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On the Unitarity of the Stueckelberg Wave Equation and Measurement as Bayesian Update from Maximum Entropy Prior Distribution

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Abstract

The Stueckelberg wave equation is solved for unitary solutions, which links the eigenvalues of the Hamiltonian directly to the oscillation frequency. As it has been showed previously that this PDE relates to the Dirac operator, and on the other hand it is a linearized Hamilton-Jacobi-Bellman PDE, from which the Schrödinger equation can be deduced in a nonrelativistic limit, it is clear that it is the key equation in relativistic quantum mechanics. We give a stationary solution for the quantum telegraph equation and a Bayesian interpretation for the measurement problem. The stationary solution is understood as a maximum entropy prior distribution and measurement is understood as Bayesian update. We discuss the interpretation of the single electron experiments in the light of finite speed propagation of the transition probability field and how it relates the interpretation of quantum mechanics more broadly.

Keywords: Telegrapher's equation; Stueckelberg equation; relativistic quantum mechanics; Bayesian inference

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1. Introduction

Since quantum mechanics was incepted in 1925-1926 by Werner Heisenberg and Erwin Schrödinger, it has been 100 years and we still do not understand, what is the correct or proper interpretation of quantum phenomena. The double-slit experiment is the canonical example. Actually, the first double-slit experiment with single electrons was conducted only in 1989 by Hitachi, [1]. The observations indicate that single electrons behave randomly, but as particles, and the ensemble behaves as waves. The experiments with single photons by for example Hamamatsu seem to indicate that photons and electrons behave the same. Therefore, from the instrumentalist point of view, one can understand electrons and photons as pointlike particles. If the single electron was an ordinary water-like wave, then even one single electron should produce interference fringes on the screen, but this has not been observed. Instead, it is observed that when electrons are repeatedly shot in the double-slit apparatus, they build up an interference pattern alike light. This was reconfirmed experimentically in 2012 in [2]. These results indicate that quantum mechanics should be seen as a statistical theory or as an ensemble theory akin to statistical mechanics.

In stochastic analysis (as a branch of advanced mathematics) we deal with continuous random processes. In statistical physics, we deal with statistical ensembles. These are created by mathematical diffusions. Quantum mechanics can be formulated through diffusion theory, when we consider probability waves and state vectors in a Hilbert space. The stochastic interpretation of quantum mechanics was incepted by Hungarian physicist Imre Fényes in 1952, [3]. It is perhaps even more surprising that it was Erwin Schrödinger himself, who linked Markov processes and the Schrödinger equation in the first place, in 1931, see [4]. Reinhold Fürth derived uncertainty relations for diffusion processes already in 1933, [5]. Edward Nelson continued this tradition in the 1960's, [6]. Extension to relativistic regimes have been developed since, for example in [7].

Formally speaking, the key here is the infinitesimal generator of the continuous Markov process: in abstract, we deal with duality between partial differential equations and stochastic processes. We can map the Schrödinger equation to a certain Markov diffusion. Then we can analyze the diffusion process through the backward and forward Kolmogorov equations, see [8]. A special type of stochastic process is the diffusion process. These processes are characterized by continuous sample paths (no jumps). Moreover, they are Markov processes. This means that they do not possess any memory. A typical (time-homogeneous) diffusion process would look like this:

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t$$

With such a diffusion, we can associate a second order elliptic differential operator called the infinitesimal generator: the associated infinitesimal generator for the diffusion is:

$$\mathcal{L} = b(x) \cdot \nabla + \frac{1}{2}Tr\left(\sigma\sigma' D^2\right)$$

where D^2 is the Hessian and σ is the diffusion matrix. The function b is the drift function and sigma (diagonal matrix) represents the amplitude of the noise from independent Brownian motions. The generator is associated with something called the Markov semigroup. The semigroup essentially tells us that we can divide a probabilistic transition from A to C in two parts: from A to B and from B to C (Chapman-Kolmogorov equations). This corresponds to the multiplication of Markov chain matrices. The generator is just the infinitesimal version corresponding to Markov semigroups for continuous diffusions. If we can define the operator adjoint of the infinitesimal generator in space L^2, we can in particular demand that the operator is self-adjoint. This gives us the class of reversible diffusions, see [8]. These are of great interest to us. We can show that self-adjoint generators correspond to gradient drifts (with constant diffusion matrices). This in turn gives us ergodicity and stationary distributions for the diffusion, which allows us to calculate ensemble averages. These averages are important as they correspond to time averages for ergodic processes. In terms of the generator, a stationary distribution must satisfy:

$$\mathcal{L}^*\rho = 0$$

Where $\mathcal{L}^*$ is the operator adjoint of the infinitesimal generator of the diffusion and ρ is the transition probability density from some point to another at x. If we consider statistical mechanics, we can study a collection of particles and we perhaps wish to know what is the equilibrium distribution of particles. We may thus model the test particle through the diffusion as above. Then, as the Fokker-Planck PDE is of the form:

$$\frac{\partial\rho}{\partial t} = \mathcal{L}^*\rho$$

, this distribution gives us the thermodynamics of equilibrium, as the transition probability ρ reaches a stationary state. This is due to the property of the drift and diffusions, they have to obey a property called detailed balance. The inverse problem is also interesting: given a stationary distribution, find a reversible diffusion which converges to it (fast). The stationary distribution corresponds actually to the linear combination of the stationary states of the time-independent eigenstates in quantum mechanics. This will be shown in this research article. In statistical mechanics it is the Gibbs ensemble. E.T. Jaynes would recognize a maximum entropy distribution, [9]. The linear approximation of the Hamilton-Jacobi-Bellman and Fokker-Planck equations gives the distribution, when the second order effects are ignored, they give also the Heisenberg uncertainty relations, [10]. The use of the maximum entropy distribution or time-independent stationary distribution as the prior distribution can be seen as an instance of objective Bayesianism put forward such writers as J.M. Keynes and Harold Jeffreys. The primitive form is the principle of indifference: "A person's credences in any two propositions should be equal if her total evidence no more supports one than the other (the evidential symmetry version), or if she has no sufficient reason to have a higher credence in one than in the other (the insufficient reason version)" [11] . Jeffreys expanded this into an operational form in [12] and Jaynes generalized it

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to use maximum entropy distributions as prior probability distributions [13]. In quantum mechanics, we might for example be unaware on what shell the electron is in an atom, so that the stationary distribution is some linear combination of the energy eigenstates via the eigenfunction expansion of the Hamiltonian operator.

Our aim is to show that a realistic and objective interpretation of quantum mechanics is indeed possible via the diffusion approach. The transition probability field corresponds to the wave behaviour of the quantum particle, and its dynamics is given by the quantum telegraph equation, for which the stationary states correspond to stationary maximum entropy distribution. Wave function collapse is then just the Bayesian update of the distribution, when the observer obtains new information via measuring the observable energy and thus the corresponding energy eigenstate. Karl Popper, known for his falsification thesis in demarcation of science and pseudoscience, was of the view that quantum mechanics without the observer makes sense. We are of the view that indeed this objectivist and realistic approach is appropriate, and that the Copenhagen orthodoxy is wrong, as put forward by Niels Bohr, John von Neumann and others.

2. Quantum Formalism and Conservation of Probability

We take the Stueckelberg wave equation as the key object in relativistic quantum mechanics. This PDE can be obtained as a linear approximation to a certain stochastic optimal control PDE. The stochastic random walk model in spacetime, as presented in [7] proposes that the test particle follows a Markov diffusion on a Minkowski spacetime. In [7] it was shown, that the Stueckelberg wave equation, invented in 1941-1942, can be derived as a linearized Hamilton-Jacobi-Bellman equation for a relativistic and stochastic optimal control problem on a Minkowski spacetime, as a linear limit. The Stueckelberg wave equation can be seen as a four-dimensional generalization of the Schrödinger wave equation. The ontology of the theory is such that the spacetime is randomly oscillating at Planck scales, and the test particle is affected by Brownian noise and some effective potential. It is plausible, that these properties link to random metrics of a spacetime, [14]. The drift for the spacetime diffusion is obtained as an optimal control, and it is a four-gradient of the value function of the stochastic optimal control model. The drift of the diffusion is thus a gradient, and we can consider the respective Kolmogorov forward and backward equations. The forward equation, or the Fokker-Planck equation posits that the transition probability density follows a partial differential equation. It is essentially a statement for conservation of probability, that is, it is a continuity equation for the transition probability.

Consider a square-integrable wave function $\phi \in L^2$ on some domain in a complex Hilbert space of functions. The Stueckelberg wave equation [15] is:

$$i\hbar\frac{\partial\phi}{\partial\tau} = \frac{\hbar^2}{2m}\Box\phi - V(\mathbf{x})\phi. \tag{1}$$

Where $\phi(\tau, t, x, y, z)$ is the square-integrable wave function on Hilbert space and $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \Delta$ is the normal d'Alembertian and Δ is the normal Laplacian operator. $V(x, y, z)$ is some potential function, for example quadratic or other well-known real valued potential. In [16], it was shown that the square of the Dirac operator yields exactly a telegrapher's equation, where the first order term is analytically continued, that is, the derivative operator first order in time contains the imaginary unit. It is therefore clear, that this type of PDE has an immanent role in relativistic quantum mechanics.

We leave the Stueckelberg wave equation aside for a moment and instead we consider the continuity equation for probability:

$$\frac{\partial\rho}{\partial\tau} + \nabla_\mu(\rho U^\mu) = 0. \tag{2}$$

Assuming a self-adjoint Hamiltonian, we may write the Born rule:

$$\rho(x, y, z, t, \tau) = \phi^*\phi. \tag{3}$$

We get:

$$\frac{\partial \rho}{\partial \tau} = \frac{\partial \phi^*}{\partial \tau}\phi + \phi^*\frac{\partial \phi}{\partial \tau}. \tag{4}$$

We substitute from the Stueckelberg equation above:

$$\frac{\partial \phi^*}{\partial \tau} = \frac{i\hbar}{2m}\Box\phi^* - \frac{i}{\hbar}V(x)\phi^* \tag{5}$$

and

$$\frac{\partial \phi}{\partial \tau} = -\frac{i\hbar}{2m}\Box\phi + \frac{i}{\hbar}V(x)\phi. \tag{6}$$

Therefore, by substituting we get the following:

$$\frac{\partial \phi^*}{\partial \tau}\phi + \phi^*\frac{\partial \phi}{\partial \tau} = \frac{i\hbar}{2m}\phi\Box\phi^* - \frac{i}{\hbar}\phi V(x)\phi^* - \frac{i\hbar}{2m}\phi^*\Box\phi + \frac{i}{\hbar}\phi^* V(x)\phi. \tag{7}$$

As the potential terms cancel out, we have just:

$$\frac{\partial \phi^*}{\partial \tau}\phi + \phi^*\frac{\partial \phi}{\partial \tau} = \frac{i\hbar}{2m}\phi\Box\phi^* - \frac{i\hbar}{2m}\phi^*\Box\phi. \tag{8}$$

This is just the continuity equation for probability, when the probability flux is given by:

$$J^\mu = \rho U^\mu = -\frac{i\hbar}{2m}(\phi\nabla^\mu\phi^* - \phi^*\nabla^\mu\phi). \tag{9}$$

The continuity equation for probability density is then:

$$\frac{\partial \rho}{\partial \tau} + \nabla_\mu J^\mu = 0. \tag{10}$$

The continuity equation for the transition probability can be understood as a Fokker-Planck equation for Markov diffusions on the Minkowski spacetime. The natural question is then, do we have a stationary distribution for the transition probability density? Indeed we do, the separable wave function of the form:

$$\phi = T(t)f(x,y,z). \tag{11}$$

Inserting the separable solution into the continuity equation, one notices that the four-divergence of probability flux vanishes. This is easy to demonstate. Suppose the wave function is of the form $\phi = e^{iEt}f(x,y,z)$. Then we have that:

$$\Box\phi = -\frac{1}{c^2}(iE)^2\phi + \Delta\phi. \tag{12}$$

and

$$\Box\phi^* = -\frac{1}{c^2}(-iE)^2\phi^* + \Delta\phi^*. \tag{13}$$

Thus :

$$\nabla_\mu J^\mu = \frac{i\hbar}{2m}(\phi\Delta\phi^* - \phi^*\Delta\phi). \tag{14}$$

Now as for the separable solution (see next section) the spatial part of the wave function obeys the Hamiltonian eigenvalue equation, the expression in the brackets disappears. We have thus shown that the four-divergence of the probability flux vanishes for such stationary solutios of the Stueckelberg wave equation.

As the Stueckelberg wave equation possesses finite speed of propagation, it is clear that the transition probability field, given by the continuity equation has a finite speed of propagation as well, due to Born rule. Therefore, we may understand the interference pattern due to the probility wave interference, even though individual electrons behave as particles. The statistical ensemble is formed,

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as many electrons are shot through the double slit. This statistical interpretation is fully in line with the approach by Ballentine, [17] and Popper, [18]. It is worth noting that the path integral formulation by Richard Feynman is also closely linked to Fokker-Planck equations, [19].

3. Quantum Telegrapher's Equation and Unitarity

In order to show explicitly how to obtain the stationary states from the Stueckelberg wave equation, consider the following: As the Hamilton-Jacobi-Bellman PDE is backwards, we reverse the proper time $\tau \longrightarrow -\tau$ to obtain

$$i\hbar\frac{\partial\phi}{\partial\tau} = -\frac{\hbar^2}{2m}\Box\phi + V(\mathbf{x})\phi. \tag{15}$$

As on a Minkowski spacetime, proper time τ can be related to coordinate time t in a frame by:

$$d\tau = \sqrt{1-\frac{v^2}{c^2}}dt, \tag{16}$$

where v is the velocity of the particle in that frame, we obtain immediately a Telegrapher's equation

$$\frac{i\hbar}{\sqrt{1-\frac{v^2}{c^2}}}\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{2m}\Box\phi + V(\mathbf{x})\phi. \tag{17}$$

or more clearly written

$$\frac{i\hbar}{\sqrt{1-\frac{v^2}{c^2}}}\frac{\partial\phi}{\partial t} - \frac{\hbar^2}{2mc^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\hbar^2}{2m}\Delta\phi + V(\mathbf{x})\phi. \tag{18}$$

This can be recast using the self-adjoint, unbounded and linear Hamiltonian operator $\mathcal{H}$:

$$\frac{i\hbar}{\sqrt{1-\frac{v^2}{c^2}}}\frac{\partial\phi}{\partial t} - \frac{\hbar^2}{2mc^2}\frac{\partial^2\phi}{\partial t^2} = \mathcal{H}\phi. \tag{19}$$

We obtain the Schrödinger equation immediately by taking the non-relativistic limit $c \longrightarrow \infty$ and we have:

$$i\hbar\frac{\partial\phi}{\partial t} = \mathcal{H}\phi. \tag{20}$$

Back to the Telegrapher's equation. The benefit is now that we can study the temporal part by using the separation of variables method. The eigenvalues for the Hamiltonian are real, and the eigenfunctions give a complete orthonormal basis on this Hilbert space, which is convenient, as we express the solution as an infinite series, when the system is confined on a compact interval. Consider the following separation of variables: $\phi = T(t)f(\mathbf{x})$. The spatial part is the familiar eigenvalue equation for the Hamiltonian operator:

$$\mathcal{H}f = \lambda f. \tag{21}$$

The obtained equation can be solved by separating variables, we have for the temporal part:

$$\beta T' + \alpha T'' = \lambda T \tag{22}$$

Try $T(t) = e^{rt}$ we have

$$\beta rT + \alpha r^2 T - \lambda T = 0 \tag{23}$$

Thus we have the quadratic

$$\alpha r^2 + \beta r - \lambda = 0 \tag{24}$$

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The quadratic formula gives:

$$r = \frac{-\beta \pm \sqrt{\beta^2 + 4\alpha\lambda}}{2\alpha} \tag{25}$$

Therefore for the unitary solution such that r is purely imaginary, we demand that the discriminant is non-positive

$$\beta^2 + 4\alpha\lambda \leq 0 \tag{26}$$

Which in turn relates the eigenvalues of the Hamiltonian operator as follows (note that as 4α is negative, we need to switch the inequality)

$$\lambda \geq -\frac{\beta^2}{4\alpha}. \tag{27}$$

As we have $\alpha = -\frac{\hbar^2}{2mc^2}$, $\beta = \frac{i\hbar}{\sqrt{1-\frac{v^2}{c^2}}}$ We have thus a set of eigenvalues, bounded from below. Unitarity now ensures that norms are conserved, and the probability is conserved. In [16], it was shown that the telegrapher's equation is obtained from the Dirac equation. We can consider a general solution for the quantum telegraph equation by considering the eigenfunction expansion. As the spectral theorem provides that the Hamiltonian eigenfunctions form an orthonormal basis in our complex Hilbert space, and that the eigenvalues are real, the general solution can be expressed as

$$\phi = \sum_{j=1}^{\infty} < \phi, f_j > f_j \left(e^{r_{j+}t} + e^{r_{j-}t} \right), \tag{28}$$

So that we have "positive energy" and "negative energy" solutions, just like in Dirac's approach. Given that for the orthonormal eigenfunctions f_j we have $< f_j, f_j >= 1 \; \forall j$. Given Parseval's theorem, we have

$$\sum_{j=1}^{\infty} | < \phi, f_j > |^2 = 1, \tag{29}$$

so that we may interpret the squares of the coefficients as the probabability of measuring an eigenvalue λ_j with the corresponding eigenfunction f_j. From the physical point of view, the discrete energy eigenvalues represent discrete relativistic masses, and the probability of measuring such eigenstate is proportional to the inner product $< \phi, f_j >$. Operating in the stationary solution with the Hamiltonian operator, we have

$$\mathcal{H}\phi = \sum_{j=1}^{\infty} < \phi, f_j > \lambda_j f_j \left(e^{r_{j+}t} + e^{r_{j-}t} \right), \tag{30}$$

Taking an inner product $< \phi, \mathcal{H}\phi >$ we obtain the expected energy for the quantum system. Therefore energy is a discrete random variable, with probability given as $| < \phi, f_j > |^2$.

The general solution above is the stationary solution, for which we have performed an eigenfunction expansion or Fourier expansion.

4. Measurement

What about measurement and collapse of the wave function? Consider the following interpretation for quantum state measurement: we assume that the general solution or the stationary solution for the quantum telegraph equation is the prior distribution, so that we update the probability according to Bayes' rule. We are looking for example the conditional probability of finding the system in an eigenstate f_j, when we have some stationary distribution. What is then $P(f_j|\phi)$? As the quantum states are elements of the Hilbert space, we can consider the orthogonal projection from the stationary state ϕ to an eigenstate f_j:

$$P_{f_j} = \frac{< \phi, f_j >}{< f_j, f_j >} f_j \tag{31}$$

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Suppose now for pedagogical reasons that the wave function is not normalized so that the eigenstates are not normalized. Then using Parseval's theorem for orthogonal functions the probability of obtaining eigenstate f_j is the ratio:

$$\frac{|<\phi, f_j>|^2 <f_j, f_j>}{\sum_{j=1}^{\infty} |<\phi, f_j>|^2 <f_j, f_j>}. \tag{32}$$

$$P(f_j|\phi) = \frac{|<\phi, f_j>|^2 <f_j, f_j>}{\sum_{j=1}^{\infty} |<\phi, f_j>|^2 <f_j, f_j>} = \frac{|<\phi, f_j>|^2 <f_j, f_j>}{<\phi, \phi>}, \tag{33}$$

or in a more useful form:

$$|<\phi, f_j>|^2 = \frac{P(f_j|\phi)}{<f_j, f_j>} <\phi, \phi> \tag{34}$$

On the other hand, it is according to Bayes' formula:

$$P(\phi|f_j) = \frac{P(f_j|\phi)}{P(f_j)} P(\phi), \tag{35}$$

from which we can see that the Bayesian update uses essentially produces the individual coefficients of the eigenfunction expansion, when we treat the stationary general solution as the prior distribution. The wave function general solution ϕ can be interpreted as a maximum entropy prior distribution in Bayesian terms, where the updated probability is given by the measurement, collapsing on the eigenstate f_j. This interpretation of measurent has much in common with QBism, discussed for example in [20]. Measurement then produces the posterior probability, which is the eigenfunction expansion Fourier-coefficient squared.

5. Conclusions

Niels Bohr has said: "There is no quantum world. There is only an abstract quantum description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature." [21]. This view can be seen as a Kantian approach to natural science. According to Kant, we cannot ever observe the reality, only our perception of reality. Naturally then quantum mechanics is necessarily an instrumentalist theory. Trying to interpret quantum mechanics is still important, as wrong ways of thinking might stall the development of further working theories. This view of scientific research programmes has been put forward by the Hungarian philosopher of science, Imre Lakatos.

The quantum telegraph equation is deduced from the well-known and manifestly covariant Stueckelberg wave equation. The telegraph equation separates, and gives unitary solutions when the real eigenvalues of the Hamiltonian operator are tied to the relativistic mass of the particle. Finally, the measurement is shown to match mathematically to Bayesian update, when the prior distribution is taken as the stationary state of the system, a thermodynamic equilibrium or maximum entropy distribution, expressed as an infinite series of Hamiltonian eigenfunctions. The measurement and collapse to an eigenstate can be seen through Bayesian inference, see [20]. The stationary or separable solution corresponds to a stationary distribution, where transition probability does not depend explicitly on time, even though the wave function does have an exponential phase factor. According to Jaynes, this maximum entropy distribution should be used as the prior in Bayesian inference and update, [9]. The stationary distribution is a Gibbs distribution or a maximum entropy distribtution. It also corresponds to David Bohm's quantum equilibrium, [22].

The aim of this research is to demonstrate and convince the reader that quantum mechanics is:

1. An instrumental theory in the sense that the Markov diffusion model on Minkowski spacetime is an instrumental theory. Perhaps there is a deeper ontology, where the spacetime fluctuations at small scales are due to some deeper property of geometry. The key equation of relativistic quantum mechanics is the Stueckelberg covariant wave equation. It governs the dynamics of

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the transition probability for the test particle. The non-relativistic limit gives us the familiar time-dependent Schrödinger equation.

2. An objective and realist theory in the sense of Karl Popper. There exists a reality independent of the observer, and quantum mechanics does not require an observer. The wavefunction is epistemic, not ontological in its nature. It carries information essentially about the transition probability of the test particle, given the Hamiltonian operator and initial conditions. The flow of probability is nevertheless propagating with finite speed, and thus information travels in the spacetime as probability waves.
3. There is no wave function collapse. Quantum mechanics is essentially statistical mechanics on Minkowski spacetimes. The Copenhagen interpretation is not appropriate, as measurement from the maximum entropy distribution can be understood as update of the distribution, when the energy level of the particle is revealed. This corresponds to projection to the eigenstate in the Hilbert space of functions. In practice we might for example wonder, on what shell the electron is, it might be anywhere, we measure the energy and we find out the shell. The exact position is still uncertain, as described by the eigenstate and the respective eigenfunction.

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