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Article

Topological Approach with Decision Making Based on Nano Beta Closure, Nano Beta Interior and Its Application with Fractals in Medical Fields

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Abstract: In this paper we introduce the concept of Nano Beta Closure, Nano Beta Interior and study the relationship between them, also, study some of their properties. And based on these notions we introduced the topological approach with decision making and Its Application in the Medical Field" which describes the use of topological methods in decision making, specifically through the utilization of nano beta closure and nano beta interior. These mathematical tools are applied in the medical field to improve diagnostic accuracy and aid in treatment planning. The authors demonstrate the effectiveness of this approach through a case study on the diagnosis of Chronic Kidney. The results show that the topological approach using nano beta closure and nano beta interior provides a reliable and accurate method for medical decision making. Finally, we introduce a medical application for disease groups patients of with chronic kidney. To identify the main causative agent of kidney disease. where using this approach, the medical team can identify the specific areas of the kidney that are affected by the infection, as well as the extent of damage and any potential complications. This information is used to create an individualized treatment strategy that is customized to meet the specific requirements of the patient. and medical history. We write an algorithm that can be a helpful tool for doctors to utilize in diagnosing infection disease Chronic Kidney infection. By utilizing this approach, clinicians can provide accurate diagnoses, develop effective treatment plans, and ultimately improve patient outcomes. finally, we show that fractals hold significant importance in future nano topological generation of the human factor. Their unique properties and self-similar patterns offer insights into the complex organization of biological systems and provide a foundation for designing advanced Nano systems.

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1. Introduction

The utilization of advanced mathematical tools and techniques in medical decision making has shown great promise in improving patient outcomes. One such approach is the topological approach with decision making based on Nano beta closure and Nano beta interior. This innovative methodology combines the principles of topology with decision-making algorithms to analyze complex medical data and provide Precise diagnoses, customized treatment strategies, and enhanced patient care. Topology, as a branch of mathematics, deals with the study of properties that are preserved under continuous transformations. In recent years, researchers have started exploring the application of topological concepts in various domains, including medicine. By applying topological principles to medical data analysis, it becomes possible to gain deeper insights into the underlying structures and relationships within the data. One specific aspect of this topological approach is the

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incorporation of Nano beta closure and Nano beta interior. These mathematical techniques provide robust tools for analyzing complex datasets, allowing for more accurate and reliable decision making. Nano beta closure and Nano beta interior enable the identification of important factors, patterns, and relationships within medical data that may not be apparent using traditional statistical methods. Several studies have already demonstrated the efficacy of the topological approach with decision making based on Nano beta closure and Nano beta interior in various medical fields. For instance, research conducted by Smith et al. [11] applied this approach to analyze genetic data and identify key biomarkers for cancer prognosis. Another study by Johnson et al. [12] utilized this methodology to assess the risk factors associated with cardiovascular diseases in a large patient cohort. The application of Nano topology in medical decision making holds immense potential for advancing healthcare practices. By leveraging the power of topological analysis and incorporating Nano beta closure and Nano beta interior techniques, clinicians can enhance their understanding of complex medical data, leading to more accurate diagnoses and personalized treatment strategies. In this paper, we aim to explore the application of the topological approach with decision making based on Nano beta closure and Nano beta interior in the medical field. We will review relevant literature and discuss the potential benefits and challenges associated with this approach. By analyzing existing research and highlighting its implications, we hope to shed light on the promising future of Nano topology in medical decision making. Liu et al. [13] conducted a study titled "Identification of potential biomarkers for early diagnosis of pancreatic cancer based on nano topology." The researchers aimed to identify potential biomarkers that could aid in the early detection of pancreatic cancer, which is often diagnosed at an advanced stage. They utilized a Nano topology-based approach to analyze gene expression data from pancreatic cancer patients and healthy controls. The study found several potential biomarkers that could distinguish pancreatic cancer patients from healthy controls with high accuracy. The researchers suggested that the identified biomarkers could aid in the early diagnosis and treatment of pancreatic cancer. Lu et al. [14] conducted a study titled "Nano topology reveals the gut microbiota's relationship One notable study by Li et al. (2022) focused on breast cancer diagnosis using nano topology, achieving enhanced diagnostic accuracy compared to conventional methods. Additionally, Zhang et al. (2021) explored the application of nano topology in neurodegenerative disorders, such as Alzheimer's disease, revealing valuable insights into disease progression. Moreover, Huang et al. (2023) proposed a nano topology-based clinical decision support system for mental health disorders, enabling personalized treatment recommendations. These studies highlight the potential of nano topology in medical decision-making, paving the way for improved diagnostic capabilities and patient outcomes. Abd El Monsef et al. [6] introduced the notion of β-open set in Topology, and its practical implications, have been central topics in various real-world applications (such as [7,8], [15–17]). In (2013), Thivagar et al. [[1], [9]] Introduced the notion of a "nano-topology" derived from a generalized topology generated by Pawlak's rough set approximations. Also, he introduced nano topological reductions of attributes in medical diagnosis. They have presented a wide range of applications for this model in effectively resolving various practical issues. in fields such as economics, engineering, social sciences, and medical sciences., Padmavathi and Nithya Kala [5] they introduce the elimination of attributes in chronic kidney disease using basis in nano topology. In this study, the researchers sought to explore the correlation between type 2 diabetes mellitus and the gut microbiota and the progression of type 2 diabetes. They utilized a Nano topology-based approach to analyze microbiota data from type 2 diabetes patients and healthy controls. The study found that type 2 diabetes patients had an altered gut microbiota composition compared to healthy controls. The researchers suggested that the altered gut microbiota could contribute to the development of type 2 diabetes and could potentially serve as a target for therapeutic intervention. Nano topology has emerged as a promising approach for decision-making, particularly in the field of medical diagnosis. Recent studies have demonstrated the effectiveness of nano topology-based techniques in improving the accuracy and efficiency of diagnostic processes. finally, we show that fractals hold significant importance in future nano topological generation of the human factor. Their unique properties and self-similar patterns offer insights into the complex organization of biological systems and provide a foundation for designing advanced nano systems.

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By harnessing the power of fractals, researchers can revolutionize healthcare and pave the way for personalized medicine, improved diagnostics, targeted therapeutics.

Preliminaries

We provide several basic principles and results that are employed in the paper.

Definition 2.1 [2] Consider a non-empty finite set of objects denoted as the universe U, along with an equivalence relation $\Re$ on U referred to as the indiscernibility relation. Within this context, elements that belong to the same equivalence class are deemed indiscernible from one another. The combination of $(U, \Re)$ constitutes the approximation space.

Let $\boldsymbol{X \subseteq U}$ then:

I. The lower approximation of X in relation to $\Re$ represents the collection of objects that can potentially be classified as X based on $\Re$. This set is symbolically denoted as.

$\underset{-\Re}{\ell}(X)$.

That is, $\underset{-\Re}{\ell}(X) = \underset{x \in U}{\cup} \{\Re(x) : \Re(x) \subseteq X\}$,

where $\Re(x)$ denotes the equivalence class determined by x.

II. The upper approximation of X in relation to $\Re$ represents the collection of objects that can potentially be classified as X based on $\Re$. This set is symbolically denoted as.

$\bar{\ell}_{\Re}(X)$. That is, $\bar{\ell}_{\Re}(X) = \underset{x \in U}{\cup} \{R(x): R(x) \cap X = \emptyset\}$.

Definition 2.2 [1] If $(U, \tau_R(X))$ in the context of X a nano space is established. Within this space if $A \subseteq X$, the nano interior of A is defined as the amalgamation of all nano open subsets contained within A is defined as the union of all nano open subsets of A and it is denoted by $int^N(A)$. That is, $int^N(A)$ is the largest nano open subset of A. Similarly, the nano closure of A is defined as the intersection of all nano closed sets include A, denoted as $_{Ncl(A)}$. In other words, cl^N signifies the smallest nano closed set that contains A.

Definition 2.3 [3] A subset A of a nano space $(U, \tau_R(X))$ is nano β-open in U if $A \subseteq cl^N(int^N(cl^N(A)))$. The set of all nano β-open sets of U is denoted by $\beta^N O(U)$.

2. Pawlak rough set theory

Definition 3.1 [4] Let $\mathcal{A}$ and $\mathcal{B}$ be two sets. We can define a binary relation R between $\mathcal{A}$ and $\mathcal{B}$ as a subset of the Cartesian product $\mathcal{A} \times \mathcal{B}$, consisting of ordered pairs $(\mathfrak{f}, \mathfrak{w}) \in \Re$, where $\mathfrak{f}$ belongs to A and $\mathfrak{w}$ belongs to B.

It is worth noting the relationship between two entities in a binary form. $\Re$ can also be defined from $\mathcal{A}$ to itself, in which case $\Re$ is referred to as A relation between two elements in a binary form. on $\mathcal{A}$. Moreover, we can represent the ordered pair $(\mathfrak{f}, \mathfrak{w})$ as $\mathfrak{f}\Re\mathfrak{w}$, indicating that $\mathfrak{w}$ is related to $\mathfrak{f}$ through the relation $\Re$. Consequently, $\mathfrak{f}$ is considered a forerunner of ω, and $\mathfrak{w}$ is regarded as a follower of $\mathfrak{f}$. We can describe the set $\theta\Re = \{\mathfrak{w} \in \mathcal{A}: \mathfrak{f}\Re\mathfrak{w}\}$ (or $\Re\mathfrak{f} = \{\mathfrak{w} \in A: \mathfrak{w}\Re\mathfrak{f}\}$) as the after set (or fore set) of the element $\mathfrak{f} \in \mathcal{A}$.

Definition 3.2 [4] A binary relation R defined on a set A is considered:

Reflexive: if for any $\mathfrak{f} \in \mathbb{A}, \mathfrak{f}\mathcal{R}\mathfrak{f}$.

(i) Symmetric: if for every $\mathfrak{f}, \mathfrak{w} \in \mathcal{A}$ and $\theta\Re\omega$, then $\mathfrak{w}\mathcal{R}\mathfrak{f}$.

(ii) Transitive: if for all $\mathfrak{f}, \mathfrak{w}, \mathfrak{P} \in \mathcal{A}$, $\mathfrak{f}\Re\mathfrak{w}$ and $\mathfrak{w}\Re\mathfrak{P}$, then $\mathfrak{f}\mathcal{R}\mathfrak{P}$.

(iii) Similarity or tolerance: if $\mathcal{R}$ exhibits both reflexivity and symmetry.

(iv) Pre-order or dominance: if $\mathcal{R}$ demonstrates reflexivity and transitivity.

(v) Equivalence: if $\mathcal{R}$ possesses the properties of reflexivity, symmetry, and transitivity.

Definition 3.3 If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $\mathcal{A} \subseteq U$ then its called:

(i) Nano β-closure is the intersection of all nano β-closed subsets of U containing A and it is denoted by $\bar{\mathcal{A}}^{N\beta}$ such that $\bar{\mathcal{A}}^{N\beta} = \cap \{\mathcal{F}: \beta^N C(U): \mathcal{A} \subseteq \mathcal{F}\}$.

(ii) Nano β-interior is the union of all nano β-open subsets of U contained in A and it is denoted by $\mathcal{A}^{0N\beta}$ such that $\mathcal{A}^{0N\beta} = \cup \{\mathfrak{G}: \beta^N O(U): \mathfrak{G} \subseteq \mathcal{A}\}$.

Theorem 3.1. If $A \subset X$, then the following hold.

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(i) $\mathcal{A}N\beta$ nano β-closed.
(ii) If $F \subset X$ is β -closed and $\mathcal{A} \subset F$ then $\mathcal{A}N\beta \subset F$.
(iii) A set $\mathcal{A}$ is considered to be β -closed if and only if $\mathcal{A}= \mathcal{A}N\beta$

Proof.
(i) Since the intersection of nano β-closed is nano β-closed and $\bar{\mathcal{A}}^{N\beta}$ is nano β-closed.
(ii) Obvious from the Definition 2.10.
(iii) It is obvious from (i).

Theorem 3.2 If $(T_{\mathcal{R}}(X), \mathfrak{U})$ is nano topological space and, $\mathcal{A}, \mathfrak{B} \subset \mathfrak{U}$ be nano subsets of $\mathfrak{U}$ then the next statements are valid.
(i) If $\mathcal{A} \subset \mathfrak{B}$ then $\bar{\mathcal{A}}^{N\beta} \subset \bar{\mathfrak{B}}^{N\beta}$.
(ii) $\left[\bar{\mathcal{A}}^{N\beta}\right]^{-N\beta} = \bar{\mathcal{A}}^{N\beta}$.
(iii) $\bar{\mathcal{A}}^{N\beta} \cup \bar{\mathfrak{B}}^{N\beta} \subset [\mathcal{A} \cup \mathfrak{B}]^{-N\beta}$.
(iv) $[\mathcal{A} \cap \mathfrak{B}]^{-N\beta} \subset \bar{\mathcal{A}}^{N\beta} \cap \bar{\mathfrak{B}}^{N\beta}$.
(v) $\bar{\emptyset}^{N\beta} = \emptyset$.

Proof.
(i) As $\bar{\mathcal{A}}^{N\beta} = \cap \{\mathcal{F}: \beta^N \mathcal{C}(\mathfrak{U}): \mathcal{A} \subseteq \mathcal{F}\}$ and $\mathcal{A} \subset \mathfrak{B}$ then $\bar{\mathcal{A}}^{N\beta} = \cap \{\mathcal{F}: \beta^N \mathcal{C}(\mathfrak{U}): \mathcal{A} \subseteq \mathcal{F}\} \subset = \cap \{\mathcal{F}: \beta^N \mathcal{C}(\mathfrak{U}): \mathfrak{B} \subseteq \mathcal{F}\} = \bar{\mathfrak{B}}^{N\beta}$.
(ii) As $\bar{\mathcal{A}}^{N\beta}$ is nano β-closed then from theorem 2.1 part (iii) we obtain $\left[\bar{\mathcal{A}}^{N\beta}\right]^{-N\beta} = \bar{\mathcal{A}}^{N\beta}$.
(iii) Since $\mathcal{A} \subset \mathcal{A} \cup \mathfrak{B}$ and $B \subset \mathcal{A} \cup \mathfrak{B}$ then $\bar{\mathcal{A}}^{N\beta} \subset [\mathcal{A} \cup \mathfrak{B}]^{-N\beta}$ (i) and $\bar{\mathfrak{B}}^{N\beta} \subset [\mathcal{A} \cup \mathfrak{B}]^{-N\beta}$ (ii) from (i) and (ii) we get $\bar{\mathcal{A}}^{N\beta} \cup \bar{\mathfrak{B}}^{N\beta} \subset [\mathcal{A} \cup \mathfrak{B}]^{-N\beta}$.
(i) As $\mathcal{A} \cap \mathfrak{B} \subset \mathcal{A}$ and $\mathcal{A} \cap \mathfrak{B} \subset \mathfrak{B}$ subsequently, $[\mathcal{A} \cap \mathfrak{B}]^{-N\beta} \subset \bar{\mathcal{A}}^{N\beta}$ (i) and $[\mathcal{A} \cap \mathfrak{B}]^{-N\beta} \subset \bar{\mathfrak{B}}^{N\beta}$ (ii) from (i) and (ii) we get $[\mathcal{A} \cap \mathfrak{B}]^{-N\beta} \subset \bar{\mathcal{A}}^{N\beta} \cap \bar{\mathfrak{B}}^{N\beta}$.
(ii) It is clear from that $\emptyset$ and $\mathfrak{U}$ are nano β-closed, hence $\bar{\emptyset}^{N\beta} = \emptyset$.

Then the equality sign of the statements (iii) and (iv) of the above Theorem 3.1 doesn't hold as the following examples.

Example 3.1 let $\mathfrak{U} = \{a, b, c, d\}$, $\frac{\mathfrak{U}}{R} = \{\{a\}, \{b\}, \{c, d\}\}, X = \{a, c\}$ and let the nano topological space on X is $T_{\mathcal{R}}(X) = \{\mathfrak{U}, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}\}$, ${T_{\mathcal{R}}}^{C}(X) = \{\mathfrak{U}, \emptyset, \{b\}, \{b, c, d\}, \{a, b\}\}$.
(i) If $\mathcal{A} = \{a, c\}, \mathfrak{B} = \{d\}$ then $\mathcal{A} \cup \mathfrak{B} = \{a, c, d\}$. $\mathcal{A}^{-N\beta} = \{a, c\}$, $\mathfrak{B}^{-N\beta} = \{d\}$ but $(\mathcal{A} \cup \mathfrak{B})^{-N\beta} = \mathfrak{U}$. Then $\bar{\mathcal{A}}^{N\beta} \cup \bar{\mathfrak{B}}^{N\beta} \neq [\mathcal{A} \cup \mathfrak{B}]^{-N\beta}$

Example 3.2 let $\mathfrak{U} = \{a, b, c, d\}$, $\frac{\mathfrak{U}}{R} = \{\{a, c\}, \{b\}, \{d\}\}, X = \{b, c, d\}$ and let the nano topological space on X is $T_{\mathcal{R}}(X) = \{\mathfrak{U}, \emptyset, \{b, d\}, \{b, c, d\}, \{c\}\}$, ${T_{\mathcal{R}}}^{C}(X) = \{\mathfrak{U}, \emptyset, \{a\}, \{a, c\}, \{a, b, d\}\}$
(i) If $\mathcal{A} = \{a, b\}, \mathfrak{B} = \{b, c, d\}$ then $\mathcal{A} \cap \mathfrak{B} = \{a\}$. $\mathcal{A}^{-N\beta} = \{a, b, d\}$, $\mathfrak{B}^{-N\beta} = \{a, c\}$ but $(\mathcal{A} \cap \mathfrak{B})^{-N\beta} = \{a, c\}$, and $\mathcal{A}^{-N\beta} \cap \mathfrak{B}^{-N\beta} = \{a\}$. And hence $[\mathcal{A} \cap \mathfrak{B}]^{-N\beta} \neq \bar{\mathcal{A}}^{N\beta} \cap \bar{\mathfrak{B}}^{N\beta}$

Theorem 3.3 If $(T_{\mathcal{R}}(X), \mathfrak{U})$ is nano topological space and, $\mathcal{A} \subset \mathfrak{U}$ be nano subsets of $\mathfrak{U}$ then the next statements are valid.
(i) $[\mathfrak{U} - \mathcal{A}]^{-N\beta} = \mathfrak{U} - \mathcal{A}^{0N\beta}$.
(ii) $[\mathfrak{U} - \mathcal{A}]^{0N\beta} = \mathfrak{U} - \mathcal{A}^{-N\beta}$.

Proof.
(i) Since $\mathcal{W} \subset \mathcal{A}$ if and only if $= \mathfrak{U} - \mathcal{A} \subset \mathfrak{U} - \mathcal{W}$, $\mathcal{W}$ is nano β-open if and only if $\mathfrak{U} - \mathcal{W}$ is nano β-closed. Thus $[\mathfrak{U} - \mathcal{A}]^{-N\beta} = \cap \{\mathfrak{U} - \mathcal{W}, \mathcal{W}$ is nano β-open set and $\mathcal{W} \subset \mathcal{A}\} = \mathfrak{U} - \cup \{\mathcal{W}, \mathcal{W}$ is nano β-open set and $\mathcal{W} \subset \mathcal{A}\} = \mathfrak{U} - \mathcal{A}^{0N\beta}$.
(ii) If we replace $\mathcal{A}$ by $\mathfrak{U} - \mathcal{A}$ in (i) we obtain $[\mathfrak{U} - (\mathfrak{U} - \mathcal{A})]^{-N\beta} = \mathfrak{U} - (\mathfrak{U} - \mathcal{A})^{0N\beta}$ which implies that $[\mathcal{A}]^{-N\beta} = \mathfrak{U} - (\mathfrak{U} - \mathcal{A})^{0N\beta}$ and hence $[\mathfrak{U} - \mathcal{A}]^{0N\beta} = \mathfrak{U} - \mathcal{A}^{-N\beta}$.

Theorem 3.4 Assuming $(U, \tau_R(X))$ is a nano topological space with respect to X. Then the following statements holds:
(i) $x \in (\mathcal{F})^{oN\beta}$ if there exist is nano $\beta - \text{open}$ $\mathcal{W}$ such that $x \in \mathcal{W} \subseteq \mathcal{F}$."
(ii) $(\mathcal{F})^{oN\beta}$ is nano β – open set.
(iii) If $\mathcal{G}$ is a nano $\beta - \text{open}$ set and $\mathcal{G} \subset \mathcal{F}$. Then $\mathcal{G} \subset (\mathcal{F})^{oN\beta}$.
(iv) $\mathcal{F}$ is nano $\beta - \text{open}$ set if and only if $\mathcal{F} = (\mathcal{F})^{oN\beta}$.

Proof:

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(i) Let $x \in (\mathcal{F})^{oN\beta}$, then $x \in \cup\{(\mathcal{W}_i : \mathcal{W}_i \subset \mathcal{F}\ \ \mathcal{W}_i \in N\beta O(\mathfrak{U})\}$. Hence there exist nano β – open set $(\mathcal{W}_{io}) \in N\beta O(\mathfrak{U})$ such that "$x \in \mathcal{W}_{io} \subset \mathcal{F}$.

(ii) Obvious

(iii) Assuming $\mathcal{G}$ is "a nano β – open set then $\mathcal{G} = (\mathcal{G})^{oN\beta}$ since $\mathcal{G} \subset \mathcal{F}$. This implies that $(\mathcal{G})^{oN\beta} \subset (\mathcal{F})^{oN\beta}$ and hence $\mathcal{G} \subset (\mathcal{F})^{oN\beta}$

(iv) Obvious.

Theorem 3.5. Assuming $(\mathfrak{U}, \tau_R(X))$ is a nano topological space with respect to$X$ and (M, I) is a nano subset of $\mathfrak{U}$ Then.

(i) $(M^C)^{-N\beta} = (M^{No\beta})^C$.

(ii) $((M)^C)^{No\beta} = ((M)^{-N\beta})^C$.

Proof.

(i) " Since $\mathcal{W} \subset \mathcal{M}$ if and only if $\mathcal{M}^C \subset W^c$, $\mathcal{W}$ is nano-β -closed-set if and only if $\mathcal{W}^c$is a nano soft-β -closed set, therefore $(M^{\ C})^{-N\beta}$ and $M^{-N\beta} = U - \cup\{\mathcal{W} \in N\beta O(\mathfrak{U}) : \mathcal{W} \subset M\} = (M^{-No\beta})^C$.

(ii) If we replace Mby $\mathcal{M}^C$in (1) we get $((\mathcal{M}^C)^C)^{-N\beta} = ((\mathcal{M})^C)^{oN\beta})^C$ which tends to $(\mathcal{M})^{-N\beta} = ((\mathcal{M}^C)^{oN\beta})^C$ and hence $((\mathcal{M})^C)^{oN\beta} = ((\mathcal{M})^{-N\beta})^C$.

Proposition 3.1 The intersection of nano α-open sets and nano β -open-set is nano β -open-set.

Proof. Let $\mathcal{A} \subset \mathfrak{U}$ be an nano α-open sets and $\mathcal{H} \subset \mathfrak{U}$ be nano open set , then $\mathcal{A} \cap \mathcal{H} \subset \mathcal{A}^{No-o} \cap \mathcal{H}^{-No-} \subset \mathcal{A}^{No-} \cap \mathcal{H}^{-No-} \subset (\mathcal{A}^o \cap \mathcal{H}^{-o})^{N-} \subset (\mathcal{A}^o \cap \mathcal{H}^{-})^{No-} \subset (\mathcal{A} \cap \mathcal{H})^{N-o-}$

Corollary 3.1. The union of nano α-closed sets and nano β -closed-set is nano β -closed-set

Theorem 3.6. If A is subset of nano topological space, then $\mathcal{A}^{No} \subset \mathcal{A}^{N\alpha o} \subset \mathcal{A}^{Npo} \subset \mathcal{A}^{oN\beta} \subset \mathcal{A} \subset \mathcal{A}^{-N\beta} \subset \mathcal{A}^{Np-} \subset \mathcal{A}^{N\alpha-} \subset \mathcal{A}^{N-}$.

Theorem 3.7 If $(T_{\mathcal{R}}(X), \mathfrak{U})$ is nano topological space and, $\mathcal{A}, \mathfrak{V} \subset \mathfrak{U}$ be nano subsets of $\mathfrak{U}$ then the next statements are valid.

(i) If $\mathcal{A} \subset \mathfrak{V}$ then $\mathcal{A}^{oN\beta} \subset \mathfrak{B}^{oN\beta}$.

(ii) $[\mathcal{A}^{oN\beta}]^{oN\beta} = \mathcal{A}^{oN\beta}$.

(iii) $\mathcal{A}^{oN\beta} \cup \mathfrak{B}^{oN\beta} \subset [\mathcal{A} \cup \mathfrak{B}]^{-N\beta}$.

(iv) $[\mathcal{A} \cap \mathfrak{B}]^{oN\beta} \subset \mathcal{A}^{oN\beta} \cap \mathfrak{B}^{oN\beta}$.

(v) $X^{oN\beta} = X$.

Proof.

(i) Since $\mathcal{A} \subset \mathfrak{V}$ then $\mathfrak{U} - \mathfrak{V} \subset \mathfrak{U} - \mathcal{A}$ since from Theorem 3.7 part (i) we get $(\mathfrak{U} - \mathfrak{V})^{-N\beta} \subset (\mathfrak{U} - \mathcal{A})^{-N\beta}$ and from Theorem 3.4 we obtain $\mathfrak{U} - (\mathfrak{V})^{0N\beta} \subset \mathfrak{U} - (\mathcal{A})^{0N\beta}$. Thus $\mathcal{A}^{oN\beta} \subset \mathfrak{B}^{oN\beta}$.

(ii) Since $[\mathcal{A}^{oN\beta}]^{oN\beta} = [[\mathfrak{U} - \mathcal{A}]^{-N\beta}]^{oN\beta} = \mathfrak{U} - \left[\mathfrak{U} - \left[\mathfrak{U} - [\mathfrak{U} - \mathcal{A}]^{-N\beta}\right]\right]^{-N\beta} = \mathfrak{U} - [[\mathfrak{U} - \mathcal{A}]^{-N\beta}]^{-N\beta} = \mathcal{A}^{oN\beta}$.

(iii) Obvious from (i).

(iv) Obvious from (i).

(v) It is obvious from the Theorem 3.4 part (iv), since X in nano β – open set then $X^{oN\beta} = X$

Then the equality sign of the statements (iii) and (iv) of Theorem 3.7 doesn't hold as the following example.

Example 3.3 let $\mathfrak{U} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathcal{d}\}$, $\frac{\mathfrak{U}}{R} = \{\{\mathfrak{a}\}, \{\mathfrak{b}\}, \{\mathfrak{c}, \mathcal{d}\}\}$, $X = \{\mathfrak{a}, \mathfrak{c}\}$ and let the nano topological space on X is $T_{\mathcal{R}}(X) = \{\mathfrak{U}, \emptyset, \{\mathfrak{a}\}, \{\mathfrak{c}, \mathcal{d}\}, \{\mathfrak{a}, \mathfrak{c}, \mathcal{d}\}\}$, $T_{\mathcal{R}}{}^C(X) = \{\mathfrak{U}, \emptyset, \{\mathfrak{b}\}, \{\mathfrak{b}, c, d\}, \{\mathfrak{a}, \mathfrak{b}\}\}$.

(i) If $\mathfrak{A} = \{\mathfrak{a}, \mathfrak{b}\}, \mathfrak{B} = \{\mathfrak{b}, \mathfrak{c}, \mathcal{d}\}$ then $\mathfrak{A} \cap \mathfrak{B} = \{\mathfrak{b}\}$. $\mathfrak{A}^{oN\beta} = \{\mathfrak{a}, \mathfrak{b}\}$, $\mathfrak{B}^{oN\beta} = \{\mathfrak{b}, \mathfrak{c}, \mathcal{d}\}$ but $(\mathfrak{A} \cap \mathfrak{B})^{oN\beta} = \phi$. Thus $[\mathfrak{A} \cap \mathfrak{B}]^{oN\beta} \neq \mathfrak{A}^{oN\beta} \cap \mathfrak{B}^{oN\beta}$.

Example 3.4 let $\mathfrak{U} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathcal{d}\}$, $\frac{\mathfrak{U}}{R} = \{\{\mathfrak{a}, \mathfrak{c}\}, \{\mathfrak{b}\}, \{\mathcal{d}\}\}$, $X = \{\mathfrak{b}, \mathfrak{c}, \mathcal{d}\}$ and let the nano topological space on X is $T_{\mathcal{R}}(X) = \{\mathfrak{U}, \emptyset, \{\mathfrak{b}, \mathcal{d}\}, \{\mathfrak{b}, \mathfrak{c}, \mathcal{d}\}, \{\mathfrak{c}\}\}$, $T_{\mathcal{R}}{}^C(X) = \{\mathfrak{U}, \emptyset, \{\mathfrak{a}\}, \{\mathfrak{a}, \mathfrak{c}\}, \{\mathfrak{a}, \mathfrak{b}, \mathcal{d}\}\}$

(i) If $\mathfrak{A} = \{\mathcal{d}\}$, $\mathfrak{B} = \{\mathfrak{a}, \mathfrak{b}\}$ then $\mathfrak{A} \cup \mathfrak{B} = \{\mathfrak{a}, \mathfrak{b}, \mathcal{d}\}$. $\mathcal{A}^{oN\beta} = \phi$, $\mathfrak{B}^{oN\beta} = \phi$ but $(\mathfrak{A} \cup \mathfrak{B})^{oN\beta} = \{a, b, d\}$. Then $\mathfrak{A}^{oN\beta} \cup \mathfrak{B}^{oN\beta} \neq [\mathfrak{A} \cup \mathfrak{B}]^{-N\beta}$.

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3. Prime neighborhood and its properties and generalized rough sets based on prime neighborhoods

In this section, we delve into the notion of a "prime neighborhood," which is derived from a previously introduced binary relation. We extensively analyze the characteristics of this concept. Expanding on the idea of the generalized neighborhood (prime-neighborhood), we establish novel generalized rough sets known as prime-approximations. We thoroughly examine the properties of these prime approximations to demonstrate their role as a broader extension of Pawlak's rough sets and certain other related generalizations.

Definition 4.1 Suppose we have a binary relation $\mathfrak{N}$ defined on the universe $\mathfrak{U}$. For every $\alpha \in \mathfrak{U}$, we can define its prime right neighborhood using the following expression:

$\mathfrak{N}_p(a) = \{g \in \mathfrak{U}: \mathfrak{N}_r(\alpha) \supseteq \mathfrak{N}_r(g)\}$.

Definition 4.2 Suppose we have a binary relation $\mathfrak{N}$ on $\mathfrak{U}$. We can propose the prime-lower and prime-upper approximations of the subset $\mathfrak{W} \subseteq \mathfrak{U}$, denoted as follows:

$\underline{\mathfrak{S}}_p(\mathfrak{W}) = \{\mathfrak{A} \in \mathfrak{U}: \mathfrak{N}_p(\mathfrak{A}) \subseteq \mathfrak{W}\}$ and $\overline{\mathfrak{S}}_p(\mathfrak{W}) = \{\mathfrak{A} \in \mathfrak{U}: \mathfrak{N}_p(\mathfrak{A}) \cap \mathfrak{W} \neq \varphi\}$.

Definition 4.3 Suppose we have a binary relation $\mathfrak{N}$ on $\mathfrak{U}$. We can define the prime-positive, prime-negative, and prime-boundary regions of a subset $\mathfrak{W} \subseteq \mathfrak{U}$,, as well as the accuracy of the prime-approximations, in the following manner:

$\mathcal{P}os_p(\mathfrak{W}) = \underline{\mathcal{L}}_p(\mathfrak{W})$, $\mathcal{N}eg_p(\mathfrak{W}) = \mathfrak{U} - \overline{\mathcal{U}}_p(\mathfrak{W})$, $\mathcal{B}nd_p(\mathfrak{W}) = \overline{\mathfrak{S}}_p(\mathfrak{W}) - \underline{\mathfrak{S}}_p(\mathfrak{W})$ and

$\kappa_p(\mathfrak{W}) = \frac{|\underline{\mathfrak{S}}_p(\mathfrak{W})|}{|\overline{\mathfrak{S}}_p(\mathfrak{W})|}$, where $\overline{\mathfrak{S}}_p(\mathfrak{W}) \neq \varphi$.

Definition 4.4 Consider $\mathfrak{U}$ as a finite set, and let's assume that we have the lower approximation $\underline{\mathfrak{S}}_p(\mathfrak{W})$ and $\overline{\mathfrak{S}}_p(\mathfrak{W})$ and the upper approximation for a subset $\mathfrak{W} \subseteq \mathfrak{U}$. The class $\mathfrak{T}_p^{\mathcal{G}n} = \{\mathfrak{U}, \varphi, \underline{\mathfrak{S}}_p(\mathfrak{W}), \overline{\mathfrak{S}}_p(\mathfrak{W}), \mathfrak{BN}_p(\mathfrak{W})\}$, where $\mathcal{B}nd_p(\mathfrak{W})$ represents the boundary region of $\mathfrak{W} \subseteq \mathfrak{U}$, forms a topology on $\mathfrak{U}$. Consequently, $\mathcal{T}^{\mathcal{G}n}$ is commonly referred to as a "generalized nano-topology" or briefly, $\boldsymbol{\mathcal{G}n}$-topology which is derived from the generalized rough approximations of $\mathfrak{W} \subseteq \mathfrak{U}$.

4. Utilizing Nano Topology for Attribute Reduction in Medical Diagnosis Applications

In this section, we utilize the simplification of attributes within a knowledge framework to identify significant indicators of a prevalent health condition known as "Chronic Kidney." By employing prime neighborhoods, we aim to pinpoint the notable signs that prompt patients to seek medical advice from doctors. Table 5.1 provides an overview of the indicators associated with chronic kidney, including Family History, Hypertension, Obesity, and Diabetes, which may occur individually or in combination.

Table 5. 1 Information decision data.

Patients	Family History ($\mathcal{F}$)	Hypertension ($\mathcal{H}$)	Obesity ($\mathcal{O}$)	Diabetes ($\mathcal{D}$)	Decision
$\mathcal{P}_1$	1	1	0	0	1
$\mathcal{P}_2$	0	1	0	0	1
$\mathcal{P}_3$	1	0	1	1	1
$\mathcal{P}_4$	0	1	1	0	0
$\mathcal{P}_5$	1	0	0	1	0
$\mathcal{P}_6$	1	1	1	0	1

Based on the information presented in the table, we can derive the symptoms exhibited by each patient as follows:

$V(p_1) = \{\mathcal{D}, \mathcal{H}\}$, $V(p_2) = \{\mathcal{H}\}$, $V(p_3) = \{\mathcal{D}, \mathcal{O}, \mathcal{F}\}$, $V(p_4) = \{\mathcal{H}, \mathcal{O}\}$, $V(p_5) = \{\mathcal{D}, \mathcal{F}\}$, $V(p_6) = \{\mathcal{D}, \mathcal{H}, \mathcal{O}\}$, Now, we can generate the following relation:

$p_i R p_j \Leftrightarrow V(p_i) \supseteq V(p_j)$.

$\mathfrak{R} = (p_1, p_1), (p_1, p_2), (p_2, p_2), (p_3, p_3), (p_3, p_5), (p_4, p_4), (p_4, p_2), (p_5, p_5), (p_6, p_6),$

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$(\mathcal{P}_6, \mathcal{P}_2), (\mathcal{P}_6, \mathcal{P}_1), (\mathcal{P}_6, p_4)\}$.

Therefore, the right vicinity of every element in $\mathfrak{U}$ with respect to this relationship can be described as follows:

$\mathfrak{R}_r(\mathcal{P}_1) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\mathfrak{R}_r(\mathcal{P}_2) = \{\mathcal{P}_2\}$, $\mathfrak{R}_r(\mathcal{P}_3) = \{\mathcal{P}_3, \mathcal{P}_5\}$, $\mathfrak{R}_r(\mathcal{P}_4) = \{\mathcal{P}_2, \mathcal{P}_4\}$,

$\mathfrak{R}_r(\mathcal{P}_5) = \{\mathcal{P}_5\}$,and $\mathfrak{R}_r(\mathcal{P}_6) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_4, \mathcal{P}_6\}$.

Accordingly, the initial neighborhoods are:

$\mathfrak{R}_p(\mathcal{P}_1) = \{\mathcal{P}_1\}$, $\mathfrak{R}_p(\mathcal{P}_2) = \{\mathcal{P}_2\}$, $\mathfrak{R}_p(\mathcal{P}_3) = \{\mathcal{P}_3\}$, $\mathfrak{R}_p(\mathcal{P}_4) = \{\mathcal{P}_2, \mathcal{P}_4\}$,

$\mathfrak{R}_p(\mathcal{P}_5) = \{\mathcal{P}_5\}$, and $\mathfrak{R}_p(\mathcal{P}_6) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_6\}$,

Now, referring to Table 4.1, we can identify two specific cases:

Situation 1: ((Individuals afflicted with Chronic kidney)

The set of Individuals afflicted with **Chronic kidney** is $X = \{p_1, p_2, p_3, p_6\}$, then we get:

$\underline{\mathfrak{S}}_p(X) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\overline{\mathfrak{S}}_p(X) = \{\mathcal{P}_1, \mathcal{P}, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}$ and $\mathfrak{BN}_p(X) = \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}$.

Therefore, the $\mathcal{G}n$-topology and its basis of X are:

$\mathfrak{T}_p^{\mathcal{G}n} = \{\mathfrak{U}, \varphi, \{\mathcal{P}_1, \mathcal{P}_2\}, \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}, \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}\}$ and $\mathcal{B}_p^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{P}_1, V_2\}, \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}\}$

Step 1 Upon the exclusion of the attribute "Family History" is removed: Therefore, the symptoms of every patient are: Case.

$V(\mathcal{P}_1) = \{\mathcal{H}\}$, $V(\mathcal{P}_2) = \{\mathcal{H}\}$, $V(\mathcal{P}_3) = \{\mathcal{O}, \mathcal{F}\}$, $V(\mathcal{P}_4) = \{\mathcal{H}, \mathcal{O}\}$, $V(\mathcal{P}_5) = \{\mathcal{F}\}$, $V(\mathcal{P}_6) = \{\mathcal{H}, \mathcal{O}\}$,

Therefore, the right vicinity of every element in $\mathfrak{U}$ with respect to this relationship can be described as follows:

$\mathfrak{R}_r(\mathcal{P}_1) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\mathfrak{R}_r(\mathcal{P}_2) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\mathfrak{R}_r(\mathcal{P}_3) = \{\mathcal{P}_3, \mathcal{P}_5\}$, $\mathfrak{R}_r(\mathcal{P}_4) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_4, \mathcal{P}_6\}$,

$\mathfrak{R}_r(\mathcal{P}_5) = \{\mathcal{P}_5\}$, and $\mathfrak{R}_r(\mathcal{P}_6) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_4, \mathcal{P}_6\}$.

Accordingly, the prime neighborhoods are:

$\mathfrak{R}_p(\mathcal{P}_1) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\mathfrak{R}_p(\mathcal{P}_2) = \{\mathcal{P}_1, p_2\}$, $\mathfrak{R}_p(\mathcal{P}_3) = \{\mathcal{P}_3, \mathcal{P}_5\}\mathfrak{R}_p(\mathcal{P}_4) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_4, \mathcal{P}_6\}$, $\mathfrak{R}_p(\mathcal{P}_5) = \{\mathcal{P}_5\}$,and $\mathfrak{R}_p(\mathcal{P}_6) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_4, \mathcal{P}_6\}$.

The set of Individuals afflicted with **Chronic kidney** is $X = \{p_1, p_2, p_3, p_6\}$, then we get:

$\underline{\mathfrak{S}}_p(X) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\overline{\mathfrak{S}}_p(X) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}$ and $\mathcal{B}_p^{\mathcal{G}n}(X) = \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}$.

Therefore, the $\mathcal{G}n$-topology and its basis of X are:

$\mathfrak{T}_{p\ A-\{\mathcal{F}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \varphi, \{\mathcal{P}_1, \mathcal{P}_2\}, \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}, \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}\}$ and $\mathcal{B}_p^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{P}_1, \mathcal{P}_2\}, \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}\}$ and then we get $\mathfrak{T}_{p\ A-\{\mathcal{F}\}}^{\mathcal{G}n} = \mathfrak{T}_p^{\mathcal{G}n}$ and the base is $\mathcal{B}_{p\ A-\{\mathcal{D}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{P}_1, \mathcal{P}_2\}, \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}\} = \mathcal{B}_p^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{P}_1, \mathcal{P}_2\}, \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}\}$

Step 2: Upon excluding the attribute "Hypertension" employing a similar approach as observed in Step 1,

$V(\mathcal{P}_1) = \{\mathcal{D}\}$, $V(\mathcal{P}_2) = \{\}$, $V(\mathcal{P}_3) = \{\mathcal{D}, \mathcal{O}, \mathcal{F}\}$, $V(\mathcal{P}_4) = \{\mathcal{O}\}$, $V(\mathcal{P}_5) = \{\mathcal{D}, \mathcal{F}\}$, $V(\mathcal{P}_6) = \{\mathcal{D}, \mathcal{O}\}$,

$\mathfrak{R}_r(\mathcal{P}_1) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\mathfrak{R}_r(\mathcal{P}_2) = \{\mathcal{P}_2\}$, $\mathfrak{R}_r(\mathcal{P}_3) = \mathfrak{U}$, $\mathfrak{R}_r(p_4) = \{\mathcal{P}_2, \mathcal{P}_4\}$,

$\mathfrak{R}_r(\mathcal{P}_5) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_5\}$, and $\mathfrak{R}_r(\mathcal{P}_6) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_4, \mathcal{P}_6\}$.

we get the prime neighborhoods are:

$\mathfrak{R}_p(\mathcal{P}_1) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\mathfrak{R}_p(\mathcal{P}_2) = \{\mathcal{P}_2\}$, $\mathfrak{R}_p(\mathcal{P}_3) = \mathfrak{U}$, $\mathfrak{R}_p(\mathcal{P}_4) = \{\mathcal{P}_2, \mathcal{P}\}$, $\mathfrak{R}_p(p_5) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_5\}$,and $\mathfrak{R}_p(\mathcal{P}_6) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_4, \mathcal{P}_6\}$

The set of Individuals afflicted with **Chronic kidney** is $X = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_6\}$, then we get:

$\underline{\mathfrak{S}}_p(X) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\overline{\mathfrak{S}}_p(X) = \mathfrak{U}$, and $\mathcal{B}_p^{\mathcal{G}n}(X) = \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_6\}$.

Consequently, the Gn-topology and its foundation on X can be summarized as:

$\mathfrak{T}_{p\ A-\{\mathcal{H}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \varphi, \{\mathcal{P}_1, \mathcal{P}_2\}, \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_6\}\} \neq \mathfrak{T}_p^{\mathcal{G}n}$ and

$\mathfrak{B}_{A-\{\mathcal{H}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \{p_1, p_2\}, \{p_3, p_4, p_5, p_6\}\} \neq \mathcal{B}_p^{\mathcal{G}n}$.

Step 3: After eliminating the attribute "Obesity" By a similar approach as observed in ,Step 1, we get the prime neighbourhoods are:

$\mathfrak{R}_p(\mathcal{P}_1) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\mathfrak{R}_p(\mathcal{P}_2) = \{\mathcal{P}_2\}$, $\mathfrak{R}_p(\mathcal{P}_3) = \{\mathcal{P}_3, \mathcal{P}_5\}$, $\mathfrak{R}_p(\mathcal{P}_4) = \{\mathcal{P}_2, \mathcal{P}_4\}$, $\mathfrak{R}_p(\mathcal{P}_5) = \{\mathcal{P}_5\}$,and $\mathfrak{R}_p(\mathcal{P}_6) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_4, \mathcal{P}_6\}$

The set of Individuals afflicted with **Chronic kidney** is $X = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_6\}$, then we get:

$\underline{\mathfrak{S}}_p(X) = \{\mathcal{P}_1, \mathcal{P}_2\}$, $\overline{\mathfrak{S}}_p(X) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}, \mathcal{P}_6\}$ and $\mathcal{B}_p^{\mathcal{G}n}(X) = \{\mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_6\}$.

Consequently, the Gn-topology and its foundation on X can be summarized as:

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$:\mathfrak{T}_{p}^{\mathcal{G}n}{}_{A-\{\mathcal{O}\}} = \{\mathfrak{U}, \varphi, \{p_1, p_2\}, \{p_1, p_2, p_3, p_4, p_6\}, \{p_3, p_4, p_6\}\} = \mathfrak{T}_{p}^{\mathcal{G}n}$ and

$\mathcal{B}_{p}^{\mathcal{G}n}{}_{A-\{\mathcal{O}\}} = \mathcal{B}_{p}^{\mathcal{G}n}\{\mathfrak{U}, \{p_1, p_2\}, \{p_3, p_4, p_6\}\}$.

Step 4: After eliminating the attribute "Diabetes" By a similar approach as observed in **Step 1**, we get the prime neighborhoods are:

$\mathfrak{R}_p(p_1) = \{p_1, p_2, p_5\}$, $\mathfrak{R}_p(p_2) = \{p_2\}$, $\mathfrak{R}_p(p_3) = \{p_3, p_5\}$, $\mathfrak{R}_p(p_4) = \{p_2, p_4\}$, $\mathfrak{R}_p(p_5) = \{p_5\}$,and $\mathfrak{R}_p(p_6) = \mathfrak{U}$

The set of Individuals afflicted with **Chronic kidney** is $X = \{p_1, p_2, p_3, p_6\}$, then we get:

$\underline{\mathfrak{S}}_p(X) = \{p_2\}$, $\overline{\mathfrak{S}}_p(X) = \{p_1, p_2, p_3, p_4, p_6\}$ and $\mathcal{B}_{p}^{\mathcal{G}n}(X) = \{p_1, p_3, p_4, p_6\}$.

Therefore, the $\mathcal{G}n$-topology and its basis of X are:

$\mathfrak{T}_{p}^{\mathcal{G}n}{}_{A-\{\mathcal{D}\}} = \{\mathfrak{U}, \varphi, \{p_1, p_2\}, \{p_1, p_2, p_3, p_4, p_6\}, \{p_1, p_3, p_4, p_6\}\} \neq \mathfrak{T}_{p}^{\mathcal{G}n}$ and

$\mathcal{B}_{p}^{\mathcal{G}n}{}_{A-\{\mathcal{D}\}} \neq \mathcal{B}_{p}^{\mathcal{G}n}\{\mathfrak{U}, \{p_1, p_2\}, \{p_1, p_3, p_4, p_6\}\}$.

Therefore, based on Steps (1-4), it is evident that the CORE, consisting of "Diabetes (D)" and "Hypertension (H)," represents the influential factors in determining the occurrence of **Chronic kidney** infection.

Situation 2: (Patients are not infected with Chronic kidney)

The set of Individuals afflicted with **Chronic kidney** is $Y = \{p_4, p_5\}$. By following the identical procedure as in **Case (1),** we achieve identical outcomes.

Observation:

Based on the CORE analysis, we have identified that Diabetes (D) and Hypertension (H) play pivotal roles in Chronic kidney infection. Therefore, these attributes are indispensable factors that significantly contribute to the development of **Chronic kidney** infection. Towards the conclusion of the paper, we present an algorithm that utilizes our methodologies for making informed decisions.

Algorithm-I	**A decision making via the prime approximations.**
Step 1:	Consider a finite universe $\mathfrak{U}$ along with a collection of attributes that represent the data in the format of an information table. In this table, the attributes are labeled as rows ($\mathcal{C}$), the objects are labeled as columns, and the attribute values are recorded as entries within the table.
Step 2:	Calculate the prime neighborhoods of every object based on Definition 4.1 using the provided information table.
Step 3:	Calculate the prime-upper approximation, prime-lower approximation, and prime-boundary for the decision set using the available data. $\mathfrak{W} \subseteq \mathfrak{U}$, namely, $\underline{\mathfrak{S}}_p(\mathfrak{W})$, $\overline{\mathfrak{S}}_p(\mathfrak{W})$ and $\mathfrak{BN}_p(\mathfrak{W})$ Accordingly, as per Definition 4.2.
Step 4:	Construct the $\mathcal{G}n$-topology $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ on $\mathfrak{U}$ induced by $\mathfrak{W}$ using Definition 4.4.
Step 5:	Construct the class the base $\mathcal{B}_{p}^{\mathcal{G}n}$ for the $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$.
Step 6:	Eliminate a feature a_1 from the condition's features ($\mathcal{C}$) and subsequently determine the prime-upper approximation, prime-lower approximation and prime-boundary for the decision set $\mathfrak{W} \subseteq \mathfrak{U}$ on $\mathcal{C} - (a_1)$.
Step 7:	Construct a $\mathcal{G}n$-topology $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ on $\mathfrak{U}$ Definition 4.4.
Step 8:	Construct the class of nano topological space $\mathfrak{T}_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}}$ for the entire table after excluding the features a_1
Step 9:	Iterate the steps 6 and 7 for all features in $\mathcal{C}$.
Step 10:	Those features in $\mathcal{C}$ for which $\mathfrak{T}_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}} \neq \mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ *or* $\mathcal{B}_{p_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}}} \neq \mathcal{B}_{p}^{\mathcal{G}n}$ form the CORE ($\mathfrak{U}$).

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```
% Step 1: Input the data table
U = [   1 1 0 0 1;    0 1 0 0 1;  1 0 1 1 1;  0 1 1 0 0; 1 0 0 1 0; 1 1 1 0 1];
% Step 2: Calculate prime neighborhoods
N_p = cell(size(U, 2), 1);
for i = 1:size(U, 2)
   N_p{i} = find(all(U(:, i:end), 2));
end
% Step 3: Calculate prime-upper, prime-lower, and prime-boundary approximations
X = U(:, 1:end-1); % Decision set X
decision = U(:, end);
prime_lower = cell(size(U, 2)-1, 1);
prime_upper = cell(size(U, 2)-1, 1);
boundary_region = cell(size(U, 2)-1, 1);
for i = 1:size(U, 2)-1
   prime_lower{i} = find(all(X(:, setdiff(1:size(U, 2)-1, i)), 2));
   prime_upper{i} = find(any(X(:, setdiff(1:size(U, 2)-1, i)), 2));
   boundary_region{i} = setdiff(prime_upper{i}, prime_lower{i});
end
% Step 4: Construct Gn-topology
T_C_Gn = cell(size(U, 2)-1, 1);
for i = 1:size(U, 2)-1
   T_C_Gn{i} = {U, [], prime_lower{i}, prime_upper{i}, boundary_region{i}};
end
% Step 5: Construct class base B_p_Gn
B_p_Gn = cell(size(U, 2)-1, 1);
for i = 1:size(U, 2)-1
   B_p_Gn{i} = unique([prime_lower{i}; boundary_region{i}]);
end

% Step 6 and 7: Eliminate features and construct Gn-topology
CORE = [];
for i = 1:size(U, 2)-1
   X_temp = X(:, setdiff(1:size(U, 2)-1, i));
   prime_lower_temp = find(all(X_temp, 2));
   prime_upper_temp = find(any(X_temp, 2));
   boundary_region_temp = setdiff(prime_upper_temp, prime_lower_temp);
   T_C_Gn_temp = {U, [], prime_lower_temp, prime_upper_temp, boundary_region_temp};

   if ~isequal(T_C_Gn_temp, T_C_Gn{i}) || ~isequal(prime_lower_temp, prime_lower{i})
      CORE = [CORE i];
   end
end

% Display the CORE features
disp("CORE features: " + num2str(CORE));
```

Note: The above code assumes that you have implemented the necessary functions to perform the calculations and constructs described in the algorithm. You will need to define and implement those functions accordingly.

To incorporate fractal elements into the given algorithm using MATLAB, we can modify the steps as follows:

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Algorithm-II	A decision making via the prime-approximations based on fractals
Step 1:	Consider a finite universe $\mathfrak{U}$ along with a collection of attributes that represent the data in the format of an information table. In this table, the attributes are labeled as rows ($\mathcal{C}$), the objects are labeled as columns, and the attribute values are recorded as entries within the table.
Step 2:	Calculate the prime neighborhoods of every object based on Definition 4.1 using the provided information table.
Step 3:	Calculate the prime-upper approximation, prime-lower approximation, and prime-boundary for the decision set using the available data. $\mathfrak{W} \subseteq \mathfrak{U}$, namely, $\underline{\mathfrak{S}}_{\mathcal{p}}(\mathfrak{W})$, $\overline{\mathfrak{S}}_{\mathcal{p}}(\mathfrak{W})$ and $\mathfrak{BN}_{\mathcal{p}}(\mathfrak{W})$ Accordingly, as per Definition 4.2.
Step 4:	Construct the $\mathcal{G}n$-topology $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ on $\mathfrak{U}$ induced by $\mathfrak{W}$ using Definition 4.4. % Fractal Modification for Step 4: % Apply fractal-based techniques to construct the Gn-topology $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ % at the nanoscale. % Use fractal dimension estimation methods, such as box counting or % Harsdorf dimension, to quantify the fractal dimension of the data. % Adjust the topology construction process based on the fractal dimension, % creating self-similar structures and connectivity patterns within $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$.
Step 5:	Construct the class the base $\mathcal{B}_{\mathcal{p}}^{\mathcal{G}n}$ for the $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$.
Step 6:	Iterate through each feature a_1 in ($\mathcal{C}$): - Eliminate feature a_1 from the condition's features $\mathcal{C}$ and obtain $\mathcal{C} - (a_1)$. Determine the prime-upper approximation, prime-lower approximation, and prime-boundary for the decision set $\mathfrak{W} \subseteq \mathfrak{U}$ on $\mathcal{C} - (a_1)$.
Step 10:	Identify the features in $\mathcal{C}$ for which $\mathfrak{T}_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}} \neq \mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ *or* $\mathcal{B}_{\mathcal{p}_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}}} \neq \mathcal{B}_{\mathcal{p}}^{\mathcal{G}n}$ form the CORE ($\mathfrak{U}$).

% Fractal Modification for Step 6:
% Apply fractal-based techniques to determine the prime-upper
% approximation, prime-lower approximation, and prime-boundary
% for the decision set W ⊆ U on C - (a_1).
% Utilize fractal analysis methods to capture the self-similarity and
% structural characteristics of the data after excluding feature a_1.
Step 7: Construct a Gn-topology T_C^Gn on U using Definition 4.4.
% Fractal Modification for Step 7:
% Apply fractal-based techniques to construct the Gn-topology T_C^Gn
% at the nanoscale after excluding feature a_1.
% Adjust the topology construction process based on the fractal dimension,
% creating self-similar structures and connectivity patterns within T_C^Gn.
Step 8: Construct the class of the nano topological space T_(U_(C-{a_1})) for the entire table after excluding feature a_1.
% Fractal Modification for Step 8:
% Apply fractal-based techniques to construct the nano topological space
% T_(U_(C-{a_1})) for the entire table after excluding feature a_1.
% Incorporate fractal analysis and topology to capture the self-similarity,
% connectivity, and properties of the data within T_(U_(C-{a_1})).
Step 9: Iterate Steps 6-8 for all features in C.

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5. Utilizing Nano Topology for Attribute Reduction in Medical Diagnosis Applications

In this section, we simplify attributes in a knowledge framework to find key indicators of "Diabetes." Using prime neighborhoods, we aim to identify important signs that lead patients to consult doctors. Table 6.1 gives an overview of Diabetes indicators, including Family History, Hypertension, Obesity, and Diabetes, occurring individually or combined.

Table 6. 1 Information decision data.

Patients	Frequent Urination ($\mathcal{F}$)	Weight Loss ($\mathcal{W}$)	Increased Hunger ($\mathcal{I}$)	Decision
$\mathcal{p}_1$	1	1	0	1
$\mathcal{p}_2$	1	0	1	0
$\mathcal{p}_3$	1	0	0	1
$\mathcal{p}_4$	0	1	1	1
$\mathcal{p}_5$	0	1	0	1
$\mathcal{p}_6$	0	0	1	1

Based on the information presented in the table, we can derive the symptoms exhibited by each patient as follows:

$V(\mathcal{p}_1) = \{\mathcal{F}, \mathcal{W}\},\ V(\mathcal{p}_2) = \{\mathcal{F}, \mathcal{I}\},\ V(\mathcal{p}_3) = \{\mathcal{F}\},\ V(\mathcal{p}_4) = \{\mathcal{W}, \mathcal{I}\},\ V(\mathcal{p}_5) = \{\mathcal{W}\},\ V(\mathcal{p}_6) = \{\mathcal{I}\},$

Now, we can generate the following relation:

$\mathcal{p}_i R \mathcal{p}_j \Leftrightarrow V(\mathcal{p}_i) \supseteq V(\mathcal{p}_j)$.

$\mathfrak{R} =$

$\{(\mathcal{p}_1, \mathcal{p}_1), (\mathcal{p}_1, \mathcal{p}_3), (\mathcal{p}_3, \mathcal{p}_3), (\mathcal{p}_1, \mathcal{p}_5), (\mathcal{p}_4, \mathcal{p}), (\mathcal{p}_2, \mathcal{p}_2), (\mathcal{p}_5, \mathcal{p}_5), (\mathcal{p}_2, \mathcal{p}_6), (\mathcal{p}_4, \mathcal{p}_5), (\mathcal{p}_4, \mathcal{p}_6), (\mathcal{p}_6, \mathcal{p}_6)\}$.

Therefore, the right vicinity of every element in $\mathfrak{U}$ with respect to this relationship can be described as follows:

$\mathfrak{R}_r(\mathcal{p}_1) = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\},\ \mathfrak{R}_r(\mathcal{p}_2) = \{\mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_6\},\ \mathfrak{R}_r(\mathcal{p}_3) = \{\mathcal{p}_3\},\ \mathfrak{R}_r(\mathcal{p}_4) = \{\mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\},$

$\mathfrak{R}_r(\mathcal{p}_5) = \{\mathcal{p}_5\}$, and $\mathfrak{R}_r(\mathcal{p}_6) = \{\mathcal{p}_6\}$.

Accordingly, the prime neighborhoods are:

$\mathfrak{R}_p(\mathcal{p}_1) = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\},\ \mathfrak{R}_p(\mathcal{p}_2) = \{\mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_6\},\ \mathfrak{R}_p(\mathcal{p}_3) = \{\mathcal{p}_3\},\ \mathfrak{R}_p(\mathcal{p}_4) = \{\mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\},$

$\mathfrak{R}_p(\mathcal{p}_5) = \{\mathcal{p}_5\}$,and $\mathfrak{R}_p(\mathcal{p}_6) = \{\mathcal{p}_6\}$, if $X = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$then , $\underline{\mathfrak{S}}_p(X) = X$, $\overline{\mathfrak{S}}_p(X) = \mathfrak{U}$ and $\mathcal{B}_p^{\mathcal{G}n}(X) = \{\mathcal{p}_2\}$.

$\mathfrak{T}_p^{\mathcal{G}n} = \{\mathfrak{U}, \varphi,\ \{\mathcal{p}_2\}, \{\mathcal{p}_1, \mathcal{p}, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}\}$ and $\mathcal{B}_p^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{p}_2\}, \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}\}$

Now, from Table 4.2, we get two cases are:

Situation 1: (Individuals afflicted with Diabetes)

Step 1: Upon the exclusion of the attribute "Frequent Urination," it follows that the symptoms exhibited by each individual are as follows:

$V(p_1) = \{\mathcal{H}\},\ V(p_2) = \{\mathcal{H}\},\ V(p_3) = \{\mathcal{O}, \mathcal{F}\},\ V(p_4) = \{\mathcal{H}, \mathcal{O}\},\ V(p_5) = \{\mathcal{F}\},\ V(p_6) = \{\mathcal{H}, \mathcal{O}\},$

Therefore, the right vicinity of every element in $\mathfrak{U}$ with respect to this relationship can be described as follows:

$\mathfrak{R}_r(\mathcal{p}_1) = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\},\ \mathfrak{R}_r(\mathcal{p}_2) = \{\mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_6\},\ \mathfrak{R}_r(\mathcal{p}_3) = \{\mathcal{p}_3\},\ \mathfrak{R}_r(\mathcal{p}_4) = \mathfrak{U},$

$\mathfrak{R}_r(\mathcal{p}_5) = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\}$, and $\mathfrak{R}_r(p_6) = \{\mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_6\}$.

Accordingly, the prime neighborhoods are:

$\mathfrak{R}_p(\mathcal{p}_1) = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\}$, $\mathfrak{R}_p(\mathcal{p}_2) = \{\mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_6\}$, $\mathfrak{R}_p(\mathcal{p}_3) = \{\mathcal{p}_3\}, \mathfrak{R}_p(\mathcal{p}_4) = \mathfrak{U}$, $\mathfrak{R}_p(\mathcal{p}_5) = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\}$,and $\mathfrak{R}_p(\mathcal{p}_6) = \{\mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_6\}$.

The set of infected patients with **Diabetes** is $X = \{p_1, p_3, p_4, p_5, p_6\}$, then we get:

$\underline{\mathfrak{S}}_p(X) = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\}$, $\overline{\mathfrak{S}}_p(X) = \mathfrak{U}$, and $\mathcal{B}_p^{\mathcal{G}n}(X) = \{\mathcal{p}_2, \mathcal{p}_4, \mathcal{p}_6\}$.

Therefore, the $\mathcal{G}n$-topology and its basis of X are:

$\mathfrak{T}_{p\ A-\{\mathcal{F}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \varphi,\ \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\}, \{\mathcal{p}_2, \mathcal{p}_4, \mathcal{p}_6\}\}$ and $\mathcal{B}_p^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\}, \{\mathcal{p}_2, \mathcal{p}_4, \mathcal{p}_6\}\}$ and then we get $\mathfrak{T}_{p\ A-\{\mathcal{F}\}}^{\mathcal{G}n} \neq \mathfrak{T}_p^{\mathcal{G}n}$ and the base is $\mathcal{B}_{p\ A-\{\mathcal{F}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_5\}, \{\mathcal{p}_2, \mathcal{p}_4, \mathcal{p}_6\}\} \neq \mathcal{B}_p^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{p}_2\}, \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}\}$

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Step 2: Upon excluding the attribute "Weight Loss," employing a similar approach as observed in **Step 1**,

$V(\mathcal{p}_1) = \{\mathcal{F}\}$, $V(\mathcal{p}_2) = \{\mathcal{D}, \mathcal{I}\}$, $V(\mathcal{p}_3) = \{\mathcal{I}\}$, $V(\mathcal{p}_4) = \{\mathcal{I}\}$, $V(\mathcal{p}_5) = \{\}$, $V(\mathcal{p}_6) = \{\mathcal{I}\}$,

$\mathfrak{R}_r(\mathcal{p}_1) = \{\mathcal{p}_1, \mathcal{p}\}$, $\mathfrak{R}_r(\mathcal{p}_2) = \mathfrak{U}$, $\mathfrak{R}_r(\mathcal{p}_3) = \{\mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$, $\mathfrak{R}_r(p_4) = \{\mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$,

$\mathfrak{R}_r(\mathcal{p}_5) = \{\mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$, and $\mathfrak{R}_r(p_6) = \{\mathcal{p}_3, \mathcal{p}_4, \mathcal{p}, \mathcal{p}_6\}$.

we get the prime neighborhoods are:

$\mathfrak{R}_p(\mathcal{p}_1) = \{\mathcal{p}_1\}$, $\mathfrak{R}_p(\mathcal{p}_2) = \mathfrak{U}$, $\mathfrak{R}_p(\mathcal{p}_3) = \{\mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$, $\mathfrak{R}_p(\mathcal{p}_4) = \{\mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$, $\mathfrak{R}_p(\mathcal{p}_5\} = \{\mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$,and $\mathfrak{R}_p(\mathcal{p}_6) = \{\mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$

The set of infected patients with **Diabetes** is $X = \{p_1, p_3, p_4, p_5, p_6\}$, hen we get:

$\underline{\mathfrak{S}}_p(X) = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$, $\overline{\mathfrak{S}}_p(X) = \mathfrak{U}$, and $\mathcal{B}_p^{\mathcal{G}n}(X) = \{\mathcal{p}_2\}$.

Consequently, the Gn-topology and its foundation on X can be summarized as :

$\mathfrak{T}_{p\ A-\{\mathcal{W}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \varphi, \{\mathcal{p}_2\}, \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}\} = \mathfrak{T}_p^{\mathcal{G}n}$ and

$\mathfrak{B}_{A-\{\mathcal{W}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{p}_2\}, \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}\} = \mathcal{B}_p^{\mathcal{G}n}$.

Step 3: After eliminating the attribute "Increased Hunger" following the same procedure as in:

By a similar approach as observed in **Step 1**, we get the prime-neighborhoods are:

$\mathfrak{R}_p(\mathcal{p}_1) = \mathfrak{U}$, $\mathfrak{R}_p(\mathcal{p}_2) = \{\mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_6\}$, $\mathfrak{R}_p(\mathcal{p}_3) = \{\mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_6\}$, $\mathfrak{R}_p(\mathcal{p}_4) = \{\mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$, $\mathfrak{R}_p(\mathcal{p}_5) = \{\mathcal{p}, \mathcal{p}_5, \mathcal{p}_6\}$,and $\mathfrak{R}_p(\mathcal{p}_6) = \{\mathcal{p}_6\}$

Algorithm-III	A decision making via the prime-approximations.
Step 1:	Consider a finite universe $\mathfrak{U}$ along with a collection of attributes that represent the data in the format of an information table. In this table, the attributes are labeled as rows ($\mathcal{C}$), the objects are labeled as columns, and the attribute values are recorded as entries within the table.
Step 2:	Calculate the prime neighborhoods of every object based on Definition 4.1 using the provided information table.
Step 3:	Calculate the prime-upper approximation, prime-lower approximation, and prime-boundary for the decision set using the available data. $\mathfrak{W} \subseteq \mathfrak{U}$, namely, $\underline{\mathfrak{S}}_p(\mathfrak{W})$, $\overline{\mathfrak{S}}_p(\mathfrak{W})$ and $\mathfrak{BN}_p(\mathfrak{W})$ Accordingly, as per Definition 4.2.
Step 4:	Construct the $\mathcal{G}n$-topology $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ on $\mathfrak{U}$ induced by $\mathfrak{W}$ using Definition 4.4.
Step 5:	Construct the class the base $\mathcal{B}_p^{\mathcal{G}n}$ for the $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$.
Step 6:	Eliminate an feature a_1 from the condition's features ($\mathcal{C}$) and subsequently determine the prime-upper approximation, prime-lower approximation and prime-boundary for the decision set $\mathbb{X} \subseteq \mathfrak{U}$ on $\mathcal{C} - (a_1)$.
Step 7:	Construct the class of nano topological space $\mathfrak{T}_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}}$ for the entire table after excluding the features a_1
Step 8:	Iterate the steps 6 and 7 for all features in $\mathcal{C}$.
Step9:	Those features in $\mathcal{C}$ for which $\mathfrak{T}_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}} \neq \mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ *or* $\mathcal{B}_{p_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}}} \neq \mathcal{B}_p^{\mathcal{G}n}$ forms the CORE ($\mathfrak{U}$).

The set of infected patients with **Diabetes** is $\mathfrak{W} = \{\mathcal{p}_1, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5, \mathcal{p}_6\}$, then we get:

$\underline{\mathfrak{S}}_p(\mathfrak{W}) = \{\mathcal{p}_4, \mathcal{p}_6\}$, $\overline{\mathfrak{S}}_p(\mathfrak{W}) = \mathfrak{U}$ and $\mathcal{B}_p^{\mathcal{G}n}(X) = \{\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_5\}$.

Therefore, the $\mathcal{G}n$-topology and its basis of X are:

$\mathfrak{T}_{p\ A-\{\mathcal{I}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \varphi, \{\mathcal{p}_4, \mathcal{p}_6\}, \{\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_5\}\} \neq \mathfrak{T}_p^{\mathcal{G}n}$ and

$\mathcal{B}_{p\ A-\{\mathcal{I}\}}^{\mathcal{G}n} = \{\mathfrak{U}, \{\mathcal{p}_4, \mathcal{p}\}, \{\mathcal{p}_1, \mathcal{p}, \mathcal{p}, \mathcal{p}\}\} \neq \mathcal{B}_p^{\mathcal{G}n}$.

Hence, from **Steps (1-3)**, we observe that: the CORE is $\{\mathcal{J}, \mathcal{F}\}$, that is the impact factors to determine breast cancer infection are "Frequent Urination" and Increased Hunger.

Situation 2: (Individuals afflicted with Diabetes)

The group of individuals diagnosed with **Diabetes** is represented by where, $Y = \{\mathcal{p}_2\}$. Following identical procedures as described in **Case (1)**, we achieve identical outcomes.

Observation: According to the findings of the CORE analysis, it has been determined that "Frequent Urination" and "Heightened Appetite" are significant factors contributing to the development of Diabetes. Hence, these characteristics serve as essential indicators in understanding

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the influential factors associated with **Diabetes** infection. In the concluding segment of this study, we introduce an algorithm that incorporates our proposed methodologies, aiming to facilitate efficient decision-making processes.

```
% Step 1: Input the finite universe U and the set of attributes representing the data
% as an information table
U = ... % Define the finite universe U
attributes = ... % Define the set of attributes
informationTable = ... % Define the information table
% Step 2: Calculate the prime neighborhoods of every object
primeNeighborhoods = calculatePrimeNeighborhoods(informationTable);
% Step 3: Calculate the prime-upper approximation, prime-lower approximation, and
% prime-boundary for the decision set X⊆U
decisionSet = ... % Define the decision set X⊆U
primeUpperApproximation = calculatePrimeUpperApproximation(decisionSet,
primeNeighborhoods);
primeLowerApproximation = calculatePrimeLowerApproximation(decisionSet,
primeNeighborhoods);
primeBoundary = calculatePrimeBoundary(decisionSet, primeNeighborhoods);
% Step 4: Construct the Gn-topology T_C^Gn on U induced by X
topology = constructGnTopology(U, decisionSet);
% Step 5: Construct the class the base B_p^Gn for the T_C^Gn
classBase = constructClassBase(topology);
% Step 6: Eliminate a feature a_1 from the condition's features (C) and
% determine the prime-upper approximation, prime-lower approximation, and prime-boundary
% for the decision set X⊆U on C - (a_1)
removedFeatures = ... % Define the features to be removed
for i = 1:length(removedFeatures)
newDecisionSet = removeFeatureFromDecisionSet(decisionSet, removedFeatures(i));
newPrimeUpperApproximation = calculatePrimeUpperApproximation(newDecisionSet,
primeNeighborhoods);
newPrimeLowerApproximation = calculatePrimeLowerApproximation(newDecisionSet,
primeNeighborhoods);
newPrimeBoundary = calculatePrimeBoundary(newDecisionSet, primeNeighborhoods);
% Step 7: Construct a Gn-topology T_C^G non U
```

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% Fractal Modification for Step 6:
% Apply fractal-based techniques to determine the prime-upper
% approximation, prime-lower approximation, and prime-boundary
% for the decision set X ⊆ U on C - (a_1).
% Utilize fractal analysis methods to capture the self-similarity and
% structural characteristics of the data after excluding feature a_1.
Step 7: Construct a Gn-topology T_C^Gn on U using Definition 4.4.
% Fractal Modification for Step 7:
% Apply fractal-based techniques to construct the Gn-topology T_C^Gn
% at the nanoscale after excluding feature a_1.
% Adjust the topology construction process based on the fractal dimension,
% creating self-similar structures and connectivity patterns within T_C^Gn.
Step 8: Construct the class of the nano topological space T_(U_(C-{a_1})) for the entire table after excluding feature a_1.
% Fractal Modification for Step 8:
% Apply fractal-based techniques to construct the nano topological space
% T_(U_(C-{a_1})) for the entire table after excluding feature a_1.
% Incorporate fractal analysis and topology to capture the self-similarity,
% connectivity, and properties of the data within T_(U_(C-{a_1})).
Step 9: Iterate Steps 6-8 for all features in C.

To incorporate fractal elements into the given algorithm using MATLAB, we can modify the steps as follows:

Algorithm-IV	**A decision making via the prime-approximations based on fractals**
Step 1:	Take in the finite universe $\mathfrak{U}$ and the set of attributes representing the data in the form of an information table, where attributes are labeled as rows ($\mathcal{C}$), objects are labeled as columns, and the attribute values are recorded as entries in the table.
Step 2:	Calculate the prime neighborhoods of every object based on Definition 4.1 using the provided information table.
Step 3:	Calculate the prime-upper approximation, prime-lower approximation, and prime-boundary for the decision set using the available data. $\mathbb{X} \subseteq \mathfrak{U}$, namely, $\underline{\mathfrak{S}}_{\mathcal{P}}(\mathbb{X})$, $\overline{\mathfrak{S}}_{\mathcal{P}}(\mathbb{X})$ and $\mathfrak{BN}_{\mathcal{P}}(\mathbb{X})$ respectively according to Definition 4.2.
Step 4:	Construct the $\mathcal{G}n$-topology $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ on $\mathfrak{U}$ induced by $\mathbb{X}$ using Definition 4.4. % Fractal Modification for Step 4: % Apply fractal-based techniques to construct the Gn-topology $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ % at the nanoscale. % Use fractal dimension estimation methods, such as box counting or % Harsdorf dimension, to quantify the fractal dimension of the data. % Adjust the topology construction process based on the fractal dimension, % creating self-similar structures and connectivity patterns within $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$.
Step 5:	Construct the class the base $\mathcal{B}_{\mathcal{P}}^{\mathcal{G}n}$ for the $\mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$.
Step 6:	Iterate through each feature a_1 in ($\mathcal{C}$): - Eliminate feature a_1 from the condition's features $\mathcal{C}$ and obtain $\mathcal{C} - (a_1)$. Determine the prime-upper approximation, prime-lower approximation, and prime-boundary for the decision set $\mathbb{X} \subseteq \mathfrak{U}$ on $\mathcal{C} - (a_1)$.
Step 10:	Identify the features in $\mathcal{C}$ for which $\mathfrak{T}_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}} \neq \mathfrak{T}_{\mathcal{C}}^{\mathcal{G}n}$ *or* $\mathcal{B}_{\mathcal{P}_{\mathfrak{U}_{\mathcal{C}-\{a_1\}}}} \neq \mathcal{B}_{\mathcal{P}}^{\mathcal{G}n}$ forms the CORE ($\mathfrak{U}$).

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% Fractal Modification for Step 6:
% Apply fractal-based techniques to determine the prime-upper
% approximation, prime-lower approximation, and prime-boundary
% for the decision set X ⊆ U on C - (a_1).
% Utilize fractal analysis methods to capture the self-similarity and
% structural characteristics of the data after excluding feature a_1.
Step 7: Construct a Gn-topology T_C^Gn on U using Definition 4.4.
% Fractal Modification for Step 7:
% Apply fractal-based techniques to construct the Gn-topology T_C^Gn
% at the nanoscale after excluding feature a_1.
% Adjust the topology construction process based on the fractal dimension,
% creating self-similar structures and connectivity patterns within T_C^Gn.
Step 8: Construct the class of the nano topological space T_(U_(C-{a_1})) for the entire table after excluding feature a_1.
% Fractal Modification for Step 8:
% Apply fractal-based techniques to construct the nano topological space
% T_(U_(C-{a_1})) for the entire table after excluding feature a_1.
% Incorporate fractal analysis and topology to capture the self-similarity,
% connectivity, and properties of the data within T_(U_(C-{a_1})).
Step 9: Iterate Steps 6-8 for all features in C.

6. Conclusion

The utilization of the topological approach with decision making based on nano beta closure and nano beta interior has proven to be a valuable tool in medical decision making across various fields, including medicine. This approach empowers clinicians to gain a deeper understanding of complex medical data, leading to accurate diagnoses, personalized treatment plans, and improved patient outcomes. By surpassing the limitations of traditional methods, nano beta closure and nano beta interior provide reliable and accurate mathematical tools for analyzing complex data, enabling informed decision making in healthcare. Chronic renal disease, also known as persistent renal failure, refers to the gradual deterioration of kidney function over an extended period. As this condition progresses, harmful levels of electrolytes, fluids, and wastes accumulate in the body, necessitating targeted management strategies to slow down kidney damage. In advanced stages, chronic renal disease has the potential to progress to end-stage renal failure, necessitating artificial filtration or kidney transplantation for survival. This paper aims to identify risk factors associated with chronic renal disease by analyzing patient data. Additionally, the rising incidence of diabetes poses a significant concern in the medical field, highlighting the need for effective methods to identify influential factors and improve patient outcomes. The application of nano beta closure and nano beta interior offers a reliable and accurate approach to medical decision making, overcoming the limitations of traditional methods. Finally, we have developed two an algorithm one to breast cancer and other for diabetes that can assist healthcare practitioners in accurately diagnosing diseases and making informed decisions. Moreover, fractals offer a promising avenue for tissue engineering and regenerative medicine. By mimicking the self-similar structures and hierarchical organization of tissues, researchers can fabricate nanoscale scaffolds that promote cellular growth and tissue regeneration. Fractal-based approaches may also facilitate the development of artificial organs and bioengineered constructs with enhanced functionality and integration within the human body.

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the entire manuscript concept and finish the preparatory work for the paper. **Elhadi, Al-Qahtani, and Al Qarni** analyzed the existing work of Topological modeling, and revised the paper.

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