviton with the spin of 2. In other words, the photon with the spin of 1 is created from the pair annihilation of the fermion and anti-fermion with the spin of 1/2. And the graviton with spin of 2 is created from the pair annihilation of the boson and anti-boson with the spin of 1. The properties of the elementary fermions and photon are summarized in the published paper [11]. The properties of the force carrying bosons and graviton are discussed in the published paper [11]. Please note that the graviton and force carrying bosons have the Planck length size because these particles are the 1-D time fluctuations with the minimum space size of Planck size. And the photon has the undefined size that could be extended to the infinite size because the photon has the zero rest mass. In other words, the size of the photon as the electromagnetic wave cannot be defined. And the elementary fermions have the size bigger than the Planck length. It is because the fermions have the space fluctuations and time fluctuations. The elementary fermions have the sizes depending on their rest masses. Heavier elementary fermion has the larger size.

The electric (E) wave and magnetic (B) wave in the 3-dimensional Euclidean space are unified within the electromagnetic (EM) wave that is based on the photons in Figs. 7, 8, 9 and 10. The E field is the E wave and B field is the B wave. The E and B fields can have the opposite or same moving directions from the moving directions of the E wave and B wave. The E field and B field



In the quantum field theory, the directions of the EM (photons), GEM and G (gravitons) waves are on the line connecting two matters. The E and G force fields are related to the E wave (GEM wave or 2EM wave with the enhanced E wave) and G wave (GEM wave with the enhanced G wave), respectively. The E and G fields have the same directions or the opposite directions depending on the source charge sign. The E and G waves are emitted from the wave source and absorbed into another matter. Therefore, two matters exchange the G waves (gravitons) for the G interaction and exchange the 2EM waves (photons) for the EM interaction.

Fig. 10. Photon and graviton are unified as the gravityelectromagnetic (GEM) wave. See Figs. 4, 7, 8 and 11, too.

is unified into the EM field in Fig. 7, 8, 9 and 10. The gravitational (G) field is newly proposed as the time fluctuation in the present work. Therefore, the electric (E) field, magnetic (M) field and gravitational (G) field are unified in the 4-dimensional (space-time) Euclidean space as the so called gravityelectromagnetic (GEM) wave that is based on the so called graviphoton in Figs. 7, 8, 9 and 10. The GEM wave makes the oscillations between the EM and G waves in Fig. 9. The EM wave and G wave are separately related to the EM force and G force, respectively. And, it is also

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clear that the EM and G waves are originated from the GEM wave in Figs. 7, 8, 9 and 10. These three waves can be expressed as the well-known wave equations.

In Fig. 11, the 2EM waves of photons are exchanged between two matters for the EM interaction. The 2EM wave is emitted from the source and absorbed to another matter. Here, the photon is the virtual force carrying boson. But the electric fields are emitted from the source with the positive charge and absorbed to the source with the negative charge. In Fig. 10, the different direction

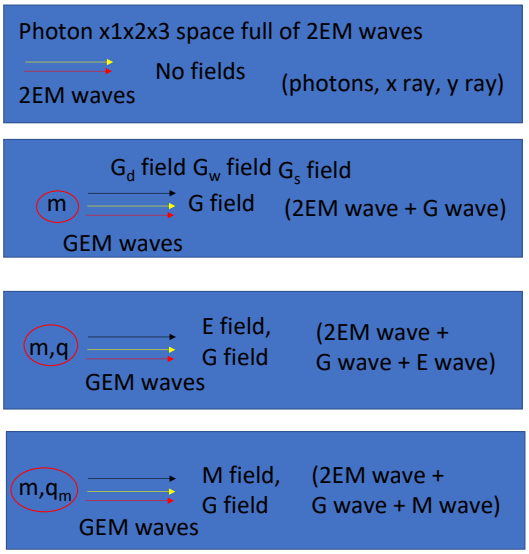

Fig. 11. All interactions are unified by using the gravityelectromagnetic (GEM) wave. See Fig. 4.

problem of the electric fields and EM waves at the source is resolved by taking the assumption that the electric field and 2EM wave consist of the same photons of 2EM wave in Fig. 11. The concepts of waves and fields are compared in Fig. 4. Basically, the electric field is the same to the enhanced E wave in the 2EM wave or GEM wave except the different definition of the direction. The direction of the electric filed is defined by the Coulomb force direction depending on the charge of the source matter and the direction of the 2EM wave is defined by the direction of the 2EM wave emitted from the matter. The same concept is applied to the gravitational force. In Figs. 10 and 11, the G waves of gravitons are exchanged between two matters for the G interaction. And the G wave is emitted from the source matter and absorbed into another mater. Here the graviton is the virtual force carrying boson. In our universe with the positive rest mass, the G field is always toward the source matter but the G wave is always emitted from the source matter. The G field and G wave are made of the same gravitons except the different definition of the direction. In the point of view of the gravitational force, the G field is defined to be the same to the G wave except the different definition of the direction in Fig. 4. In the point of view of the outgoing wave from the source, the direction of the gravitational wave is assigned. In Figs. 10 and 11, the G force field and EM force field are originated from the same GEM wave.

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3. Space-time evolution and big bang in terms of TQSM

The big bang theory has been developed to show the beginning moment of our matter universe. The 4-D space and time were created at one moment (big bang) in the past as shown in Fig. 1. This space and time creation means there was nothing before the big bang. Even the time did not exist before the big bang moment. It seems to me that the god created everything of the space and time at the big bang moment. The CPT symmetry does not exist because there is nothing before the big bang. Because our universe is the matter dominated universe, the CP symmetry problem is unsolved. Also, the time (T) symmetry before and after the big bang cannot be applied to our universe. I think that these CP and T symmetry problems are the serious problems. For the explanation of the geometrical structure of our universe, the 4-D Minkowski space was introduced in the special and general relative theories. Then the introduced time should be varied depending on the relative velocity of two space-time frames. This is called as the relative time of t which corresponds to the relative time of t_l in the present 3-D quantized space model (TQSM). We know in the real life that the massive particles and matters are made of the 3-D space volume at the instant time of t. It means that the instant time of t has the very small-time gap of $c\Delta t_q$ when we observe the 3-D space volume at the time of t. This is the 3-D quantized x1x2x3 space in Fig. 2. The well-known big bang theory assumes that this 3-D quantized space is flowing along the relative time axis of ct_l. In this case the time and space are warped together. There is nothing else except our universe of 3-D quantized space. The well-known big bang theory, special and general relativity theories and standard model are developed based on this single 3-D quantized space on the 4-D Minkowski space.

Therefore, I have thought of the origin of the big bang based on the 4-D Euclidean space rather than the 4-D Minkowski space. Let us think of the 3-D Euclidean quantized space which is overlapped on the 4-D Euclidean quantized space in Figs. 2 and 4. Then the 3-D quantized x1x2x3 space is moving along the 4th dimension axis in the 4-D Euclidean space. The 4th dimension axis is the absolute time axis of ct. This 3-D quantized space is assigned as the x1x2x3 space and the 4-D Euclidean space is assigned as the x0y0z0ct space and time. It is assumed that the 3-D quantized x1x2x3 space is moving along the x4 axis with the constant speed of c because the 3-D quantized x1x2x3 space is the photon space. Then, from $c = \Delta x4/\Delta t$, $x4 = ct$, the x4 axis is the absolute time axis of ct. Under this assumption, the mathematical 4-D Euclidean space is evolved to the physical 3-D quantized space based on the 4-D Euclidean space as shown in Fig. 2. From this physical concept, the 4-D momenta in Fig. 3 is defined for the x1x2x3 space. Then, Note that, in Fig. 3, the 4-D momenta can be expressed as $p_t = m_0 c\frac{x4}{ct}$, $p_x = m_0 c\frac{x}{ct}$ and $p_{tl} = m_0 c\frac{d}{ct}$. The 4th dimension axis is the absolute time axis of ct. Note that the negative time, negative mass (energy) and negative momenta are possible. $E_0 < 0$ and $m_0 < 0$ for $p_t < 0$ in Fig. 3. And the C, P and T symmetry are defined for our universe based on the 4-D Euclidean space. The T symmetry is defined from the absolute time axis of ct not from the relative time of ct_l. The negative time momentum means negative mass and negative energy. Our matter universe with the positive time momentum and anti-matter partner universe with the negative time momentum should be created following the CPT symmetry as shown in Figs. 2 and 3. Therefore, there is only the 4-D Euclidean space before the big bang and there is the 3-D quantized space based on the 4-D Euclidean space after the big bang. Because our universe has the positive time momentum and positive energy (mass) and the partner universe has the negative time momentum and negative energy (mass), these two universes are created from the nothing with the zero time momentum and zero energy

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(mass). Therefore, in Figs. 2 and 3, the CPT symmetry explains why our universe is the matter universe and where the energy of our universe comes from. Now we have the complete birth history of our universe in terms of the present 3-D quantized space model (TQSM).

Note that the time, t in the Lorentz transformations corresponds to t_l in the present work. In other words, the time axis of ct in the present work is the fourth dimension axis in the 4-dimensional Euclidean space in Figs. 2, 3, 4 and 5. And the time axis of ct in the Lorentz transformations of the Minkowski space is the distance axis of ct_l in the present 4-dimensional Euclidean space. Therefore, the direction of the ct_l time axis depends on the corresponding particle velocity of $v = x/t_l$. This means that the observed time of t_l depends on the corresponding velocity of v as shown in the Lorentz transformations of the special relativity. Therefore, the time of t_l is called as the relative time in the present work. The relation between the relative time of t_l and the observed space distance of x is described as a function of v in the Lorentz transformations of the special relativity. But the time axis direction of ct does not depend on the particle velocity. It is just the fourth dimension axis in the 4-dimensional Euclidean space. Therefore, the time of t is called as the absolute time in the present work. It is clear from the definition of the time momentum ($p_t = E_0/c = m_0c$). Then from $c = \Delta x4/\Delta t$, $x4 = ct$, the x4 axis is the absolute time axis of ct. The photon with the constant speed (c) is the flat space with the zero charge in Fig. 3. It is defined that our x1x2x3 universe is the photon space which moves along the positive x4 axis with the constant speed of c in the 4-dimensional Euclidean space of x0y0z0x4. In this definition, the x4 axis becomes the absolute time axis of ct in the 4-dimensional Euclidean space of x0y0z0x4 in Figs. 2, 3, 4 and 5. Then, this photon space has the microscopic time width of $c\Delta t_q$ which could be the Planck length size. But the time width of the local photon space can be changed depending on the photon energy. This photon space is called as the x1x2x3 space in Figs. 2, 3, 4 and 5.

The photon consists of many internal photons. These internal photons form the photon wave of the electromagnetic wave by the interferences. And the x4 axis is the ct time axis. The photon has the constant velocity of $c= \Delta x4/\Delta t$ along the x4 axis in Figs. 3 and 5. And the internal photons have the constant velocity of $c= \Delta x/\Delta t_l$ along the x axis on the x1x2x3 space in Fig. 5. So, the moving distance ($\Delta x = c\Delta t_l$) of the internal photon on the x1x2x3 space is the same to the moving distance ($\Delta x4 = c\Delta t$) of the whole photon space along the ct axis in Fig. 5. This means that the whole photon space can be treated as the rest particle with the velocity of c along the ct axis. This axis is the absolute time axis. Also, each internal photon can be considered as the moving particle on the x1x2x3 space with the velocity of c. This internal photon is the electromagnetic (2EM) wave which is closely related to the electric and magnetic fields. It is thought that each photon and electromagnetic wave can consist of several internal photons with the less energies. Therefore, the x1x2x3 space corresponding to each photon can be always defined based on the 4-dimensional Euclidean space as shown for our universe in Fig. 5. Here our universe is the x1x2x3 photon (flat) space. In Fig. 5, the rest particle, moving particle and photon are compared. The photon has the zero rest mass. Therefore, even though the internal photon is moving along the x space direction with the constant speed of c, the photon space is not changed. It is expressed as A=B in Fig. 5. In other words, the electromagnetic wave does not change the space. But the moving massive particle changes the space as shown in Fig. 5. It indicates that the electromagnetic wave is the space fluctuations which does not change the photon space itself. Also, the photon moves along the time axis of ct like the rest particle does. Therefore, ct is equal to ct_l for the photon in Fig. 5. This means that the photon has both properties of the particle (photon) and wave (electromagnetic wave). This explains the particle-wave duality of the photon.

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In Figs. 2, 3, 4 and 5, our universe is the x1x2x3 photon space positioned on the mother x0y0z0x4 space. This is the 3-dimensional quantized x1x2x3 space. Everything in the present work is based on the 4-dimension Euclidean space. Then, our universe is moving with the photon velocity of c

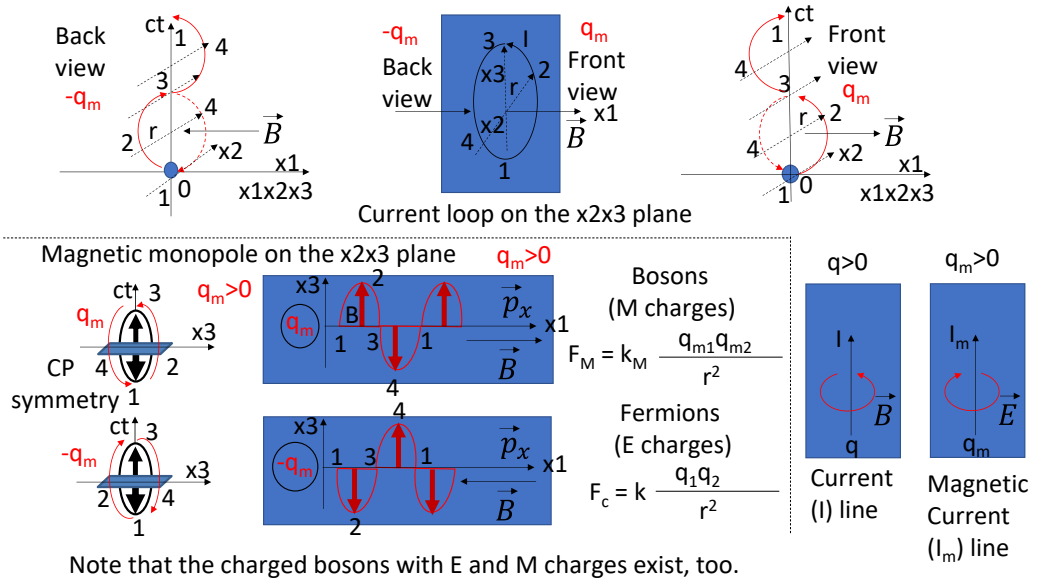

Fig. 12. Magnetic monopoles (q_m) are defined as the bosons. The magnetic dipole of the electric charge current loop with the counterclockwise direction is discussed in terms of the time (ct) and space (x1x2x3).

along the x4 axis which is called as the time axis of ct. Please note that the absolute time lapse is defined as the $\Delta t = \Delta x4/c$. This is the absolute time which is not dependent on the particle velocity. The relative time of t_l is dependent on the particle velocity as shown in the special relativity theory. The time of t in the special relativity theory corresponds to the relative time of t_l in the present work. The relative time of t_l is defined as $\Delta t_l = \Delta l/c = \Delta x/v$. Our universe can be locally warped to create the particles in Fig. 5. Then the rest mass energy and electric charge (EC) are introduced as the warped space of our x1x2x3 universe. The 3-dimensional quantized space is the photon space. The particle is the warped space with the velocity (v) slower than the photon velocity of c. Then the rest mass of the particle is defined as the 4-dimensional volume of the warped space. Under this definition of the particle mass, it is proposed that the electric charge is positive for the positively warped space and negative for the negatively warped space in the x1x2x3 space. Because other lepton charges (LC) and color charges (CC) exist in the real world, we need two more 3-D quantized spaces of the x4x5x6 and x7x8x9 spaces [11]. The x4x5x6 and x7x8x9 spaces are for the LC and CC charges, respectively. The present work including only the EC x1x2x3 space for simplicity can be easily extended to the researches including all three spaces of the x1x2x3 space, LC x4x5x6 space and CC x7x8x9 space. In the extended case, the charge (q) of (EC) can be, easily, replaced with the charge (q) of (EC, LC, CC). I will include three charges of EC, LC, and CC in the following paper.

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4. Bosons defined as the magnetic monopoles

The electric monopoles are discovered as the electric charges (EC). And the gravitational monopoles are observed as the masses. However, the magnetic monopoles have never been discovered. This is one of the unsolved physics questions. The magnetic dipole has been defined by the observation of the magnetic fields, which are originated from the electric charge current loop as shown in Fig. 12. In Fig. 12, the current loop produces the magnetic field (B). The direction of the magnetic B field is decided by the current (I) direction. In Fig. 12, the front view of the loop shows the emitted magnetic B field because the current has the counterclockwise direction. And the back view of the loop has the magnetic B field with the inward direction because the current has the clockwise direction. Therefore, the front view and back view of the loop act like the positive magnetic charge (q_m) and negative magnetic charge ($-q_m$), respectively. The electric charge current loop is drawn in the 3-D Euclidean space which is the flat photon space. This current

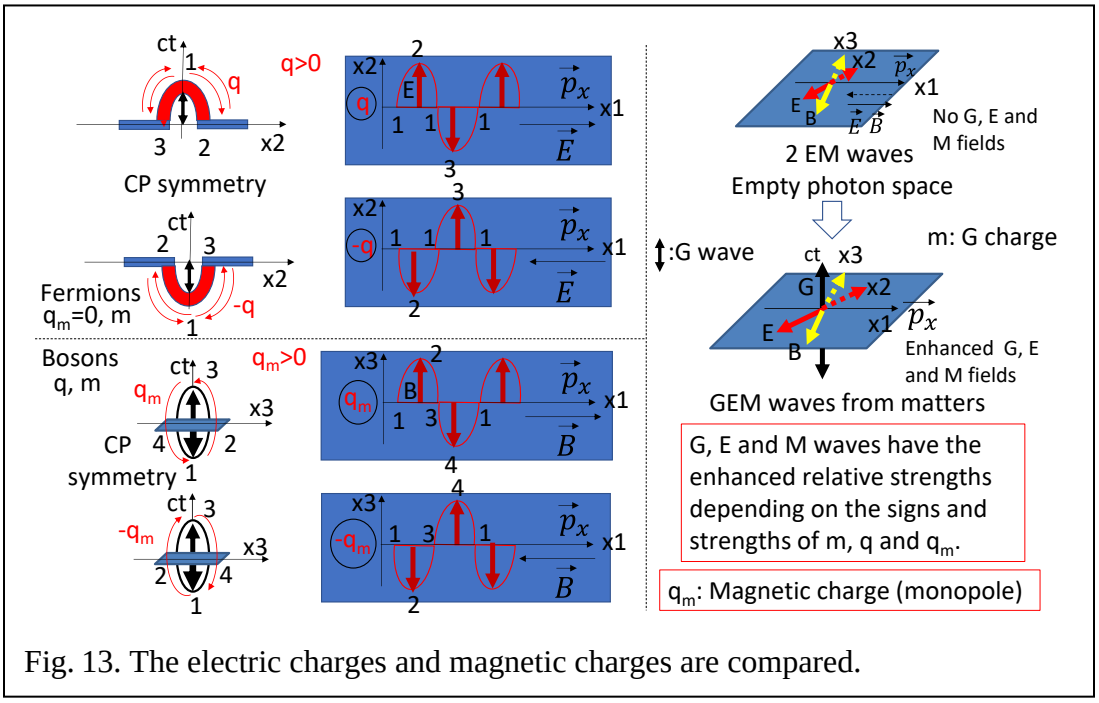

Fig. 13. The electric charges and magnetic charges are compared.

loop can be drawn in the time (ct) and space (x1x2x3), too. Then the front view and back view of the loop are drawn on the 4-D Euclidean space. The back view of the current loop has the clockwise current flow and the front view of the current loop has the counterclockwise current flow. This can be clearly seen in Fig. 12.

Th current loop is the closed loop in the x2x3 space plane. Also, the current loop is the closed loop in the ct (time)-x2x3 space. In Fig. 12, the closed loop in the time(ct)-x2x3 space is drawn on the ct-x2 coordinate system. This closed current loop corresponds to the closed wrapped space on the time(ct)-x1x2x3 space in Fig. 13. In Fig. 6, the elementary fermions have the open warped spaces and the elementary bosons including the graviton have the closed warped spaces. As well known, the fermions have the electric charges which are called as the electric monopole. The elementary bosons have the closed warped spaces in Fig. 6. This closed warped space of the boson corresponds

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to the closed current loop. Therefore, the closed warped space of the boson can make the space fluctuations along the x3 axis in Fig. 12. This space fluctuations are called as the magnetic (B) field. The magnetic dipole of the current loop shows that the sign of the probable magnetic charge depends on the direction of the current flow. In other words, the magnetic charge cannot be absolutely decided on the current loop because the current loop as the magnetic dipole cannot separate the clockwise view and counterclockwise view. This means that the current loop exists only as the magnetic dipole. However, the boson has the clockwise or counterclockwise direction on the time (ct) and x1x2x3 space. In Fig. 12, the boson fluctuations are drawn on the ct-x3 plane for the explanation. The bosons with the counterclockwise fluctuations are defined as the magnetic monopole with the positive magnetic charge (q_m). Then, the bosons with the clockwise fluctuations are the magnetic monopole with the negative magnetic charge ($-q_m$). These sign definitions of the magnetic charges are made because the relation between the magnetic charge sign and the magnetic field direction is the same to the relation between the electric charge sign and electric field direction. The charged bosons have both the electric charges and magnetic charges. See section 5 for more details about the magnetic monopoles.

5. Three monopoles of electric charge, magnetic charge, and gravitational mass

In Fig. 13, the electric charges and magnetic charges are compared. The electric waves have the close relations with the electric fields as shown in Figs. 4 and 11. The electric and magnetic waves are the x1x2x3 space fluctuations in Figs. 4 and 11. And the gravitational waves are the time fluctuations in Figs. 4 and 11. The electric field is originated from the space fluctuations due to

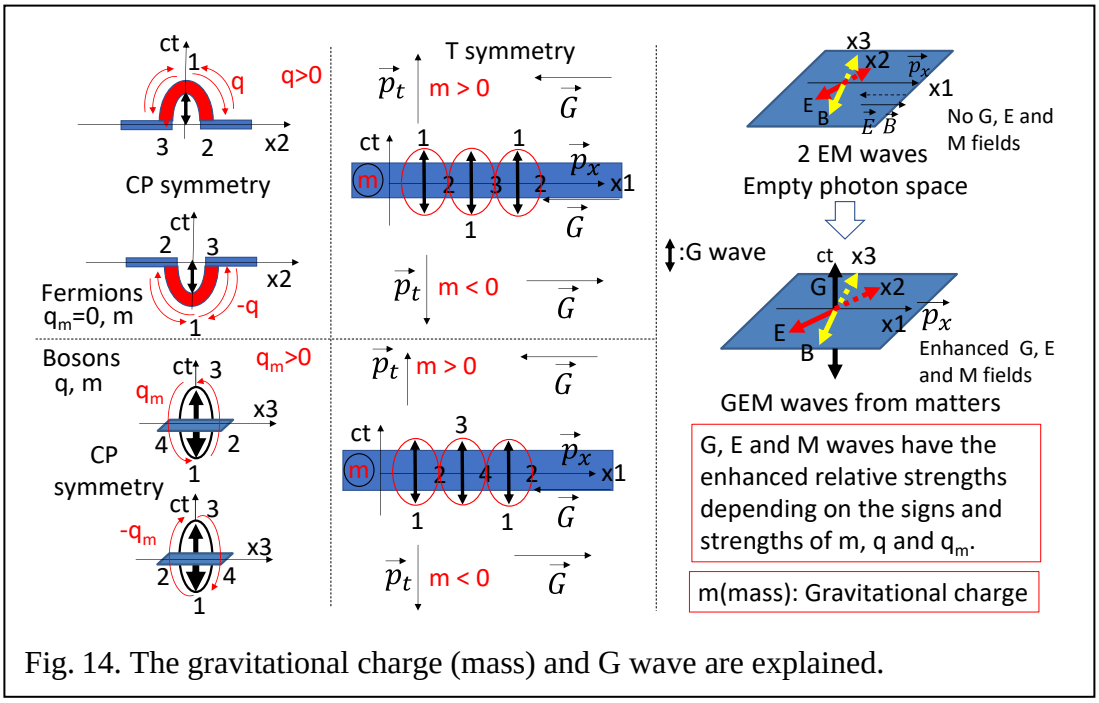

Fig. 14. The gravitational charge (mass) and G wave are explained.

the electric charge in Fig. 13. The magnetic field is originated from the space fluctuations. In Fig. 13, the flat space is the photon space with the 2EM waves. When the photons are moving on the flat space, the space fluctuations take place but the time fluctuations do not happen. But when the

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photons are moving on the warped space, both the space fluctuations and time fluctuations take place. These additional space and time fluctuations are added to the photons of the 2EM waves with the zero G, E and M fields. Then, the GEM wave is created with the enhanced E and G fields in Fig. 11. For example, the photons inside the warped space of the fermion are moving to create the E wave and E field in Fig. 13. This E wave are added to the preexisting 2EM wave to create the 2EM wave with the enhanced E wave in Figs. 11 and 13. Also, because the fermion has the mass, the G wave is produced and added to the 2EM wave to make the GEM wave. The positive charge is defined for the warped space toward the positive ct direction. The negative charge is defined for the warped space toward the negative ct direction. This is understood under the CP symmetry. Here, it is assumed that the E field from the electric charge is spread out through the 2EM wave (photons) of the photon space as shown in Fig. 11.

The same ideas are applied to the magnetic fields produced from the bosons in Fig. 13. The photons are moving around the closed warped space of the boson. When the photons are moving along the counterclockwise direction on the ct-x3 plot, it is called as the positive magnetic charge. This is understood under the CP symmetry. And when the photons are moving along the clockwise direction on the ct-x3 plot, it is defined as the negative magnetic charge. When the photons are moving on the warped space, both the space fluctuations and time fluctuations take place. These additional space and time fluctuations are added to the 2EM waves with the zero G, E and M fields.

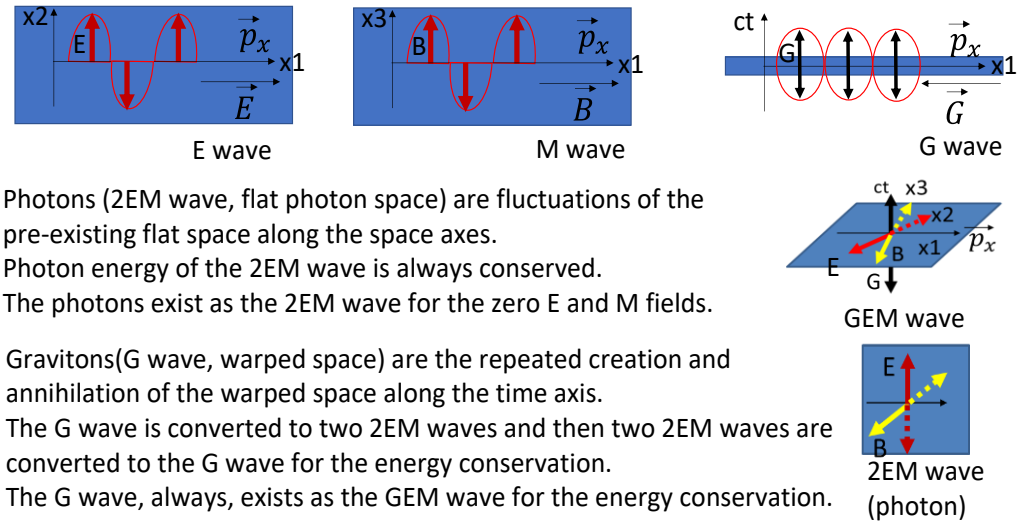

Fig. 15. The photons and gravitons are compared.

Then, the GEM wave is created with the enhanced M and G fields. For example, the photons inside the warped space of the boson are moving to create the M wave and M field in Fig. 13. This M wave are added to the preexisting 2EM wave to create the 2EM wave with the enhanced E wave. Also, because the boson has the mass, the G wave is produced and added to the 2EM wave to make the GEM wave. Here, it is assumed that the M field from the magnetic charge is spread out through the 2EM wave (photons) of the photon space as shown in Fig. 11.

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In Fig. 14, the gravitational wave is explained. The G wave is always connected with the massive particle. In other words, the massive particles create and emit the G waves. Because the new G waves are created, the 2EM waves are created in connection with the G wave to make the GEM wave for the energy conservation. This new GEM wave is added to the preexisting 2EM wave to make the GEM wave with the enhanced energy and strength. Here, it is assumed that the G field from the gravitational mass is spread out through the 2EM wave (photons) of the photon space. The mass has the positive sign when it has the positive time momentum as shown in Figs. 3 and 14. The negative mass has the negative time momentum. Our universe has the positive time momentum, and the particles within our universe have the positive masses. The positive mass has

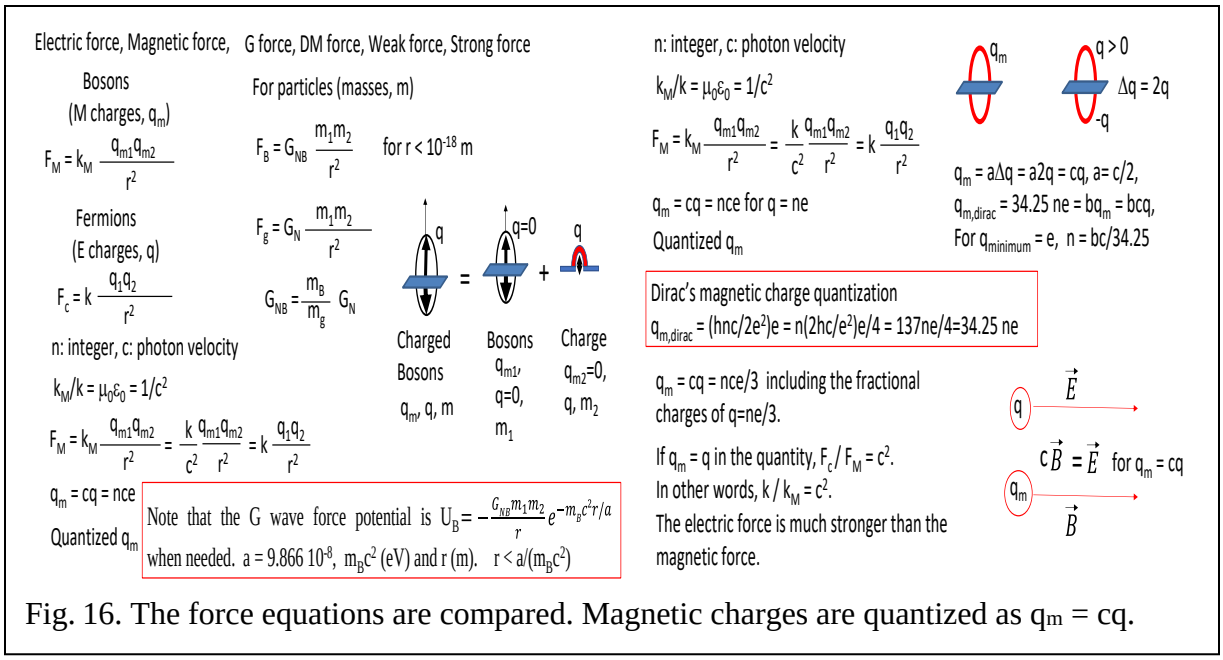

Fig. 16. The force equations are compared. Magnetic charges are quantized as $q_m = cq$.

the inward G field and the negative mass has the outward G field as shown in Fig. 14. The gravitational field is understood under the T symmetry.

In Fig. 15, the photons and gravitons are compared. The photons can exist as 2EM wave. But the gravitons can exist not as the G wave but as the GEM wave because of the energy conservation as shown in Figs. 9, 15 and 11. Now we have the GEM wave that can explain all forces. This means that all forces have the similar force equations. We know that the electric force equation (Coulomb force equation) and gravitational force equation have the similar force equation forms. The same principle can be applied to the rest of forces. The similar force equations with the different force strength coefficients and different force ranges can be used to all of forces as shown in Figs. 8 and 16. Therefore, all forces are separated into the G wave forces and the EM forces in terms of the present 3-D quantized space model as shown in Fig. 17. The mesons are the magnetic monopoles. Therefore, it will be interesting to search for the magnetic fields which can be observed from the experiments with the mesons. As shown in Figs. 8 and 16, if all forces have the similar force equations, the hadronization of the baryons and mesons should be explained. The question about the hadronization of the baryons and mesons will be discussed in the following paper. The

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magnetic charges are quantized as $q_m = cq$ in Fig. 16. The present magnetic charge quantization is compared with the Dirac's magnetic charge quantization [43].

6. Force fields of the photon space and the particles

Particles are made of the elementary fermions and our universe is based on the x1x2x3 photon space. The elementary bosons are responsible for the forces between the particles. The forces can

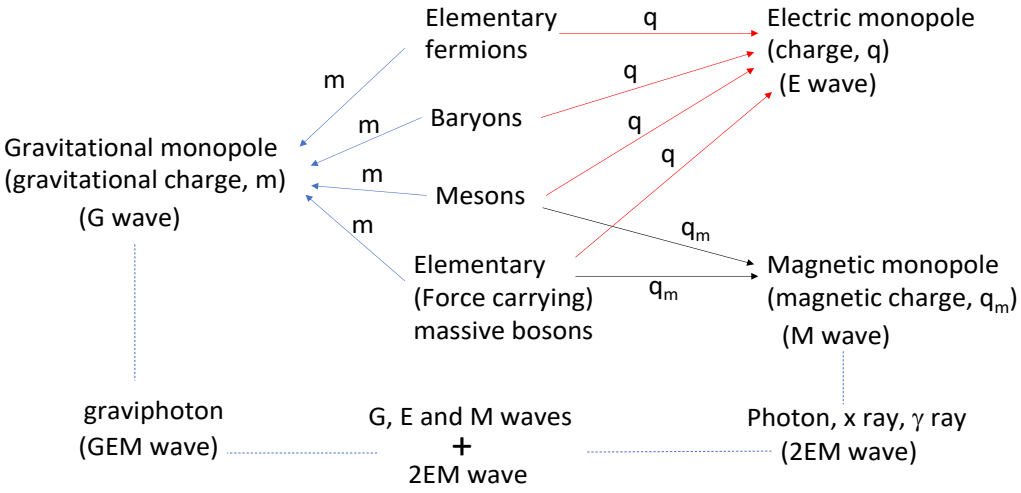

Fig. 17. Three kinds of the monopoles are compared.

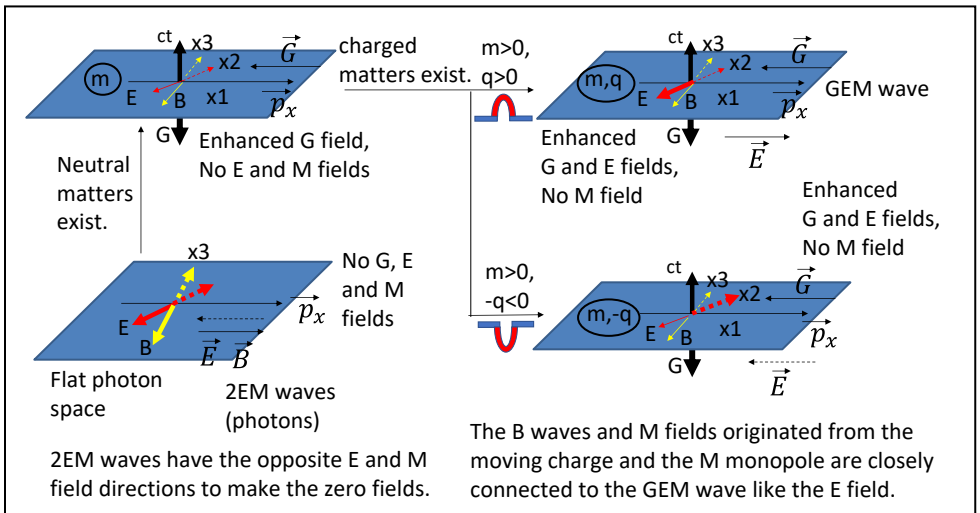

Fig. 18. 2EM waves are the general intermediators of all forces. See Fig. 11.

be represented by the force fields like the G field, E field and M field. The photon space is full of the photons (2EM waves). The photon space without the particles have no force fields. Because the photon space without the particles does not have the force fields, the 2EM wave (photons) needs to be formed. Therefore, the photon space is full of the 2EM waves which are called as the x rays and gamma rays.

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In the real universe, there are many particles which produce the G fields, E fields and M fields as shown in Fig. 17. In this case, these G, E and M fields are added to the basic 2EM waves to make the GEM waves. These G, E and M fields are moving with the 2EM waves with the speed of light. In other words, the G, E and M force fields get on the transportation ship of the 2EM waves to be spread out as shown in Fig. 11. This indicates that the photons (2EM waves) are the general intermediators carrying all fields between two particles. The 2EM waves are called as the photon, x ray or gamma ray. This is drawn again in Fig. 19. The fields of the force carrying bosons are

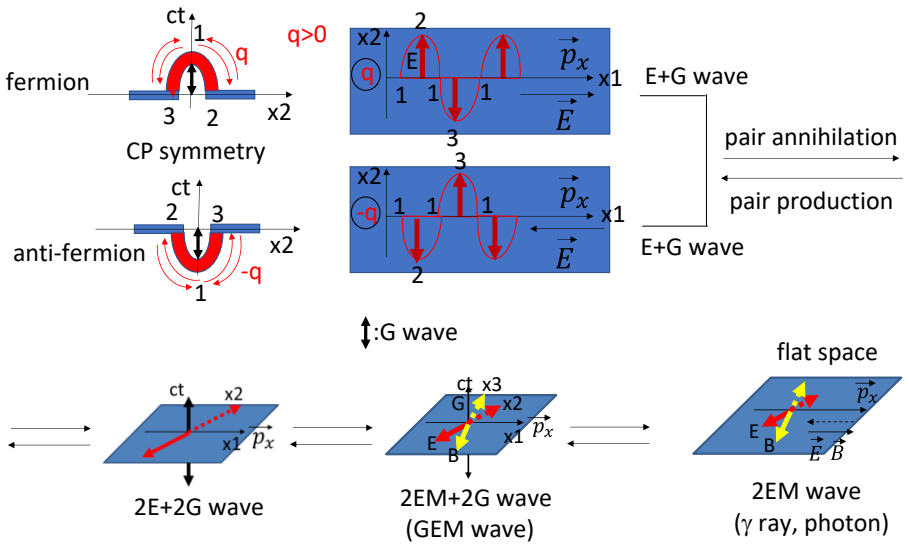

Fig. 19. The pair production and annihilation of the fermion and anti-fermion is explained by using the G, E and M waves. See Figs. 9 and 14.

combined into the GEM waves. Because all force fields are spread out along with the similar GEM waves, all force equations have the similar forms with the different strength coefficients in Figs. 8 and 16. This means that all forces are unified by the GEM waves.

As one example, the pair creation and annihilation of the fermion and anti-fermion to the photon (γ ray) is shown in terms of the G, E and M waves in Fig. 19. The pair creation of the particle and anti-particle is discussed in Fig. 9, too. The particle and anti-particle can be created from the gamma ray or photon. This is the process that the flat photon space without the G field is changed to the warped space with the G field. The reverse process is possible. In other words, the G wave can be converted to the 2EM wave and the 2EM wave can be converted to the G wave in terms of the wave mechanics. Each fermion corresponds to the EM wave. Because a fermion and an antifermion exist, two EM waves are needed to describe the fermion and anit-fermion in one wave. This means that the photon is the 2EM wave but not the EM wave. Also, because the photon does not have the E and M fields, the photon needs to be the 2EM wave which have both positive and negative components of the E and M fields.

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7. Summary and conclusions

In the present paper, three unsolved physical topics are discussed in terms of the 3-D quantized space model (TQSM) based on the 4-D Euclidean space. These three topics are what happened before and after the big bang moment, whether all forces can be unified in a consistent way and whether the magnetic monopoles can be discovered. The new ideas could be needed to solve these three unsolved physical questions. The electromagnetic (EM) wave has been successfully used to unify the magnetic force and electric force. This method is applied to unify other forces. First, the magnetic wave and electric wave are the space fluctuations. We do not observe the rest mass of the photon. We observe the massive particles with the 3-D volumes. The massive particles and massless photons occupy the 3-D volume on the 3-D Euclidean space. Then, what is the difference between the massless photon and massive particles? The answer to this question comes from the 4-D Euclidean space. It is thought that the massive particles take the 4-D warped space that is the warped version of the flat photon space along the time axis of ct. The 4-D volume of the 4-D warped space is the rest mass energy of the massive particle. Under this new idea, the flat photon space has the zero-rest mass energy. The flat photon space has the 3-D quantized space of the physical x1x2x3 space with the very small-time width of $c\Delta t_q$. Therefore, the mathematical 4-D Euclidean space is the unquantized space without the photons (space fluctuations). And the physical 4-D Euclidean space is the 4-D quantized space-time that is the 3-D quantized space of the physical x1x2x3 space with the very small-time width of $c\Delta t_q$. This physical space has the photons (space fluctuations) with the 2EM wave. This is called as the space-time evolution in the present work from the mathematical space to the physical space. Please see Figs. 2 and 4 for the graphical descriptions.

In the 4-D Euclidean space, all axes have the positive and negative directions. However, the space momenta along the 3-D space axes have been studied in the physical world. For the time axis, only the positive axis in the 4-D Minkowski space has been taken into consideration because we observe only the positive time direction. From the viewpoint of the 4-D momenta on the 4-D Euclidean space in Figs. 2, 3 and 4, the time axis should have the positive and negative time directions. If the negative time direction is allowed in the physical point of view, the well-known big bang theory should be changed to include the negative time direction in Figs. 2, 3 and 4. In this case, the partner universe with the negative time momentum is allowed. Note that the negative energy and negative mass are allowed from the negative time momentum. This means that the big bang is the pair creation of our matter universe and partner anti-matter universe which are the 3-D quantized spaces. This new interpretation completes the big bang theory in terms of the conserved CPT symmetry. It explains why our universe is the matter universe.

If we agree that the gravitational (G) wave is the fluctuation along the time axis on the 4-D Euclidean space, it is easy to unify the EM wave and G wave. The unified form of the EM and G waves is called as the gravityelectromagnetic (GEM) wave. The boson corresponding to GEM wave is called as the graviphoton with the intrinsic spin of 2. Also, it is assumed that the dark

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matter force bosons, weak force bosons and strong force bosons are originated from the gravitons. These four forces are called as the G wave forces. These G wave forces have the similar force equations between two massive particles (matters) in Fig. 8. These G wave forces are the attractive forces between the positive masses within our universe. If these assumptions are right, the GEM wave will unify all five forces. This is called as the super unification theory (SUT). I agree that much further and deeper researches are still needed on the new unification theory.

In the present work, the photons and gravitons are compared. The photons can exist as 2EM wave. But the gravitons can exist not as the G wave but as the GEM wave because of the energy conservation. Now we have the GEM wave that can explain all forces. This means that all forces have the similar force equations. We know that the electric force equation (Coulomb force equation) and gravitational force equation have the similar force equation forms. The same principle can be applied to the rest of forces. The similar force equations with the different force strength coefficients and different force ranges can be used to all of forces as shown in Figs. 8 and 16. Therefore, all forces are separated into the G wave forces and the EM forces in terms of the present 3-D quantized space model as shown in Fig. 17.

And the force carrying bosons and mesons are, for the first time, proposed as the possible candidates of the magnetic monopoles like the fermions and baryons are the electric monopoles in Fig. 17. Therefore, it will be interesting to search for the magnetic fields which can be observed from the experiments with the mesons. As shown in Figs. 8 and 16, if all forces have the similar force equations, the hadronization of the baryons and mesons should be explained. The question about the hadronization of the baryons and mesons will be discussed in the following paper. The magnetic charges are quantized as $q_m = cq$ in Fig. 16. The present magnetic charge quantization is compared with the Dirac's magnetic charge quantization [43].

In the real universe, there are many particles which produce the G fields, E fields and M fields as shown in Fig. 17. In this case, these G, E and M fields are added to the basic 2EM waves to make the GEM waves. These G, E and M fields are moving with the 2EM waves. In other words, the G, E and M force fields get on the transportation ship of the 2EM waves to be spread out as shown in Fig. 11. This indicates that the photons (2EM waves) are the general intermediators carrying all fields between two particles. The 2EM waves are called as the photon, x ray or gamma ray. This is drawn again in Fig. 18. The fields of the force carrying bosons are combined into the GEM waves. Because all force fields are spread out along with the similar GEM waves, all force equations have the similar forms with the different strength coefficients in Figs. 8 and 16. This means that all forces are unified by the GEM waves.

It is concluded that three unsolved questions of what happened before and after the big bang moment, whether all forces can be unified in a consistent way, and whether the magnetic monopoles can be discovered are solved in terms of the 3-D quantized space model. I wish the present results can inspire people to study on the present topics with their own new ideas.

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