Brief Report

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Empirical Mass Formula for Charged Leptons

Eckehard W. Mielke *

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Empirical Mass Formula for Charged Leptons

Eckehard W. Mielke

Departamento de Física, Universidad Autónoma Metropolitana Iztapalapa, Apartado Postal 55-534, C.P. 09340, México-City, México; ekke@xanum.uam.mx

Abstract: An explicit mass formula for charged leptons is proposed which employes the $(2\ell+1)$ degeneracy of the $SU(2)$ Laplacian of quantum mechanics. Possible cosmological implications are briefly discussed.

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1. Introduction

The immense difference between the individual masses of charged leptons and the appearance of generations in particle physics still appears to be a mystery. This short paper intents to shed some light on the appearance of generations in particle physics. A mass formula for charged leptons is proposed which interpolates between De Broglie–Vigier ideas on "hidden" internal degrees of freedom and (gauge) group concepts.

2. Product Formula

Here we propose the *product* formula

$$m_N = \Pi_{\ell=0}^{N-1} 2^{(2\ell-1)}(2\ell+1)^2 \,\text{MeV} \tag{1}$$

for the charged leptons, where ℓ is an angular momentum eigenvalue and $N \geq 3$ the number of generations. This is only aproximate and in the need of "radiative corrections".

On the other hand, hypothetical *topological excitations* of the electron would also lead to a product formula

$$\tilde{m}_N = 4\pi(16\pi/3)^{N-1}\, m_e \tag{2}$$

for $N \geq 2$ in terms of the volumes of apropiate fibre bundles [4].

3. Next Generation Lepton Masses?

In view of (1), the electron starts with $m_e = 0.5$ MeV, about the experimentally established value. Accordingly, the tau lepton is $2 \times 9 = 18$ times heavier than the muon which itselve is $8 \times 25 = 200$ more massive than the elctron, cf. Ne'eman et al. [10], p. 247.

Thus we find the masses $0.5, 100, 1800$ MeV which are rather close to the experimental values of the known charged leptons.

If a next generation lepton $L^{\pm}$ would exist, according to our formula (1), it would be $32 \times 49 = 1568$ times heavier than the tau, i.e., $m_L = 5.6448$ TeV. This is well above the lower bound of 100.8 GeV from current searches in particle accelerators, cf. the Particle Data Group [9].

A hypothetical 5th generation lepton would aquire an "astronomically" large mass of $128 \times 81 = 10{,}368$ times the predicted value of the $L^{\pm}$ as it has been dubbed here. Even in the decay of cosmic rays, this PeV range would remain rather difficult to detect even in the future.

Let us compare this with the rather precise lepton mass formula of Barut

$$\tilde{m}_\ell/m_e = 1 + \frac{1}{20\alpha}(2\ell+1)\ell(\ell+1)[3\ell(\ell+1)-1] \tag{3}$$

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in the equivalent representation [7] for $\ell = N - 1$ and α Sommerfeld's fine structure constant. However, it would predict a rather low mass of about $m_L \simeq 10$ GeV and should have already been seen in current searches for next generation charged leptons.

4. New Cosmos?

Our rather exotic formula (1) would indicate an internal "hidden" sub-structure of leptons, as considered earlier by Vigier et al. [1]. As in the angular momentum operator of $SU(2) \approx SO(3)$, the eigenfunctions of the electron excitations would be $2\ell + 1$ degenerate.

On the other hand, if nature restricts herself to *precisely* three generations, the known leptons could be accomodated in irreducible spinor representations of a $SO(10)$ unification of gauge groups. This may also indicate a lower dimensional topology of the early Universe and a Chern-Simons like gravitational term [2], cf. also the Mielke–Baekler model [8]. This might also facilitate the conformal representation of the initial condition ("Big Bang") in Penrose's model [11] of Conformal Cyclic Cosmology. It is not clear if conformal transformations a la Weyl could resolve discrepancies [3] in the cosmological parameters like the Hubble constant H_0 or the recent observations of massive galaxies at high redshift via the James Weph Space Telescope (JSWT).

The scale-invariant normalized sum and product of known charged lepton masses can be related to vacuum expectations values $<>$ of a hypothetical $U(3)$ nonet of scalar fields. Then the emperical mass spectrum can be understood as originating from a specific choice of scalar potentials [6]. In this scheme, possible higher generations are so far lacking, however.

A beyond 3 generation standard model would require also heavy quarks and cannot co-exist with relatively light Higgs particle of 125 GeV in the standard model, cf. Holdom [5].

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