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Posted Date: 29 October 2025

doi: 10.20944/preprints202510.2218.v1

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A Thermodynamic Derivation of the Cosmological Constant Λ and Resolution of the Vacuum Catastrophe from a New Quantum Scale

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Abstract

A thermodynamic and quantum derivation of the vacuum energy density $u_\Lambda = \Lambda c^4/(8\pi G)$ is presented from first principles, resolving the long-standing vacuum catastrophe without recourse to Planck-scale physics. Using the Bekenstein–Hawking entropy and Gibbons–Hawking temperature of the de Sitter horizon, we apply $E = TS$ to show that u_Λ arises naturally as a maximum entropy bound of the universe. An independent derivation from zero-point energy follows by introducing a physically motivated cutoff at the Lambda scale $L_\Lambda = (\hbar G/\Lambda c^3)^{1/4}$, a new quantum–thermodynamic scale defined by $G, \hbar, c$, and Λ. *The resulting* Λ*-units are unique: vacuum-matching to de Sitter horizon thermodynamics fixes the remaining affine freedom in the dimensional analysis. In this gauge, c and* $\hbar$ *take unit value, while G and* Λ *appear symmetrically with their hierarchy encoded by the dimensionless gravitational fine-structure constant* $\alpha_\Lambda \equiv c^3/(G\hbar\Lambda)$. This unifies thermodynamic and quantum perspectives, eliminating the 10^{120}–fold discrepancy in vacuum energy predictions. We validate the framework across diverse domains—including the Casimir effect, boson and fermion gases, and electromagnetic radiation—each saturating at the same vacuum bound. The results support the Law of Entropic Constraint, in which gravity, inertia and electromagnetism are subject to horizon-encoded information limits. The Planck scale is revealed as incomplete without the inclusion of Λ. The Λ system of units emerges as its natural completion—superseding the Planck scale, just as Planck units superseded Stoney's once $\hbar$ was recognized as fundamental.

Keywords: cosmological constant problem; dark energy; Bekenstein–Hawking entropy; de Sitter space-time; Gibbons–Hawking temperature; gravitational fine-structure constant; Casimir effect; horizon thermodynamics; natural units; vacuum energy density; zero-point energy

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1. Introduction

The cosmological constant Λ has undergone a remarkable transformation. Introduced by Einstein to stabilize the universe and later abandoned, it has returned as a cornerstone of modern cosmology with the discovery of accelerated expansion [1–3]. Yet its physical meaning remains obscure [4]. The "vacuum catastrophe"— quantum field theory's (QFT) estimate of a vacuum energy density exceeding cosmological bounds by 10^{120}—reveals a deep gap in how quantum mechanics, gravity, and thermodynamics interrelate.

Thermodynamic treatments of spacetime (e.g., Jacobson [5], Verlinde [6], Padmanabhan [7]) suggest that horizon entropy and temperature encode gravitational dynamics, but they do not pin down the observed vacuum density nor the role of Λ as a fundamental constant. If Λ is fundamental, physics must explain why it fixes the vacuum energy and how it enters the basic scales of nature [8,9].

This paper makes four contributions:

1. **Thermodynamic Derivation of the Vacuum Energy.**
Using the Gibbons–Hawking temperature of the de Sitter horizon [10], the Bekenstein–Hawking entropy [11,12], and the Clausius relation $E = TS$ (or $\delta Q = T\,dS$) [5,13], we obtain directly the vacuum energy density in general relativity (GR),

$$u_\Lambda = \frac{\Lambda c^4}{8\pi G}. \tag{1}$$

identifying Λ as a finite entropy bound of the universe (throughout, u denotes a generic energy density)[10,14,15].

2. **A New Quantum Scale—the Λ-Scale.** As Λ represents a finite information capacity of the universe, we argue the Planck scale is incomplete without it [16]. Combining $\{G, \hbar, c, \Lambda\}$ selects a unique scale (Figure 1.) We *introduce* the dimensionless gravitational fine-structure constant (GFSC),

$$\alpha_\Lambda \equiv \frac{c^3}{G\hbar\Lambda} \tag{2}$$

an entropic bound that anchors quantum theory to cosmology and resolves the vacuum catastrophe without invoking Planck-scale physics. Details and the dimensional analysis are given in App. A. For historical context see Appendix B.

Planck scale	Lambda scale
$L_P = \sqrt{\frac{\hbar G}{c^3}}$	$L_\Lambda = \sqrt[4]{\frac{\hbar G}{\Lambda c^3}}$
$T_P = \sqrt{\frac{\hbar G}{c^5}}$	$T_\Lambda = \sqrt[4]{\frac{\hbar G}{\Lambda c^7}}$
$M_P = \sqrt{\frac{\hbar c}{G}}$	$M_\Lambda = \sqrt[4]{\frac{\hbar^3 \Lambda}{c\, G}}$
$\Theta_P = \sqrt{\frac{\hbar\, c^5}{G\, k_B^2}}$	$\Theta_\Lambda = \sqrt[4]{\frac{\hbar^3 \Lambda\, c^7}{G\, k_B^4}}$

Figure 1. The Lambda scale emerges once the cosmological constant Λ is recognized as a fundamental constant of nature, fixing the total entropy of the universe. The Planck system, whilst being a self-consistent set of natural units, is thermodynamically sterile: it omits the vacuum contribution. Once included the Λ-scale supersedes the Planck scale as the physically complete framework, embedding vacuum thermodynamics in its foundations. [**Notation.** We denote the Λ–base units by L_Λ (length), T_Λ (time), M_Λ (mass), and Θ_Λ (temperature). In formulas, T denotes a physical temperature; the temperature unit is Θ_Λ. Dimensions remain $[L]$, $[T]$, $[M]$]. See Appendix A.

3. **Cross-Domain Validation of the Λ-scale.**
Applications— Force interactions in Λ units (Sec. 4), electromagnetic radiation (Sec. 5), Casimir systems (Sec. 6) and boson/fermion gases (Sec. 7) — saturate the same bound, supporting the Law of Entropic Constraint (Sec. 9).
4. **A Quantum Derivation of u_Λ from the Zero-Point Energy (ZPE):** Crucially, *we also provide an explicit zero-point (ZPE) density-of-states derivation of* u_Λ in Eq.(1), based on the Λ-scale in Figure 1 (see Sec.8 and Appendix C), not via boson/fermion cancellations but through a finite mode budget. Without Planck-cutoff assumptions, ultraviolet divergences are eliminated, yielding a finite and radiatively stable vacuum energy [4,17].

2. Deriving the Vacuum Energy from Horizon Thermodynamics

2.1. Thermodynamics of Causal Event Horizons

The thermodynamic structure of causal event horizons — a defining feature of GR, partitions spacetime into accessible and inaccessible regions. This division gives rise to observable thermodynamic properties: temperature, entropy, and energy. The universality of their expressions and the deeper thermodynamic structure are governed by the four laws of black hole thermodynamics [11,12,18]. See Figure 2.

Law	**Rindler Horizon**	**Black Hole (Schwarzschild)**	**de Sitter Horizon**
Zeroth	Uniform proper acceleration $\Rightarrow$ constant κ for all Rindler observers	$\kappa = c^4/(4GM)$ constant over the event horizon	$\kappa = c^2\sqrt{\Lambda/3}$ constant for all observers
First	$\delta E = TdS$, with Unruh temperature $T = \hbar a/(2\pi c k_B)$	$dM = \frac{\kappa}{8\pi G} dA$ (plus $\Omega dJ, \Phi dQ$ for Kerr–Newman)	$E = TS$, with $T = \hbar c/(2\pi k_B R_\Lambda)$
Second	Horizon area grows with accelerated matter crossing the Rindler wedge	Hawking area theorem: $dA \geq 0$	de Sitter entropy $S \propto A$ fixed; horizon persists even in empty space
Third	$a \to 0$ unattainable $\Rightarrow$ $T \to 0$ unreachable	Extremal black holes ($\kappa \to 0$) cannot be formed in finite steps	$\Lambda \to 0 \Rightarrow T \to 0$ unreachable by smooth dynamics

Figure 2. The four laws of black hole thermodynamics, shown here for Rindler, Schwarzschild, and de Sitter horizons. Originally formulated for stationary black holes, these laws reveal a profound connection between gravity, entropy, and temperature. They are now understood to reflect the *causal structure of spacetime* itself, applying to any causal boundary [10,12,19,20]

This viewpoint has been developed by many authors over the past decades [5–7,10–12,18,20]. The three archetypal cases are shown schematically in Figures 3–6.

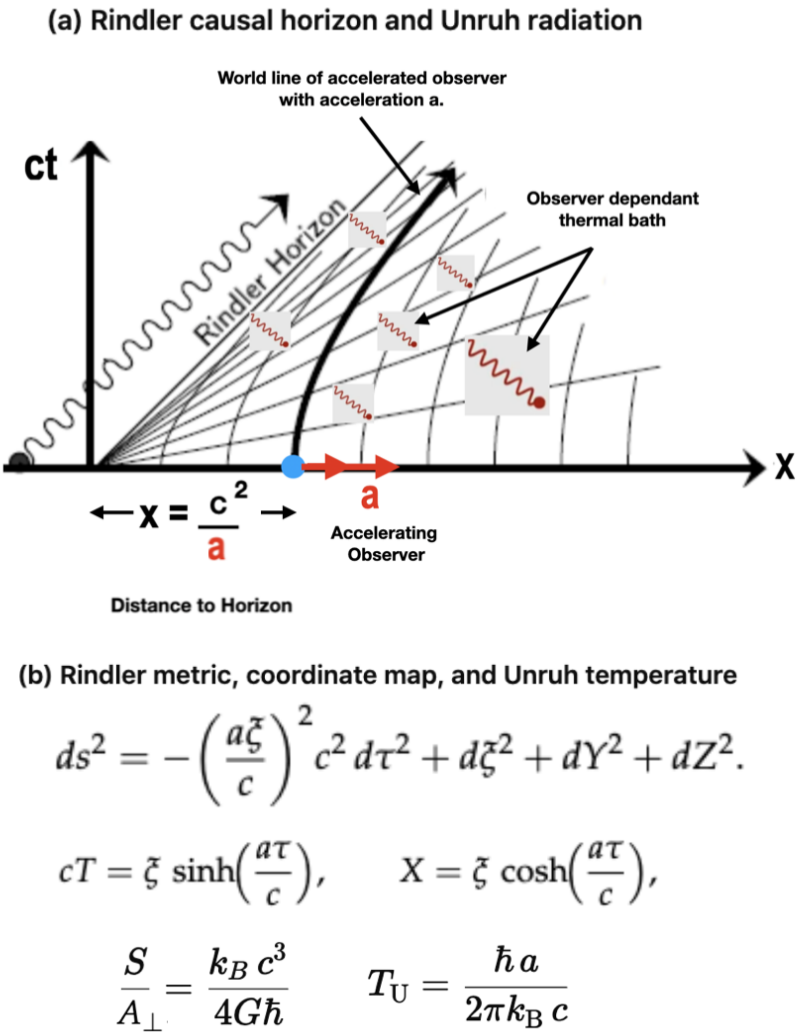

Figure 3. Rindler wedge for a uniformly accelerated observer with proper acceleration a. A causal horizon forms at proper distance $x = c^2/a$. For this observer the Minkowski vacuum is a thermal equilibrium state with Unruh temperature $T_U = \hbar a/(2\pi k_B c)$, so a comoving detector responds as if immersed in a thermal bath—while inertial observers register no flux (observer dependence). For the Rindler horizon we quote entropy per unit transverse area, $S/A_\perp$.

An accelerated observer in flat spacetime perceives a *Rindler horizon* and detects a thermal bath at the Unruh temperature [20]. See Figure 3.

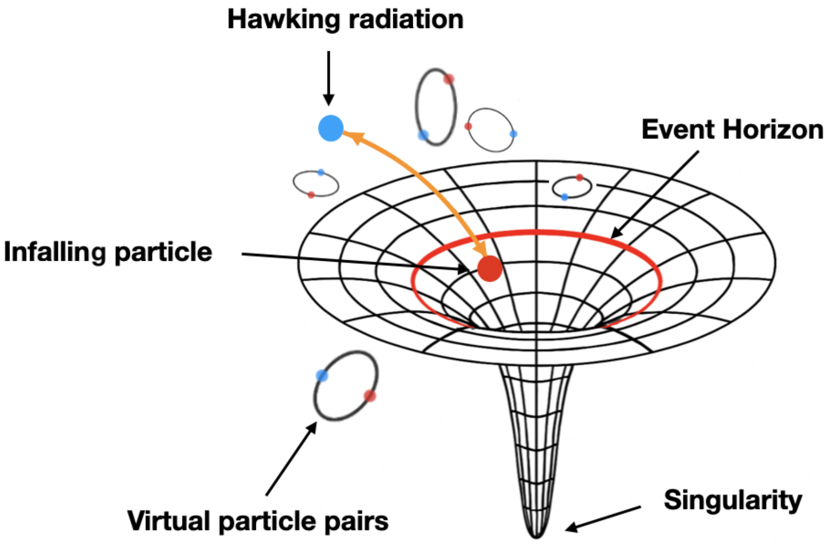

Figure 4. Hawking radiation. Quantum fluctuations at a black hole horizon produce entangled pairs; one particle escapes as radiation while its partner falls in. The effect illustrates the universal thermality of horizons.

A black hole horizon radiates at the Hawking temperature, its entropy fixed by the Bekenstein–Hawking area law [11,12] See Figures 4 and 5.

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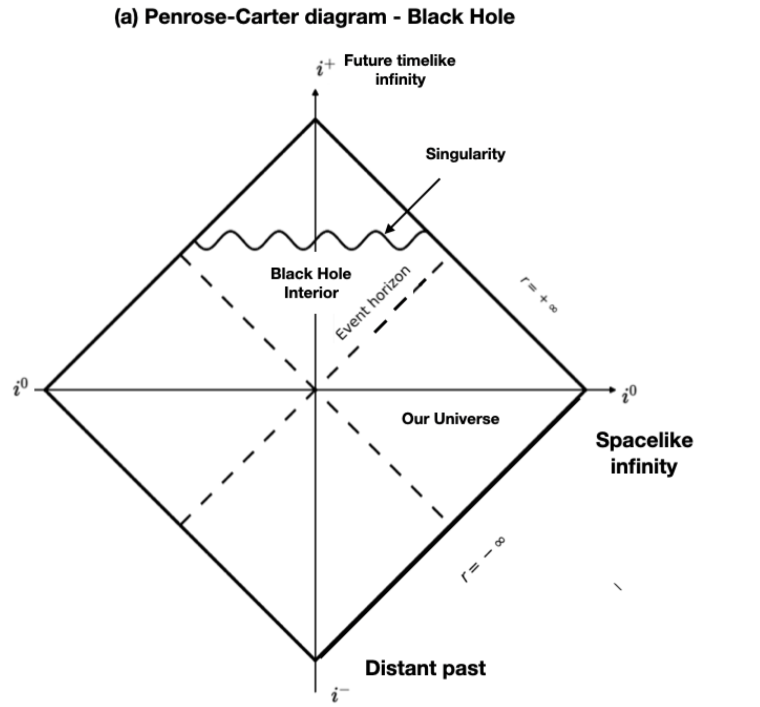


(b) Schwarzschild relations

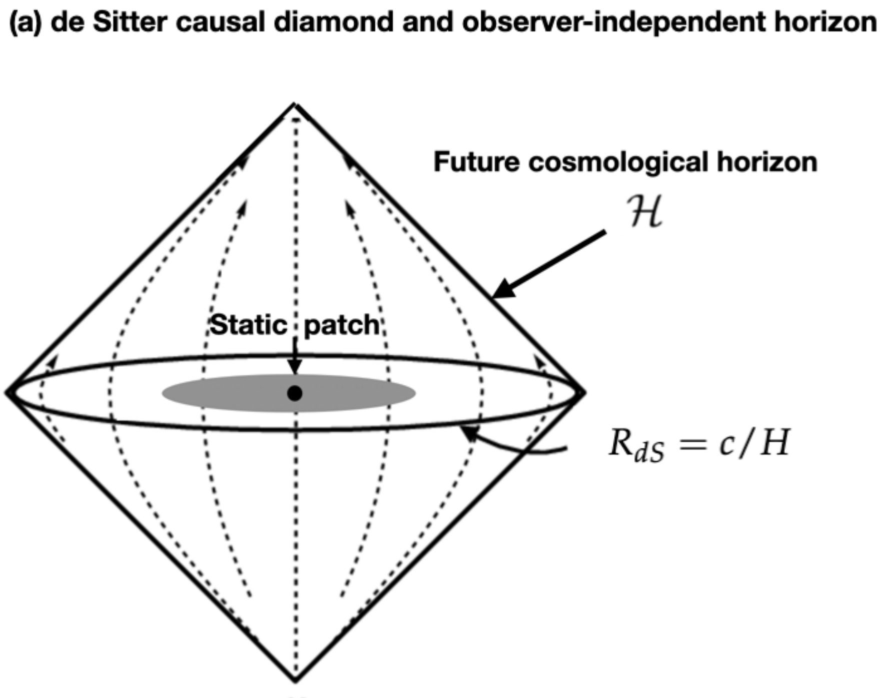
$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \qquad S = \frac{k_B c^3 A}{4G\hbar} \qquad A = 4\pi r_s^2 = \frac{16\pi G^2 M^2}{c^4}.$$

$$r_s = \frac{2GM}{c^2}$$

Figure 5. Schwarzschild spacetime. Penrose–Carter diagram showing the causal structure of a black hole: singularity (wavy line), event horizon (dashed line), and infinities.

Finally, de Sitter spacetime, relevant to our universe with positive Λ, possesses a cosmological horizon with Gibbons–Hawking temperature and entropy proportional to its area [10]. See Figure 6.



(b) de Sitter horizon as a universal entropy bound

$$ds^2 = -\left(1 - \frac{\Lambda r^2}{3}\right)c^2dt^2 + \left(1 - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$T_{\mathrm{GH}} = \frac{\hbar c}{2\pi k_B R} = \frac{\hbar c}{2\pi k_B} \cdot \sqrt{\frac{\Lambda}{3}}.$$

$$S = \frac{k_B c^3 A}{4G\hbar} = \frac{3\pi k_B c^3}{G\Lambda\hbar}, \quad A = 4\pi R_{\mathrm{dS}}^2.$$

$$R_{\mathrm{dS}} = \sqrt{\frac{3}{\Lambda}}$$

Figure 6. de Sitter horizon. The causal diamond of a comoving observer defines the finite entropy accessible in an expanding universe, establishing a universal bound on information capacity. *$\mathcal{H}$ labels the (future) horizon; H is the Hubble rate. The de Sitter radius is $R_{\mathrm{dS}} = c/H = \sqrt{3/\Lambda}$.*

This principle - that causal event horizons are inextricably tied to an entropy and temperature - holds irrespective of the horizon's origin — whether due to acceleration, mass, or vacuum energy — and it is the universality of this horizon thermodynamics that forms the basis for our derivation of the vacuum energy density. The common structure across these three spacetimes is illustrated in Table 1.

 doi:10.20944/preprints202510.2218.v1

Table 1. Thermodynamic properties of three causal horizons. All obey the universal relations $T \propto a$ and $S \propto$ area. For the Rindler horizon we report entropy per unit transverse area $A_\perp$ (as in Fig. 3); for Schwarzschild and de Sitter we use the total area A. They differ only in how the effective radius R is set: by proper acceleration ($\chi = c^2/a$, Rindler), by mass ($R = r_s = 2GM/c^2$, Schwarzschild), or by the vacuum Λ ($R = R_{\rm dS} = \sqrt{3/\Lambda}$, de Sitter).

Spacetime	**Horizon Type**	**Horizon Radius** R	**Temperature** T	**Entropy** S
Rindler	Acceleration	observer-dependent	$T = \dfrac{\hbar a}{2\pi k_B c}$	$S = \dfrac{k_B c^3}{4G\hbar} A_\perp$
Schwarzschild	Event Horizon	$R_S = \dfrac{2GM}{c^2}$	$T = \dfrac{\hbar c^3}{8\pi G M k_B}$	$S = \dfrac{k_B c^3}{4G\hbar} A$
de Sitter	Cosmological	$R_{\rm dS} = \sqrt{3/\Lambda}$	$T = \dfrac{\hbar c}{2\pi k_B R_{\rm dS}}$	$S = \dfrac{k_B c^3}{4G\hbar} A$

Because our universe is asymptotically approaching de Sitter space, its horizon provides the correct setting to apply the identity $E = TS$, yielding a finite and universal expression for the vacuum energy density $u_{\rm vac}$ in the context of horizon thermodynamics [19,21,22], and consistent with quantum–informational perspectives [23].

2.2. *Deriving the Vacuum Energy u_{vac} from Horizon Thermodynamics*

The integrated form of the First Law of Thermodynamics $E = TS$, has its deeper roots in the Second Law of Thermodynamics, formalized by Rudolf Clausius in the mid-19th century [13,24],

$$\delta S = \frac{\delta Q}{T} \tag{3}$$

We now make use of the thermodynamic identity, applied to the cosmological horizon of de Sitter spacetime.

2.3. *Horizon Energy E*

To obtain the total energy content E of the de Sitter universe, we begin with the entropy S of the de Sitter horizon:

$$S = \frac{k_B c^3 A}{4G\hbar}, \quad A = 4\pi R^2, \quad R = \sqrt{\frac{3}{\Lambda}}. \tag{4}$$

Substituting the de Sitter radius into this expression:

$$A = 4\pi\left(\frac{3}{\Lambda}\right) = \frac{12\pi}{\Lambda} \tag{5}$$

$$S = \frac{k_B c^3}{4G\hbar} \cdot \frac{12\pi}{\Lambda} = \frac{3\pi k_B c^3}{G\hbar\Lambda} \tag{6}$$

Note that Eq.(6) has the GFSC defined in Eq. (2) embedded within it, making this a maximum bound on entropy. Next, we use the Gibbons–Hawking temperature:

$$T = \frac{\hbar c}{2\pi k_B R} = \frac{\hbar c}{2\pi k_B} \cdot \sqrt{\frac{\Lambda}{3}}. \tag{7}$$

Combining:

$$E = TS = \left(\frac{\hbar c}{2\pi k_B}\sqrt{\frac{\Lambda}{3}}\right)\left(\frac{3\pi k_B c^3}{G\hbar\Lambda}\right) = \frac{3}{2}\frac{c^4}{G} \cdot \frac{1}{\sqrt{3\Lambda}} \tag{8}$$

2.4. *Horizon Volume and Energy Density*

The spatial volume enclosed by the de Sitter horizon is:

$$V = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(\frac{3}{\Lambda}\right)^{3/2}. \tag{9}$$

The vacuum energy density is then:

$$u_\Lambda = \frac{E}{V} = \left(\frac{3c^4}{2G}\cdot\frac{1}{\sqrt{3\Lambda}}\right)\bigg/\left(\frac{4\pi}{3}\left(\frac{3}{\Lambda}\right)^{3/2}\right) \tag{10}$$

Simplifying:

$$u_\Lambda = \frac{3c^4}{2G}\cdot\frac{1}{\sqrt{3\Lambda}}\cdot\frac{3}{4\pi}\cdot\left(\frac{\Lambda}{3}\right)^{3/2} = \frac{\Lambda c^4}{8\pi G}. \tag{11}$$

This elegant result is deeply significant. In GR Eq. (11) emerges only from dimensional consistency of the Einstein field equations, here the observed vacuum energy density is derived without invoking any matter fields or action principles. Instead, it arises from applying *thermodynamics* to the geometry of de Sitter spacetime.

3. From Thermodynamics to a New Quantum Scale in Nature

QFT has not yet produced a reliable prediction of the observed vacuum energy density when regulated at Planck scales because it is constructed without reference to Λ. Being blind to the large-scale curvature of spacetime, it is a miscalibrated scale. A new quantum scale is therefore needed, one that includes Λ ab initio. Only then can we meaningfully measure the vacuum without triggering a theoretical breakdown [7].

Our thermodynamic derivation of Λ fixes the vacuum energy density as a finite, horizon–regulated quantity. It is this that justifies why Λ should be treated not as an arbitrary parameter but as a fundamental constant of nature, on the same footing as c and $\hbar$. Each of these constants encodes a fundamental limiting principle: c defines the maximum velocity of causal signals, $\hbar$ defines the minimum quantum of action, and Λ defines the maximum entropy – the finite information capacity of the universe. [10–12].

A dimensional analysis involving $\{c, \hbar, G, \Lambda\}$ yields a new set of natural units in Appendix A, illustrated in Figure 1 and applied in Figure 7. For historical context see Appendix C.

$$u_\Lambda = M_\Lambda L_\Lambda^{-1} T_\Lambda^{-2}$$

$$u_\Lambda = \left(\frac{\hbar^3\Lambda}{Gc}\right)^{\frac{1}{4}} \cdot \left(\frac{\Lambda c^3}{\hbar G}\right)^{\frac{1}{4}} \cdot \left(\frac{\Lambda c^7}{\hbar G}\right)^{\frac{1}{2}}$$

$$u_\Lambda = \left(\frac{\hbar^2\Lambda^2 c^2}{G^2}\right)^{\frac{1}{4}} \cdot \left(\frac{\Lambda c^7}{\hbar G}\right)^{\frac{1}{2}}$$

$$u_\Lambda = \frac{\Lambda c^4}{G}$$

Using the Lambda base units one obtains the dimensional form of the energy density of the vacuum in GR.

Figure 7. An energy density u_Λ in J/m^3 in Λ base units will have the dimensions $M_\Lambda L_\Lambda^{-1} T_\Lambda^{-2}$ the result has exactly the dimensional form as the energy density of the vacuum that emerges in GR (without geometric normalisation), demonstrating the elegance and simplicity through which the vacuum catastrophe is abolished. The geometric factor $1/(8\pi)$ arises only upon matching to the GR source term (horizon geometry / Einstein equations); see Section 8 and Appendix C. From the standpoint of the Λ-scale the vacuum catastrophe is a physicist's ghost.

This new quantum scale, unlike the Planck scale, avoids the break down when measuring the quantum vacuum. It signifies a new unity between GR, QFT and thermodynamics, in which the vacuum catastrophe is revealed as illusory, and nothing more than a physicist's ghost. see Figure 7.

4. Forces in Λ-Units: How Interactions Respect the Entropy Bound

Within the Λ-framework, horizons defined by Λ impose a finite entropy bound on spacetime (see Section. 2.1); deviations from geodesic motion couple to this structure, producing an entropic response. Expressed in Λ-units, inertial F_Λ^i, gravitational F_Λ^G and electromagnetic forces F_Λ^E, reduce to the scales shown in Figure 8:

Inertial Force	Gravitational Force	Electromagnetic Force
$F = ma$	$F_G = \frac{Gm_1m_2}{r^2}$	$F_E = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$
$a_\Lambda = \left(\frac{c^{11}\Lambda}{\hbar G}\right)^{1/2}$	$m_\Lambda = \left(\frac{\hbar^3\Lambda}{cG}\right)^{1/4}$	$\alpha_E = \frac{e^2}{4\pi\varepsilon_0\hbar c}$
$m_\Lambda = \left(\frac{\hbar^3\Lambda}{cG}\right)^{1/4}$	$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4}$	$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4}$
$\boxed{F_\Lambda^{(i)} = m_\Lambda a_\Lambda = \left(\frac{\hbar\Lambda c^5}{G}\right)^{1/2}}$	$\boxed{F_\Lambda^{(G)} = \frac{Gm_\Lambda^2}{L_\Lambda^2} = \Lambda\hbar c}$	$\boxed{F_\Lambda^{(E)} = \alpha_E\left(\frac{\hbar\Lambda c^5}{G}\right)^{1/2}}$

Figure 8. Force scales in Λ-units. The second row lists the base quantities required for each case: (a_Λ, m_Λ) for inertia, (m_Λ, L_Λ) for gravity, and (α_E, L_Λ) for electromagnetism. Here $\alpha_E \equiv e^2/(4\pi\varepsilon_0\hbar c) \approx 1/137$ is the electromagnetic fine-structure constant (FSC).

We see that Λ is present in all the force terms (Figure 8) and that Newton's constant G, drops out of the expression for the gravitational force term, leaving a 'gravitational force' without any need for G,

$$F = \frac{Gm_1m_2}{r^2} \Longrightarrow F_\Lambda^{(G)} = \Lambda\hbar c. \tag{12}$$

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One finds from the definition of the GFSC in Eq. (2),

$$G = \frac{1}{\alpha_\Lambda} \cdot \frac{c^3}{\hbar\Lambda} \tag{13}$$

substituting Eq. (13) into the inertial force term F^i_Λ we find,

$$F^i_\Lambda = (\frac{\hbar\Lambda c^5}{G})^{1/2} = (\alpha_\Lambda \hbar^2 \Lambda^2 c^2)^{1/2} \tag{14}$$

$$F^i_\Lambda = \sqrt{\alpha_\Lambda} \cdot F^G_\Lambda \tag{15}$$

Thus inertial forces share the same form as gravitational ones but scaled by a factor $\sqrt{\alpha_\Lambda}$. It is remarkable that the electromagnetic sector (a gauge interaction) respects the same entropic bound, taking just a fraction, α_E, of the inertial force.

Gravity looks weak at macroscales because the gravitational unit $F_\Lambda^{(G)}$ is suppressed relative to $F_\Lambda^{(i)}$ by $\alpha_\Lambda^{-1/2}$, while gauge forces scale with their own dimensionless couplings (e.g. α_E). In all cases the common thread is that all forces and their resulting motions are constrained by a finite-information capacity of the vacuum.

5. Electromagnetic Radiation and the Vacuum Bound

Electromagnetism, like inertia and gravity, is constrained by the vacuum's entropy bound. In Λ-units, the Coulomb force [25] is revealed not as arbitrarily strong but as a fraction of the vacuum's maximal force, scaled by the fine-structure constant α_E. This reframes electromagnetic interactions as vacuum-limited couplings.

Gauge invariance still permits shifting the zero of potential, but the Λ-framework fixes the maximum field strength by horizon thermodynamics. Thus, electromagnetism joins inertia and gravity as an expression of the same entropic constraint.

To illustrate this explicitly, we derive the Poynting flux [26,27] in Λ-units, showing how energy transport in electromagnetic waves saturates at the same thermodynamic limit set by Λ. See Table 2.

Table 2. Comparison of force quanta in Stoney, Planck, and Λ units. In the Λ-framework, all force quanta are scaled relative to the entropy-bound limit.

Force Type	**Stoney Units**	**Planck Units**	**Lambda Units**
Inertial	$\frac{c^4}{G}$	$\frac{c^4}{G}$	$\left(\frac{\hbar\Lambda c^5}{G}\right)^{1/2}$
Gravitational	$\frac{c^4}{G}$	$\frac{c^4}{G}$	$\Lambda\hbar c$
Electromagnetic	$\frac{c^4}{G}$	$\alpha_E \frac{c^4}{G}$	$\alpha_E \left(\frac{\hbar\Lambda c^5}{G}\right)^{1/2}$

5.1. The Poynting Flux in Λ-Units

In standard notation the time–averaged Poynting flux for a plane electromagnetic wave in vacuum is

$$\langle S_{EM} \rangle = \tfrac{1}{2}\, \varepsilon_0 c\, E_0^2. \tag{16}$$

The electric field of a point charge at distance $r = L_\Lambda$ in Λ units is [25,28]:

$$E_\Lambda = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e}{L_\Lambda^2} \tag{17}$$

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The corresponding flux has a magnitude given by:

$$\langle \mathbf{S}_{\rm EM} \rangle = \frac{1}{2}\varepsilon_0 c E_\Lambda^2 \tag{18}$$

$$= \frac{1}{2}\varepsilon_0 c \left(\frac{e}{4\pi\varepsilon_0 L_\Lambda^2} \right)^2 = \frac{e^2 c}{32\pi^2 \varepsilon_0 L_\Lambda^4} \tag{19}$$

Expressing e^2/ε_0 using the fine-structure constant [29]:

$$\alpha_E = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \Rightarrow \frac{e^2}{\varepsilon_0} = 4\pi\alpha_E \hbar c \tag{20}$$

So:

$$\langle \mathbf{S}_{\rm EM} \rangle = \frac{\alpha_E \hbar c^2}{8\pi L_\Lambda^4} \tag{21}$$

Substituting for the Lambda length as,

$$L_\Lambda^4 = \frac{\hbar G}{\Lambda c^3} \Rightarrow \frac{1}{L_\Lambda^4} = \frac{\Lambda c^3}{\hbar G} \tag{22}$$

into Eq.(21) for $\langle \mathbf{S}_{\rm EM} \rangle$ we obtain:

$$\langle \mathbf{S}_{\rm EM} \rangle = \frac{\alpha_E \hbar c^2}{8\pi} \cdot \frac{\Lambda c^3}{\hbar G} = \frac{\alpha_E \Lambda c^5}{8\pi G} = \alpha_E c \cdot u_\Lambda \tag{23}$$

The average radiative flux is thus:

$$\boxed{\langle S_{\rm EM} \rangle = \alpha_E \, c \, u_\Lambda}$$

This is a striking result. It shows that the classical electromagnetic flux density is directly proportional to the vacuum energy density of spacetime — scaled by the fine-structure constant, a quantum measure of the coupling strength of light to matter.

Our Poynting analysis reveals the EM energy/flux, a gauge field, respects the bound set by u_Λ.

6. The Casimir Effect and the Quantum Vacuum

The Casimir force between conducting plates has long been taken as evidence of vacuum fluctuations arising from a modified mode spectrum between the plates [30,31]. However, the precise connection between such boundary–dependent zero-point energies and the homogeneous cosmological vacuum remains an open question in contemporary physics [4,32,33].

The standard expression for the pressure is,

$$P_{\rm Casimir} = -\frac{\pi^2 \hbar c}{240 d^4} \tag{24}$$

where d is the plate separation. The minus sign denotes vacuum *tension* (negative pressure): suppressed modes between the plates make the interior zero-point pressure lower than outside, producing an inward force. This fixes the *sign* (tension) of the vacuum in a bounded geometry. See Figure 9.

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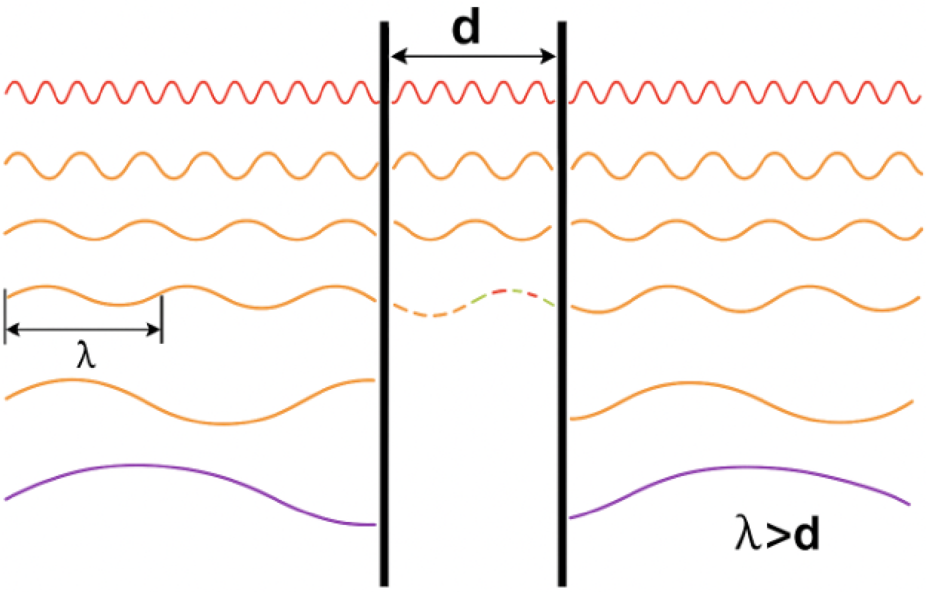

Figure 9. The Casimir effect arises from the suppression of vacuum fluctuations between closely spaced plates, yielding a greater vacuum pressure outside the plates compared to that inside, resulting in a measurable attractive force. As a direct manifestation of zero-point energy, this phenomenon provides a laboratory probe into the structure of the vacuum — and when correctly interpreted, links directly to the cosmological constant Λ.

Introducing the Λ-length cutoff

$$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4}, \tag{25}$$

and substituting into Eq.(24), shows that the Casimir pressure is a fixed fraction of the vacuum bound,

$$P_{Casimir} = -\frac{\pi^3}{30} \cdot \frac{\Lambda c^4}{8\pi G} = -\frac{\pi^3}{30} \cdot u_\Lambda \tag{26}$$

Thus a laboratory-scale quantum effect validates the same vacuum limit that emerges thermodynamically from de Sitter horizons. It is precisely a negative pressure that gravitates in GR, yielding a cosmic repulsion and an accelerated universal expansion. See Figure 10.

$$P_{\mathrm{Cas}}(d) = -\frac{\pi^2 \hbar c}{240\, d^4} \quad \overset{?}{\Rightarrow} \quad u_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

$$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4}$$

$$P_{\mathrm{Cas}}(d) = -\frac{\pi^2 \hbar c}{240 L_\Lambda^4} = -\frac{\pi^2 \hbar c}{240} \cdot \frac{\Lambda c^3}{\hbar G} = -\left(\frac{\pi^3}{30}\right) \cdot \frac{\Lambda c^4}{8\pi G}$$

$$P_{\mathrm{Cas}}(L_\Lambda) = -\frac{\pi^3}{30} \cdot u_\Lambda$$

$$u_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

Figure 10. Connecting the standard Casimir pressure with GR's vacuum density. At the Λ–length $L_\Lambda = (\hbar G/\Lambda c^3)^{1/4}$, Eq. 24 gives $P_{\mathrm{Casimir}}(L_\Lambda) = -(\pi^3/30)\, u_\Lambda \approx -1.0335\, u_\Lambda$, i.e. the same negative pressure (tension) as the GR vacuum ($p_\Lambda = -u_\Lambda$). The Λ-scale shows they are the same quantity up to a dimensionless factor of order unity, making the quantum vacuum and the cosmological vacuum one and the same in this framework.

6.1. From Quantum Fluctuations to Measurable Force

The Λ length sits in the Casimir window. With CODATA values,

$$\begin{aligned} L_\Lambda &= \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4} \approx 3.9 \times 10^{-5}\,\mathrm{m} = 39\,\mu\mathrm{m}, \\ \nu_\Lambda &\equiv \frac{c}{L_\Lambda} \approx 7.7\ \mathrm{THz}. \end{aligned} \tag{27}$$

which lies squarely in the mid–IR/THz, precisely the range where Casimir forces have been probed (e.g. Lamoreaux's torsion–pendulum and subsequent torsional studies [31,34]). This coincidence is not incidental in our framework.

Evaluating the ideal ($T \to 0$) plate–plate pressure at the Λ length yields a quartic correspondence with the GR vacuum density. This suggests, in principle, a laboratory route to infer Λ using real materials, with corrections incorporated through the standard Lifshitz framework [35]. Figures 11 and 12 outline how such a determination of Λ could be made experimentally from micron–scale force measurements, far above the Planck length .

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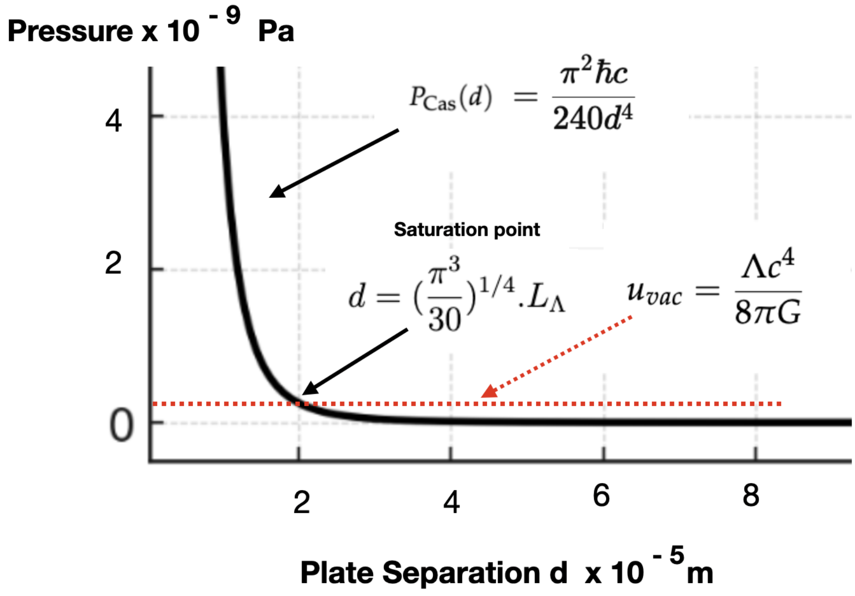

Figure 11. Casimir pressure versus plate separation. The ideal law $P_{\rm Cas}(d) = \pi^2\hbar c/(240\, d^4)$ (black) meets the vacuum ceiling (red) at a crossover distance d_*. Measuring d_* determines the cosmological constant, $\Lambda_{\rm Cas} = (\pi^3\hbar G/c^3)\, d_*^{-4}$. For ideal reflectors the crossover is $d_* = (\pi^3/30)^{1/4} L_\Lambda$ with $L_\Lambda = (\hbar G/\Lambda c^3)^{1/4}$; material/thermal corrections are handled in the Lifshitz framework.

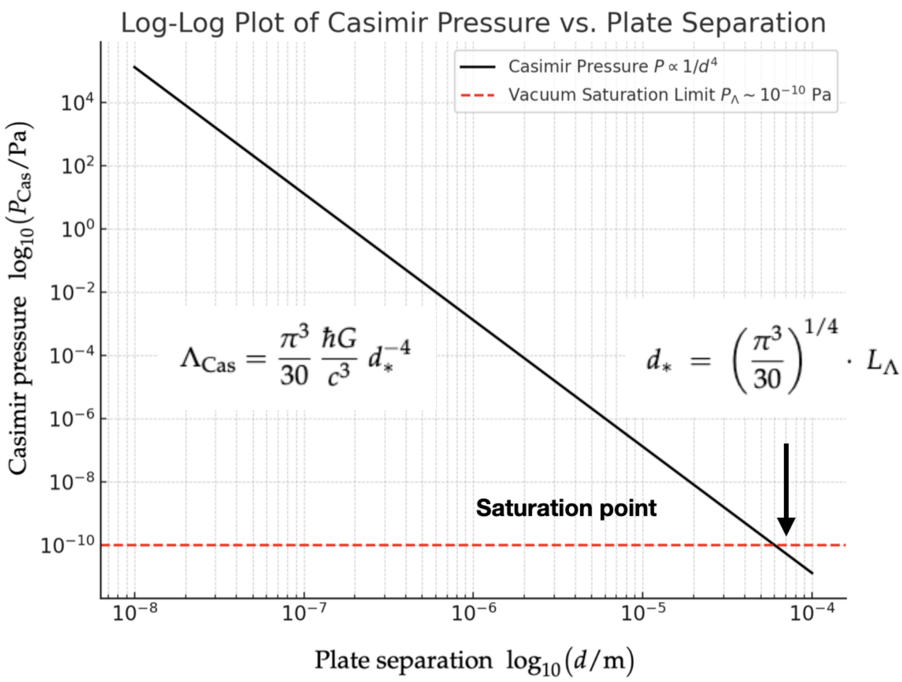

Figure 12. Log–log view of $P_{\rm Cas}$ vs. d. The black line shows the d^{-4} trend (slope -4); the red line marks the vacuum ceiling. Their intersection defines d_*, from which $\Lambda_{\rm Cas} = (\pi^3\hbar G/c^3)\, d_*^{-4}$ follows. Arrow indicates the micron–tens-of-microns crossover scale [31,35].

6.2. On the Origins of the Casimir Effect

The interpretation of Casimir forces has long been debated: zero–point fluctuations versus retarded van der Waals interactions. The Lifshitz framework [35] shows these are equivalent descriptions, with the Casimir expression emerging as the limiting case of Lifshitz theory for ideal reflectors ($T \to 0$).

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7. Quantum Gases: Bosons and Fermions in Thermal Equilibrium with the Vacuum

Both bosonic and fermionic gases exhibit the same thermodynamic law: their equilibrium energy densities scale quartically with temperature, $u(T) \propto T^4$ [36,37]. This Stefan–Boltzmann scaling reflects a deeper geometric constraint imposed by causal horizons. For massless bosons such as photons, the cosmic microwave background (CMB) energy density is u_γ where,

$$u_\gamma = \frac{\pi^2}{15} \frac{(k_B T)^4}{\hbar^3 c^3}, \tag{27}$$

while fermionic gases such as relic neutrinos form the cosmic neutrino background (CνB), is denoted by u_ν; N_ν denotes the number of light neutrino species (we take $N_\nu = 3$),

$$u_\nu = \frac{7}{8} N_\nu \frac{\pi^2}{15} \frac{(k_B T)^4}{\hbar^3 c^3}. \tag{28}$$

Substituting the Λ–temperature,

$$\Theta_\Lambda = \left(\frac{\hbar^3 \Lambda c^7}{G k_B^4} \right)^{1/4}, \tag{28}$$

into the boson and fermion energy densities, reveals both as fixed fractions of the vacuum energy, see Figure 13. Bosons saturate the bound rapidly, while fermions are Pauli-suppressed, but both approach finite values consistent with a capped vacuum.

Bosons	**Fermions**
$u_\gamma = \frac{\pi^2}{15} \frac{(k_B \Theta_\Lambda)^4}{\hbar^3 c^3}$	$u_\nu = 3 \cdot \frac{7}{8} \cdot \frac{\pi^2}{15} \frac{(k_B \Theta_\Lambda)^4}{\hbar^3 c^3}$
$u_\gamma = \frac{8\pi^3}{15} \frac{\Lambda c^4}{8\pi G}$	$u_\nu = \frac{7\pi^3}{5} \frac{\Lambda c^4}{8\pi G}$
$u_\gamma = \frac{8\pi^3}{15} u_\Lambda$	$u_\nu = \frac{7\pi^3}{5} u_\Lambda$

Figure 13. Bosons (photons, u_γ) and fermions (relic neutrinos, u_ν) at the Λ-temperature Θ_Λ: $u \propto T^4$, and at $T = \Theta_\Lambda$ both become fixed fractions of u_Λ ($N_\nu = 3$ shown).

The usual QFT bookkeeping—adding bosonic $+\frac{1}{2}\hbar\omega$ and fermionic $-\frac{1}{2}\hbar\omega$—does not capture the physics [4,38]. The vacuum density is not a delicate boson–fermion cancellation; it is a finite entropy–energy budget set by Λ.

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8. Quantum Derivation of the Vacuum Density (ZPE with a Λ cutoff)

We write the zero–point energy density in the standard (mode–count) form

$$u_{\rm ZPE}(\omega_c) = \frac{\hbar}{2} \int_0^{\omega_c} \omega\, g(\omega)\, d\omega, \tag{29}$$

where $g(\omega) = \omega^2/(\pi^2 c^3)$ for a relativistic linear dispersion $\omega = ck$. Evaluated with a Planck–scale ultraviolet cutoff $\omega_P = 2\pi/t_P = 2\pi\sqrt{c^5/(\hbar G)}$ this gives

$$u_{\rm ZPE}^{\rm (Planck)} = \frac{\hbar}{8\pi^2 c^3}\, \omega_P^4 = \frac{2\pi^2\, c^7}{\hbar G^2} \qquad \text{(overestimates by } \sim 10^{120}\text{)}. \tag{30}$$

The Λ scale and the physical cutoff

The Λ units fix a natural length, time, and (linear) frequency

$$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4}, \qquad T_\Lambda = \frac{L_\Lambda}{c}, \qquad \nu_\Lambda = \frac{1}{T_\Lambda} = \left(\frac{\Lambda c^7}{\hbar G}\right)^{1/4}. \tag{31}$$

Horizon thermodynamics selects a universal numerical factor so that UV saturation occurs at

$$\omega_c \equiv \omega_\Lambda^* = 2\pi \nu_\Lambda^*, \qquad \nu_\Lambda^* = \frac{\zeta}{T_\Lambda}, \tag{32}$$

where $T_\Lambda = L_\Lambda/c$ and ζ is a pure number fixed by vacuum matching. Using

$$T_\Lambda = \left(\frac{\hbar G}{\Lambda c^7}\right)^{1/4}, \qquad \zeta = (16\pi^3)^{-1/4},$$

one gets

$$\omega_c = 2\pi \frac{\zeta}{T_\Lambda} = \pi^{1/4}\left(\frac{\Lambda c^7}{\hbar G}\right)^{1/4} = \frac{\pi^{1/4}}{T_\Lambda}. \tag{33}$$

The explicit integral with the matched upper limit

Inserting $\omega_c = \omega_\Lambda^*$ into (29) (with $g(\omega) = \omega^2/\pi^2 c^3$) gives

$$u_\Lambda = \frac{\hbar}{8\pi^2 c^3}\, \omega_c^4 = \frac{\hbar}{8\pi^2 c^3}\left(\pi^{1/4}\right)^4 \left(\frac{\Lambda c^7}{\hbar G}\right) = \frac{\Lambda c^4}{8\pi G}. \tag{34}$$

Thus the same quartic ZPE integral, evaluated at the Λ cutoff ω_Λ^*, reproduces exactly the GR vacuum density.

Massive fields decouple at the Λ scale

For a species of mass m with angular frequency ω_k,

$$\omega_k = \sqrt{c^2 k^2 + (mc^2/\hbar)^2}, \tag{35}$$

and the same $k_{\max}$, the energy density becomes

$$u^{(m)} = \frac{s\hbar}{4\pi^2} \int_0^{k_{\max}} k^2 \sqrt{c^2k^2 + (mc^2/\hbar)^2}\, dk = \mathcal{O}\!\left(\frac{mc^2}{12\pi^2}\, k_{\max}^3\right) \ll u^{\rm massless}, \tag{36}$$

with degeneracy s. Hence heavy fields are parametrically suppressed by powers of $k_{\max}/(mc/\hbar)$. At the Λ cutoff ($\hbar c/L_\Lambda \sim$ meV), only effectively massless modes (photons; plausibly gravitons) contribute appreciably.

 doi:10.20944/preprints202510.2218.v1

Interpretation.

The result above is not a fine–tuned cancellation but a horizon–imposed saturation of the mode count: a finite entropy/energy budget set by Λ. Radiative stability follows naturally—high–frequency loops do not shift u_Λ [4,5,7,39].

Repeating the calculation with Planck's ($T = 0$) spectrum reproduces

$$u_{\text{ZPE}}^{(\text{Planck})} = \frac{\pi\hbar}{c^3}\nu_c^4, \tag{37}$$

with $\nu_c = \nu_\Lambda^*$ (equivalently, $u_\Lambda = \hbar\omega_c^4/(8\pi^2 c^3)$ for $\omega = 2\pi\nu$), providing a useful consistency check; see Appendix C.

9. Law of Entropic Constraint (LoEC)

Across horizon thermodynamics, Casimir, quantum gases and EM flux, all ceilings track a single dimensionless constant:

$$\boxed{\alpha_\Lambda \equiv \frac{c^3}{G\hbar\Lambda} \sim 10^{123}}. \tag{38}$$

Fixed by horizon matching (Sec. 2.1, App. A), a direct corollary is the vacuum ceiling

$$u_\Lambda = \frac{\Lambda c^4}{8\pi G}, \qquad u \leq u_\Lambda. \tag{39}$$

The cosmological constant and corresponding u_Λ are *small* because the entropy of the universe is so *large*, bounded by Eq. (38). With Boltzmann's relation (penned by Planck himself) [40,41],

$$S = k_B \ln W, \tag{40}$$

we have

$$W = \exp\left(\frac{S_{\max}}{k_B}\right) \approx \exp(10^{123}). \tag{41}$$

This is an immensely large entanglement entropy [5], meaning the number of accessible microstates available to the quantum vacuum is truly colossal. The smallness of u_Λ is therefore a consequence of the largeness of the GFSC, α_Λ. Furthermore, this finite number of microstates, according to the LoEC, cannot be created or destroyed arbitrarily: the *total capacity* is conserved [14]. A universe in which $\Lambda \to 0$ removes the bound, $S_{\text{dS}} \to \infty$, and the Λ-scale would disappear.

Our preceding analysis has shown that seemingly independent domains converge on the same vacuum bound u_Λ. This unification points to a deeper principle: the vacuum encodes a maximum entropy, and all physical processes respect this limit.

Inertia emerges from entanglement across local Rindler horizons, while gravity reflects curvature-induced entropy bounds in de Sitter space.

If Λ sets the universe's entropy capacity, models that treat dark energy as a time-varying $\Lambda(t)$ are in tension with horizon thermodynamics, since $S_{\text{dS}} \propto 1/\Lambda$ would drift unless compensated by entropy production. Likewise, varying c or $\hbar$ would shift the same bound. The Hubble-tension [42,43] is therefore more plausibly traced to measurement systematics or new matter-sector physics than to variations in $\Lambda, c, \hbar$. Gravity need not be fundamental: the Λ–framework suggests it is the macroscopic imprint of information loss across causal horizons.

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Box 9.1: The Law of Entropic Constraint (LoEC): Summary

All physical processes are constrained by a maximum entropy determined by the vacuum's causal structure, underpinned by the dimensionless GFSC.

$$\alpha_\Lambda \equiv \frac{c^3}{G\hbar\Lambda} \sim 10^{123} \tag{LoEC}$$

Its key consequences are:

- A finite vacuum energy density arises from the finite maximum entropy of a de Sitter causal patch. This provides a universal *ceiling* on coarse–grained energy densities accessible within a causal region: $u \le u_\Lambda \equiv \Lambda c^4/(8\pi G)$.
- Inertial, gravitational, and electromagnetic interactions respect the same horizon–thermodynamic bound.
- Consistency across sectors—bosons, fermions, electromagnetic flux, and the vacuum—means their *extremal* configurations saturate the same ceiling u_Λ; generic states satisfy $u < u_\Lambda$.
- In equilibrium, the LoEC favors a constant cosmological term Λ. Allowing $\Lambda(t)$ would require compensating entropy production (i.e., a non–equilibrium extension of the bookkeeping).
- Gravity can be interpreted as an *emergent* macroscopic response of geometry to entropy flow across causal horizons (à la Jacobson), rather than an independent fundamental force scale.

10. Discussion

Einstein's aesthetic dilemma concerning the cosmological constant has haunted theoretical physics for more than a century. In extending his 1915 field equations [44],

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{42}$$

with a cosmological constant [45],

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{43}$$

Einstein complained that he had "always had a bad conscience," finding it "ugly indeed that the field law of gravitation should be composed of two logically independent terms" [46]. His discomfort reflected the sense that Λ had been appended without justification. Yet historical precedent shows this unease was misplaced. Newton had already noted in the *Principia* that spherical symmetry admits not only the familiar inverse-square force law but also a linear term proportional to R [47]. See Figure 14.

doi:10.20944/preprints202510.2218.v1

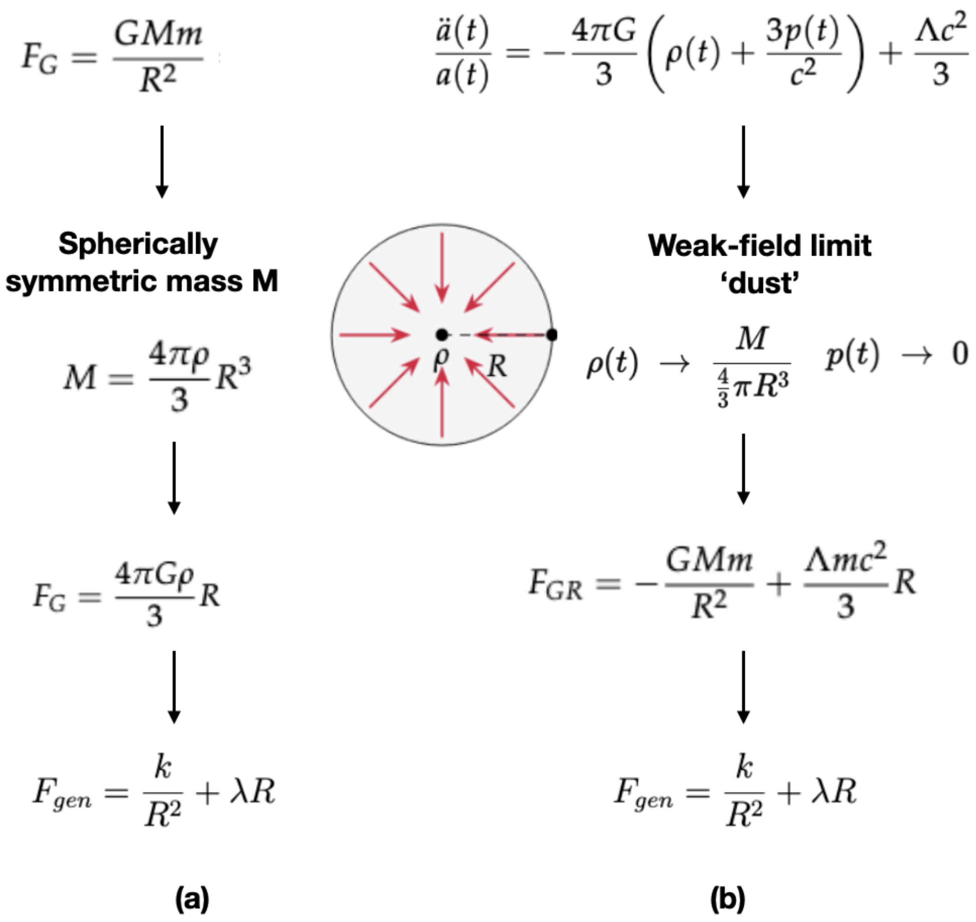

Figure 14. Newton's cosmological constant. (a) Newton showed that spherical symmetry permits two central force laws: an inverse–square attraction, $F_G \propto -1/R^2$, and a linear (harmonic) term $\propto R$. Both combine naturally into the general two–term form $F_{\text{gen}} = -k/R^2 + \lambda R$. (b) In general relativity, the second Friedmann equation in the weak–field dust limit yields the force law $F_{GR} = -GMm/R^2 + (\Lambda mc^2/3)R$, which likewise reduces to the unified form $F_{\text{gen}} = -k/R^2 + \lambda R$. *Note.* The $+\lambda R$ term is a Hooke-like *tensional* contribution (units: force/length, i.e. $[\lambda] = \mathrm{N\,m^{-1}}$). In GR with a cosmological constant Λ it evaluates to $\lambda = \Lambda mc^2/3$, i.e. the linear repulsive force is set by the vacuum energy density. Thus what Einstein regarded as an "ugly addition" of two terms is in fact a structural feature common to both Newtonian and relativistic gravity.

What appeared to Einstein as a blemish was in fact the natural echo of a symmetry already recognised in classical gravity.

The thermodynamic interpretation of general relativity reveals why the cosmological term is inevitable. Jacobson demonstrated that the Einstein equations without Λ are equivalent to the Clausius relation $\delta Q = T\,dS$ [5], expressing the balance of heat and entropy flow across local horizons.

Box 10.1: Thermodynamic Bookkeeping ↔ Einstein equation (with Λ)

$$\textbf{First law:} \quad dU = \delta Q - p\,dV \quad (\delta Q = T\,dS)$$

$$\textbf{Einstein eq.:} \quad G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda\, g_{\mu\nu}$$

Term-by-term correspondence:

$$\underbrace{dU}_{\text{internal energy}} \leftrightarrow \underbrace{G_{\mu\nu}}_{\text{internal curvature (geometry)}}$$

$$\underbrace{\delta Q\ (= TdS)}_{\text{heat / energy flux}} \leftrightarrow \underbrace{\frac{8\pi G}{c^4} T_{\mu\nu}}_{\text{energy–momentum flux}}$$

$$\underbrace{p\,dV}_{\text{work}} \leftrightarrow \underbrace{\Lambda\, g_{\mu\nu}}_{\text{vacuum term}}$$

Interpretation.

- $G_{\mu\nu}$ plays the role of *internal energy* stored in curvature—the geometric response.
- $\frac{8\pi G}{c^4} T_{\mu\nu}$ maps to the *heat/energy flux* across a causal screen; via $\delta S = \delta Q/T$ it accounts for entropy flow (see Sec. 2.1).
- The Λ term acts like a uniform *vacuum pressure/tension*. Writing $p_\Lambda = -\frac{\Lambda c^4}{8\pi G}$, the geometric piece $-\Lambda g_{\mu\nu}$ corresponds to the work term $-\,p_\Lambda dV$, matching the first law $dU = \delta Q - p\,dV$.
- The vacuum has equation of state $w = -1$ ($p = -\rho c^2$), so the geometric term $-\Lambda g_{\mu\nu}$ corresponds to the work piece with $p_\Lambda = -\Lambda c^4/(8\pi G)$.

Once the cosmological constant is restored, the equations assume the structure of the full first law of thermodynamics,

$$\Delta U = T\,\Delta S - p\,\Delta V, \tag{44}$$

with the $\Lambda g_{\mu\nu}$ contribution supplying the missing $p\,dV$ work term. In thermodynamic language, vacuum energy has the unique equation of state $p_\Lambda = -\rho_\Lambda c^2$, so the cosmological constant precisely mimics the work function of the vacuum. As Padmanabhan [7] emphasised, this endows spacetime with the ability to perform work and completes the thermodynamic analogy. Far from being an arbitrary appendage, Λ is the key to upgrading Jacobson's Clausius identity into the full first law, closing the circle of the thermodynamic derivation presented in Section 2.2.

Once Λ is recognised as a *fundamental constant*, a new natural scale inevitably follows. The Λ-length,

$$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4} = 3^{-1/4}\,(L_P\,R_{\rm dS})^{1/2}, \tag{45}$$

Equation (45) shows that L_Λ is simply the geometric mean scale derived from the Planck length L_P and the de Sitter horizon radius $R_{\rm dS}$ (up to the fixed factor $3^{-1/4}$). This ties together quantum uncertainty, causality, and horizon thermodynamics in a single invariant.

A central point of comparison is with the traditional Planck scale. In Planck units, the vacuum energy density expected from zero-point fluctuations is catastrophically large, exceeding observation by some 120 orders of magnitude. This mismatch makes the observed value of

$$u_\Lambda = \frac{\Lambda c^4}{8\pi G} \tag{46}$$

appear absurdly small [48–50]. Yet this impression arises only when Planck units are taken as the measure of naturalness [51]. Once Λ is admitted as a fundamental constant, a new unit system emerges in which u_Λ is not anomalously tiny but exactly the saturation value set by horizon thermodynamics. The Λ-length provides the correct cutoff, and the resulting vacuum energy is radiatively stable [4,52]. In this framework the Planck scale is revealed as inadequate for "weighing the vacuum," while the Λ-scale resolves the paradox without fine-tuning. What appears as an inexplicable smallness is reinterpreted as the fundamental benchmark of information capacity. Thus the vacuum catastrophe is not a failure of physics but the signal that the Planck yardstick breaks down when applied to the quantum structure of empty space.

Several further consequences flow directly from treating Λ as fundamental and recognising the Λ-scale as basic:

- **Bound on entropy.** Λ imposes a universal entropy bound of spacetime. The Law of Entropic Constraint (LoEC) asserts that all physical processes are restricted by this bound, encoded in the de Sitter horizon entropy $S \propto 1/\Lambda$. The vacuum energy u_Λ is a necessary corollary of the bound and the GFSC encodes it.
- **Finite and stable vacuum energy.** The LoEC forbids $\Lambda(t)$, since the de Sitter entropy bound would otherwise change in time. Time-varying models (quintessence/phantom) [53–56] are therefore not a true cosmological constant ($w \neq -1$) and proposals to solve the Hubble tension [57] with $\Lambda(t)$ contradict both LoEC and the horizon–thermodynamic derivation.
- **Quantum derivation of vacuum energy.** In Λ-units, the zero-point energy sum saturates at the Λ-scale reproducing the GR vacuum density in Eq.(46), and yielding the radiatively stable derivation that Zel'dovich and Sakharov first sought, unifying the quantum and cosmological vacua as a single entity [17,58].
- **Cross-domain consistency.** Independent systems all saturate the Λ-bound in the same way. Bosonic and fermionic gases at the Λ-temperature approach fixed fractions of u_Λ; Casimir pressures with separation L_Λ reproduce the vacuum density of GR; electromagnetic fluctuations scale as $\alpha_E u_\Lambda$ via the Poynting analysis. These diverse phenomena confirm that the same entropy ceiling governs matter, radiation, and the vacuum alike [30,59,60].

These consequences show that the Λ-framework addresses simultaneously the vacuum catastrophe, the stability of the vacuum, and the constancy of couplings. In each case, the objections that once plagued the cosmological constant dissolve when it is treated as the natural thermodynamic completion of the first law and the basis of a new quantum scale.

In this light, Einstein's dilemma disappears. The two-term structure that he found unsightly is the unavoidable expression of deeper principles. Just as Newton showed that two distinct force laws are permitted by symmetry, the thermodynamic derivation shows that two terms are demanded by consistency: the Einstein tensor encodes the response of spacetime geometry, while the Λ-term enforces its finite information capacity through the Λ-scale. Einstein's infamous "blunder" is thereby reinterpreted as an inevitable consequence of symmetry and thermodynamics. Recognising Λ as fundamental not only restores logical unity to the field equations but also reframes them as the complete first law of spacetime dynamics, with the Λ-scale as the bridge between quantum theory and cosmology.

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Planck versus Λ scales

The Planck system of units was constructed heuristically by equating a Schwarzschild radius with a Compton wavelength, and has long been assumed to define the natural scale of quantum gravity. Yet this construction is ad hoc and plagued by pathologies. It predicts a vacuum density overshooting observation by 10^{120} (the "vacuum catastrophe") [4], a runaway instability at every instant [61,62], and a large conceptual gap ("Planck desert") between M_P and Standard Model scales [63,64]. Most significantly, as highlighted in Table 3, the Planck system is thermodynamically sterile: its base set $\{G, \hbar, c\}$— is purely mechanical, with k_B introduced only by convention—the Λ base set $\{G, \hbar, c, \Lambda\}$ by contrast, embeds a thermodynamic constant ab initio: Λ fixes the de Sitter entropy bound, so information capacity is intrinsic rather than appended. Its base units follow not from heuristic dimensional analysis but from the Clausius relation $E = TS$, ensuring consistency with the observed vacuum energy density. In this sense, the Λ framework subsumes and supersedes the Planck scale as a complete natural system, unifying quantum, thermodynamic, and gravitational perspectives.

Table 3. Comparison of Planck and Λ natural unit systems. Planck units arise by heuristic dimensional analysis of $\{G, \hbar, c\}$, while Λ-units are fixed by the thermodynamics of causal horizons (Bekenstein–Hawking, Gibbons–Hawking), embedding entropy intrinsically and uniquely reproducing the GR vacuum energy density.

Feature	**Planck System** $(G, \hbar, c)$	Λ **System** $(G, \hbar, c, \Lambda)$
Core base units	Length $L_P = \sqrt{\hbar G/c^3}$, Time $T_P = \sqrt{\hbar G/c^5}$, Mass $M_P = \sqrt{\hbar c/G}$.	Length $L_\Lambda = (\hbar G/\Lambda c^3)^{1/4}$, Time $T_\Lambda = (\hbar G/\Lambda c^7)^{1/4}$, Mass $M_\Lambda = (\hbar c/\sqrt{G\Lambda})^{1/2}$.
Method of derivation	Heuristic dimensional analysis of $\{G, \hbar, c\}$. Unique by construction but thermodynamically sterile.	Thermodynamics of causal horizons (Bekenstein–Hawking, Gibbons–Hawking). Vacuum energy density u_Λ uniquely fixed, with entropy intrinsic.
Role of k_B	Present in Planck's original set but *drops out* of L_P, M_P, t_P. Thermodynamics not built-in. Temperature only via a bolt-on Planck temperature.	Thermodynamics embedded in base units $L_\Lambda, M_\Lambda, T_\Lambda$. The Bekenstein–Hawking relation gives a fundamental entropy unit $S_\Lambda = 3\pi k_B\, \alpha_\Lambda$.
Entropy/ thermodynamics	Absent from core units. No natural unit of entropy. Essentially a mechanical system.	Entropy is intrinsic. Causal horizons saturate a finite entropy bound, embedding thermodynamics at the foundation.
Cosmological constant	Assumes $\Lambda = 0$ (relic of a non-accelerating cosmos). No scale associated with vacuum energy.	$\Lambda > 0$ fundamental. Uniquely fixes the vacuum energy density $u_\Lambda = \Lambda c^4/(8\pi G)$, linking quantum and thermodynamic structure.

11. Conclusions

The analysis presented suggests, Λ is not a 'blunder' but the missing piece of the cosmic–quantum jigsaw puzzle [45,46,65]. Its genuine inclusion at the foundations of physics reveals the vacuum not as a place of violent discontinuity, but as a regulated structure where spacetime remains smooth and continuous even at the smallest scales. Far from heralding the breakdown of the laws of physics, the Λ-scale completes them. The supposed realm of quantum gravity, long confined to the Planck scale, is displaced by some thirty orders of magnitude into a domain where thermodynamic and quantum principles converge. Taken together, our results recast Λ from an expendable add-on to a fundamental constant that fixes the universe's information capacity [14]. The Λ–framework turns the divergent zero-point sum into a finite non-zero value because the de Sitter horizon enforces geometric saturation: only modes that "fit" contribute, while heavy, short-wavelength modes are suppressed [66]. Λ thereby emerges as a thermodynamic necessity, with the maximum entropy

$$S_{\max} = \frac{A_{dS}}{4G\hbar}, \tag{47}$$

providing the global bound.

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In this light, the Law of Entropic Constraint (LoEC), underpinned by the gravitational fine-structure constant

$$\alpha_\Lambda = \frac{c^3}{G\hbar\Lambda}, \tag{48}$$

functions as a conservation-like principle: entropy can be redistributed locally through unitary dynamics, but the total number of accessible microstates is bounded globally. This explains why de Sitter spacetime lacks global energy conservation and unitarity, even while local conservation laws remain intact [22,67].

Historically, this framework fulfils what Planck originally sought. His aim was to resolve the UV catastrophe, via an exact law of entropy conservation [68,69]. Instead his journey initiated the quantum revolution. The Λ–framework, by contrast, subsumes and supersedes the Planck system, providing the entropy law he sought through the LOEC.

Most significantly, this perspective resolves Einstein's misgivings about the "ugly" addition of two terms [46]. It provides the Zel'dovich–Sakharov program [17,58] with a successful, radiatively stable derivation of vacuum energy, and extends Jacobson's thermodynamic derivation from local Rindler horizons to the full de Sitter horizon [5]. In doing so, the Λ–framework unifies quantum, gravitational, and thermodynamic perspectives.

In the gravitational context, thermodynamic constraints are not merely consequences of geometry; they give rise to it [5,6]. Spacetime's structure may be the most elegant entropy-management architecture the universe has ever devised, with Λ setting its intrinsic scale and, in the process, revealing gravity's innate quantum nature.

Author Contributions: Conceptualization, methodology, formal analysis, investigation, visualization, writing—original draft, and writing—review & editing, all by the author.

Funding: The author conducted this research independently and received no external funding.

Institutional Review Board Statement: Not applicable. This theoretical study involved no human or animal subjects.

Informed Consent Statement: Not applicable.

Data Availability Statement: All data supporting the findings of this study are contained within the article and its appendices (and any supplementary materials).

Acknowledgments: The author gratefully acknowledges Dr. John F. Cooper for insightful discussions and constructive comments that improved the manuscript.

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Conflicts of Interest: The author declares no conflict of interest.

Use of Artificial Intelligence Tools: Artificial intelligence tools (ChatGPT, OpenAI, 2025) were used to assist in formatting, language refinement, and LaTeX code organization. All scientific content, data analysis, derivations, and interpretations are solely the author's original work.

Abbreviations

The following abbreviations are used in this manuscript:

Table 4. Abbreviations used in this manuscript.

Symbol	Meaning / Definition
Λ	Cosmological constant (inverse length squared; also sets the entropy/area scale).
u_Λ	Vacuum energy density, $u_\Lambda = \Lambda c^4/(8\pi G)$.
L_Λ, T_Λ, M_Λ	Λ length, time, mass scales: $L_\Lambda = \sqrt[4]{\hbar G/(\Lambda c^3)}$, $T_\Lambda = \sqrt[4]{\hbar G/(\Lambda c^7)}$, $M_\Lambda = \sqrt[4]{\hbar^2 c^2/(G\Lambda)}$.
Θ_Λ	Λ–temperature unit: $\Theta_\Lambda = \sqrt[4]{\hbar^3 \Lambda c^7/(G\, k_B^4)}$.
L_P, T_P, M_P	Planck length, time, mass: $L_P = \sqrt{\hbar G/c^3}$, $T_P = \sqrt{\hbar G/c^5}$, $M_P = \sqrt{\hbar c/G}$.
α_E	Electromagnetic fine–structure constant, $\alpha_E = e^2/(4\pi\varepsilon_0\hbar c) \approx 1/137$ (HEP convention sets $\varepsilon_0 = 1$, giving $\alpha_E = e^2/4\pi$).
α_Λ	Gravitational fine–structure constant, $\alpha_\Lambda = c^3/(G\hbar\Lambda)$.
GR, QFT	General Relativity; Quantum Field Theory.
R_{dS}, H	de Sitter horizon radius; Hubble rate ($H = \sqrt{\Lambda c^2/3}$).
T_{GH}, T_U, T_H	Gibbons–Hawking, Unruh, Hawking temperatures.
S_{BH}, S_{dS}	Bekenstein–Hawking and de Sitter horizon entropies.
κ	Surface gravity (sets $T = \hbar\kappa/(2\pi k_B)$).
ν_Λ^*, ω_Λ^*	Matched cutoff (linear and angular) frequencies.
ZPE	Zero–point energy.
LoEC	Law of Entropic Constraint.
PoE	(Einstein's) Principle of Equivalence.
DOS	Density of states.
CMB, CNB, u_γ, u_ν	Cosmic Microwave / Neutrino Background; symbols for the photon and neutrino energy densities.
Ω_H, Φ_H	Horizon angular velocity; horizon electric potential (work coefficients).
dJ, dQ	Infinitesimal changes in black–hole angular momentum and electric charge.

Appendix A Derivation of the Λ-Scale

We build natural units from $\{G, \hbar, \Lambda, c\}$ by writing any target unit X as a monomial

$$X \propto G^p\, \hbar^q\, \Lambda^r\, c^s. \tag{A1}$$

with exponents (p, q, r, s) to be determined from dimensional balance in base units $[L, T, M]$ (cf. Planck's 1899 dimensional construction; here extended to include Λ [16]):

$$[G] = L^3T^{-2}M^{-1}, \qquad [\hbar] = L^2T^{-1}M, \qquad [\Lambda] = L^{-2}, \qquad [c] = LT^{-1}. \tag{A2}$$

Hence the dimension of $G^p\hbar^q\Lambda^r c^s$ is

$$[X] = L^{3p+2q-2r+s}\; T^{-2p-q-s}\; M^{-p+q}. \tag{A3}$$

The three exponents are exactly the three linear forms that will become the rows of the matrix in the formal derivation.

Appendix A.0.1 Length L_Λ: solve the linear system by inspection.

For a length unit we require $L^1T^0M^0$, i.e.

$$\begin{cases} 3p + 2q - 2r + s = 1, \\ -2p - q - s = 0, \\ -p + q = 0, \end{cases} \quad \Longrightarrow \quad \begin{cases} q = p, \\ s = -3p, \\ 2p - 2r = 1 \;\Rightarrow\; r = p - \frac{1}{2}. \end{cases} \tag{A4}$$

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Thus the general solution is a one-parameter family

$$(p,q,r,s) = \left(p,\ p,\ p-\tfrac{1}{2},\ -3p\right). \tag{A5}$$

Two instructive choices:

$$\begin{aligned} &\textbf{Planck} \quad p=\tfrac{1}{2}: \quad L \propto G^{1/2}\hbar^{1/2}c^{-3/2} = \ell_P = \left(\hbar G/c^3\right)^{1/2}, \\ &\boldsymbol{\Lambda}\textbf{-scale} \quad p=\tfrac{1}{4}: \quad L \propto G^{1/4}\hbar^{1/4}\Lambda^{-1/4}c^{-3/4} = L_\Lambda = \left(\tfrac{\hbar G}{\Lambda c^3}\right)^{1/4}. \end{aligned}$$

Appendix A.0.2 Time T_Λ: identical logic.

For a time unit we require $L^0T^1M^0$, hence

$$\begin{cases} 3p+2q-2r+s=0, \\ -2p-q-s=1, \\ -p+q=0, \end{cases} \quad\Longrightarrow\quad \begin{cases} q=p, \\ s=-3p-1, \\ 2p-2r-1=0 \ \Rightarrow\ r=p-\tfrac{1}{2}. \end{cases} \tag{A6}$$

So the family is

$$(p,q,r,s) = \left(p,\ p,\ p-\tfrac{1}{2},\ -3p-1\right), \tag{A7}$$

Two instructive choices:

$$\begin{aligned} &\textbf{Planck} \quad p=\tfrac{1}{2}: \quad T \propto G^{1/2}\hbar^{1/2}c^{-5/2} = t_P = \left(\hbar G/c^5\right)^{1/2}, \\ &\boldsymbol{\Lambda}\textbf{-scale} \quad p=\tfrac{1}{4}: \quad T \propto G^{1/4}\hbar^{1/4}\Lambda^{-1/4}c^{-7/4} = \frac{L_\Lambda}{c} = T_\Lambda. \end{aligned}$$

Appendix A.0.3 Mass M_Λ: again, solving by inspection.

For a mass unit we require $L^0T^0M^1$, i.e.

$$\begin{cases} 3p+2q-2r+s=0, \\ -2p-q-s=0, \\ -p+q=1, \end{cases} \quad\Longrightarrow\quad \begin{cases} q=p+1, \\ s=-3p-1, \\ 2p+1-2r=0 \ \Rightarrow\ r=p+\tfrac{1}{2}. \end{cases} \tag{A8}$$

Therefore

$$(p,q,r,s) = \left(p,\ p+1,\ p+\tfrac{1}{2},\ -3p-1\right). \tag{A9}$$

Two instructive choices:

$$\begin{aligned} &\textbf{Planck} \quad p=-\tfrac{1}{2}: \quad M \propto G^{-1/2}\hbar^{1/2}c^{1/2} = M_P = \left(\hbar c/G\right)^{1/2}, \\ &\boldsymbol{\Lambda}\textbf{-scale} \quad p=-\tfrac{1}{4}: \quad M \propto G^{-1/4}\hbar^{3/4}\Lambda^{1/4}c^{-1/4} = \frac{\hbar}{c\,L_\Lambda} = M_\Lambda. \end{aligned}$$

By–inspection solutions (one–parameter families). For $X \propto G^p \hbar^q \Lambda^r c^s$ the required exponents for the base units are:

$$\begin{aligned} &\text{Length } L: \ (p,q,r,s) = (p,\ p,\ p-\tfrac{1}{2},\ -3p), && \Rightarrow\ L \propto G^p \hbar^p \Lambda^{p-1/2} c^{-3p}, \\ &\text{Time } T: \ (p,q,r,s) = (p,\ p,\ p-\tfrac{1}{2},\ -3p-1), && \Rightarrow\ T \propto G^p \hbar^p \Lambda^{p-1/2} c^{-3p-1}, \\ &\text{Mass } M: \ (p,q,r,s) = (p,\ 1+p,\ p+\tfrac{1}{2},\ -1-3p), && \Rightarrow\ M \propto G^p \hbar^{1+p} \Lambda^{p+1/2} c^{-1-3p}. \end{aligned}$$

Two instructive choices.

$$\textbf{Planck} \quad p = \tfrac{1}{2} \text{ in } L, T, \quad p = -\tfrac{1}{2} \text{ in } M: \ \begin{cases} L = \ell_P = (\hbar G/c^3)^{1/2}, \\ T = t_P = (\hbar G/c^5)^{1/2}, \\ M = M_P = (\hbar c/G)^{1/2}. \end{cases}$$

$$\Lambda\textbf{–scale} \quad p = \tfrac{1}{4} \text{ in } L, T, \quad p = -\tfrac{1}{4} \text{ in } M: \ \begin{cases} L_\Lambda = \left(\hbar G/(\Lambda c^3)\right)^{1/4}, \\ T_\Lambda = L_\Lambda / c, \\ M_\Lambda = \hbar/(c\, L_\Lambda). \end{cases}$$

The three blocks above are the three row-constraints we shall later collect into the 3×4 matrix. Solving each target dimension (L, T, M) exposes the same one-dimensional affine freedom: dimensional analysis alone leaves a free parameter p (a nullspace direction). In practice, this means there is a whole family of possible "natural units." Choosing $p = \frac{1}{2}$ reproduces the Planck units, while $p = \frac{1}{4}$ yields the Λ-units with their characteristic quarter-powers.

This inspection method already shows that dimensional analysis by itself cannot uniquely fix the scale: the affine parameter simply shifts weight among $\{G, \hbar, \Lambda, c\}$. To determine which member of the family Nature selects, one must go beyond inspection. [16,70].

Appendix A.1 Deriving and Fixing the Λ-Unit System

To formalize the inspection analysis above, we now write out the dimensional algebra explicitly. Using our four constants are defined in (A2), we consider a monomial

$$X \ = \ G^p\, \hbar^q\, \Lambda^r\, c^s, \tag{A10}$$

and demand $[X] = M^b L^a T^c$. Matching exponents of M, L, T gives the linear system

$$\begin{pmatrix} -1 & 1 & 0 & 0 \\ 3 & 2 & -2 & 1 \\ -2 & -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} b \\ a \\ c \end{pmatrix}. \tag{A11}$$

This 3×4 matrix has rank 3, hence a **1-dimensional nullspace**. A basis vector is

$$\boldsymbol{n} = (1,1,1,-3)^\top, \qquad \begin{pmatrix} -1 & 1 & 0 & 0 \\ 3 & 2 & -2 & 1 \\ -2 & -1 & 0 & -1 \end{pmatrix} \boldsymbol{n} = \boldsymbol{0}. \tag{A12}$$

Therefore any particular solution (p_0, q_0, r_0, s_0) can be shifted by $\lambda\, \boldsymbol{n}$ for arbitrary λ:

$$(p,q,r,s) = (p_0, q_0, r_0, s_0) + \lambda\,(1,1,1,-3). \tag{A13}$$

Vacuum matching anchor and uniqueness.

Dimensional analysis leaves the affine family (A13). *Physics fixes λ by the vacuum–matching postulate* (QFT cutoff = GR vacuum):

$$\frac{\hbar c}{L^4} = \frac{\Lambda c^4}{G}. \tag{A14}$$

In exponent space (order $G, \hbar, \Lambda, c$) this requires

$$L^4 \propto G^1 \hbar^1 \Lambda^{-1} c^{-3} \quad \Longrightarrow \quad 4(p,q,r,s) = (1,1,-1,-3),$$

so the unique length exponents are

$$(p,q,r,s) = \left(\tfrac{1}{4}, \tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{3}{4}\right), \qquad L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4}. \tag{A15}$$

With the kinematic anchors $L/T = c$ and $ML^2/T = \hbar$ this yields

$$T_\Lambda = \frac{L_\Lambda}{c}, \qquad M_\Lambda = \frac{\hbar}{c\, L_\Lambda}. \tag{A16}$$

Conclusion: with (M, L, T) as bases and four constants $(G, \hbar, c, \Lambda)$, every target dimension has a **1-parameter family** of exponent quadruples. This is the mathematical origin of the "infinitely many (p,q,r,s)".

Appendix A.2 Planck, Stoney, and Λ as "nullspace + anchors"

A unit system becomes **unique** once we add enough **physical anchors** to kill the nullspace freedom(s). With the vacuum matching (A14) selecting $\lambda = 0$, the dimensionless values of the constants in Λ-units are summarized in Table A1.

Table A1. Dimensionless values in Λ-units. Anchors enforce $\tilde{c} = \tilde{\hbar} = 1$. Vacuum matching selects the symmetric point, yielding $\tilde{G}\,\tilde{\Lambda} = \alpha_\Lambda^{-1}$ and $\tilde{G} = \tilde{\Lambda} = \alpha_\Lambda^{-1/2}$, where $\alpha_\Lambda \equiv c^3/(G\hbar\Lambda)$ is the GFSC.

Constant	Base dimensions	$\tilde{X}$ in Λ-units	Note
c	$L\,T^{-1}$	1	anchor $L/T = c$
$\hbar$	$M\,L^2\,T^{-1}$	1	anchor $ML^2/T = \hbar$
G	$L^3\,M^{-1}\,T^{-2}$	$\alpha_\Lambda^{-1/2}$	paired with $\tilde{\Lambda}$; product fixed
Λ	L^{-2}	$\alpha_\Lambda^{-1/2}$	symmetric with $\tilde{G}$

Planck units $\{G, \hbar, c\}$ [16].

Here $N = 3$ constants for 3 bases $(M, L, T) \Rightarrow$ no nullspace. Uniqueness comes for free once we impose the usual identifications $L/T = c$ and $McL = \hbar$.

Stoney units $\{G, c, e\}$ (+ EM convention) [71].

Relativity ($L/T = c$), gravity (G), and an electromagnetic normalization (choice of Coulomb constant / rationalization) fix the set. One recovers c, G and a natural charge/action scale (e.g. $e^2/4\pi\varepsilon_0 c$ in SI) by construction.

Λ-units $\{G, \hbar, c, \Lambda\}$.

Now $N = 4$ for 3 bases $\Rightarrow$ **one** free parameter (the λ in (A13)). Two standard anchors are kept:

$$\frac{L}{T} = c, \qquad M\,c\,L = \hbar. \tag{A17}$$

A **single Λ-specific anchor** removes the remaining freedom:

$$\textit{Vacuum matching (thermo/GR = QFT cutoff):} \quad \frac{\hbar c}{L^4} = \frac{\Lambda c^4}{G}. \tag{A18}$$

This fixes the length scale uniquely:

$$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4}, \qquad T_\Lambda = \frac{L_\Lambda}{c}, \qquad M_\Lambda = \frac{\hbar}{c\,L_\Lambda} = \left(\frac{\hbar^3 \Lambda}{G\,c}\right)^{1/4}. \tag{A19}$$

Thus the Λ-triple is **unique** once (A18) is adopted.

Equivalent algebraic symmetry. For length, write $L \propto G^p \hbar^q \Lambda^r c^s$. Solving (A11) for $[L]$ gives the family

$$p = q = -\frac{s}{3}, \qquad r = p - \frac{1}{2}. \tag{A20}$$

Imposing the simple **equal-weight symmetry** $p = q = -r$ ("give quantum and gravity equal weight, oppose Λ with equal magnitude") yields

$$p = q = \tfrac{1}{4}, \quad r = -\tfrac{1}{4}, \quad s = -\tfrac{3}{4}, \tag{A21}$$

i.e. exactly (A19). This symmetry condition is just the exponent form of the vacuum matching (A18).

Appendix A.3 Why most choices do not *return c and ħ*

Because (p, q, r, s) live on the affine line (A31), generic choices give

$$\frac{L}{T} \neq c, \qquad M\,\frac{L}{T}\,L \neq \hbar.$$

The anchors (A17) pick out the subfamily consistent with relativity and quantum kinematics; the Λ-anchor (A18) then selects **one** point on that subfamily—giving (A19). This mirrors explicit counterexamples: other valid exponent choices produce a different velocity scale and a different action scale.

Appendix A.4 Temperature and k_B

Historically, Planck [16] introduced a **fourth base dimension** Θ and used k_B to tie temperature to energy. Two consistent options:

1. **With k_B (four bases M, L, T, Θ).** Add $k_B : [ML^2T^{-2}\Theta^{-1}]$. Then a natural Λ-temperature follows from horizon thermodynamics:

$$\Theta_\Lambda \sim \frac{\hbar c}{k_B L_\Lambda} = \frac{1}{k_B}\left(\frac{\hbar^3 \Lambda c^7}{G}\right)^{1/4}. \tag{A22}$$

   Up to order-unity factors, this is the de Sitter/Gibbons–Hawking scale.
2. **Without k_B (modern HEP convention).** Set $k_B = 1$ so temperature is measured in energy; Θ is not an independent base dimension and (A19) suffices.

Appendix A.5 Practical "algorithm" (no trial & error)

1. Write the dimensional matrix (A11); solve once to get a particular solution and the null vector $\boldsymbol{n} = (1, 1, 1, -3)$.
2. Enforce the two standard anchors (A17) so that unit speed is c and unit action is $\hbar$.
3. Impose the single Λ-anchor (A18) ⇒ fix the free parameter λ.
4. Read off the exponents for $L_\Lambda, T_\Lambda, M_\Lambda$ as in (A19); if using k_B, define Θ_Λ via (A22).

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Remark on electromagnetism.

$\alpha_E = e^2/4\pi\varepsilon_0\hbar c$ is dimensionless and **not** set by units. One may choose EM conventions (e.g. Heaviside–Lorentz [27,72,73]) so that ε_0 is absorbed, but the Λ-unit **uniqueness** rests entirely on the mechanical anchors plus the Λ-postulate, not on EM conventions.

Therefore: Admitting Λ as a fundamental constant introduces one nullspace degree of freedom in the dimensional algebra; a single, physically motivated **vacuum-matching** postulate removes it and yields a **unique** Λ-scale. This both explains the infinity of formal solutions and why the physically relevant Λ-triple emerges uniquely.

Table A2. Comparison of natural unit systems. Enough anchors remove nullspace freedom.

System	Constants used	Base dims.	Anchors imposed	Unique outcome
Stoney (1881)	$\{G, c, e\}$ (with ε_0)	(M, L, T, Q)	(i) $L/T = c$; (ii) G; (iii) EM normalisation (Coulomb const.)	Recovers c, G and natural charge/action scale ($e^2/4\pi\varepsilon_0 c$ in SI)
Planck (1899)	$\{G, \hbar, c\}$ (opt. k_B)	(M, L, T) or (M, L, T, Θ)	(i) $L/T = c$; (ii) $McL = \hbar$; (opt. iii) k_B for Θ	Unique (L_P, T_P, M_P); with k_B, Planck temperature Θ_P
Λ-scale (present)	$\{G, \hbar, c, \Lambda\}$ (opt. Θ with k_B)	(M, L, T)	(i) $L/T = c$; (ii) $McL = \hbar$; (iii) $\hbar c/L^4 = \Lambda c^4/G$	Unique $(L_\Lambda, T_\Lambda, M_\Lambda)$; with k_B, $\Theta_\Lambda \sim \hbar c/(k_B L_\Lambda)$

Once $\{G, \hbar, \Lambda, c\}$ are admitted, dimensional analysis alone leaves a one-parameter family of monomials. A single, physically motivated postulate—matching the QFT cutoff to the GR vacuum density—selects a unique member: the Λ-scale. What looks like algebraic freedom is therefore resolved by thermodynamics, which also explains why the physically relevant Λ-triple emerges uniquely.

Appendix A.6 The Λ Scale as a Geometric Mean

An elegant property of the Λ scale is that it lies midway, on a logarithmic scale, between the Planck length L_P and the de Sitter horizon radius R_{dS}. This can be shown directly by inspection. Define the familiar quantities:

$$L_P = \sqrt{\frac{\hbar G}{c^3}}, \qquad R_{dS} = \sqrt{\frac{3}{\Lambda}}, \qquad L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4}. \tag{A23}$$

Taking the geometric mean of L_P and R_{dS} gives

$$\sqrt{L_P\, R_{dS}} = \left(\frac{\hbar G}{c^3} \cdot \frac{3}{\Lambda}\right)^{1/4} = 3^{1/4} L_\Lambda. \tag{A24}$$

Hence,

$$L_\Lambda = \frac{1}{3^{1/4}} \sqrt{\ell_P R_{dS}}. \tag{A25}$$

If instead we adopt the common convention $R_\Lambda \equiv \Lambda^{-1/2}$ (dropping the factor $\sqrt{3}$), then L_Λ is *exactly* the geometric mean:

doi:10.20944/preprints202510.2218.v1

	Λ–scale	Planck scale
Length	$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{1/4} = 3.9 \times 10^{-5}\,\mathrm{m}$	$L_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35}\,\mathrm{m}$
Time	$T_\Lambda = \left(\frac{\hbar G}{\Lambda c^7}\right)^{1/4} = 1.3 \times 10^{-13}\,\mathrm{s}$	$T_P = \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44}\,\mathrm{s}$
Mass	$M_\Lambda = \left(\frac{\hbar^3 \Lambda}{cG}\right)^{1/4} = 9.0 \times 10^{-39}\,\mathrm{kg}$	$M_P = \sqrt{\frac{\hbar c}{G}} = 2.2 \times 10^{-8}\,\mathrm{kg}$
Temperature	$\Theta_\Lambda = \left(\frac{\hbar^3 \Lambda c^7}{G k_B^4}\right)^{1/4} = 5.8 \times 10^{1}\,\mathrm{K}$	$\Theta_P = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.4 \times 10^{32}\,\mathrm{K}$

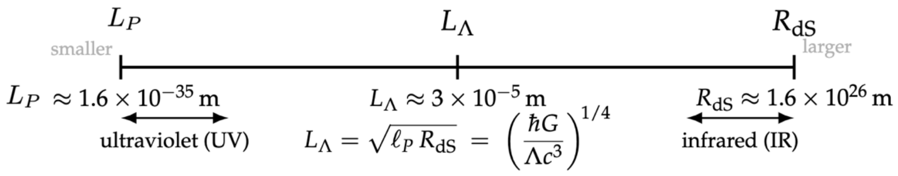

Figure A1. Hierarchy of fundamental length scales. The Planck length L_P anchors the UV, the de Sitter horizon R_{dS} anchors the IR, and the emergent Λ–scale L_Λ sits at the geometric mean, $L_\Lambda = \sqrt{L_P R_{\mathrm{dS}}}$. Numerical values shown here are evaluated using CODATA values, with $\Lambda = 1.1 \times 10^{-52} m^{-2}$ inferred from late–time cosmology.

Appendix A.7 Dimensionless G and Λ in Λ–units and the role of α_Λ

$$\tilde{X} \equiv \frac{X}{[X]_\Lambda}. \tag{A26}$$

The corresponding Λ–unit dimensions are

$$[c]_\Lambda = \frac{L}{T}, \qquad [\hbar]_\Lambda = McL, \tag{A27}$$

$$[G]_\Lambda = \frac{L^3}{MT^2}, \qquad [\Lambda]_\Lambda = L^{-2}. \tag{A28}$$

For gravity and the cosmological constant, define the dimensionless values $\tilde{G} \equiv G/(L^3/(MT^2))$ and $\tilde{\Lambda} \equiv \Lambda L^2$. Evaluated in the $\lambda = 0$ (vacuum–matched) Λ–units,

$$\tilde{G} = G\,\frac{M_\Lambda T_\Lambda^2}{L_\Lambda^3} = \left(\frac{G\hbar\Lambda}{c^3}\right)^{1/2} = \alpha_\Lambda^{-1/2}, \qquad \tilde{\Lambda} = \Lambda L_\Lambda^2 = \left(\frac{G\hbar\Lambda}{c^3}\right)^{1/2} = \alpha_\Lambda^{-1/2}, \tag{A29}$$

so that

$$\boxed{\tilde{G}\,\tilde{\Lambda} = \alpha_\Lambda^{-1}}. \tag{A30}$$

More generally, moving along the null direction by λ (so $L(\lambda) = L_\Lambda \alpha_\Lambda^{-\lambda}$, $T(\lambda) = T_\Lambda \alpha_\Lambda^{-\lambda}$, $M(\lambda) = M_\Lambda \alpha_\Lambda^{\lambda}$) gives

$$\tilde{G}(\lambda) = \alpha_\Lambda^{2\lambda - \frac{1}{2}}, \qquad \tilde{\Lambda}(\lambda) = \alpha_\Lambda^{-2\lambda - \frac{1}{2}}, \qquad \tilde{G}(\lambda)\tilde{\Lambda}(\lambda) = \alpha_\Lambda^{-1}. \tag{A31}$$

The vacuum–matching postulate (A14) fixes the remaining affine freedom by selecting the symmetric point $\lambda = 0$. In the resulting vacuum–matched Λ units the quarter–power base scales give

$$\tilde{c} = 1, \qquad \tilde{\hbar} = 1, \qquad \tilde{G} = \tilde{\Lambda} = \alpha_\Lambda^{-1/2},$$

where $\alpha_\Lambda \equiv c^3/(G\hbar\Lambda)$ is a dimensionless invariant.Along the null rescaling direction one may shift weight between G and Λ while keeping the product fixed:

$$\tilde{G}(\lambda)\,\tilde{\Lambda}(\lambda) = \alpha_\Lambda^{-1}.$$

Consequences.

- Vacuum matching enforces $\lambda = 0$, so the chosen units are the symmetric quarter–power ones, with
$$\tilde{G} = \tilde{\Lambda} = \alpha_\Lambda^{-1/2}, \qquad \tilde{c} = 1, \qquad \tilde{\hbar} = 1. \tag{A32}$$
- One may re–scale along the null direction (choose $\lambda \neq 0$) to make *either* $\tilde{G} = 1$ (take $\lambda = \frac{1}{4}$) *or* $\tilde{\Lambda} = 1$ (take $\lambda = -\frac{1}{4}$), but not both simultaneously, since the product is invariant:
$$\tilde{G}(\lambda)\,\tilde{\Lambda}(\lambda) = \alpha_\Lambda^{-1}. \tag{A33}$$

This parameter is the gravitational analogue of the electromagnetic fine–structure constant α: it is dimensionless and invariant under unit choices. Here α_Λ encodes the simultaneous scaling of both G and Λ alongside c and $\hbar$. In vacuum–matched Λ–units one has $\tilde{c} = \tilde{\hbar} = 1$ and $\tilde{G} = \tilde{\Lambda} = \alpha_\Lambda^{-1/2}$.

Closing summary. With four constants for three base dimensions, admitting Λ leaves a one–parameter nullspace freedom in the dimensional algebra. The kinematic anchors $L/T = c$ and $ML^2/T = \hbar$ fix $\tilde{c} = \tilde{\hbar} = 1$, and the vacuum–matching postulate
$$\frac{\hbar c}{L^4} = \frac{\Lambda c^4}{G} \tag{A32}$$
removes the remaining freedom, selecting the quarter-power length $L_\Lambda = \big(\hbar G/(\Lambda c^3)\big)^{1/4}$ in (A32) and thereby determining the full Λ unit set. In these units the dimensionless gravitational and cosmological constants share a single parameter,
$$\tilde{G} = \tilde{\Lambda} = \alpha_\Lambda^{-1/2}, \qquad \alpha_\Lambda \equiv \frac{c^3}{G\,\hbar\,\Lambda} \tag{A33}$$
so $\tilde{G}\,\tilde{\Lambda} = \alpha_\Lambda^{-1}$ is the concise condition for G and Λ to join c and $\hbar$ in the Λ system.

EM note. The electromagnetic fine-structure constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ is dimensionless and invariant under unit choices; the uniqueness of the Λ units arises from the mechanical anchors plus the vacuum-matching postulate (A32), independent of the EM sector.

 doi:10.20944/preprints202510.2218.v1

Appendix B Natural Units and the Lessons from History

George Stoney [71] introduced the first system of natural units, based on G, c, e and ϵ_0, it was an early attempt to unify gravitation and electromagnetism. See Figure B1.

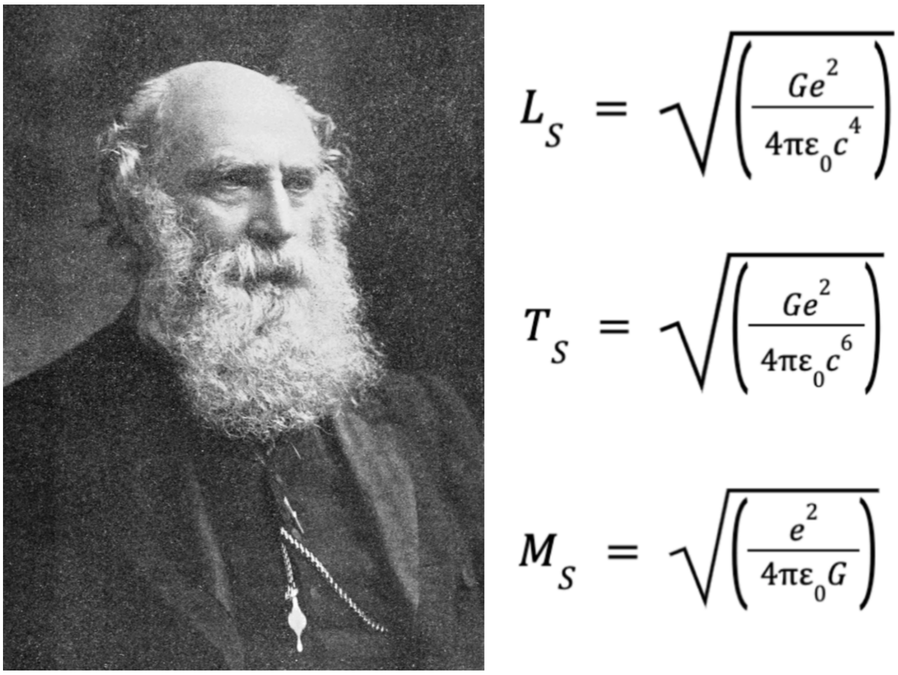

Figure B1. George Johnstone Stoney devised his system of natural units, prior to the discovery of Planck's quantum of action. Although not penned by Stoney, the derived unit of angular momentum ℓ_s is just $M_S L_S^2 T_S^{-1}$ and given by equation (B1). Whilst the FSC entered physics in 1916 by Sommerfeld, it could have emerged much earlier with the discovery of $\hbar$. *Image created by the author using public-domain source material.*

In this first system of natural units, the fundamental natural Stoney unit of angular momentum ℓ_S is given by,

$$\ell_S = [M_S][L_S]^2[T_S]^{-1} \tag{B1}$$

$$\ell_S = \left(\frac{e^2}{4\pi\epsilon_0 G}\right)^{\frac{1}{2}} \left(\frac{Ge^2}{4\pi\epsilon_0 c^4}\right) \left(\frac{4\pi\epsilon_0 c^6}{Ge^2}\right)^{1/2} = \frac{e^2}{4\pi\epsilon_0 c} \tag{B2}$$

$$\ell_S = \frac{e^2}{4\pi\varepsilon_0 c} = 7.69 \times 10^{-37}\ \mathrm{J\,s}. \tag{B35}$$

Once Planck uncovered $\hbar$, his quantum of action [41], denoting a universal lower bound on angular momentum ℓ_P where,

$$\ell_P = [M_P][L_P]^2[T_P]^{-1} \tag{B3}$$

$$\ell_P = \left(\frac{\hbar c}{G}\right)^{1/2} \left(\frac{\hbar G}{c^3}\right)^{1} \left(\frac{c^5}{\hbar G}\right)^{1/2} = \hbar \tag{B4}$$

$$\ell_P = 1.055 \times 10^{-34} Js \tag{B5}$$

the Stoney system became subsumed into the Planck scale. See Figure B2.

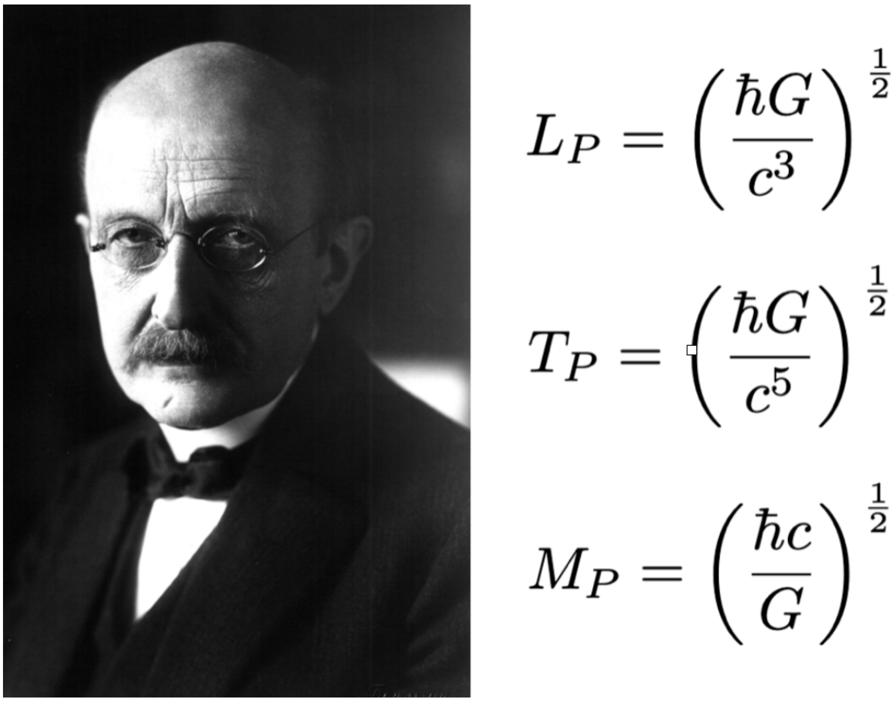

Figure B2. Max Planck's discovery of his quantum of action, $\hbar$ subsumed the existing Stoney system of natural units, into the Planck scale. Yet, the scale is the theoretical source of the catastrophic energy density of the vacuum (See Section 8); a universal expansion rate in which the universe should double in size every $10^{-44}s$ [4,33,38,52]. The observation of an accelerated expansion[1,2] and the realisation that $\Lambda \neq 0$, suggests such Planckian pathologies result from an incomplete quantum scale, without Λ, setting a fundamental entropic bound and limit in nature (See Section 2.2). Just as the emergence of the FSC, α_E reflected the emergence of Planck's new scale from the Stoney system, the dimensionless GFSC, α_Λ becomes its gravitational analogue, pointing to a more fundamental quantum scale in nature, the Λ-scale. *Image created by the author using public-domain source material.*

The ratio between the two angular momenta, both in units of kgm^2s^{-1} is simply the FSC given by,

$$\frac{\ell_S}{\ell_P} = \frac{\frac{e^2}{4\pi\epsilon_0 c}}{\hbar} = \frac{e^2}{4\pi\epsilon_0\hbar c} = \alpha_E \approx \frac{1}{137} \quad \text{(B6)}$$

Although the world of physics was introduced to the FSC in 1916 when Sommerfeld observed the fine splitting of hydrogen spectral lines [74], it could have in theory been introduced much earlier by Planck himself once his quantum of action was discovered. The FSC is therefore a ratio of two like dimensioned quantities - angular momentum in Stoney units and that of $\hbar$ itself, not some 'magical' relationship between several, seemingly unrelated constants [75,76]. We find the Λ-scale, see Figure B3, becomes its natural successor.

 doi:10.20944/preprints202510.2218.v1

$$L_\Lambda = \left(\frac{\hbar G}{\Lambda c^3}\right)^{\frac{1}{4}}$$

$$T_\Lambda = \left(\frac{\hbar G}{\Lambda c^7}\right)^{\frac{1}{4}}$$

$$M_\Lambda = \left(\frac{\hbar^3 \Lambda}{G c}\right)^{\frac{1}{4}}$$

Figure B3. The Lambda scale cannot emerge from dimensional analysis alone. An infinite number of possibilities represented by the affine family (A13) is the consequence. Whilst the Planck system maps three constants $G, \hbar, c$ on to three base dimensions [M],[L],[T], uniqueness comes for free. However, the requirement of mapping four constants $G, \hbar, c$ and Λ on to the same three base dimensions, leaves an extra degree of freedom (λ) in (A13). Physics fixes λ by the vacuum–matching postulate: the requirement that the resulting ZPE density, matches the vacuum energy from GR (See Eq. A14). The inclusion of Einstein's cosmological constant becomes a thermodynamic completion of his gravitational vision, represented in Box 10.1. Rather than a historical blunder, it is simultaneously a necessity and a constraint, an inevitable consequence of its thermodynamic foundation. *Image created by the author using public-domain source material.*

In GR, Λ has the dimensions of an inverse length squared or simply an inverse unit area in m^{-2}, thus,

$$[\Lambda] = [L]^{-2} \tag{B7}$$

In the Planck system, a unit inverse area is the inverse of the Planck length squared:

$$[L_P]^{-2} = \frac{1}{\frac{\hbar G}{c^3}} = \frac{c^3}{\hbar G} \tag{B8}$$

Thus the ratio of the two is also a comparison of like dimensions, both inverse unit areas, yields the GFSC α_Λ,

$$\frac{[L_P]^{-2}}{\Lambda} = \frac{c^3}{G\Lambda\hbar} = \alpha_\Lambda \approx 10^{123} \tag{B9}$$

In gravitational physics, the cosmological constant is simply interpreted as a vacuum energy density. However, thermodynamics suggests the energy of a system is tied to its entropy (E = TS), thus Λ is also a measure of entropy, that transforms the Bekenstein-Hawking bound into a fundamental upper limit of entropy for the Λ-universe.

$$S_{BH} \leq k_B \frac{c^3}{4G\hbar} A = k_B \frac{c^3}{4G\Lambda\hbar} = k_B \frac{\alpha_\Lambda}{4} \tag{B10}$$

Where α_Λ is just the cosmological constant in its dimensionless form. The consequences of Lambda's introduction into physics finds a historical parallel with the shift from Stoney to Planck units, once $\hbar$ was discovered [16]. If Λ is considered a fundamental constant of nature and is incorporated

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into physics, then a new 'natural scale' emerges - the Λ-scale. Like its Planck predecessor whose base and derived quantaties differ by factors of $\sqrt{\alpha_E}$ respectively from their Stoney counterparts, see Figure B4, the Lambda base and derived units differ from their Planck counterparts by $\sqrt[4]{\alpha_\Lambda}$, see Figure B5.

Just as α_E encodes a limit on action, α_Λ encodes a universal bound on gravitational entropy. The Lambda scale is thus not an arbitrary extension, but the natural successor to the Planck system — and the only one consistent with a finite vacuum entropy.

$$\frac{L_P}{L_S} = \frac{\sqrt{\frac{\hbar G}{c^3}}}{\sqrt{\frac{Ge^2}{4\pi\epsilon_0 c^4}}} = \sqrt{\frac{4\pi\epsilon_0\hbar c}{e^2}} = \frac{1}{\sqrt{\alpha_E}}$$

$$\frac{T_P}{T_S} = \frac{\sqrt{\frac{\hbar G}{c^5}}}{\sqrt{\frac{Ge^2}{4\pi\epsilon_0 c^6}}} = \sqrt{\frac{4\pi\epsilon_0\hbar c}{e^2}} = \frac{1}{\sqrt{\alpha_E}}$$

$$\frac{M_P}{M_S} = \frac{\sqrt{\frac{\hbar c}{G}}}{\sqrt{\frac{e^2}{4\pi\epsilon_0 G}}} = \sqrt{\frac{4\pi\epsilon_0\hbar c}{e^2}} = \frac{1}{\sqrt{\alpha_E}}$$

$$\alpha_E = \frac{e^2}{4\pi\epsilon_0\hbar c} \sim \frac{1}{137}$$

Figure B4. The Planck scale differs from their Stoney counterparts by factors of the square root of the FSC, $1/\sqrt{\alpha_E}$.

With c, $\hbar$, and now Λ, we arrive at a complete triad of fundamental constants — each defining a domain of limiting behavior: causality, action, and information capacity. A coherent system of natural units must reflect all three. The Lambda system grounded in thermodynamics and quantum field theory, achieves precisely this. See Figure B5. It not only resolves the vacuum catastrophe, but redefines the conditions under which all physical laws — including gravity — must operate (LoEC).

$$\frac{L_P}{L_\Lambda} = \frac{\sqrt{\frac{\hbar G}{c^3}}}{\sqrt[4]{\frac{\hbar G}{\Lambda c^3}}} = \sqrt[4]{\frac{G\Lambda\hbar}{c^3}} = \frac{1}{\sqrt[4]{\alpha_\Lambda}}$$

$$\frac{T_P}{T_\Lambda} = \frac{\sqrt{\frac{\hbar G}{c^5}}}{\sqrt[4]{\frac{\hbar G}{\Lambda c^7}}} = \sqrt[4]{\frac{c^3}{G\Lambda\hbar}} = \sqrt[4]{\alpha_\Lambda}$$

$$\frac{M_P}{M_\Lambda} = \frac{\sqrt{\frac{\hbar c}{G}}}{\sqrt{\frac{\hbar^3\Lambda}{cG}}} = \sqrt[4]{\frac{c^3}{G\Lambda\hbar}} = \sqrt[4]{\alpha_\Lambda}$$

$$\alpha_\Lambda = \frac{c^3}{G\Lambda\hbar} \sim 10^{123}$$

Figure B5. The Planck scale differs from the Λ-scale in an analogous manner to the Planck to Stoney ratios, but this time by factors involving the quartic root of the GFSC, $1/\sqrt[4]{\alpha_\Lambda}$.

 doi:10.20944/preprints202510.2218.v1

Historically, Stoney's units $\{c, G, e\}$ were subsumed once $\hbar$ was recognized as fundamental: adopting the Planck charge $q_P = \sqrt{4\pi\varepsilon_0 \hbar c}$ rewrites electromagnetism in terms of the dimensionless coupling $\alpha_E = e^2/4\pi\epsilon_0\hbar c = (e/q_P)^2$. Analogously, adding Λ furnishes an IR ingredient: vacuum matching selects the Λ length $L_\Lambda = \left(\hbar G/(\Lambda c^3)\right)^{1/4}$ and packages gravity with the dimensionless $\alpha_\Lambda = c^3/G\Lambda\hbar$. The Λ–framework thus extends Planck's by incorporating horizon thermodynamics.

In summary, once Λ is recognised as a fundamental constant fixing a finite upper bound on the universe's entropy [10,11], the Lambda scale emerges naturally as the successor to the Planck system. Dimensional analysis identifies L_Λ as the geometric mean of the Planck length L_P and the de Sitter radius $R_{\rm dS}$ [7,17]. This property, combined with the thermodynamic requirement that Λ be constant, makes L_Λ an inevitable new quantum length. Unlike the Planck scale, which overestimates the vacuum energy, the Lambda scale matches the observed value, aligning GR with thermodynamics and QFT, resolving the vacuum catastrophe [4].

Appendix C Vacuum energy from Planck's spectrum at $T = 0$ (consistency check)

Planck's spectral energy density including the zero–point term [60,77,78] is

$$u(\nu, T)\, d\nu = \frac{8\pi h\, \nu^3}{c^3}\left(\frac{1}{e^{h\nu/kT} - 1} + \frac{1}{2}\right) d\nu. \tag{C1}$$

At $T = 0$ the thermal part vanishes and the zero–point contribution remains:

$$u(\nu, 0)\, d\nu = \frac{4\pi h}{c^3}\, \nu^3\, d\nu. \tag{C2}$$

Integrating up to a sharp UV cutoff ν_c gives

$$u_{\rm vac}(\nu_c) = \int_0^{\nu_c} \frac{4\pi h}{c^3} \nu^3\, d\nu = \frac{\pi h}{c^3}\, \nu_c^4. \tag{C3}$$

Equivalent k–space/mode–counting form. Quantizing the electromagnetic field, each mode contributes $\frac{1}{2}\hbar\omega$ and the density of states per unit volume is $g(\omega)\, d\omega = \omega^2/(\pi^2 c^3)\, d\omega$. Then

$$u_{\rm vac}(\omega_c) = \int_0^{\omega_c} \frac{\hbar\omega}{2}\, g(\omega)\, d\omega = \frac{\hbar}{2\pi^2 c^3} \int_0^{\omega_c} \omega^3 d\omega = \frac{\hbar\, \omega_c^4}{8\pi^2 c^3} = \frac{\pi h}{c^3} \nu_c^4, \tag{C4}$$

since $\omega = 2\pi\nu$ and $h = 2\pi\hbar$.

Matching to GR at the Λ scale.

The GR vacuum energy density is

$$u_\Lambda = \frac{\Lambda c^4}{8\pi G}. \tag{C5}$$

Using the Λ–scale cutoff (fixed by horizon thermodynamics / vacuum matching; see Sec. 8 and Eq. A32)

$$\nu_c = \nu_\Lambda^* = \left(\frac{\Lambda c^7}{16\pi^3 \hbar G}\right)^{1/4} = \left(\frac{1}{16\pi^3}\right)^{1/4} \frac{1}{T_\Lambda}, \qquad T_\Lambda = \left(\frac{\hbar G}{\Lambda c^7}\right)^{1/4}. \tag{C6}$$

Substituting (C6) into (C3) yields

$$u_{\rm vac} = \frac{\pi h}{c^3}\left(\frac{\Lambda c^7}{16\pi^3 \hbar G}\right) = \frac{\pi(2\pi\hbar)}{c^3}\, \frac{\Lambda c^7}{16\pi^3 \hbar G} = \frac{1}{8\pi}\, \frac{\Lambda c^4}{G} = \boxed{\frac{\Lambda c^4}{8\pi G}}. \tag{C7}$$

Hence the zero–point integral at the Λ cutoff reproduces the GR vacuum density exactly. Thus both the thermodynamic (Section 2.2) and canonical routes (Section 8) agree.

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