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Article

The Green's type matrices for consumption reduction in a heterogeneous population model of TCLs with diffusion

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Abstract: We consider a control problem for a diffusive PDE model of heterogeneous population of thermostatically controlled loads (TCLs) aiming to balance the aggregate power consumption within a given amount of time. Using the Green's function approach, the problem is formulated as an approximate controllability problem for a residue depending on control parameters nonlinearly. A sufficient condition for approximate controllability is derived in terms of initial temperature distribution, operation time of TCLs and threshold value of the aggregate power consumption. Case studies allow to reveal the advantages of the proposed solution from numerical calculations point of view.

Keywords: PDE; Power Consumption; TCLs; Control; Minimization

1. Introduction

The usage of renewable energy sources is becoming more efficient as an alternative to power sources. Such advancement is exciting but performs significant challenges to the power industry. One of the main challenges is inherent variability in achieving significant penetration of renewable energy in modern power supply system [1]. Hence, a necessity of efficient power supply strategies, as well as balancing schemes in presence of intermittent power source occurs. One of best solutions is the elaboration of programmable calculating devices and power control means. Usually, thermostatically controlled loads (TCLs) are the typical control means in such systems and it may be a better option to provide necessary generation-balancing services. Because it is feasible for loads to compensate for power imbalances more rapidly than thermal generators [2]. By controlling aggregated TCLs, a promising opportunity can be obtained to mitigate the mismatch between power generation and demand, thus enabling renewable energy penetration and improving grid reliability [3].

TCLs account for more than one-third of electricity consumption in the United States [4]. Nowadays, the model of TCLs has gained extensive attention. Several methods have been used to model TCL populations. In [5], multi-agent reinforcement learning was developed for modeling of TCLs. Model accuracy scales linearly with the number of agents and gives evidence for the increased agency to further sensing, domain knowledge or data gathering time. In reference [6], multi-state operating reserve model of aggregate TCLs has been presented for power system short-term reliability evaluation. This analytical model for characterizing dynamics of the operating reserve is firstly developed based on the migration of TCLs' room temperature inside temperature hysteresis band. A frequency control technique is used in the literature [7] based on decision tree concept by conducting TCLs at smart grids. A new controlling action is introduced here to limit the probabilities of separating loaded feeders from smart grids due to maloperation of under-frequency load shedding relays. In [8], authors propose a two-dimensional state-queuing modeling approach for the TCLs that increases the state vector to a two-dimensional state matrix. This method improves the accuracy of the state-queuing model by adding temperature-varying-rate information.

Loads are usually regulated by switches between on and off regimes, as it is the case for example air conditioners. Switches are used when the thermostat temperature approaches the minimal or maximal admissible values. Thus, large populations of TCLs can be manipulated by small deviations of set-point temperature without causing any considerable inconvenience to consumers. In particular, a common temperature setpoint offset needs to be manipulated for controlling the aggregate power requirement of a population of TCLs [9]. This opportunity allows to develop efficient elaboration schemes aiming to reduce power consumption by a proper choice of the set-point temperature and the minimal and maximal admissible values of the temperature.

Currently, there exist multiple mathematical models allowing to describe the basic principle of TCL units. The simplest models are the hybrid ODE models that have been investigated in many previous works including [9–11]. In general statement, the hybrid ODE model provides a relationship for the conditioned mass temperature, Θ_i as follows [11]:

$$\dot{\Theta}_i = \frac{1}{R_i C_i}\left[\Theta_{\infty,i}(t) - \Theta_i(t) - S_i(t) R_i P_i + w(t)\right], \quad i = 1, 2, \ldots, N, \quad t > 0, \tag{1}$$

where $\Theta_{\infty,i}(t)$ is the external temperature, $C_i \in \mathbb{R}^+$ is the thermal capacitance (kWh/°C), $R_i \in \mathbb{R}^+$ is the thermal resistance (°C/kW), and $P_i \in \mathbb{R}$ (kW) is the cooling power supplied by the TCLs when switched on, the discrete function $S_i \in \{0, 1\}$ represents the hardware *on/off* state, and w represents unpredictable heat gains or losses due to occupancy: all the above quantities with a subscript i are for the i^{th} load.

Except differential equation (1), Θ_i is subject to a natural constraint

$$\Theta_{min,i} \le \Theta_i(t) \le \Theta_{max,i}, \quad i = 1, 2, \ldots, N, \quad t > 0. \tag{2}$$

Here, $\Theta_{min,i}$ and $\Theta_{max,i}$ are the minimal and maximal admissible temperatures. Then, the set-point temperature of the i^{th} load, $\Theta_{sp,i}$, is determined in terms of $\Theta_{min,i}$ and $\Theta_{max,i}$ as follows:

$$\Theta_{sp,i} = \frac{\Theta_{max,i} + \Theta_{min,i}}{2}.$$

On the other hand, introducing the width of the temperature band of the i^{th} load, σ_i, we have

$$\Theta_{min,i} = \Theta_{sp,i} - \frac{\sigma_i}{2}, \quad \Theta_{max,i} = \Theta_{sp,i} + \frac{\sigma_i}{2}. \tag{3}$$

More sophisticated, but more realistic models are described by PDEs. Let the states u and v denote the density of TCL units per temperature Θ and time t in the *on* and *off* states, respectively. Assume a homogeneous population of TCLs, i.e., assume that the parameters R_i, C_i, P_i, $\Theta_{min,i}$, and $\Theta_{max,i}$ are equal for all TCLs omitting the subscript i from the corresponding symbols. Then, the couple (u, v) satisfies the following system of first order PDEs [3,12]:

$$[\partial_t - \alpha\lambda(\Theta,t)\,\partial_\Theta - \alpha]\, u(\Theta,t) = 0, \quad [\partial_t + \alpha\mu(\Theta,t)\,\partial_\Theta - \alpha]\, v(\Theta,t) = 0 \tag{4}$$

$$\lambda(\Theta_{max},t)\, u(\Theta_{max},t) = \mu(\Theta_{max},t)\, v(\Theta_{max},t) \tag{5}$$

$$\mu(\Theta_{min},t)\, v(\Theta_{min},t) = \lambda(\Theta_{min},t)\, u(\Theta_{min},t) \tag{6}$$

where

$$\alpha = \frac{1}{RC} > 0, \quad \lambda(\Theta,t) = -\left(\mu(\Theta,t) - RP\right) > 0, \quad \mu(\Theta,t) = \Theta_\infty(t) - \Theta > 0.$$

Remark 1. *Despite the fact that system (4) is coupled via boundary conditions (5), (6), there exist various methods for exact and numerical determination of (u, v).*

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At this, some important characteristics as numbers of loads in the *on* and *off* states and aggregate power consumption are evaluated in terms of (u, v) as follows:

$$n_{on}\left(t\right)=\int_{\Theta_{min}}^{\Theta_{max}}u\left(\Theta,t\right)\,\mathrm{d}\Theta,\;\;n_{off}\left(t\right)=\int_{\Theta_{min}}^{\Theta_{max}}v\left(\Theta,t\right)\,\mathrm{d}\Theta,$$

$$\kappa\left(t\right)=\frac{P}{k}\int_{\Theta_{min}}^{\Theta_{max}}u\left(\Theta,t\right)\,\mathrm{d}\Theta, \tag{7}$$

where k is the performance coefficient.

The quantities $\Theta_{sp,i}$ and σ_i are usually considered as control parameters in order to control the aggregate power consumption (7). For a relevant study, see, e.g., [4].

2. A PDE model considering diffusion phenomenon

Numerical simulation of various TCL populations reveals a diffusive phenomenon. Therefore, in [3] a diffusion equation has been proposed for a proper analysis of heterogeneous population models. The model equations read as

$$\mathcal{D}_{\lambda}\left[u\right]=0,\;\;\mathcal{D}_{\mu}\left[v\right]=0,\;\;t>0,\;\Theta\in\left(\Theta_{min},\Theta_{max}\right), \tag{8}$$

where

$$\mathcal{D}_{\lambda}\equiv\partial_t-\beta\partial_{\Theta\Theta}-\alpha\lambda\left(\Theta,t\right)\partial_{\Theta}-\alpha,\;\;\mathcal{D}_{\mu}\equiv\partial_t-\beta\partial_{\Theta\Theta}+\alpha\mu\left(\Theta,t\right)\partial_{\Theta}-\alpha,$$

and β is a diffusivity coefficient with unit $(°\mathrm{C})^3/\mathrm{s}$.

Homogeneous system (8) is equipped with the following boundary conditions:

$$\lambda u-\mu v=0,\;\;\Theta=\Theta_{min},\Theta_{max},\;\;t>0, \tag{9}$$

$$\partial_{\Theta}u+\partial_{\Theta}v=0,\;\;\Theta=\Theta_{min},\Theta_{max},\;\;t>0. \tag{10}$$

The following initial conditions are also given:

$$u\left(\Theta,0\right)=u_0\left(\Theta\right),\;\;v\left(\Theta,0\right)=v_0\left(\Theta\right). \tag{11}$$

Remark 2. *Note that system (8) is coupled via boundary conditions (9)–(10), ensuring that the total number of TCLs is conserved. Unlike the model without a diffusive term (see Remark 1), analysis of (8)–(11) is sophisticated.*

2.1. Solution via matrices of Green's type

Let us derive the closed form solution of (8)–(11) by means of the concept of Green's function. To this end, we introduce new functions

$$\overline{u}\left(t\right)=\theta\left(t-0\right)u\left(t\right),\;\;\overline{v}\left(t\right)=\theta\left(t-0\right)v\left(t\right),$$

where θ is the Heaviside function, and $t-0$ in its argument indicates that the point $t=0$ is approached from the right. Then, (8) is reduced to

$$\mathcal{D}_{\lambda}\left[\overline{u}\right]=u_0\left(\Theta\right)\delta\left(t\right),\;\;\mathcal{D}_{\mu}\left[\overline{v}\right]=v_0\left(\Theta\right)\delta\left(t\right),\;\;t>0,\;\Theta\in\left(\Theta_{min},\Theta_{max}\right). \tag{12}$$

Boundary conditions (9), (10) refrain their forms.

Following to [14], the general solution of (12)–(11) can be represented as follows:

$$\begin{aligned} \begin{pmatrix} \overline{u} \\ \overline{v} \end{pmatrix} &= \int_0^t \int_{\Theta_{min}}^{\Theta_{max}} \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} u_0(\vartheta)\,\delta(\tau) \\ v_0(\vartheta)\,\delta(\tau) \end{pmatrix} \mathrm{d}\vartheta\,\mathrm{d}\tau = \\ &= \int_{\Theta_{min}}^{\Theta_{max}} \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \Bigg|_{\tau=0} \begin{pmatrix} u_0(\vartheta) \\ v_0(\vartheta) \end{pmatrix} \mathrm{d}\vartheta, \end{aligned} \tag{13}$$

where $G_{ij} = G_{ij}(\Theta, \vartheta, t, \tau)$, $i, j = 1, 2$, are the elements of the matrix of Green's type. An efficient numerical scheme for computing G_{ij} has been developed by Melnikov in [14].

Remark 3. *Note that solution (13) is linear with respect to the initial state* (u_0, v_0).

2.2. *Aggregate power consumption*

The closed-form solution (13) allows to express the aggregated power consumption in terms of system parameters. Substituting the firs raw of (13) into (7), we will obtain

$$\kappa(t) = \frac{P}{k} \int_{\Theta_{min}}^{\Theta_{max}} \int_{\Theta_{min}}^{\Theta_{max}} \left[u_0(\vartheta)\, G_{11}(\Theta, \vartheta, t, 0) + v_0(\vartheta)\, G_{12}(\Theta, \vartheta, t, 0)\right] \mathrm{d}\vartheta\,\mathrm{d}\Theta. \tag{14}$$

3. Control of the diffusive PDE model

In this section, we consider the problem of controlling the amount of the aggregate power consumption (7) by choosing the quantities Θ_{sp} and σ accordingly. Such a problem for the simplest model described by (4) has been considered, e.g., in [?]. The problem is to choose appropriate values for Θ_{sp} and σ in order to ensure

$$\mathcal{R}(\Theta_{sp}, \sigma) = \max_{0 \le t \le T} \kappa(t) - \kappa_0 \le \varepsilon, \tag{15}$$

with a desired accuracy $\varepsilon > 0$. Here, $\kappa_0 \ge 0$ is a given threshold value of admissible consumption, T is the operation time of the TCLs. In other words, we choose Θ_{sp} and σ in such a way that the total power consumption during the operation interval $[0, T]$ does not exceed a desired amount.

Remark 4. *Even though we call* ε *an accuracy, it does not imply that it should be a small quantity. Indeed, as soon as* $\kappa_0 = 0$, ε *can not be smaller than the minimal possible value of* κ *in* $[0, T]$. *On the other hand, when* $\kappa_0 > 0$, *meaningful values of* $\varepsilon \ll 1$.

Remark 5. *Note that in the terminology of [13], this is an approximate controllability problem.*

Since the mathematical model is linear in (u, v), for our purpose, we involve the Green's function approach developed in [13]. Substituting (14) into (15), we make the dependence $\mathcal{R} = \mathcal{R}(\Theta_{sp}, \sigma)$ explicit.

Remark 6. *It should be noted that* $\mathcal{R}$ *depends on* Θ_{sp}, σ *through the limits of integration, as well as through* G_{11} *and* G_{12}. *Thus, the minimization of* $\mathcal{R}$ *is not that straightforward.*

The main result of the paper can be stated in the following theorem.

Theorem 1. *If for given* u_0, v_0, T, κ_0 *and* ε, *there exist a bounded constant* $C(\Theta_{sp}, \sigma) > 0$ *such that*

$$|u_0(\vartheta)\, G_{11}(\Theta, \vartheta, t, 0) + v_0(\vartheta)\, G_{12}(\Theta, \vartheta, t, 0)| \le C(\Theta_{sp}, \sigma), \tag{16}$$

uniformly for $0 \leq t \leq T$, $\Theta_{min} \leq \Theta, \vartheta \leq \Theta_{max}$, *then*

$$\mathcal{R}\left(\Theta_{sp}, \sigma\right) \leq \frac{P}{k}\sigma^2 C\left(\Theta_{sp}, \sigma\right) - \kappa_0. \tag{17}$$

From Theorem 1 it directly follows

Corollary 1. *For approximate controllability of (8)–(11) it is sufficient that for given* u_0, v_0, T, κ_0 *and* ε,

$$\frac{P}{k}\sigma^2 C\left(\Theta_{sp}, \sigma\right) - \kappa_0 \leq \varepsilon. \tag{18}$$

Remark 7. *Depending on values of* κ_0, *(18) may not be satisfied by any values of* Θ_{sp}, σ *with a given* ε. *See also Remark 4. In such cases, inequality (15) must be evaluated.*

Proof. When (16) holds, then the estimate

$$\begin{aligned}
&\frac{P}{k}\int_{\Theta_{min}}^{\Theta_{max}}\int_{\Theta_{min}}^{\Theta_{max}}\left[u_0\left(\vartheta\right)G_{11}\left(\Theta,\vartheta,t,0\right)+v_0\left(\vartheta\right)G_{12}\left(\Theta,\vartheta,t,0\right)\right]\,\mathrm{d}\Theta\,\mathrm{d}\vartheta \leq \\
&\quad\leq \frac{P}{k}\int_{\Theta_{min}}^{\Theta_{max}}\int_{\Theta_{min}}^{\Theta_{max}}\left|u_0\left(\vartheta\right)G_{11}\left(\Theta,\vartheta,t,0\right)+v_0\left(\vartheta\right)G_{12}\left(\Theta,\vartheta,t,0\right)\right|\,\mathrm{d}\Theta\,\mathrm{d}\vartheta \leq \\
&\quad\leq \frac{P}{k}C\left(\Theta_{sp},\sigma\right)\int_{\Theta_{min}}^{\Theta_{max}}\int_{\Theta_{min}}^{\Theta_{max}}\mathrm{d}\Theta\,\mathrm{d}\vartheta \leq \frac{P}{k}C\left(\Theta_{sp},\sigma\right)\left(\Theta_{max}-\Theta_{min}\right)^2
\end{aligned}$$

is obtained immediately. Taking into account (3) in (15), we arrive at (17). □

4. Simulations

In this section, we provide results of numerical simulation of the diffusive system above. Following to [?], we choose the system parameters according to Table 1. Involving the numerical scheme for computing the elements of the matrix of Green's type G_{ij}, $i, j = 1, 2$, developed in [14], we compute (u, v) according to (13). Then, κ is computed using (14).

Recalling Remark 3, note that in this case the solution of the diffusion model (u, v) is linear in N

Quantity	Value	Dimension
α	0.05	1/h
P	14	kW
R	2	°C/kW
k	2.5	
Θ_∞	32	°C
β	0.01	$(°C)^3/s$
N	{800, 1000, 1500}	TCLs
$u_0(\Theta)$	0	TCLs/°C
$v_0(\Theta)$	$\frac{30N}{16\sigma}\left[1-\frac{4}{\sigma^2}\left(\Theta-\Theta_{sp}\right)^2\right]^2$	TCLs/°C
σ	$(0.5, 1.5)$	°C
Θ_{sp}	$(17, 21)$	°C

Table 1. Values of inputs used during simulation

Numerical evaluation of G_{ij}, $i, j = 1, 2$ shows that for the values of parameters represented in Table 1, there always exists a constant C such that (16) holds. Therefore, Theorem 1 is valid and (17) holds. Moreover, it has been observed that for these values, sufficient condition (18) holds implying that the diffusive model above is approximate controllable.

Numerical analysis shows that for any fixed t, κ is a convex function of $\left(\Theta_{sp}, \sigma\right)$ with a unique global minimum. Moreover, for fixed $\Theta_{sp} \in [17, 21]$, κ increases when σ increases in $[0.5, 1.5]$, and

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that for fixed $\sigma \in [0.5, 1.5]$, κ decreases when Θ_{sp} increases in $[17, 21]$. Therefore, the global minimum of κ is achieved at $(\Theta_{sp}, \sigma) = (21, 0.5)$. Figure 1 expresses the behavior of the minimal aggregate consumption against the number of TCLs when $\kappa_0 = \frac{1}{2} \max_{0 \le t \le T} \kappa(t)$. As it is expected, κ increases with increase of N. It is also seen from Figure 1 that as time increases, the minimal aggregate consumption stabilizes around corresponding κ_0.

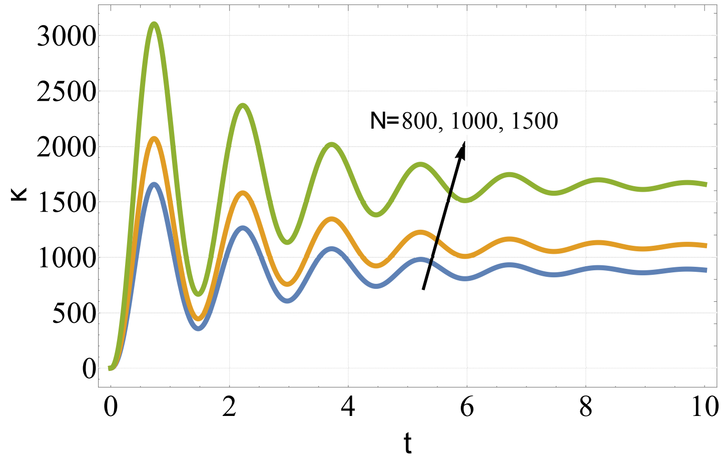

Figure 1. The minimal aggregate power consumption against time for $N = 800, 1000$ and 1500 TCLs

Now let us examine the corresponding solution (u, v). In the contour plots below (Figures 2-4), dark blue regions correspond to near-zero values and the lighter the color of the domain, the higher the corresponding value. For $N = 800$, as Figure 2 shows, for $t < 1$, in the whole range of $\Theta \in [\Theta_{min}, \Theta_{max}]$, u is in the dark blue region meaning that the density of TCLs in the on state is low. A similar region occurs near $t = 2$. In between these regions, u approaches its maximal value in the range of $\Theta \in [20.2, \Theta_{max}]$. Then, the values of u oscillate between lower and higher values until it is stabilized near $\tilde{u} = 400$ for $t \ge 9$. A similar behavior is observed also for v with the difference that v is stabilized near $\tilde{v} = 500$ TCLs/°C much earlier, for $t \ge 5$.

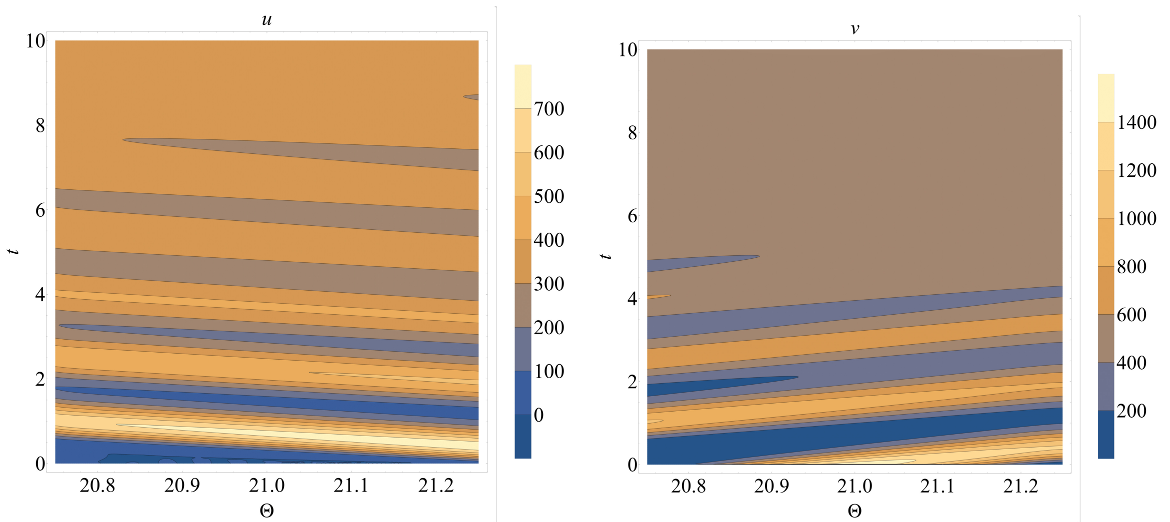

Figure 2. Contour plots of the distributions u and v for $N = 800$

A similar behavior is observed also in the case of $N = 1000$, but unlike the previous case, in this case, the stabilization of both u and v occurs much later. However, when $N = 1500$, v is stabilized surprisingly fast, for $t \ge 6.5$.

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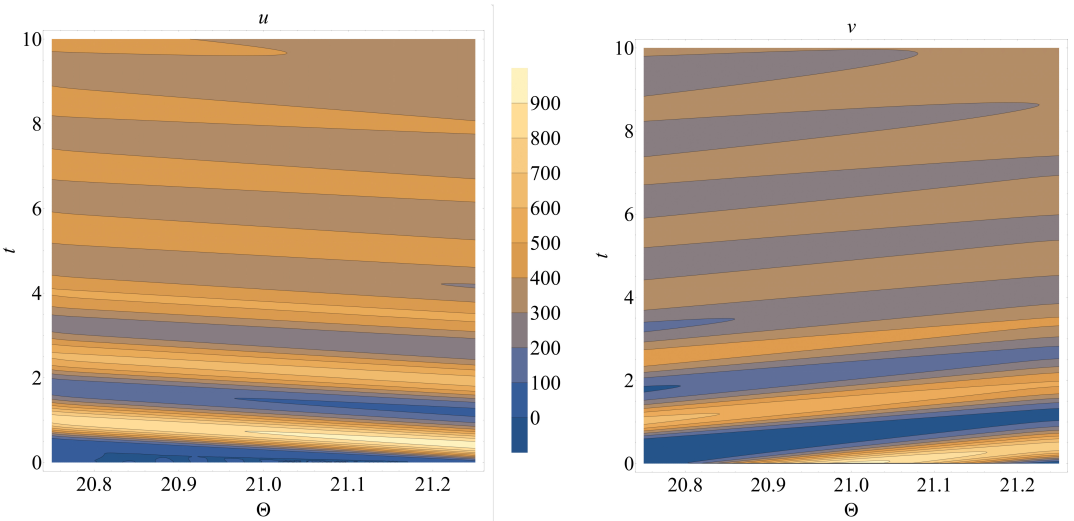

Figure 3. Contour plots of the distributions u and v for $N = 1000$

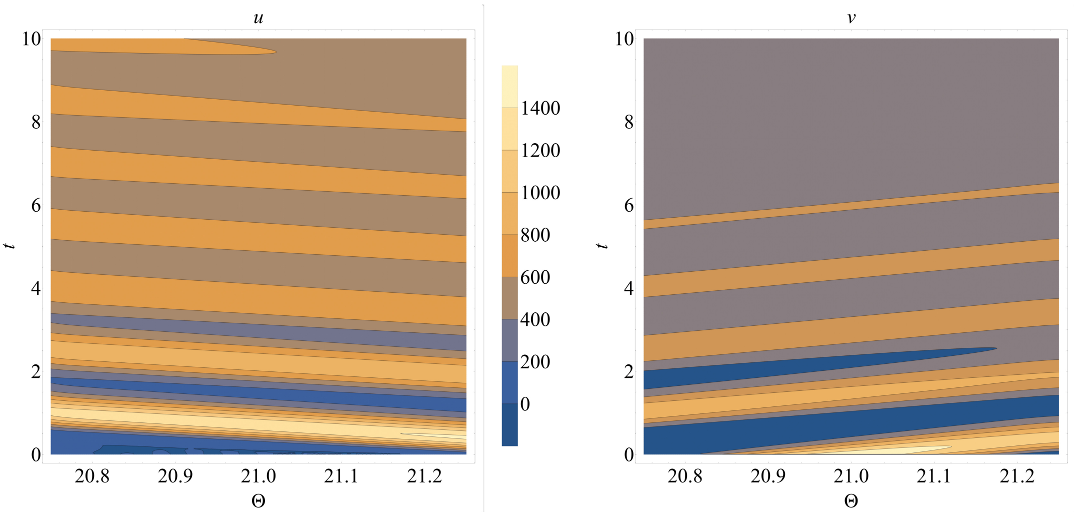

Figure 4. Contour plots of the distributions u and v for $N = 1500$

5. Conclusions

A diffusive behavior in modelling of smart grids of TCLs has been observed recently. The diffusion naturally affects the aggregate power consumption which is a very important criterion in estimating the efficiency of any smart grid of TCLs. In this paper, we consider the controllability property of the diffusive model of TCLs described by a one-dimensional diffusion equation with variable coefficients. The analysis of the governing system is sophisticated by the fact that the system is coupled through the boundary conditions. Involving the Melnikov method for numerical computation of the corresponding Green's matrix, we derive a sufficient condition for approximate controllability of the grid providing minimal aggregate power consumption. The set-point value and the bandwidth of the temperature act as control parameters.

Numerical analysis shows that in the considered range of the parameter values, the population model of TCLs with diffusion is approximately controllable. An extensive numerical analysis reveals that the power consumption is a convex function of control parameters having a unique global minimum. The global minimum is found at the point with maximal set-point and minimal bandwidth values. Corresponding densities of TCLs in on and off states are examined numerically.

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