

Article

Not peer-reviewed version

Constructing Physics from Measurements

[Alexandre Harvey-Tremblay](#) *

Posted Date: 20 August 2025

doi: 10.20944/preprints202404.1009.v22

Keywords: foundations of physics



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a Creative Commons CC BY 4.0 license, which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Article

Constructing Physics From Measurements

Alexandre Harvey-Tremblay

Independent Scientist, Montréal, Canada; aht@protonmail.ch

Abstract

We reformulate fundamental physics from a list of axioms to the solution of a single optimization problem. Any experiment begins with a preparation, evolves, and ends with a measurement. We maximize the relative Shannon entropy of the final measurement with respect to its preparation, subject to a measurement constraint. Solving this for the natural constraint—the most permissive constraint admitting translations, boosts, rotations, and reorientations—yields a unified theory whose functional form contains general relativity (on spacetime) and Yang–Mills (on internal spaces). Mathematical consistency further restricts valid solutions to 3+1 dimensions, automatically selecting spacetime. Thus the forces, symmetries, and dimensionality of modern physics emerge as the unique solution to a single optimization posed over all experiments consistent with nature’s constraint.

Keywords: foundations of physics

1. Introduction

Statistical mechanics (SM), in the formulation developed by E.T. Jaynes [1,2], is founded on an entropy optimization principle. Specifically, the Boltzmann entropy is maximized under the constraint of a fixed average energy $\bar{E}$:

$$\bar{E} := \sum_i \rho_i E_i \tag{1}$$

The Lagrange multiplier equation defining the optimization problem is:

$$\mathcal{L} := -k_B \sum_i \rho_i(\beta) \ln \rho_i(\beta) + \lambda \left(1 - \sum_i \rho_i(\beta) \right) + k_B \beta \left(\bar{E} - \sum_i \rho_i(\beta) E_i \right) \tag{2}$$

where λ and β are Lagrange multipliers enforcing the normalization and average energy constraints. Solving this optimization problem for ρ_i yields the Gibbs measure:

$$\rho_i(\beta) = \frac{1}{Z(\beta)} \exp(-\beta E_i), \tag{3}$$

where $Z(\beta) = \sum_j \exp(-\beta E_j)$ is the partition function.

For comparison, quantum mechanics (QM) is not formulated as the solution to an optimization problem, but rather consists of a collection of axioms [3,4]:

1. **State Space:** Every physical system is associated with a complex Hilbert space, and its state is represented by a ray (an equivalence class of vectors differing by a non-zero scalar multiple) in this space.
2. **Observables:** Physical observables correspond to Hermitian (self-adjoint) operators acting on the Hilbert space.
3. **Dynamics:** The time evolution of a quantum system is governed by the Schrödinger equation, where the Hamiltonian operator represents the system’s total energy.

4. **Measurement:** Measuring an observable projects the system into an eigenstate of the corresponding operator, yielding one of its eigenvalues as the measurement result.
5. **Probability Interpretation:** The probability of obtaining a specific measurement outcome is given by the squared magnitude of the projection of the state vector onto the relevant eigenstate (Born rule).

Physical theories have traditionally been constructed in two distinct ways. Some, like QM, are defined through a set of mathematical axioms that are first postulated and then verified against experiments. Others, like SM, emerge as solutions to optimization problems with experimentally-verified constraints.

We propose to generalize the optimization methodology of E.T. Jaynes to encompass all of physics, aiming to derive a unified theory from a single optimization problem.

To that end, we introduce the following constraint:

Axiom 1 (Nature).

$$\bar{H} := \int_V w(t, \vec{x}) \operatorname{tr} H(\vec{x}) d^p \vec{x}$$

where $H(\vec{x})$ is an operator, $\bar{H}$ is the average of its trace, p is the number of spatial dimensions (e.g. three in spacetime), and $w(t, \vec{x})$ is an information density¹.

This constraint, as it replaces the scalar energy E_i with the operator $H(\vec{x})$, extends E.T. Jaynes' optimization method to encompass non-commutative observables and other operators required for fundamental physics.

We then construct an optimization problem:

Definition 1 (Physics). *Physics is the solution to:*

$$\underbrace{\mathcal{L}[w]}_{\text{an optimization problem}} := \underbrace{- \int_V w(t, \vec{x}) \ln \frac{w(t, \vec{x})}{w(0, \vec{x})} d^p \vec{x}}_{\text{on the entropy of a measurement relative to its preparation}} + \underbrace{t \left(\bar{H} - \int_V w(t, \vec{x}) \operatorname{tr} H(\vec{x}) d^p \vec{x} \right)}_{\text{as constrained by nature}}$$

where t is the Lagrange multiplier² enforcing the natural constraint.

This definition constitutes our complete proposal for reformulating fundamental physics—no additional principles will be introduced. By replacing the Boltzmann entropy with the relative Shannon entropy, the optimization problem extends beyond thermodynamic variables to encompass any type of experiment. This generalization occurs because relative entropy captures the relationship between a final measurement and its initial preparation. Finally, the natural constraint defines the domain of applicability of the theory.

The crucial insight is that because our formulation maintains complete generality in the structure of experiments while optimizing over all possible natural theories, the resulting solution holds true, by

¹ An information density is a non-negative, unnormalized measure, $w(t, \vec{x}) \geq 0$. This contrasts with a probability density, which is additionally normalized to unity. An information density is the most general structure whose entropy is real-valued. The explicit form of w will be identified in Theorem 1 by solving the optimization problem. The unnormalized nature of w aligns with foundational concepts in Quantum Field Theory (QFT), where unnormalized field amplitudes and their associated conserved (but not unit-normalized) charge densities are primary, rather than single-particle probability amplitudes. The framework will later show how quantities suitable for probabilistic interpretation (Theorem 8), such as the Dirac current, emerge from the dynamics.

² As we solve the optimization problem, we will find that the Lagrange multiplier t takes on the role of time, yielding dynamical equations. This is similar to the Lagrange multiplier β in statistical mechanics taking on the role of temperature via $\beta = 1/(k_B T)$ after solving the optimization problem for the Gibbs measure.

construction, for all realizable experiments within its domain. This approach reduces our reliance on postulating axioms through trial and error, and simplifies the foundations of physics.

Specifically, when we employ the *natural constraint* —the most permissive constraint for this problem (Section 2.2)— the solution spawns its largest domain, yielding the Dirac theory in the language of spacetime algebra (STA) for the linear case, and general relativity (acting on spacetime) as well as Yang-Mills (acting on internal spaces of spacetime) for the non-linear case.

As we will see, Definition 1 automatically restricts the valid solutions to the specific case of 3+1 dimensions. In other dimensional configurations, various obstructions arise making the solution violate the axioms of probability theory. We will demonstrate the obstructions in Section 2.1 then we will investigate the unobstructed case in Section 2.2. These obstructions are desirable because they provide a mechanism for the observed dimensionality of our universe as solely satisfying the real-valued and non-negativity structure of the solution.

2. Results

Theorem 1. *The general solution of the optimization problem (Definition 1) is:*

$$w(t, \vec{x}) = \det(\exp(-tH(\vec{x})))w(0, \vec{x})$$

Proof. We solve Definition 1 by taking the variation of the Lagrange multiplier equation with respect to w . (To improve the legibility, we will drop the explicit parametrization in $w(t, \vec{x}) \rightarrow w$ and $H(\vec{x}) \rightarrow H$ in the proof.)

$$0 = \delta w \mathcal{L}[w] \quad (4)$$

$$= -\delta w \int_V w \ln \frac{w}{w'_0} dx + t \left(\delta w \bar{H} - \delta w \int_V w \operatorname{tr} H dx \right) \quad (5)$$

$$= -\ln \frac{w}{w'_0} \delta w - \delta w - t \operatorname{tr}(H) \delta w \quad (6)$$

$$= -\ln w \delta w + \ln w'_0 \delta w - \delta w - t \operatorname{tr}(H) \delta w \quad (7)$$

$$= -\ln w + \ln w'_0 - 1 - t \operatorname{tr}(H) \quad (8)$$

$$\Rightarrow \ln w = -1 - t \operatorname{tr}(H) + \ln w'_0 \quad (9)$$

$$\Rightarrow w = \exp(-1) \exp(-t \operatorname{tr} H) w'_0 \quad (10)$$

$$w = \exp(-t \operatorname{tr} H) w_0 \quad (w_0 := \exp(-1) w'_0) \quad (11)$$

$$w = \det(\exp(-tH)) w_0 \quad (\det \exp M \equiv \exp \operatorname{tr} M) \quad (12)$$

Finally, restoring the explicit parametrization, we end with:

$$w(t, \vec{x}) = \det(\exp(-tH(\vec{x})))w(0, \vec{x}) \quad (13)$$

□

The steps of this proof are standard for solving an entropy optimization problem in statistical mechanics, and simply mimic the steps in the usual derivation of the Gibbs measure, as shown in Annex A.

In section 2.2, we will show that when $H(\vec{x})$ is taken to be the most permissive constraint in 3+1D—a covariant derivative over all possible transformations of the elements of STA, this solution entails the hamiltonian form of the Dirac equation expressed in the language of STA. For now (Section 2.1), we will investigate the obstructions.

2.1. Dimensional Obstructions

We begin by exploring the dimensional obstructions of this solution. We found that all geometric configurations except the 3+1D case are obstructed. By obstructed, we mean that the solution does not satisfy the non-negativity requirement of its interpretation as an information density (i.e. this would entail negative probabilities, non-real probabilities, or other definability problems).

<i>Dimensions</i>	<i>Does a solution exist?</i>	
$H(\vec{x}) \in \text{Cl}(0)$	Obstructed (Undefinable)	(14)
$H(\vec{x}) \in \text{Cl}(0, 1)$	Obstructed (Undefinable)	(15)
$H(\vec{x}) \in \text{Cl}(1, 0)$	Obstructed (Undefinable)	(16)
$H(\vec{x}) \in \text{Cl}(2, 0)$	Obstructed (Undefinable)	(17)
$H(\vec{x}) \in \text{Cl}(1, 1)$	Obstructed (Negative probabilities)	(18)
$H(\vec{x}) \in \text{Cl}(0, 2)$	Obstructed (Non-real probabilities)	(19)
$H(\vec{x}) \in \text{Cl}(3, 0)$	Obstructed (Non-real probabilities)	(20)
$H(\vec{x}) \in \text{Cl}(2, 1)$	Obstructed (Non-real probabilities)	(21)
$H(\vec{x}) \in \text{Cl}(1, 2)$	Obstructed (Non-real probabilities)	(22)
$H(\vec{x}) \in \text{Cl}(0, 3)$	Obstructed (Ill-defined probabilities)	(23)
$H(\vec{x}) \in \text{Cl}(4, 0)$	Obstructed (Non-real probabilities)	(24)
$H(\vec{x}) \in \text{Cl}(3, 1)$ (spacetime)	Dirac Equation (Section 2.2)	(25)
$H(\vec{x}) \in \text{Cl}(2, 2)$	Obstructed (Undefinable)	(26)
$H(\vec{x}) \in \text{Cl}(1, 3)$	Obstructed (Non-real probabilities)	(27)
$H(\vec{x}) \in \text{Cl}(0, 4)$	Obstructed (Non-real probabilities)	(28)
$H(\vec{x}) \in \text{Cl}(5, 0)$	Obstructed (Non-real probabilities)	(29)
$\vdots$	$\vdots$	
$H(\vec{x}) \in \text{Cl}(6, 0)$	Suspected Obstructed (No observables)	(30)
$\vdots$	$\vdots$	

Let us now demonstrate the obstructions mentioned above.

Theorem 2 (Non-real probabilities). *The quantity $\det(\exp(-tH))$ resulting from the optimization procedure, when evaluated for the matrix representations of the Clifford algebras in this category, is either complex-valued or quaternion-valued, making them unsuitable.*

Proof. These Clifford algebras are classified as follows:

$$\text{Cl}(0, 2) \cong \mathbb{H} \quad (31)$$

$$\text{Cl}(3, 0) \cong \mathbb{M}_2(\mathbb{C}) \quad (32)$$

$$\text{Cl}(0, 3) \cong \mathbb{H}^2 \quad (33)$$

$$\text{Cl}(1, 2) \cong \mathbb{M}_2(\mathbb{C}) \quad (34)$$

$$\text{Cl}(4, 0) \cong \mathbb{M}_2(\mathbb{H}) \quad (35)$$

$$\text{Cl}(1, 3) \cong \mathbb{M}_2(\mathbb{H}) \quad (36)$$

$$\text{Cl}(0, 4) \cong \mathbb{M}_2(\mathbb{H}) \quad (37)$$

$$\text{Cl}(5, 0) \cong \mathbb{M}_2^2(\mathbb{H}) \quad (38)$$

Evaluating the function $\det(\exp(-tH))$ derived from the entropy maximization procedure for operators H associated with these algebras yields values in $\mathbb{C}$ (for Clifford algebras isomorphic to $\mathbb{M}_2(\mathbb{C})$) or $\mathbb{H}$ (for Clifford algebras isomorphic to $\mathbb{H}$ or $\mathbb{M}_2(\mathbb{H})$). Since w must be real and non-negative, these are obstructed. $\square$

Theorem 3 (Negative probabilities). *The even sub-algebra of this dimensional configuration allows for negative probabilities, making it unsuitable.*

Proof. This category contains one dimensional configuration:

$\text{Cl}(1,1)$ Let $\psi = a + be_0e_1$, then:

$$\det(a + be_0e_1) = (a + be_0e_1)^\dagger(a + be_0e_1) = (a - be_0e_1)(a + be_0e_1) = a^2 - b^2e_0e_1e_0e_1 = a^2 - b^2 \quad (39)$$

which is valued in $\mathbb{R}$.

In this case the probability can be negative. $\square$

Theorem 4 (Ill-defined probabilities). *This Clifford algebra is isomorphic to direct sums of matrix algebras, rather than a single matrix algebra. Consequently, the determinant operation, as required by the solution form $w = \det(\exp(-tH))w_0$, is ill-defined or inapplicable in this context, making this algebra unsuitable. It is possible to introduce the Dieudonné determinant valid for direct sums of matrices, as expressed in a block matrix, but this changes the definition of w we have given.*

Proof. This Clifford algebra is classified as follows:

$$\text{Cl}(2,1) \cong \mathbb{M}_2^2(\mathbb{R}) \quad (40)$$

The notion of determinant is ill-defined as we are dealing with a direct sum of matrices instead of a singular matrix. $\square$

Theorem 5 (Non-definability). *The optimization problem is not definable for these dimensional configurations.*

Proof. This category contains five dimensional configurations:

$\text{Cl}(0)$ Definition 1 requires 1 time parameter for the Lagrange multiplier, and at least 1 space parameter for the integration measure. This configuration has neither.

$\text{Cl}(0,1)$ Definition 1 requires 1 time parameter for the Lagrange multiplier, and at least 1 space parameter for the integration measure. This configuration has the time parameter, but lacks a space parameter.

$\text{Cl}(1,1)$ Definition 1 requires 1 time parameter for the Lagrange multiplier, and at least 1 space parameter for the integration measure. This configuration has the space parameter, but lacks a time parameter.

$\text{Cl}(2,1)$ Definition 1 requires 1 time parameter for the Lagrange multiplier, and at least 1 space parameter for the integration measurement. This configuration has two space parameters, but lacks the time parameter.

$\text{Cl}(2,2)$ Definition 1 requires 1 time parameter for the Lagrange multiplier, and at least 1 space parameter for the integration measurement. This configuration has two space parameters, but has more time parameters than Lagrange multipliers.

$\square$

In Annex C, we provide two conjectures suggesting that all dimensions above 6D are also obstructed. We recommend to read the next section of this paper before reading Annex C, as we utilize concepts that we introduce in the following section for these conjectures.

2.2. Spinors + Dirac Equation

We will now investigate this solution in the context of $\text{Cl}(3, 1)$. We begin with a definition of the determinant for a multivector.

2.2.1. The Multivector Determinant

Our goal here will be to express the determinant of a real 4×4 matrix as a multivector self-product. To achieve that, we begin by defining a general multivector of $\text{Cl}(3, 1)$:

$$\mathbf{u} := a + \mathbf{x} + \mathbf{f} + \mathbf{v} + \mathbf{b} \quad (41)$$

where a is a scalar, $\mathbf{x}$ a vector, $\mathbf{f}$ a bivector, $\mathbf{v}$ is pseudo-vector and $\mathbf{b}$ a pseudo-scalar. Explicitly,

$$\mathbf{u} := a \quad (42)$$

$$+ t\gamma_0 + x\gamma_1 + y\gamma_2 + z\gamma_3 \quad (43)$$

$$+ f_{01}\gamma_0\gamma_1 + f_{02}\gamma_0\gamma_2 + f_{03}\gamma_0\gamma_3 + f_{12}\gamma_1\gamma_2 + f_{13}\gamma_1\gamma_3 + f_{23}\gamma_2\gamma_3 \quad (44)$$

$$+ p\gamma_1\gamma_2\gamma_3 + q\gamma_0\gamma_2\gamma_3 + v\gamma_0\gamma_1\gamma_3 + w\gamma_0\gamma_1\gamma_2 \quad (45)$$

$$+ b\gamma_0\gamma_1\gamma_2\gamma_3 \quad (46)$$

Definition 2 (Real-Majorana Algebra Isomorphism). *The map $\varphi : \text{Cl}(3, 1) \rightarrow \text{Mat}(4, \mathbb{R})$ defined by:*

$$\varphi(1) := \text{diag}(1, 1, 1, 1) \quad (47)$$

$$\varphi(\gamma_0) := \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \quad \varphi(\gamma_1) := \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad (48)$$

$$\varphi(\gamma_2) := \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \varphi(\gamma_3) := \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (49)$$

$$\varphi(\gamma_\mu\gamma_\nu) := \varphi(\gamma_\mu)\varphi(\gamma_\nu) \quad (50)$$

$$\varphi(\gamma_\mu\gamma_\nu\gamma_\kappa) := \varphi(\gamma_\mu)\varphi(\gamma_\nu)\varphi(\gamma_\kappa) \quad (51)$$

$$\varphi(\gamma_0\gamma_1\gamma_2\gamma_3) := \varphi(\gamma_0)\varphi(\gamma_1)\varphi(\gamma_2)\varphi(\gamma_3) \quad (52)$$

extends linearly and multiplicatively to an isomorphism between $\text{Cl}(3, 1)$ and the algebra of real 4×4 matrices.

Definition 3 (Matrix Representation).

$$\varphi(\mathbf{u}) = \begin{bmatrix} a + f_{02} - q - z & b - f_{13} + w - x & -f_{01} + f_{12} - p + v & f_{03} + f_{23} + t + y \\ -b + f_{13} + w - x & a + f_{02} + q + z & f_{03} + f_{23} - t - y & f_{01} - f_{12} - p + v \\ -f_{01} - f_{12} + p + v & f_{03} - f_{23} + t - y & a - f_{02} + q - z & -b - f_{13} - w - x \\ f_{03} - f_{23} - t + y & f_{01} + f_{12} + p + v & b + f_{13} - w - x & a - f_{02} - q + z \end{bmatrix}$$

To manipulate and analyze multivectors in $\text{Cl}(3, 1)$, we introduce several important operations, such as the multivector conjugate, the pseudo-blade conjugate, and the multivector determinant.

Definition 4 (Multivector Conjugate—in $\text{Cl}(3, 1)$).

$$\mathbf{u}^\ddagger := a - \mathbf{x} - \mathbf{f} + \mathbf{v} + \mathbf{b}$$

Definition 5 (Pseudo-Blade Conjugate—in $\text{Cl}(3, 1)$). *The pseudo-blade conjugate of $\mathbf{u}$ is*

$$\mathbf{u}^\dagger := a + \mathbf{x} + \mathbf{f} - \mathbf{v} - \mathbf{b}$$

Lundholm [5] proposes a number of multivector norms, and shows that they are the *unique* forms which carries the properties of the determinants such as $N(\mathbf{uv}) = N(\mathbf{u})N(\mathbf{v})$ to the domain of multivectors:

Definition 6. *The self-products associated with low-dimensional Clifford algebras are:*

$$\text{Cl}(0, 1) : \quad \mathbf{u}^* \mathbf{u} \quad (53)$$

$$\text{Cl}(2, 0) : \quad \mathbf{u}^\dagger \mathbf{u} \quad (54)$$

$$\text{Cl}(3, 0) : \quad (\mathbf{u}^\dagger \mathbf{u})^* \mathbf{u}^\dagger \mathbf{u} \quad (55)$$

$$\text{Cl}(3, 1) : \quad (\mathbf{u}^\dagger \mathbf{u})^\dagger \mathbf{u}^\dagger \mathbf{u} \quad (56)$$

$$\text{Cl}(4, 1) : \quad ((\mathbf{u}^\dagger \mathbf{u})^\dagger \mathbf{u}^\dagger \mathbf{u})^* ((\mathbf{u}^\dagger \mathbf{u})^\dagger \mathbf{u}^\dagger \mathbf{u}) \quad (57)$$

where $\mathbf{u}^*$ is a conjugate that reverses the sign of pseudo-scalar blade (i.e. the highest degree blade of the algebra).

We can now express the determinant of the matrix representation of a multivector via a self-product. This choice is unique:

Theorem 6 (The Multivector Determinant—in $\text{Cl}(3,1)$).

$$(\mathbf{u}^\dagger \mathbf{u})^\dagger \mathbf{u}^\dagger \mathbf{u} \equiv \det \varphi(\mathbf{u})$$

Proof. As this form naively expands into $16^4 = 65536$ product terms, we utilize a computer-assisted proof of this equality in Annex B. $\square$

As can be seen from this theorem, the relationship between determinants and multivector products in $\text{Cl}(3,1)$ is a quartic form that cannot be reduced to a simpler bilinear form.

2.2.2. The Optimization Problem

The relative Shannon entropy requires measures that are everywhere non-negative. Consequently, we will first identify the largest sub-algebra of $\text{Cl}(3,1)$ whose determinant is non-negative.

Theorem 7 (Non-Negativity of Even Multivectors). *Let $\mathbf{u} = \mathbf{a} + \mathbf{f} + \mathbf{b}$ be an even multivector of $\text{Cl}(3,1)$. Then its multivector determinant $(\mathbf{u}^\dagger \mathbf{u})^\dagger \mathbf{u}^\dagger \mathbf{u}$ is non-negative.*

Proof.

$$(\mathbf{u}^\dagger \mathbf{u})^\dagger \mathbf{u}^\dagger \mathbf{u} = ((\mathbf{a} - \mathbf{f} + \mathbf{b})(\mathbf{a} + \mathbf{f} + \mathbf{b}))^\dagger (\mathbf{a} - \mathbf{f} + \mathbf{b})(\mathbf{a} + \mathbf{f} + \mathbf{b}) \quad (58)$$

$$= (a^2 + a\mathbf{f} + a\mathbf{b} - \mathbf{f}a - \mathbf{f}^2 - \mathbf{f}\mathbf{b} + \mathbf{b}a + \mathbf{b}\mathbf{f} + \mathbf{b}^2)^\dagger (a^2 + a\mathbf{f} + a\mathbf{b} - \mathbf{f}a - \mathbf{f}^2 - \mathbf{f}\mathbf{b} + \mathbf{b}a + \mathbf{b}\mathbf{f} + \mathbf{b}^2) \quad (59)$$

$$= (a^2 + \mathbf{b}^2 - \mathbf{f}^2 + 2a\mathbf{b})^\dagger (a^2 + \mathbf{b}^2 - \mathbf{f}^2 + 2a\mathbf{b}) \quad (60)$$

Let us calculate f^2 .

$$f^2 = (\mathbf{E} + I\mathbf{B})(\mathbf{E} + I\mathbf{B}) \quad (61)$$

where $\mathbf{E} := E_{01}\mathbf{e}_{01} + E_{02}\mathbf{e}_{02} + E_{03}\mathbf{e}_{03}$ and where $I\mathbf{B} := B_{12}\mathbf{e}_{12} + B_{13}\mathbf{e}_{13} + B_{23}\mathbf{e}_{23}$

$$= \mathbf{E}^2 + \mathbf{E}I\mathbf{B} + I\mathbf{B}\mathbf{E} - \mathbf{B}^2 \quad (62)$$

$$= \mathbf{E}^2 - \mathbf{B}^2 + 2I\mathbf{E} \cdot \mathbf{B} \quad (63)$$

$$= (a^2 - b^2 - \mathbf{E}^2 + \mathbf{B}^2 - 2I\mathbf{E} \cdot \mathbf{B} + 2Iab)^\dagger (a^2 - b^2 - \mathbf{E}^2 + \mathbf{B}^2 - 2I\mathbf{E} \cdot \mathbf{B} + 2Iab) \quad (64)$$

$$= ((a^2 - b^2 - \mathbf{E}^2 + \mathbf{B}^2) + 2I(\mathbf{E} \cdot \mathbf{B} - ab))((a^2 - b^2 - \mathbf{E}^2 + \mathbf{B}^2) - 2I(\mathbf{E} \cdot \mathbf{B} - ab)) \quad (65)$$

$$= (a^2 - b^2 - \mathbf{E}^2 + \mathbf{B}^2)^2 + 4(\mathbf{E} \cdot \mathbf{B} - ab)^2 \quad (66)$$

$$= (a^2 - b^2 - E_{01}^2 - E_{02}^2 - E_{03}^2 + B_{12}^2 + B_{13}^2 + B_{23}^2)^2 + 4(E_{01}B_{23} + E_{02}B_{13} + E_{03}B_{12} - ab)^2 \quad (67)$$

which is non-negative—the sum of two squares of real numbers is in $\mathbb{R}_{\geq 0}$. $\square$

To define the optimization problem in $\text{Cl}(3, 1)$, we note the following:

1. In 3+1D, we are interested in the case where the states are an element of the even sub-algebra of $\text{Cl}(3, 1)$, whose determinant is non-negative:

$$\varphi(\mathbf{u}) = \begin{bmatrix} a + f_{02} & b - f_{13} & -f_{01} + f_{12} & f_{03} + f_{23} \\ -b + f_{13} & a + f_{02} & f_{03} + f_{23} & f_{01} - f_{12} \\ -f_{01} - f_{12} & f_{03} - f_{23} & a - f_{02} & -b - f_{13} \\ f_{03} - f_{23} & f_{01} + f_{12} & b + f_{13} & a - f_{02} \end{bmatrix} \quad (68)$$

hence satisfying the non-negativity requirement of the information density $w(t, x) \geq 0$.

2. It is well-known that the even sub-algebra of $\text{Cl}(3, 1)$ is isomorphic to 3+1D spinors[6].
3. In the continuum such elements are transformed by a connection which is valued in $\mathfrak{spin}(3, 1) \oplus \mathfrak{u}(1)$:

$$\mathbf{M} \rightarrow \omega_\mu = \frac{1}{2}\omega_\mu^{ab}\gamma_{ab} + IV_\mu \quad (69)$$

4. We also consider translations ∂_x, ∂_y and ∂_z . Hence, we define the covariant derivative D_i as:

$$D_i := \partial_i + \frac{1}{2}\omega_i^{ab}\gamma_{ab} + IV_i \quad (70)$$

5. The operator $\gamma^i D_i + \gamma^0 \omega_0$ represents the set of all transformations (including translations) that can be done on the even elements of $\text{Cl}(3, 1)$ parametrized in spacetime. As such it is the **most permissive** linear³ constraint within spacetime that transforms its even multivectors.

Flat Spacetime:

The optimization problem will be as follows:

$$\mathcal{L}[w] := - \int_V w(t, \vec{x}) \ln \frac{w(t, \vec{x})}{w(0, \vec{x})} d^3\vec{x} + t \left(\overline{H} - \int_V w(t, \vec{x}) \text{tr}(\gamma^0(\gamma^i D_i + \gamma^0 \omega_0)) d^3\vec{x} \right) \quad (71)$$

The solution is:

$$w(t, \vec{x}) = \det \exp(-t\gamma^0(\gamma^i D_i + \gamma^0 \omega_0)) w(0, \vec{x}) \quad (72)$$

³ In Section 2.3 on Yang-Mills and gravity, we will use an even more general non-linear constraint.

The base field is identified from the multivector determinant, and is written as:

$$\det \phi(t, \vec{x}) = \det \exp(-t\gamma^0(\gamma^i D_i + \gamma^0 \omega_0)) \det \phi(0, \vec{x}) \quad (73)$$

where

1. $w(t, \vec{x}) := \det \phi(t, \vec{x})$
2. $w(0, \vec{x}) := \det \phi(0, \vec{x})$

As such, Equation 73 is formally equivalent to Equation 72.

The following expression, obtained by lifting the determinant⁴, satisfies the solution:

$$\phi(t, \vec{x}) = \exp(-t\gamma^0(\gamma^i D_i + \gamma^0 \omega_0)) \phi(0, \vec{x}) \quad (74)$$

where $\phi(0, \vec{x})$ is defined as:

Definition 7 (Spinor-valued Field).

$$\phi(0, \vec{x}) = \phi(0, \vec{x}) = a(\vec{x}) + \mathbf{f}(\vec{x}) + \mathbf{b}(\vec{x}) \quad (75)$$

The base field $\phi(t, \vec{x})$ leads to a variant of the Schrödinger equation obtained by taking its derivative with respect to t :

Definition 8 (Spinor-valued Schrödinger Equation).

$$\frac{\partial \phi(t, \vec{x})}{\partial t} = -\gamma^0(\gamma^i D_i + \gamma^0 \omega_0) \phi(t, \vec{x})$$

The above expression is simply the massless Dirac equation in Hamiltonian form. Specifically, the Dirac equation is obtained as follows:

$$\frac{\partial \phi(t, \vec{x})}{\partial t} = -\gamma^0(\gamma^i D_i + \gamma^0 \omega_0) \phi(t, \vec{x}) \quad (76)$$

$$\implies 0 = -\gamma^0(\gamma^i D_i + \gamma^0 \omega_0) \phi(t, \vec{x}) - \frac{\partial \phi(t, \vec{x})}{\partial t} \quad (77)$$

$$= (\gamma^i D_i + \gamma^0 \omega_0) \phi(t, \vec{x}) - \gamma^0 \frac{\partial \phi(t, \vec{x})}{\partial t} \quad (78)$$

$$= (\gamma^i D_i + \gamma^0 \omega_0) \phi(t, \vec{x}) - \gamma^0 \partial_0 \phi(t, \vec{x}) \quad (79)$$

$$= \gamma^\mu D_\mu \phi(t, \vec{x}) \quad (80)$$

$$= D\phi(t, \vec{x}) \quad (81)$$

where $\gamma^\mu D_\mu$ is the Dirac operator over all 4 spacetime coordinates. The end result is the massless Dirac equation in the language of STA.

From Noether's theorem, it is known that the Dirac equation contains a conserved charge current $J = \psi^\dagger \gamma_0 \psi$, which is the Dirac current.

Theorem 8 (Positive-Definite Inner Product). *The inner product, defined as $\gamma_0 \cdot \psi^\dagger \gamma_0 \psi$ is positive definite.*

⁴ The removal of the determinant implies an additional term $\det A = \det B \implies A = CB$, where $\det C = 1$. Furthermore, since $\det C = 1$ implies $C \in \text{SL}(4, \mathbb{R})$, it is simply a gauge choice. Here, we choose $C = 1$.

Proof. Let $\psi := a + \mathbf{f} + \mathbf{b}$ and $\mathbf{f} := \mathbf{E} + I\mathbf{B}$, then:

$$\gamma_0 \cdot (a - \mathbf{f} + \mathbf{b})\gamma_0(a + \mathbf{f} + \mathbf{b}) \quad (82)$$

$$= \gamma_0 \cdot (a\gamma_0 - \mathbf{E}\gamma_0 - I\mathbf{B}\gamma_0 + \mathbf{b}\gamma_0)(a + \mathbf{f} + \mathbf{b}) \quad (83)$$

$$= \gamma_0 \cdot (\gamma_0 a + \gamma_0 \mathbf{E} - \gamma_0 I\mathbf{B} - \gamma_0 \mathbf{b})(a + \mathbf{f} + \mathbf{b}) \quad (84)$$

$$= \gamma_0 \cdot \gamma_0(a + \mathbf{E} - I\mathbf{B} - \mathbf{b})(a + \mathbf{E} + I\mathbf{B} + \mathbf{b}) \quad (85)$$

$$= \gamma_0 \cdot \gamma_0(a^2 + a\mathbf{E} + aI\mathbf{B} + a\mathbf{b}) \quad (86)$$

$$+ \mathbf{E}a + \mathbf{E}^2 + \mathbf{E}I\mathbf{B} + \mathbf{E}\mathbf{b} \quad (87)$$

$$- I\mathbf{B}a - I\mathbf{B}\mathbf{E} - (I\mathbf{B})^2 - I\mathbf{B}\mathbf{b} \quad (88)$$

$$- \mathbf{b}a - \mathbf{b}\mathbf{E} - \mathbf{b}I\mathbf{B} - \mathbf{b}^2) \quad (89)$$

$$= (a^2 + \mathbf{E}^2 - (I\mathbf{B})^2 - \mathbf{b}^2) \quad (90)$$

$$= (a^2 + b^2 + \mathbf{E}^2 + \mathbf{B}^2) \quad (91)$$

$$= (a^2 + b^2 + f_{01}^2 + f_{02}^2 + f_{03}^2 + f_{12}^2 + f_{13}^2 + f_{23}^2) \quad (92)$$

which is positive-definite. $\square$

Consequently, the quantity $\rho := \gamma_0 \cdot \psi^\dagger \gamma_0 \psi$ can be understood as a probability density when normalized such as in single particle relativistic quantum theory.

Theorem 9 (Equivalence to David Hestenes'[6] formulation).

$$a + \mathbf{f} + \mathbf{b} \equiv \sqrt{\rho} R e^{-I\theta/2} \text{ iff } \det(a + \mathbf{f} + \mathbf{b}) \neq 0 \quad (93)$$

where R is a rotor $R^\dagger R = 1$

Proof. Let $\psi = a + \mathbf{f} + \mathbf{b}$. Then

$$\psi^\dagger \psi = a^2 - b^2 - f^2 + 2Iab \quad (94)$$

$$= a^2 - b^2 - \mathbf{E}^2 + \mathbf{B}^2 - 2I\mathbf{E} \cdot \mathbf{B} + 2Iab \quad (95)$$

which is a complex number. In polar form, $\psi^\dagger \psi = \rho e^{-I\theta}$, which implies iff $\psi^\dagger \psi \neq 0$, that $\psi = \sqrt{\rho} R e^{-I\theta/2}$. $\square$

We also note that the definition of the Dirac equation recovers David Hestenes' formulation of the same in the massless case $D\psi I\sigma_3 = m\psi\gamma_0$. Indeed, posing $m = 0 \implies D\psi = 0$.

Curved Spacetime:

In curved spacetime, we utilize the ADM formalism. We foliate spacetime in hypersurfaces Σ_t of constant t :

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \quad (96)$$

The optimization problem Lagrangian remains similar, but the constraint now acquires lapse and shift functions:

$$\mathcal{L} := - \int_{\Sigma_t} w(t, \vec{x}) \ln \frac{w(t, \vec{x})}{w(0, \vec{x})} \sqrt{h} d^3\vec{x} + t \left(\bar{H} - \int_{\Sigma_t} w(t, \vec{x}) \text{tr}(N\gamma^n(\gamma^i D_i - K) - N^i D_i) \sqrt{h} d^3\vec{x} \right) \quad (97)$$

where

1. $h := \det h_{ij}$
2. $\gamma^n := n_a \gamma^a$ is the STA representation of the the normal vector n^a to the spatial hypersurface Σ_t

3. $\gamma^i := \mathbf{e}_a^i \gamma^a$
4. $K := K_k^k$ is the trace of the extrinsic curvature
5. $D_i := \partial_i + \frac{1}{2} \omega_i^{ab} \gamma_{ab} + IV_i$ is the 3D spatial covariant derivative on the slice Σ_t .

The problem is solved in a manner similar to the flat case and leads to the Schrödinger equation:

$$\frac{\partial \phi(t, \vec{x})}{\partial t} = -(N \gamma^n (\gamma^i D_i - K) - N^i D_i) \phi(t, \vec{x}) \quad (98)$$

This is the Hamiltonian form of the massless Dirac equation $\gamma^\mu D_\mu \phi = 0$ with covariant derivative D_μ expressed with lapse and shift functions and containing a spin and pseudoscalar connection.

Field Functional:

The field functional version of the optimization problem integrates over all possible geometries:

$$\mathcal{L} := - \int \mathcal{D}e \, w[e; t] \ln \frac{w[e; t]}{w[e; 0]} + t \left(\bar{H} - \int \mathcal{D}e \, w[e; t] \text{tr} H[e] \right) \quad (99)$$

where

1. The configuration $e = \{\mathbf{e}_i^a(\vec{x}), N(\vec{x}), N^i(\vec{x})\}$ includes the spatial dreibein $\mathbf{e}_i^a(\vec{x})$, lapse $N(\vec{x})$, and shift $N^i(\vec{x})$, representing a full specification of the spatial geometry and its embedding coordinates for the slice Σ_t .
2. $h_{ij} := \delta_{ab} \mathbf{e}_i^a(\vec{x}) \mathbf{e}_j^b(\vec{x})$
3. $H[e] := \int_{\Sigma_t} (N[e] \gamma^n [e] (\gamma^i [e] D_i [e] - K[e]) - N^i [e] D_i [e]) (\vec{x}) \sqrt{h[e]}(\vec{x}) \, d^3 \vec{x}$

The optimization problem is solved in a manner similar to the previous cases and leads to the Schrödinger equation:

$$\frac{\partial \phi[e; t]}{\partial t} = -H[e] \phi[e; t] \quad (100)$$

2.3. Yang-Mills + Gravity

We now consider a generalized constraint with a sequence of terms in D of increasing powers. This can be achieved by wrapping D within a function f , and using a power series expansion. Finally, we utilize a scaling factor Λ to eliminate the dimensional units of D , leading to $f(D^2/\Lambda^2)$ as the final general form. The power of 2 is utilized to make the constraint symmetric with respect to a sign change.

The optimization problem will be as follows:

$$\mathcal{L}[w] := - \int_V w(t, \vec{x}) \ln \frac{w(t, \vec{x})}{w(0, \vec{x})} \, d^3 \vec{x} + t \left(\bar{H} - \int_V w(t, \vec{x}) \text{tr} f(D^2/\Lambda^2) \, d^3 \vec{x} \right) \quad (101)$$

where $D := \gamma^0 (\gamma^i D_i + \gamma^0 \omega_0)$, and where f and Λ are a generalization function and regularization constant, respectively.

The solution $w(t, \vec{x})$ is:

$$w(t, \vec{x}) = e^{-1} w(0, \vec{x}) \exp[-t \text{Tr} f(D^2/\Lambda^2)] \quad (102)$$

and the value of the hamiltonian functional at the maximum is

$$H = f(D^2/\Lambda^2) \quad (103)$$

and the Lagrangian is:

$$\mathcal{L} = \text{Tr}(f(D^2/\Lambda^2)) \quad (104)$$

Hence the stationary condition of the entropy-maximisation problem is identical to the spectral action introduced by Connes and Chamseddine [7].

This form conveniently encode gravity and Yang-Mills, as shown by Connes and Chamseddine. Indeed, by applying the heat-kernel (Seeley–DeWitt) expansion to the elliptic operator D^2 one obtains, for large Λ ,

$$\mathrm{Tr}(f(D^2/\Lambda^2)) = \sum_{k=0}^{\lfloor n/2 \rfloor} c_k \Lambda^{n-2k} a_{2k}(D^2) + O(\Lambda^{-\infty}),$$

with universal coefficients c_k that are the moments of f and with the Seeley–DeWitt coefficients $a_{2k}(D^2)$. As shown in [7], the first few non-vanishing terms reproduce precisely the familiar Lagrangians of gravity and gauge theory:

1. $a_0(D^2)$ yields a cosmological-constant term $\Lambda^n \int \sqrt{g} d^n x$;
2. $a_2(D^2)$ gives the Einstein–Hilbert action $\Lambda^{n-2} \int \sqrt{g} R d^n x$;
3. $a_4(D^2)$ contains the Yang–Mills kinetic term $\int \sqrt{g} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) d^n x$ together with the scalar-field kinetic and potential contributions.

Thus the spectral action automatically encodes General Relativity and Yang–Mills, with all coupling constants expressed in terms of the moments of the cut-off function f and the single scale Λ .

Consequently, the optimisation problem we have solved provides a statistical-mechanical interpretation of the Einstein–Hilbert and Yang–Mills terms.

3. Discussion

When asked what a physical theory is, a practical answer is: a set of equations that applies to all experiments realizable within a domain, with “nature” as the most general domain. We mathematically realize this operational definition as a single constrained optimization, maximizing the relative Shannon entropy of a final measurement with respect to its preparation, under a natural constraint:

$$\underbrace{\mathcal{L}[w]}_{\text{an optimization problem}} := \underbrace{- \int_V w(t, \vec{x}) \ln \frac{w(t, \vec{x})}{w(0, \vec{x})} d^p \vec{x}}_{\text{on the entropy of a measurement relative to its preparation}} + \underbrace{t \left(\bar{H} - \int_V w(t, \vec{x}) \mathrm{Tr} H(\vec{x}) d^p \vec{x} \right)}_{\text{as constrained by nature}}.$$

Relative entropy captures the essence of any experiment (preparation $\rightarrow$ readout). The “natural constraint” must support exponentials—hence an associative operator algebra closed under addition and multiplication—admit a faithful finite-dimensional matrix representation so that a trace produces a scalar for the Lagrange dual, and include all transformations admissible on the base field. This generalizes Jaynes’ MaxEnt beyond thermodynamics and fixes the domain on which solutions make sense.

From this, a short chain of necessities follows:

1. Inference forces exponentials and hence algebras. Maximizing relative entropy yields $\exp(\cdot)$ solutions, so constraints live in operator algebras with faithful matrix representations and a well-defined trace.
2. Natural constraint = maximal operator content. On the even subalgebra, the least-restrictive generator of all admissible transformations is the covariant Dirac operator; its functional generalization $f(D^2/\Lambda^2)$ yields the spectral action whose Seeley–DeWitt expansion reproduces the Einstein–Hilbert and Yang–Mills sectors (as in the Connes–Chamseddine framework).
3. Dimensional selection by positivity and definability. Because the objective is a relative entropy, the transported scalar density must be real and nonnegative. Surveying Clifford algebras, only $\mathrm{Cl}(3, 1)$ admits a nontrivial even subalgebra with a determinant-like self-product giving a real, nonnegative scalar throughout the flow; other signatures fail by non-real/negative densities or by requiring extra combining choices beyond the “natural” maximality.

4. Emergent dynamics and unification. In $Cl(3, 1)$, the even subalgebra supports a positive-definite density and Dirac dynamics; the functional constraint $f(D^2/\Lambda^2)$ gives the spectral action (GR + Yang–Mills) via the heat-kernel expansion. This fixes the form of the laws (Dirac, Einstein–Hilbert, Yang–Mills); numerical couplings reside in the choice of f and the scale Λ .

The conclusion of these necessities is that, conditional only on the ontic possibility of measurements, the most general inferable structure permitted by the mathematics of entropy-maximization is *the universe itself*. The claim is not that couplings or detailed gauge representations are fixed a priori; rather, the *form* of the laws (Dirac operator, Einstein–Hilbert term, Yang–Mills action) is fixed by the inferential principle, while numerical parameters are encoded by the choice of f and the scale Λ and are calibrated empirically. In plain terms:

The universe is an inferred structure: the minimally constrained outcome of maximally generalized maximum-entropy inference.

3.1. Proposed Interpretation of QM

An experiment begins with a measured preparation $w(0, \vec{x})$, evolves under the natural constraint (Axiom 1), and ends with a measured readout $w(t, \vec{x})$. We take the experiment (not abstract states) as ontic. Physics is therefore not a set of laws that are both axiomatic and empirically validated, but the optimal interpolation from $w(0, \vec{x})$ to $w(t, \vec{x})$ under nature’s constraint. Laws are inferred, not primitive.

3.1.1. Measure-to-measure evolution resolves interpretive tensions

Standard presentations evolve $\psi(0, \vec{x}) \rightarrow w(t, \vec{x})$, inviting a “measurement postulate.” Here, dynamics is between measures: $w(0, \vec{x}) \rightarrow w(t, \vec{x})$. Wavefunctions $\psi(0, \vec{x}) \rightarrow \psi(t, \vec{x})$ appear as the most efficient inferential intermediates selected by the optimization. This shift aligns with laboratory practice and defuses the measurement problem: the endpoints are already measures.

Core Argument:

1. We propose that a well-defined experiment begin with a measurement outcome $w(0)$, not an abstract quantum state $\psi(0)$.
2. Example: Preparing $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ requires:
 - (a) Measure systems to collapse to $|0\rangle$ or $|1\rangle$.
 - (b) Discard all systems in state $|1\rangle$.
 - (c) Apply a Hadamard gate H to $|0\rangle$.
 - (d) The preparation is complete.

Neglecting the initial measurement (a) implies that systems of unknown states are sent into the Hadamard gate—the resulting experiment is ill-defined.

Challenges and Solutions:

1. Objection 1: Preparation Without Collapse
 - (a) Issue: Traditional QM superficially appears to allow preparing $|\psi\rangle$ without collapsing it (e.g. cooling).
 - (b) Response: In practice, all preparations are validated by measurement (or an equivalent).
 - (c) Example:
 - i. Cooling various qubits $|\psi\rangle$ to $|0\rangle$ is non-invertible (one cannot return to the initial $|\psi\rangle$ because of dissipative effects). The end result is mathematically equivalent to a measurement $|0\rangle$ or $|1\rangle$ followed by a discard of $|1\rangle$.
 - ii. Creating $|+\rangle = H|0\rangle$ requires assuming the initial $|0\rangle$, validated by prior conditions.
2. Objection 2: Loss of Quantum Coherence
 - (a) Issue: If preparation starts with a measurement, how do we account for coherence (e.g., interference)?

- (b) Response: Coherence emerges operationally.
- (c) Example:
 - i. Measure systems to collapse to $|0\rangle$ or $|1\rangle$.
 - ii. Discard all systems in state $|1\rangle$.
 - iii. Apply H to many initial $|0\rangle$ -verified states.
 - iv. Aggregate final measurements ($q \in \mathbb{E}$) show interference patterns, even though individual experiments start with collapsed states.

3. Objection 3: Entanglement and Nonlocality

- (a) Issue: Entangled states require joint preparation of superpositions.
- (b) Response: Entanglement is preparable from an initial measurement like any other state.
- (c) Example:
 - i. Measure systems to collapse to $|00\rangle$, $|01\rangle$, $|10\rangle$, or $|11\rangle$.
 - ii. Discard all systems in state $|01\rangle$, $|10\rangle$, and $|11\rangle$.
 - iii. Apply a Hadamard gate to the first qubit: $(H \otimes I)|00\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$
 - iv. Apply a CNOT gate (with first qubit as control, second as target): $\text{CNOT}[\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)] = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

The final state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is an entangled state—specifically, it's one of the Bell states (sometimes denoted as $|\Phi^+\rangle$).

In all cases, skipping the initial measurement allows unknown inputs and ill-defined experiments—precisely what the variational framework precludes when it interpolates between $w(0)$ and $w(t)$.

3.1.2. Atomic experiments as the fundamental evolving objects

Definition 1 ranges over all realizable experiments; in practice we conduct them atomically. An atomic experiment is an ordered pair $(q_{\text{in}}, q_{\text{out}})$ drawn from an outcome set $\mathbb{E}$. A run of n atomic experiments produces empirical frequencies that define $w(0)$ and $w(t)$ (or normalized $\rho(0)$ and $\rho(t)$) via counting and the law of large numbers. These empirical measures are the endpoints in the optimization; the inferred field dynamics is the optimal, constraint-compatible map between them. Thus atomic experiments are the irreducible “facts,” and the usual quantum machinery (states, unitaries, Born-type bilinears) emerges as the most efficient calculus for transporting measures under the natural constraint.

3.2. Stabilizing Physics on Empirical Foundations

This framework drops primitive ontologies (fields, particles) in favor of concepts grounded directly in empirical practice. In standard theories, foundational entities are periodically replaced as data accumulate (e.g., “electron as particle” $\rightarrow$ field in QFT), introducing conceptual drift away from direct verifiability.

By contrast, we take experiments as the mathematical foundation (Definition 1). Atomic experiments—ordered pairs of outcomes from preparation to readout—are the irreducible facts; the theory is the optimal interpolation between their empirical measures under the natural constraint. Unlike fields or particles, recorded outcomes cannot be superseded by “more fundamental” entities; they are the basis against which any future concept must be tested. This stance resonates with event-centric and operational approaches that treat events/measurements as primary building blocks rather than enduring substances), and with broader efforts to frame dynamics as interpolation rather than reduction.

Consequently, the wavefunction ψ , spacetime geometry, and gauge fields are derived tools for transporting $w(0, \vec{x})$ to $w(t, \vec{x})$, not ontic primitives. The foundations thereby become robust to conceptual turnover, anchored in verifiable experimental structure. Science’s iterative practice then refines the interpolation—tightening models while preserving empirical testability and falsifiability.

3.3. Emergence of Time as a Lagrange Multiplier

Time is not postulated; it emerges as the Lagrange-dual variable conjugate to the natural constraint, in direct analogy with Jaynes’ identification of temperature from inference. In statistical mechanics, maximizing entropy at fixed mean energy gives the dual parameter $\beta = \partial S / \partial \bar{E}$, later read as $\beta = 1 / (k_B T)$. Here, maximizing relative Shannon entropy under $\bar{H} = \int w \operatorname{Tr} H$ yields

$$t = \frac{\partial S_{\text{rel}}}{\partial \operatorname{Tr} H} \tag{105}$$

At the extremum this dual variable parametrizes the flow

$$w(t, \vec{x}) \propto \operatorname{detexp} \left[-t H(\vec{x}) \right] w(0, \vec{x}), \tag{106}$$

and, at the operator level, induces Dirac evolution for amplitudes ϕ :

$$\partial_t \phi = -\mathcal{H} \phi, \quad \mathcal{H} = \gamma^0 \left(\gamma^i D_i + \gamma^0 \omega_0 \right). \tag{107}$$

Thus the operational meaning of t is fixed by the dynamics it generates: it is the affine parameter of the maximum-entropy flow transporting a preparation $w(0, \vec{x})$ to a readout $w(t, \vec{x})$. This inference-based view of time resonates with entropic dynamics accounts of quantum theory and measurement, and with broader information-theoretic approaches where temporal structure is tied to dual parameters, complexity, or entropy growth.

3.4. Physical Interpretation of the Information Density $w(t, \vec{x})$

We interpret the information density $w(t, \vec{x})$ as an entropy-carrying evidence measure that is operationally built from aggregating atomic experiments (preparation–readout pairs) and mathematically transported by the natural constraint. It is nonnegative and generally unnormalized so that the relative Shannon entropy between preparation and readout is real and finite. Physically, w is not a Born probability density (those arise downstream from conserved, positive-definite bilinears such as the Dirac charge density after normalization); rather, it is the extensive, gauge-invariant scalar that the optimization transports, furnishing an explicit bridge from laboratory measurement counts to field amplitudes, and—through its positivity, reality, and multiplicativity—selecting 3+1 as the unique consistent arena. This places w as a new, empirically grounded scalar observable of spinor configurations that both underwrites the variational principle and constrains admissible dynamics.

4. Conclusions

E.T. Jaynes fundamentally reoriented statistical mechanics by recasting it as a problem of inference rather than mechanics. His approach revealed that the equations of thermodynamics are not arbitrary physical laws but necessary consequences of maximizing entropy subject to constraints. This work extends Jaynes’ inferential paradigm to address a more fundamental question: what is a physical theory itself?

A physical theory, at its essence, is a set of equations that applies to all experiments realizable within a domain. While this definition is informal, our contribution lies in making this concept mathematical. By formulating it as an optimization problem—maximizing the relative entropy of measurement outcomes subject to the natural constraint—we transform the abstract definition of a physical theory into a precise, solvable mathematical problem.

This approach represents a profound methodological shift. Rather than constructing physical theories through trial-and-error enumerations of axioms, we derive them as necessary solutions to a well-defined optimization problem. Physics thus emerges not as a collection of independently discovered laws but as the unique optimal interpolation device between arbitrary experimental preparation and measurement under the constraint of nature.

The power of this formulation lies in its generality allowing us to recover several established physical theories from entropy optimization. Jaynes showed that statistical inference with minimal assumptions yields thermodynamics; we suggest that this same principle, properly generalized, may yield the foundation to all of physics.

Funding: Funding: This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Institutional Review Board Statement: During the preparation of this manuscript, we utilized a Large Language Model (LLM), for assistance with spelling and grammar corrections, as well as for minor improvements to the text to enhance clarity and readability. This AI tool did not contribute to the conceptual development of the work, data analysis, interpretation of results, or the decision-making process in the research. Its use was limited to language editing and minor textual enhancements to ensure the manuscript met the required linguistic standards.

Data Availability Statement: Data Availability Statement: No datasets were generated or analyzed during the current study.

Conflicts of Interest: The author declares that he has no competing financial or non-financial interests that are directly or indirectly related to the work submitted for publication.

Appendix A. SM

Here, we solve the Lagrange multiplier equation of SM.

$$\mathcal{L} := \underbrace{-k_B \sum_i \rho_i(\beta) \ln \rho_i(\beta)}_{\text{Boltzmann Entropy}} + \underbrace{\lambda \left(1 - \sum_i \rho_i(\beta)\right)}_{\text{Normalization Constraint}} + \underbrace{k_B \beta \left(\bar{E} - \sum_i \rho_i(\beta) E_i\right)}_{\text{Average Energy Constraint}} \quad (\text{A1})$$

We solve the maximization problem as follows:

$$0 = \frac{\partial \mathcal{L}(\rho_1, \dots, \rho_i, \dots, \rho_n)}{\partial \rho_i} \quad (\text{A2})$$

$$= -k_B \ln \rho_i(\beta) - k_B - \lambda - k_B \beta E_i \quad (\text{A3})$$

$$= k_B \ln \rho_i(\beta) + k_B + \lambda + k_B \beta E_i \quad (\text{A4})$$

$$\Rightarrow k_B \ln \rho_i(\beta) = -k_B - \lambda - k_B \beta E_i \quad (\text{A5})$$

$$\Rightarrow \rho_i(\beta) = \exp\left(\frac{-k_B - \lambda}{k_B}\right) \exp(-\beta E_i) \quad (\text{A6})$$

$$= \frac{1}{Z(\beta)} \exp(-\beta E_i) \quad (\text{A7})$$

The partition function, is obtained as follows:

$$1 = \sum_j \exp\left(\frac{-k_B - \lambda}{k_B}\right) \exp(-\beta E_j) \quad (\text{A8})$$

$$\Rightarrow \left(\exp\left(\frac{-k_B - \lambda}{k_B}\right)\right)^{-1} = \sum_j \exp(-\beta E_j) \quad (\text{A9})$$

$$Z(\beta) = \sum_j \exp(-\beta E_j) \quad (\text{A10})$$

Finally, the probability measure is:

$$\rho_i(\beta) = \frac{1}{\sum_j \exp(-\beta E_j)} \exp(-\beta E_i) \quad (\text{A11})$$

Appendix B. SageMath Program Showing $[u^\dagger u]_{3,4} u^\dagger u = \det \varphi(u)$

```

from sage.algebras.clifford_algebra import CliffordAlgebra
from sage.quadratic_forms.quadratic_form import QuadraticForm
from sage.symbolic.ring import SR
from sage.matrix.constructor import Matrix

# Define the quadratic form for GA(3,1) over the Symbolic Ring
Q = QuadraticForm(SR, 4, [-1, 0, 0, 0, 1, 0, 0, 1, 0, 1])

# Initialize the GA(3,1) algebra over the Symbolic Ring
algebra = CliffordAlgebra(Q)

# Define the basis vectors
e0, e1, e2, e3 = algebra.gens()

# Define the scalar variables for each basis element
a = var('a')
t, x, y, z = var('t x y z')
f01, f02, f03, f12, f23, f13 = var('f01 f02 f03 f12 f23 f13')
v, w, q, p = var('v w q p')
b = var('b')

# Create a general multivector
udegree0=a
udegree1=t*e0+x*e1+y*e2+z*e3
udegree2=f01*e0*e1+f02*e0*e2+f03*e0*e3+f12*e1*e2+f13*e1*e3+f23*e2*e3
udegree3=v*e0*e1*e2+w*e0*e1*e3+q*e0*e2*e3+p*e1*e2*e3
udegree4=b*e0*e1*e2*e3
u=udegree0+udegree1+udegree2+udegree3+udegree4

u2 = u.clifford_conjugate()*u

u2degree0 = sum(x for x in u2.terms() if x.degree() == 0)
u2degree1 = sum(x for x in u2.terms() if x.degree() == 1)
u2degree2 = sum(x for x in u2.terms() if x.degree() == 2)
u2degree3 = sum(x for x in u2.terms() if x.degree() == 3)
u2degree4 = sum(x for x in u2.terms() if x.degree() == 4)
u2conj34 = u2degree0+u2degree1+u2degree2-u2degree3-u2degree4

I = Matrix(SR, [[1, 0, 0, 0],
                [0, 1, 0, 0],
                [0, 0, 1, 0],
                [0, 0, 0, 1]])

#MAJORANA MATRICES
y0 = Matrix(SR, [[0, 0, 0, 1],
                [0, 0, -1, 0],
                [0, 1, 0, 0],
                [-1, 0, 0, 0]])

```

```
y1 = Matrix(SR, [[0, -1, 0, 0],
                 [-1, 0, 0, 0],
                 [0, 0, 0, -1],
                 [0, 0, -1, 0]])
```

```
y2 = Matrix(SR, [[0, 0, 0, 1],
                 [0, 0, -1, 0],
                 [0, -1, 0, 0],
                 [1, 0, 0, 0]])
```

```
y3 = Matrix(SR, [[-1, 0, 0, 0],
                 [0, 1, 0, 0],
                 [0, 0, -1, 0],
                 [0, 0, 0, 1]])
```

```
mdegree0 = a
mdegree1 = t*y0+x*y1+y*y2+z*y3
mdegree2 = f01*y0*y1+f02*y0*y2+f03*y0*y3+f12*y1*y2+f13*y1*y3+f23*y2*y3
mdegree3 = v*y0*y1*y2+w*y0*y1*y3+q*y0*y2*y3+p*y1*y2*y3
mdegree4 = b*y0*y1*y2*y3
m=mdegree0+mdegree1+mdegree2+mdegree3+mdegree4
```

```
print(u2conj34*u2 == m.det())
```

The program outputs

True

showing, by computer-assisted symbolic manipulations, that the determinant of the real Majorana representation of a multivector u is equal to the double-product: $[u^\dagger u]_{3,4} u^\dagger u = \det \varphi(u)$.

Appendix C. Obstructions in 6D and Above

Conjecture 1 (No observables (6D)). *The multivector representation of the norm in 6D restricts observables to the identity.*

Argument. In six dimensions and above, the self-product patterns found in Definition 6 collapse. The research by Acus et al. [7] in 6D Clifford algebra concludes that the determinant, so far defined through a self-products of the multivector, fails to extend into 6D. The crux of the difficulty is evident in the reduced case of a 6D multivector containing only scalar and grade-4 elements:

$$s(B) = b_1 B f_5(f_4(B) f_3(f_2(B) f_1(B))) + b_2 B g_5(g_4(B) g_3(g_2(B) g_1(B))) \quad (A12)$$

This equation is not a multivector self-product but a linear sum of two multivector self-products[7].

The full expression is given in the form of a system of 4 equations, which is too long to list in its entirety. A small characteristic part is shown:

$$a_0^4 - 2a_0^2 a_{47}^2 + b_2 a_0^2 a_{47}^2 p_{412} p_{422} + \langle 72 \text{ monomials} \rangle = 0 \quad (A13)$$

$$b_1 a_0^3 a_{52} + 2b_2 a_0 a_{47}^2 a_{52} p_{412} p_{422} p_{432} p_{442} p_{452} + \langle 72 \text{ monomials} \rangle = 0 \quad (A14)$$

$$\langle 74 \text{ monomials} \rangle = 0 \quad (A15)$$

$$\langle 74 \text{ monomials} \rangle = 0 \quad (A16)$$

From Equation A12, it is possible to see that no observable $\mathbf{O}$ can satisfy this equation because the linear combination does not allow one to factor it out of the equation.

$$b_1 \mathbf{O} B f_5(f_4(B) f_3(f_2(B) f_1(B))) + b_2 B g_5(g_4(B) g_3(g_2(B) g_1(B))) = b_1 B f_5(f_4(B) f_3(f_2(B) f_1(B))) + b_2 \mathbf{O} B g_5(g_4(B) g_3(g_2(B) g_1(B))) \quad (\text{A17})$$

Any equality of the above type between $b_1 \mathbf{O}$ and $b_2 \mathbf{O}$ is frustrated by the factors b_1 and b_2 , forcing $\mathbf{O} = 1$ as the only satisfying observable. Since the obstruction occurs within grade-4, which is part of the even sub-algebra it is questionable that a satisfactory theory (with non-trivial observables) is constructible in 6D, using our method. $\square$

This conjecture proposes that the multivector representation of the determinant in 6D does not allow for the construction of non-trivial observables, which is a crucial requirement for a relevant quantum formalism. The linear combination of multivector self-products in the 6D expression prevents the factorization of observables, limiting their role to the identity operator.

Conjecture 2 (No observables (above 6D)). *The norms beyond 6D are progressively more complex than the 6D case, which is already obstructed.*

These theorems and conjectures provide additional insights into the unique role of the unobstructed 3+1D signature in our proposal.

We also note that it is interesting that our proposal is able to rule out $\text{Cl}(1, 3)$ even if in relativity, the signature of the metric $(+, -, -, -)$ versus $(-, -, -, +)$ does not influence the physics. However, in Clifford algebra, $\text{Cl}(1, 3)$ represents 1 space dimension and 3 time dimensions. Therefore, it is not the signature itself that is ruled out but rather the specific arrangement of 3 time and 1 space dimensions, as this configuration yields quaternion-valued "probabilities" (i.e. $\text{Cl}(1, 3) \cong \mathbb{M}_2(\mathbb{H})$ and $\det \mathbb{M}_2(\mathbb{H}) \in \mathbb{H}$).

References

1. Jaynes, E.T. Information theory and statistical mechanics. *Physical review* **1957**, *106*, 620.
2. Jaynes, E.T. Information theory and statistical mechanics. II. *Physical review* **1957**, *108*, 171.
3. Dirac, P.A.M. *The principles of quantum mechanics*; Number 27, Oxford university press, 1981.
4. Von Neumann, J. *Mathematical foundations of quantum mechanics: New edition*; Vol. 53, Princeton university press, 2018.
5. Lundholm, D. Geometric (Clifford) algebra and its applications. *arXiv preprint math/0605280* **2006**.
6. Hestenes, D. Spacetime physics with geometric algebra. *American Journal of Physics* **2003**, *71*, 691–714.
7. Acus, A.; Dargys, A. Inverse of multivector: Beyond $p+q=5$ threshold. *arXiv preprint arXiv:1712.05204* **2017**.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.