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Article

Diffeomorphism-Invariant Bilocal Gravity for Future–Mass Projection (FMP): Noether Conservation, PPN/Poisson Limits, and Observational Touchstones

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Abstract

We formulate a covariant, bilocal action for the Future–Mass Projection (FMP) framework, in which the baryonic energy–momentum tensor couples to its own future worldline neighborhood via a causal bitensor kernel. From diffeomorphism invariance of the nonlocal action we obtain the Noether identity $\nabla_\mu T_{\text{eff}}^{\mu\nu} = 0$ with $T_{\text{eff}}^{\mu\nu} = T_b^{\mu\nu} + T_{\text{FMP}}^{\mu\nu}$. In the Minkowski limit we derive the conserved Noether charges (energy, linear and angular momentum), establish energy positivity under a positive-type kernel, and connect to the Newton/Poisson regime where the effective density $\rho_{\text{eff}} = \rho_b + \rho_F$ sources the potential. We outline Parameterized Post-Newtonian (PPN) consistency conditions, provide a practical diagnostics checklist for simulations, and situate FMP among nonlocal gravity approaches developed for late-time cosmology and galaxy dynamics. An accompanying literature review summarizes Noether theory in GR, nonlocal Lagrangian symmetries, bitensor technology, precision tests (Cassini, LLR) and data anchors (Planck CMB, DESI BAO, Euclid, SPARC).

Keywords: dark matter; Future–Mass Projection (FMP); nonlocal gravity; bilocal action; Noether theorem; diffeomorphism invariance; covariant conservation; effective stress–energy tensor; bitensors (sygne world function, parallel propagator); ward identities; positive-type kernel; Minkowski limit; Poisson/PPN limits; solar-system tests (Cassini, LLR); Planck CMB; DESI BAO; Euclid weak lensing; SPARC rotation curves; galaxy dynamics; structure growth

1. Introduction

Noether's theorems relate continuous symmetries to conservation laws and play a central role in relativistic field theory and General Relativity (GR).¹ In GR, diffeomorphism invariance yields the Bianchi identities and the covariant conservation of the (effective) stress tensor that sources the Einstein equation. *Nonlocal* or *bilocal* actions have long been explored as effective descriptions of infrared gravitational physics, cosmological acceleration, and structure growth, with various kernels or inverse d'Alembert operators [3–7]. For such actions, appropriately generalized Noether analyses and Ward identities exist and guarantee conservation laws when the kernels are constructed as covariant bitensors with suitable symmetry and causality properties [8–11].

This paper develops a diffeomorphism-invariant *bilocal* action for FMP, in which the ordinary baryonic $T_b^{\mu\nu}$ is coupled to its future light-cone domain via a parallel-propagated kernel acting on pairs of spacetime points.² Our goals are: (i) to give a compact and rigorous derivation of covariant conservation $\nabla_\mu T_{\text{eff}}^{\mu\nu} = 0$ from diffeomorphism invariance; (ii) to derive conserved Noether charges in the Minkowski limit and state positivity conditions on the kernel; (iii) to summarize Poisson/PPN limits and observational guardrails from Solar System tests (Cassini Shapiro delay, LLR bounds on $\dot{G}/G$) and cosmology/astrophysics (Planck, DESI, Euclid; SPARC rotation curves).

¹ For pedagogical discussions of Noether, energy, and diffeomorphism invariance in GR see e.g. De Haro [1], Will [2].

² We rely on the Sygne world function and parallel propagator to build bitensors; see Poisson, Pound & Vega [12].

Context and observations.

Planck 2018 CMB constraints provide the “acoustic anchor” for precision cosmology [13]. Large-scale structure experiments such as DESI (BAO & full shape) and Euclid (weak lensing, higher-order statistics) tighten distance and growth constraints [14–17]. At galaxy scales, the SPARC sample offers high-quality rotation curves and $3.6\ \mu\text{m}$ photometry for mass modeling [18]. Solar System PPN tests bound post-Newtonian parameters and time-variation of G at high precision [2,19,20].

2. A Covariant Bilocal Action for FMP

We postulate the total action

$$S[g, \Psi] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + S_b[g, \Psi] + S_{\text{FMP}}[g, T_b], \quad (1)$$

$$S_{\text{FMP}} = \frac{1}{2} \int d^4x d^4x' \sqrt{-g(x)} \sqrt{-g(x')} T_{\mu\nu}^{(b)}(x) \mathcal{K}^{\mu\nu}{}_{\rho'\sigma'}(x, x') T^{(b)\rho'\sigma'}(x'). \quad (2)$$

The kernel $\mathcal{K}^{\mu\nu}{}_{\rho'\sigma'}(x, x')$ is a *bitensor* built from the metric via the parallel propagator $\Pi^\alpha{}_{\beta'}(x, x')$ and the Synge world function $\sigma(x, x')$:

$$\mathcal{K}^{\mu\nu}{}_{\rho'\sigma'}(x, x') = \Pi^\mu{}_{\rho'}(x, x') \Pi^\nu{}_{\sigma'}(x, x') k[\sigma(x, x')] \Theta_+(x \prec x'), \quad (3)$$

satisfying (i) pair symmetry $\mathcal{K}^{\mu\nu}{}_{\rho'\sigma'}(x, x') = \mathcal{K}_{\rho'\sigma'}{}^{\mu\nu}(x', x)$, (ii) causal future support Θ_+ , and (iii) decay/positivity conditions specified below.

Varying S_{FMP} with respect to $g_{\mu\nu}$ defines

$$T_{\text{FMP}}^{\mu\nu}(x) \equiv -\frac{2}{\sqrt{-g(x)}} \frac{\delta S_{\text{FMP}}}{\delta g_{\mu\nu}(x)}, \quad T_{\text{eff}}^{\mu\nu} \equiv T_b^{\mu\nu} + T_{\text{FMP}}^{\mu\nu}. \quad (4)$$

3. Noether Identity from Diffeomorphism Invariance

Under an infinitesimal diffeomorphism generated by ξ^α , fields transform via Lie derivatives, and the action (1) is invariant by construction because $\mathcal{K}$ is a bitensor functional of g . Standard manipulations (as in GR plus bilocal terms) yield the Ward identity

$$\nabla_\mu T_{\text{eff}}^{\mu\nu} = 0, \quad (5)$$

which is Noether’s theorem for diffeomorphisms in the present nonlocal setting; see also generalized treatments of nonlocal Lagrangians [8–11].

4. Minkowski Limit: Conserved Charges and Positivity

On $g_{\mu\nu} = \eta_{\mu\nu}$, choose a temporally nonlocal, spatially local kernel

$$\mathcal{K}^{\mu\nu}{}_{\rho\sigma}(x, x') = P^{\mu\nu}{}_{\rho\sigma} k(t' - t) \delta^{(3)}(\mathbf{x} - \mathbf{x}') \Theta(t' - t), \quad P^{\mu\nu}{}_{\rho\sigma} = \frac{1}{2}(\delta_\rho^\mu \delta_\sigma^\nu + \delta_\sigma^\mu \delta_\rho^\nu). \quad (6)$$

Time/space translations and rotations imply conserved Noether charges

$$E = \int d^3x \mathcal{H}_{\text{eff}}, \quad \mathbf{P} = \int d^3x \mathcal{P}_{\text{eff}}, \quad \mathbf{J} = \int d^3x \mathbf{r} \times \mathcal{P}_{\text{eff}}, \quad (7)$$

with total energy

$$E(t) = \int d^3x T_b^{00}(t, \mathbf{x}) + \frac{1}{2} \int d^3x \int dt' T_{\mu\nu}^{(b)}(t, \mathbf{x}) P^{\mu\nu}{}_{\rho\sigma} k(t' - t) T^{(b)\rho\sigma}(t', \mathbf{x}). \quad (8)$$

If k depends only on $(t' - t)$ and is pair-symmetric, then $\dot{E} = 0$. If the temporal Fourier transform $\tilde{k}(\omega) \geq 0$ (positive type), the bilocal contribution is positive semidefinite, ensuring $E \geq \int T_b^{00}$.

5. Newton/Poisson and PPN Limits

In the weak-field limit one obtains

$$\nabla^2 \Phi = 4\pi G(\rho_b + \rho_F), \quad \rho_F(t, \mathbf{x}) = \int_0^\infty d\tau k(\tau) \Pi[\rho_b(t + \tau, \mathbf{x})], \quad (9)$$

with Π the appropriate projection from $T_{\mu\nu}^{(b)}$ onto density. The continuity equation here is the nonrelativistic form of (5). For Solar System safety, one demands small $\tilde{k}(0)$ and a short memory scale τ_0 such that $|\gamma - 1|$, $|\beta - 1|$, Shapiro delay and $\dot{G}/G$ remain within experimental bounds (Cassini; LLR) [2,19,20].

6. Observational Guardrails and Data Anchors

CMB/BAO/Lensing. Planck 2018 sets the acoustic scale and baseline parameters [13]; DESI first-year BAO and full-shape constraints sharpen distances and growth [14,15]; Euclid is poised to transform weak-lensing statistics and higher-order probes [16,17]. **Galaxies.** The SPARC sample [18] provides rotation curves and photometry for mass models against which FMP predictions can be tested. **PPN/Local tests.** Bounds from Cassini (parameter γ) and LLR ($\dot{G}/G$) provide hard priors on $\tilde{k}(0)$ and τ_0 [2,19,20].

7. Numerical Diagnostics (Go/No-Go)

We recommend the following diagnostics in Minkowski boxes and weak-field solvers:

- **Diffeo/Noether residue:** $\epsilon_{\text{div}} \equiv \int \sqrt{-g} |\nabla_\mu T_{\text{eff}}^{\mu\nu}|^2 / \int \sqrt{-g} |T_{\text{eff}}|^2 \rightarrow 0$ (target $< 10^{-8}$).
- **Energy constancy:** Evaluate (8), require $\max_t |E(t) - E(0)|/E(0) < 10^{-8}$.
- **Positivity:** Discretized quadratic form $\sum_{ij} T_i k_{ij} T_j \geq 0$ for random time series T .
- **PPN gates:** Enforce $|\gamma - 1|$, $|\beta - 1|$, $\dot{G}/G$ within Cassini/LLR bounds.
- **Poisson check:** $\|\nabla \cdot \mathbf{j}_{\text{eff}} + \partial_t \rho_{\text{eff}}\|_\infty$ below grid tolerance.

8. Discussion

The bilocal FMP action (2) supplies a clean symmetry foundation: diffeomorphism invariance $\Rightarrow$ Noether conservation (5), while the kernel axioms ensure causality, symmetry, and positivity at the level of charges. Compared with other nonlocal gravities [3–7], FMP couples directly to baryonic T_b over future-directed worldline neighborhoods and is designed for galaxy/cosmology phenomenology under tight PPN gates. Future work includes (i) confronting full $f\sigma_8(z)/S_8$ data with tuned kernels; (ii) halo/RC fits on SPARC; (iii) CLASS/CAMB-like linear response modules to propagate $\mathcal{K}$ into growth and lensing.

9. Conclusions

We provided a diffeomorphism-invariant bilocal action for FMP, a Noether derivation of covariant conservation, conserved charges and positivity in flat space, and a set of practical constraints to keep the model consistent from Solar System to cosmology. The framework is ready for numerical implementation and data-level tests.

Acknowledgments: We thank the broader literature on nonlocal gravity, Noether identities for nonlocal Lagrangians, and precision tests of GR for guidance.

Appendix A. Sketch of the Noether Derivation

Varying (2) under δ_ξ and using the bitensor nature of $\mathcal{K}$ ($\delta_\xi \mathcal{K} = \mathcal{L}_\xi \mathcal{K}$) one finds $\delta_\xi S_{\text{FMP}} = \int d^4x \sqrt{-g} \frac{1}{2} T_{\text{FMP}}^{\mu\nu} \delta_\xi g_{\mu\nu}$. Combining with the Einstein–Hilbert and matter parts yields $\delta_\xi S = \int \sqrt{-g} \nabla_\mu (T_{\text{eff}}^{\mu\nu} \xi_\nu)$. Since $\delta_\xi S = 0$ for arbitrary ξ^μ , we obtain $\nabla_\mu T_{\text{eff}}^{\mu\nu} = 0$.

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