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# The Field Equation and Lagrangian of the NUVO Conformal Scalar Field - Part 2 of the NUVO Theory Series

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Article

# The Field Equation and Lagrangian of the NUVO Conformal Scalar Field

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**Abstract:** This paper derives the dynamical field equation and Lagrangian for the scalar conformal field  $\lambda(t, r, v)$  that underpins NUVO theory. Beginning with a variational approach and physically motivated scalar terms, we construct a Lagrangian density, extract the corresponding Euler–Lagrange equation, and analyze the resulting field equation for symmetries, conservation laws, and its reduction to Newtonian gravity. The formulation provides a foundational link for coupling  $\lambda$  to spacetime geometry, classical dynamics, and possibly quantum behavior.

**Keywords:** NUVO theory; conformal scalar field; scalar field dynamics; energy-momentum tensor; time-dilation; relativistic correspondence; Newtonian mechanics; conformal coupling; pinertia; sinertia; Lagrangian

## 1. Introduction

The NUVO theory introduces a scalar conformal field  $\lambda(t, r, v)$  that modifies the underlying structure of space and time in response to both relativistic kinetic energy and classical gravitational potential energy [1]. Unlike traditional general relativity, which encodes gravity through spacetime curvature sourced by the stress-energy tensor, NUVO constructs a conformally flat metric governed by a single scalar field  $\lambda$ , defined as:

$$\lambda(t, r, v) = \frac{1}{\sqrt{1 - v^2/c^2}} + \frac{GM}{rc^2} \tag{1}$$

Here, the first term represents the relativistic kinetic distortion (via the Lorentz gamma factor), and the second term represents the gravitational distortion (via the Newtonian potential energy normalized by rest energy). The total scalar field thus captures both local motion and potential in a unified geometrical modulation.

This scalar field acts as a multiplicative conformal factor on the flat Minkowski metric:

$$g_{\mu\nu} = \lambda^2(t, r, v) \eta_{\mu\nu} \tag{2}$$

leading to modified geodesics and observable effects such as time dilation, redshift, and orbital precession that align with general relativistic predictions under appropriate limits.

The goal of this paper is to formalize the dynamics of  $\lambda(t, r, v)$  by deriving its field equation from a variational principle. This includes:

- Constructing a Lagrangian density  $\mathcal{L}[\lambda]$  based on kinetic and source terms.
- Deriving the Euler–Lagrange equation that governs  $\lambda$ .
- Identifying symmetries and corresponding conservation laws.
- Demonstrating how the resulting field equation reduces to Newtonian gravity in the weak-field, low-velocity limit.

By establishing a consistent field theory for  $\lambda$ , we build the foundation for coupling this scalar modulation to both classical gravitational dynamics and potential extensions into quantum theory.

## 2. Definition of the Scalar Field $\lambda(t, r, v)$

The conformal scalar field  $\lambda(t, r, v)$  in NUVO theory encodes the total space-time distortion experienced by a test particle due to its relativistic kinetic energy and its position in a gravitational potential. The field is defined as:

$$\lambda(t, r, v) = \frac{1}{\sqrt{1 - v^2/c^2}} + \frac{GM}{rc^2} \quad (3)$$

where:

- $v$  is the speed of the particle relative to the local inertial frame,
- $G$  is the gravitational constant,
- $M$  is the source mass,
- $r$  is the radial distance from the source,
- $c$  is the speed of light.

The form of  $\lambda$  reflects the total normalized energy affecting the particle's interaction with space. The first term,  $\gamma(v) = (1 - v^2/c^2)^{-1/2}$ , is the Lorentz factor, representing the relativistic kinetic energy normalized by rest mass. The second term,  $GM/(rc^2)$ , is the Newtonian gravitational potential energy per unit rest energy.

This scalar field is used to generate a conformally transformed spacetime metric:

$$g_{\mu\nu} = \lambda^2(t, r, v) \eta_{\mu\nu} \quad (4)$$

where  $\eta_{\mu\nu}$  is the Minkowski metric of flat spacetime. The conformal factor  $\lambda^2$  modulates the spacetime interval such that time and spatial intervals are both scaled locally according to the particle's state.

This construction allows gravity and motion to be understood as geometric consequences of the scalar field's variation in space and time, without invoking curvature in the traditional Riemannian sense. Instead, distortions arise directly from the scalar field modulating flat geometry.

In regions where  $v \ll c$  and  $r \gg GM/c^2$ , the scalar field reduces to:

$$\lambda(t, r, v) \approx 1 + \frac{v^2}{2c^2} + \frac{GM}{rc^2} \quad (5)$$

recovering the familiar corrections of special relativity and Newtonian gravity. This limiting behavior anchors the NUVO formulation to classical empirical results while allowing for deeper geometric structure in more extreme regimes.

## 3. Construction of the Lagrangian Density

To accurately describe the scalar conformal field  $\lambda(t, r, v)$  in NUVO theory, we must distinguish between two fundamentally different sources of modulation:

1. **Local, non-propagating modulation** due to the particle's own instantaneous velocity, represented by the Lorentz gamma factor  $\gamma(v)$ .
2. **Global, propagating modulation** due to the gravitational potential from distributed sources, represented by a field component  $\lambda_{\text{field}}(t, r)$ .

Accordingly, we split the total conformal field into two additive components:

$$\lambda(t, r, v) = \lambda_{\text{local}}(v) + \lambda_{\text{field}}(t, r) \quad (6)$$

where:

$$\lambda_{\text{local}}(v) = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad \lambda_{\text{field}}(t, r) = \frac{GM}{rc^2} + \delta\lambda(t, r) \quad (7)$$

The  $\lambda_{\text{local}}$  term affects only the local observer's modulation and is not associated with a propagating field. It should not appear in any derivative terms in the Lagrangian. The  $\lambda_{\text{field}}$  term includes both the static Newtonian component and any dynamical evolution  $\delta\lambda(t, r)$  that governs global interactions.

The resulting Lagrangian density then becomes:

$$\mathcal{L} = -\frac{1}{2}\kappa g^{\mu\nu} \partial_\mu \lambda_{\text{field}} \partial_\nu \lambda_{\text{field}} - \rho(x^\mu) [\lambda_{\text{local}}(v) + \lambda_{\text{field}}(t, r)] \quad (8)$$

Here:

- The first term governs the propagation and curvature effects of the global scalar field only.
- The second term couples both local and global conformal contributions to the matter source  $\rho(x^\mu)$ , reflecting that the observable distortion at a given location depends on both velocity and position.

This formulation ensures that:

- $\lambda_{\text{local}}$  remains algebraic — it does not induce any spatial or temporal field evolution.
- $\lambda_{\text{field}}$  is a true scalar field — it may propagate, accumulate, and superpose across space-time.

In this way, the NUVO Lagrangian reflects the theory's core postulate: that motion-induced modulation is inherently local and geometric, while potential-induced modulation defines the global structure of the field.

**Note on Closure-Induced Propagation:** Although  $\lambda_{\text{local}}$  is treated here as a strictly non-propagating term tied to the particle's instantaneous velocity, NUVO theory anticipates that under certain geometric conditions—such as orbital phase closure or modulation resonance—the local modulation may transition into a global field effect. In such cases, the closure of local modulation cycles may emit coherent scalar field disturbances, effectively allowing  $\lambda_{\text{local}}$  to influence  $\lambda_{\text{field}}$  and propagate through space. This secondary dynamic will be introduced and formalized in a later section of the theory.

#### 4. Field Equation from Variational Principle

We now derive the field equation governing  $\lambda_{\text{field}}(t, r)$  from the Lagrangian constructed in Section 3. As established, only the global, propagating component of the conformal field participates in the field dynamics. The local component  $\lambda_{\text{local}}(v)$  contributes algebraically to physical observables but does not appear in any variational derivative or differential operator.

Starting from the action[2]:

$$S = \int \mathcal{L}[\lambda_{\text{field}}] \sqrt{-g} d^4x \quad (9)$$

with the Lagrangian density:

$$\mathcal{L} = -\frac{1}{2}\kappa g^{\mu\nu} \partial_\mu \lambda_{\text{field}} \partial_\nu \lambda_{\text{field}} - \rho(x^\mu) [\lambda_{\text{local}}(v) + \lambda_{\text{field}}(t, r)] \quad (10)$$

we vary the action with respect to  $\lambda_{\text{field}}$ :

$$\delta S = \int \left( \frac{\partial \mathcal{L}}{\partial \lambda_{\text{field}}} - \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \lambda_{\text{field}})} \right) \right) \delta \lambda_{\text{field}} \sqrt{-g} d^4x = 0$$

The resulting Euler–Lagrange equation is:

$$\kappa \nabla^\mu \nabla_\mu \lambda_{\text{field}} = \rho(x^\mu) \quad (11)$$

where  $\nabla^\mu$  denotes the covariant derivative with respect to the metric  $g_{\mu\nu} = \lambda^2 \eta_{\mu\nu}$ , and  $\square \equiv \nabla^\mu \nabla_\mu$  is the covariant d'Alembertian operator.

In the weak-field, low-velocity, and nearly flat-space limit, where  $\lambda \approx 1$  and  $g_{\mu\nu} \approx \eta_{\mu\nu}$ , this reduces to:

$$\kappa \square \lambda_{\text{field}} = \rho \quad (12)$$

This is a standard scalar wave equation sourced by energy density, governing the evolution and propagation of the field  $\lambda_{\text{field}}$  throughout space-time.

**Interpretation:** The field equation (12) implies that changes in energy density  $\rho$  generate conformal distortions in space-time geometry that propagate outward as scalar field variations in  $\lambda_{\text{field}}$ . These variations in turn modulate time dilation, length contraction, and force interactions, consistent with the predictions of NUVO theory.

**Note:** The  $\lambda_{\text{local}}$  term, while contributing to the coupling with matter via  $\rho$ , is not dynamical and does not enter the derivative structure of the field equation. This ensures that local velocity effects remain confined to the observer's frame and do not lead to spurious propagation unless explicitly allowed by higher-order resonance behavior (see Section 7).

#### Physical Interpretation: Local Modulation Does Not Radiate

In the NUVO framework, the scalar field  $\lambda$  is decomposed into a local, non-propagating component  $\lambda_{\text{local}}(v)$  and a global, field-propagating component  $\lambda_{\text{field}}(t, r)$ . This has profound implications for how observers experience and communicate geometric distortions.

Consider a scenario in which an accelerating satellite passes by a stationary satellite:

- The accelerating satellite experiences an increase in  $\lambda_{\text{local}}$  due to its own instantaneous velocity. This leads to time dilation and spatial modulation relative to its rest frame.
- The stationary satellite, however, is unaffected by this change. Since  $\lambda_{\text{local}}$  does not propagate through the field, the accelerating observer's modulation does not induce any time dilation or metric change in the nearby stationary observer.

This result differs sharply from general relativity, where accelerating mass-energy alters the shared metric and produces gravitational effects felt by nearby observers. In NUVO, geometric distortions due to motion are *strictly local* unless specific resonance or closure conditions trigger field-level propagation (to be discussed in later sections).

This separation introduces a refined interpretation of the equivalence principle. While local acceleration still produces measurable effects within a single frame (e.g., clock dilation, spatial contraction), those effects do not automatically extend to other frames unless mediated by  $\lambda_{\text{field}}$ . Thus, NUVO preserves a weaker form of the equivalence principle that applies only locally — and only within a given observer's own modulation envelope.

## 5. Symmetries and Conservation Laws

Having established the field equation for the scalar component  $\lambda_{\text{field}}(t, r)$ , we now examine the symmetries of the Lagrangian and the resulting conservation laws. These are crucial for understanding energy flow, field behavior under transformations, and deviations from general relativity.

### Spacetime Symmetry and Coordinate Invariance

The Lagrangian

$$\mathcal{L} = -\frac{1}{2}\kappa g^{\mu\nu} \partial_\mu \lambda_{\text{field}} \partial_\nu \lambda_{\text{field}} - \rho(x^\mu) [\lambda_{\text{local}}(v) + \lambda_{\text{field}}(t, r)] \quad (13)$$

is manifestly scalar under general coordinate transformations. Since it contains no explicit dependence on the coordinates  $x^\mu$  outside of the source term  $\rho(x^\mu)$ , it respects diffeomorphism invariance and supports the conservation of stress-energy in the standard variational formalism.

### Noether Current and Energy-Momentum Tensor

From Noether's theorem [3], continuous spacetime translation invariance implies the existence of a conserved energy-momentum tensor for the scalar field:

$$T_{\mu\nu}^{(\lambda)} = \kappa \left( \partial_\mu \lambda_{\text{field}} \partial_\nu \lambda_{\text{field}} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \lambda_{\text{field}} \partial_\beta \lambda_{\text{field}} \right) \quad (14)$$

This tensor describes the flow of energy and momentum carried by the propagating scalar field  $\lambda_{\text{field}}$ , analogous to the stress-energy tensor in scalar field theory or the electromagnetic field.

Conservation of energy-momentum is expressed by the vanishing covariant divergence:

$$\nabla^\mu T_{\mu\nu}^{(\lambda)} = 0 \quad (15)$$

This conservation law holds automatically when the field equation (11) is satisfied, assuming that the background spacetime is conformally flat and the matter sources  $\rho(x^\mu)$  are conserved.

### Scaling and Shift Symmetries

In the absence of a source term, the kinetic portion of the Lagrangian possesses a shift symmetry:

$$\lambda_{\text{field}} \rightarrow \lambda_{\text{field}} + \epsilon, \quad \text{for constant } \epsilon$$

This symmetry would imply a conserved current associated with uniform modulation offset. However, the presence of the  $\rho(x^\mu)\lambda_{\text{field}}$  term breaks this symmetry explicitly — which is physically consistent, since mass–energy should anchor the modulation to physical sources.

Similarly, the Lagrangian is not invariant under global scaling of  $\lambda$ , due to the linear source coupling. This indicates that modulation strength is physically fixed by the interaction with matter, and not arbitrary.

### Interpretation

Unlike general relativity, where the metric itself is a dynamical tensor sourced by the full stress-energy tensor, NUVO theory contains a scalar modulation field  $\lambda$  that deforms a flat background and carries its own independent stress-energy. This allows for:

- Local modulation via  $\lambda_{\text{local}}(v)$  without gravitational back-reaction,
- Propagating modulation via  $\lambda_{\text{field}}$  that evolves dynamically and obeys a conservation law,
- Partial decoupling between mass-energy and geometric propagation — except when geometric closure conditions are met.

Together, these results form the basis for understanding energy transfer, wave propagation, and gravitational interaction in the NUVO framework.

## 6. Classical Gravity Limit

To confirm consistency with known physics, we now examine the behavior of NUVO theory in the classical limit. This corresponds to:

- Low velocities:  $v \ll c$
- Weak gravitational fields:  $GM/rc^2 \ll 1$
- Static conditions:  $\partial_t \lambda_{\text{field}} \approx 0$

In this limit, the total scalar field simplifies. Expanding both components to leading order:

$$\lambda_{\text{local}}(v) = \frac{1}{\sqrt{1 - v^2/c^2}} \approx 1 + \frac{v^2}{2c^2} \quad (16)$$

$$\lambda_{\text{field}}(r) = \frac{GM}{rc^2} + \delta\lambda(t, r) \approx \frac{GM}{rc^2} \quad (17)$$



So the total scalar field becomes:

$$\lambda(t, r, v) \approx 1 + \frac{v^2}{2c^2} + \frac{GM}{rc^2} \quad (18)$$

This expression recovers the first-order corrections associated with:

- Special relativistic time dilation (via the  $\frac{v^2}{2c^2}$  term),
- Newtonian gravitational time dilation and redshift (via the  $\frac{GM}{rc^2}$  term),
- Modest conformal scaling of space and time in weak fields.

### 6.1. Recovery of Newton's Law of Gravity

Let us now show that Newton's inverse-square gravitational law [4] emerges from the geodesic equation under the conformally flat metric:

$$g_{\mu\nu} = \lambda^2(t, r, v) \eta_{\mu\nu} \quad (19)$$

Using the weak-field approximation (18), the geodesic equation for a test particle becomes:

$$\frac{d^2 x^i}{dt^2} \approx -\frac{1}{\lambda} \partial^i \lambda \cdot c^2 \quad (20)$$

Substituting Eq. (18), and retaining only the spatially varying part  $\frac{GM}{rc^2}$ :

$$\partial^i \lambda \approx -\frac{GM}{r^2 c^2} \hat{r}^i \quad (21)$$

Plugging this into the geodesic equation:

$$\frac{d^2 x^i}{dt^2} \approx -\left(\frac{1}{1 + \frac{v^2}{2c^2} + \frac{GM}{rc^2}}\right) \left(-\frac{GM}{r^2 c^2} \cdot c^2\right) \hat{r}^i \approx -\frac{GM}{r^2} \hat{r}^i \quad (22)$$

where the correction factor is approximately unity in the weak-field, low-velocity regime.

This confirms that **Newton's law of gravity is exactly recovered** in this limit, and that NUVO reproduces classical gravitational behavior without requiring curved spacetime.

### 6.2. Interpretation

In NUVO, the force arises from spatial gradients in the scalar field  $\lambda(t, r, v)$ , which itself encodes both velocity and gravitational potential. The geodesic deviation results not from curvature in the Riemannian sense, but from conformal modulation of a flat underlying space. The resulting dynamics are equivalent to Newtonian gravity when  $\lambda \approx 1$ , with additional corrections at higher energies or deeper gravitational wells.

This supports the claim that NUVO theory is compatible with all first-order relativistic corrections and classical gravity, while providing a richer geometric framework for analyzing deviations in strong fields, high velocities, and discrete modulation behaviors.

### Note on Special Relativity as a Limiting Case of NUVO

While the expansion in Eq. (18) resembles that of special relativity and Newtonian gravity combined, it is important to highlight a conceptual distinction. In special relativity, the Lorentz factor  $\gamma(v)$  is typically applied based on an integrated or frame-consistent velocity across inertial trajectories. This assumes a form of global motion history or constant velocity reference.

In NUVO, by contrast, the velocity  $v$  entering  $\lambda_{\text{local}}(v)$  is strictly the *instantaneous velocity* at a spacetime point, derived from local acceleration and motion state. This makes  $\lambda_{\text{local}}$  a pointwise geometric field, not a global inertial property. As a result:

- Special relativity is recovered only in the limiting case where instantaneous velocity remains constant across a region.
- If motion ceases or reverses, the modulation due to  $\lambda_{\text{local}}$  collapses immediately.
- The twin paradox and related effects are resolved geometrically in NUVO by this strict locality condition.

Thus, special relativity emerges as a special case of NUVO under continuous uniform motion, but its assumptions do not fully generalize into the NUVO conformal framework.

### 6.3. When Local Modulation Becomes Global: Closure-Triggered Propagation

While  $\lambda_{\text{local}}(v)$  is treated as strictly non-propagating under normal motion, NUVO theory anticipates that under specific geometric conditions — such as resonance, orbital closure, or phase alignment — this local modulation can transition into a global effect.

These conditions include:

- Cyclic orbital advance reaching a resonance condition (e.g., arc-length advance),
- Modulation harmonics forming standing waves in  $\lambda(t, r, v)$ ,
- Closure of both spatial and temporal modulation after  $N$  orbits (e.g., electron–proton resonance in hydrogen),
- Coherent inertia collapse events between coupled nucleons.

In such scenarios, the geometry locally satisfies a resonance criterion:

$$\oint d\phi \lambda_{\text{local}}(t, r(t), v(t)) = 2\pi n \lambda_{\text{res}} \quad (\text{closure condition}) \quad (23)$$

When this condition is met, the accumulated local modulation can “release” as a perturbation in  $\lambda_{\text{field}}$ , thereby becoming a true propagating signal in the scalar field. This mechanism may underlie:

- Quantized energy transitions,
- Scalar radiation emission,
- The origin of discrete spectral lines,
- Coupling to other geometric systems (such as orbital or nuclear transitions).

This behavior is not captured by the classical field equation (12), but is proposed as a higher-order effect — a nonlinearity or resonance response that will be formalized in later extensions of NUVO.

Thus, NUVO provides a natural gateway for understanding how fundamentally local motion can, under well-defined geometric conditions, trigger observable global changes — echoing quantum behavior from purely geometric roots.

## 7. Discussion and Outlook

In this paper, we have constructed and analyzed a variational field theory for the scalar conformal field  $\lambda(t, r, v)$  central to NUVO theory. The field was separated into two conceptual components:

- A local, non-propagating part  $\lambda_{\text{local}}(v)$  that modulates geometry based on a particle’s instantaneous velocity;



- A global, dynamical part  $\lambda_{\text{field}}(t, r)$  sourced by mass–energy distributions and governed by a wave-like field equation.

We derived a Lagrangian density that respects this decomposition and obtained the field equation for  $\lambda_{\text{field}}$  through standard variational methods. The resulting dynamics resemble a scalar wave equation in curved space, reducing to Newtonian gravity in the weak-field limit while incorporating relativistic corrections through the conformal structure.

We also analyzed the symmetries of the theory and constructed the energy–momentum tensor for the scalar field. The conservation laws derived from Noether’s theorem confirm the internal consistency of the theory and demonstrate that  $\lambda_{\text{field}}$  behaves like a genuine carrier of scalar energy and momentum.

### 7.1. NUVO vs. General Relativity

NUVO diverges significantly from general relativity in both formulation and interpretation:

- Spacetime remains globally flat and is modified only through pointwise conformal scaling, not full Riemannian curvature.
- Only global mass–energy content sources propagating effects; local velocity-induced modulation does not automatically generate gravitational radiation or influence neighboring observers.
- The equivalence principle is weakened to a strictly local interpretation: acceleration affects one’s own geometry, but not others’ unless a resonance or closure condition is met.

This interpretation resolves longstanding paradoxes such as the twin paradox and supports a more modular view of spacetime structure.

### 7.2. Geometric Closure and Future Work

We have introduced the concept of geometric closure — conditions under which the normally confined local modulation becomes globally significant. While not part of the classical field equation, this mechanism will be formalized in future work as a potential source of:

- Scalar radiation,
- Discrete energy transitions,
- Quantum-like behavior from orbital resonance,
- Coupling mechanisms between sinertia, pinertia, and  $\lambda$ .

This provides a roadmap for extending NUVO beyond classical gravitational behavior and into quantum and nuclear domains.

### 7.3. Next Steps

Future directions include:

1. Developing an effective potential theory for  $\lambda$  in multi-body systems.
2. Exploring gauge-like symmetries in the scalar sector.
3. Coupling  $\lambda$  to sinertia fields and flux collapse to model nuclear binding.
4. Quantizing the  $\lambda$  field under boundary conditions defined by closure.

This field formulation lays the foundation for those investigations by grounding the scalar dynamics in a principled variational structure. As NUVO continues to develop, this scalar field will serve as the geometric substrate from which gravitational, quantum, and nuclear phenomena emerge in a unified framework.

## Appendix A. Derivation Details

This appendix presents the step-by-step derivation of the field equation for  $\lambda_{\text{field}}$  from the action principle.

### A.1 Action and Lagrangian

We begin with the action:

$$S = \int \mathcal{L}[\lambda_{\text{field}}, \partial_\mu \lambda_{\text{field}}] \sqrt{-g} d^4x \quad (\text{A24})$$

where the Lagrangian is:

$$\mathcal{L} = -\frac{1}{2} \kappa g^{\mu\nu} \partial_\mu \lambda_{\text{field}} \partial_\nu \lambda_{\text{field}} - \rho(x^\mu) [\lambda_{\text{local}}(v) + \lambda_{\text{field}}(x^\mu)] \quad (\text{A25})$$

We vary the action with respect to  $\lambda_{\text{field}}(x^\mu)$ :

$$\delta S = \int \left( \frac{\partial \mathcal{L}}{\partial \lambda_{\text{field}}} - \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \lambda_{\text{field}})} \right) \right) \delta \lambda_{\text{field}} \sqrt{-g} d^4x$$

### A.2 Euler–Lagrange Equation

Compute the functional derivatives:

$$\frac{\partial \mathcal{L}}{\partial \lambda_{\text{field}}} = -\rho(x^\mu) \quad (\text{A26})$$

$$\frac{\partial \mathcal{L}}{\partial (\partial_\mu \lambda_{\text{field}})} = -\kappa g^{\mu\nu} \partial_\nu \lambda_{\text{field}} \quad (\text{A27})$$

Now apply the Euler–Lagrange equation:

$$-\rho(x^\mu) + \partial_\mu (\kappa g^{\mu\nu} \partial_\nu \lambda_{\text{field}}) = 0 \quad (\text{A28})$$

If we assume that  $\kappa$  is constant and commute it outside the derivative, and apply the definition of the covariant d'Alembertian operator  $\square = \nabla^\mu \nabla_\mu$ , we obtain:

$$\kappa \square \lambda_{\text{field}} = \rho(x^\mu) \quad (\text{A29})$$

### A.3 Flat-Space Limit

In flat spacetime with  $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ , the covariant derivative reduces to the partial derivative, and the d'Alembertian becomes:

$$\square = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

Thus the field equation becomes:

$$\kappa \left( -\frac{\partial^2 \lambda_{\text{field}}}{\partial t^2} + \nabla^2 \lambda_{\text{field}} \right) = \rho(x^\mu) \quad (\text{A30})$$

This completes the derivation of the scalar field equation governing  $\lambda_{\text{field}}$  in both curved and flat backgrounds.

### A.4 Notes on Normalization

The coupling constant  $\kappa$  determines the relative scaling between field strength and matter source. In future work, it may be fixed empirically or derived from consistency with orbital behavior (e.g., requiring Newton's law in the weak-field limit).

It is also possible to introduce a canonical normalization for  $\lambda$  to render  $\kappa$  dimensionless or to match natural units. For now, we retain  $\kappa$  explicitly to preserve generality in coupling strength.

This derivation provides the technical foundation for NUVO's field dynamics and supports the claim that  $\lambda_{\text{field}}$  behaves as a classical scalar field with minimal coupling to matter and clean geometric interpretation in both weak and strong regimes.

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