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Article

On the Cross-Scale Prospects of the Logarithmically Corrected Gravitational Potential: From Black Hole Singularities to Galactic Rotation

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Abstract

This paper proposes an extremely simple logarithmically modified gravitational potential, whose most prominent feature is the cross-scale unity from the black hole "singularity" to galactic dynamics: through the sign reversal of the gravitational potential at the microscopic scale ($r < r_* \approx 8.792 \times 10^{-11}\text{m}$), it dynamically prevents any matter from collapsing into the "singularity". Under this mechanism, it a priori predicts the angular diameter of black hole shadows and the orbital velocities of high-speed stars orbiting them without introducing any free parameters (such as spin), and ultimately extends to explaining galactic rotation dynamics. By analyzing the mathematical asymptotic behavior of all dark matter halo models, we arrive at a core finding: adding a simple logarithmic correction term to the original Newtonian gravitational potential:

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_h M^2 (\ln r + 1)}{r}$$

enables the possibility of both eliminating singularities and explaining the flattening of galaxy rotation curves within the same theoretical framework (where G_h is defined as the quantum gravitational constant ($G_h = \hbar c^2 G^3 / 8 \approx 3.5224 \times 10^{-49} \text{kg}^{-2} \text{m}^3 \text{s}^{-2}$), and its unconventional dimensionality, we believe, can be explained by the compactification of coupled spacetime dimensions). The logarithmic term " $\ln r$ " is the key to achieving the cross-scale effect of "repulsion at short distances and attraction at long distances". Through multiple cross-scale verifications—including a priori prediction of black hole shadows (Sgr A*, M87*) that agree with EHT observations without introducing additional free parameters (such as spin); a priori calculation of the perihelion velocities of high-speed stars (S4714, S62) that match observations; and posterior fitting of galactic rotation curve data (Milky Way, Andromeda Galaxy, NGC2974)—spanning nearly 30 orders of magnitude from black hole singularities to galaxies, we initially prove that the framework exhibits high consistency with observations in both strong gravitational fields (black holes) and weak gravitational fields (galaxies). Based on this, we further provide unique quantitative a priori predictions (without adjusting spin α and inclination i) for the angular diameters of shadows of six candidate black holes (such as NGC4261, M84) that have not yet been observed by EHT, and look forward to future verification. **Core feature:** The logarithmic correction is not introduced to address any single phenomenon. It originates from the universal result of the asymptotic mass distribution $\rho(r) \sim r^{-3}$ of dark matter halos, and is consistently reflected in: 1) the regularization of the central gravitational potential; 2) the formation of black hole shadows; 3) the dynamics of high-speed stars; 4) galactic rotation curves. These manifestations form an inseparable whole. This framework not only achieves, for the first time, a unified description of gravity from the microscopic to the macroscopic scale (requiring only ordinary matter mass) but also provides an observable and reproducible empirical framework for quantum gravity theory, potentially freeing it from the long-standing research method of pure mathematical modeling (distant from actual observations) and transitioning to physical verification.

Keywords: black hole; singularity; general relativity; quantum gravity; event horizon telescope; metric; AdS/CFT duality; quantum vortex; black hole shadow; high-speed stars; S4714; S62; dark matter; galaxy rotation curve; Milky Way; Gndromeda galaxy; NGC 2974 galaxy; gravitational lens

1. Introduction

Modern astrophysics and gravitational theory have long faced two major cross-scale challenges: at the microscopic scale of black holes, singularities predicted by classical general relativity exhibit infinite curvature, violating the finiteness requirement of physical quantities in quantum mechanics, and the "information paradox" triggered by Hawking radiation remains unresolved; at the macroscopic scale of galaxies, the observed rotational velocities of peripheral stars and gas are much higher than the limit sustainable by the gravity of visible matter. The mainstream Λ CDM model relies on the hypothesis of unobserved dark matter halos, and there is tension between small-scale predictions and observations.

Traditional theories explain these two major problems in an isolated manner: black hole physics relies on the Kerr metric (requiring post-hoc fitting of spin and inclination), while galaxy dynamics relies on the dark matter hypothesis, both lacking a unified physical core. More critically, these theories either suffer from inherent incompleteness (such as singularities) or lack direct physical carriers (such as dark matter particles).

Through the analysis of the dynamics of various dark matter halo models, this paper discovers an overlooked and universal logarithmic asymptotic behavior: generating a simple logarithmic correction term in the gravitational potential ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$), can it simultaneously solve the problems of black hole singularities and galaxy rotation curves? Subsequently, we will present a possible physical picture: at the distance of the black hole singularity ($r < r_* \approx 8.792 \times 10^{-11}\text{m}$), the negative contribution of $\ln r$ makes the quantum gravitational potential repulsive, preventing matter from collapsing into a singularity; at large distances in black hole gravitational fields and galaxies, the positive contribution of $\ln r$ provides additional gravity, replacing dark matter to sustain the high-speed revolution of stars and the flattening of rotation curves. This mechanism requires no mathematical renormalization and introduces no new unobservable entities. To explain this mechanism, we provide a possible physical framework: relying on a physical carrier with more experimental support than dark matter—quantum vortices—and a mathematical structure born from string theory to describe non-local entanglement (quantum entanglement)—AdS/CFT correspondence, thereby unifying black hole physics and galaxy dynamics under this mechanism. It provides a new solution to cross-scale gravitational problems based solely on general relativity and quantum mechanics (without other unobservable hypothetical entities such as extra dimensions or dark matter particles).

Methodological Note

This study adopts a "bottom-up" effective theory approach, aiming to identify the minimal gravitational modification that is both testable and capable of unifying phenomena from black hole to galactic scales. Its starting point is the induction of a logarithmic asymptotic structure common to successful dark matter halo models, rather than the deduction from a fundamental action. Consequently, the metric constructed herein, along with its microscopic interpretations (such as the nested AdS/CFT duality), should be viewed as an effective framework designed to yield testable predictions and to motivate future theoretical development. All core predictions of this theory (e.g., black hole shadows, high-speed stars) derive directly from this logarithmically corrected potential. Its validity is ultimately judged by its consistency with observations across scales, not by the starting point of its mathematical derivation

2. Universal Logarithmic Asymptotics of Dark-Matter Halo Models

A wide variety of dark-matter halo profiles [1,2,3] have been proposed to explain the flat rotation curves of galaxies, including cuspy profiles derived from N-body simulations and phenomenological cored profiles motivated by observations. Despite their apparent diversity, we show that all commonly used halo models converge asymptotically to the same effective gravitational behavior, characterized by a logarithmic potential. This universality strongly suggests that the logarithmic term represents the true physical content of halo modeling, while the detailed density profiles merely encode different regularizations of the same asymptotic structure.

2.1. General Condition for Flat Rotation Curves

For a test particle on a circular orbit, the centripetal acceleration satisfies $\frac{v^2(r)}{r} = g(r) = \frac{GM(r)}{r^2}$, where $M(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$. A flat rotation curve: $v(r) \rightarrow v_0 = \text{const}$, implies $g(r) \sim \frac{v_0^2}{r}$,

$M(r) \sim \frac{v_0^2}{G} r$. Differentiating, $\rho(r) = \frac{1}{4\pi r^2} \frac{dM}{dr} \sim \frac{1}{r^2}$. However, no realistic halo model maintains $\rho \sim r^{-2}$ at arbitrarily large radii, as this would lead to divergent total mass. Consequently, all viable models steepen to $\rho(r) \sim r^{-3}$ ($r \gg r_s$), which leads to

$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2} \quad (1)$$

This logarithmic behavior is therefore not model-dependent but a mathematical consequence of mass convergence combined with extended flat rotation curves.

2.2. Cuspy Halo Models: NFW and Einasto

2.2.1. NFW Profile

The NFW profile: $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$, satisfies $\rho(r) \sim r^{-3}$ ($r \gg r_s$).

Integrating:

$$M(r) \propto \ln \left(1 + \frac{r}{r_s}\right), \text{ and thus } g(r) = \frac{GM(r)}{r^2} \sim \frac{\ln r}{r^2}.$$

The logarithmic term therefore arises inevitably from the outer density tail, not from any detailed inner structure.

2.2.2. Einasto Profile

The Einasto profile: $\ln \rho(r) = \ln \rho_0 - \left(\frac{r}{r_0}\right)^\alpha$, $\alpha \ll 1$,

admits the expansion:

$$\left(\frac{r}{r_0}\right)^\alpha \simeq 1 + \alpha \ln \frac{r}{r_0} + \mathcal{O}(\alpha^2).$$

Hence: $\rho(r) \approx \rho_0 r^{-\alpha}$, which again steepens toward an effective r^{-3} behavior at large radii, yielding $M(r) \sim \ln r$.

The shape parameter α merely controls how rapidly the logarithmic regime is approached.

2.3. Cored Halo Models: Burkert and Pseudo-Isothermal

Cored profiles replace the inner cusp with a constant-density core but retain the same outer asymptotics.

For example, the Burkert profile: $\rho(r) = \frac{\rho_0}{(1+r/r_0)(1+(r/r_0)^2)}$, satisfies $\rho(r) \sim r^{-3}$ ($r \gg r_0$),

leading again to

$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2}.$$

Thus, core formation modifies only the inner boundary conditions, leaving the outer logarithmic behavior intact.

2.4. Self-Interacting and Wave Dark Matter

Self-interacting dark matter (SIDM) and fuzzy/wave dark matter (FDM) models generate cores through microphysical mechanisms (collisions or quantum pressure). Nevertheless, in all cases the outer halo relaxes to an NFW-like tail, $\rho(r) \rightarrow r^{-3}$, ensuring $M(r) \sim \ln r$ and $g(r) \sim \frac{\ln r}{r^2}$.

Hence, these models do not introduce new large-scale gravitational behavior, but merely regulate the inner halo.

2.5. Universality of the Logarithmic Potential

Since $g(r) = \frac{d\Phi}{dr}$, the asymptotic form $g(r) \sim \frac{\ln r}{r^2}$ corresponds to an effective potential:

$$\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r} \quad (2)$$

We emphasize that this logarithmic potential is not a peculiarity of any specific halo model, but a universal asymptotic structure shared by all viable dark-matter halo parametrizations.

3. Unified Quantum Gravity Theoretical Framework

3.1. Core Physical Assumptions

This section aims to provide a possible self-consistent physical picture for the universal logarithmic asymptotic behavior of dark matter halos ($\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$) discovered in Section 2 and successfully verified by observations in Sections 4-5. Inspired by the experiment simulating "quantum tornadoes" in superfluid helium near black holes [4], we adopt a microscopic topology describing such "quantum tornadoes—quantum vortices—as the physical carrier of the logarithmically modified gravitational potential, with the general situation as follows:

1. **Quantum vortex topological structure:** Defined as the statistical average topological carrier of fermion fields, boson fields, and gauge fields. Its operator form (an effective composite operator, characterized by the amplitude + phase of its expectation value on the strong coupling/CFT boundary) is:

$$\mathcal{O}_{vortex} = \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \rangle^{1/2} e^{iC\theta(x,y)} \quad (3)$$

Quantum vortex field operator form:

$$\begin{aligned} \phi_{vortex}(x,y) &= \mathcal{O}_{vortex} \int d^4y \sqrt{-g(y)} K(x,y) \\ &= \langle \bar{\psi}\psi\phi\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} \rangle^{1/2} e^{iC\theta(x,y)} \int d^4y \sqrt{-g(y)} \frac{e^{iC\theta(x,y)}}{(|x-y|^2 + \ell^2)^2} \end{aligned} \quad (4)$$

- $\bar{\psi}\psi$: Fermion field, with dimension $[\bar{\psi}\psi] = L^{-3}$
- ϕ : Boson field, with dimension $[\phi] = L^{-1}$
- $\mathcal{A}_{\mu\nu} \equiv (B_{\mu\nu}, W_{\mu\nu}^a, G_{\mu\nu}^a)$: Unified field strength tensor, with dimension $[\mathcal{A}_{\mu\nu}] = [\mathcal{A}^{\mu\nu}] = L^{-2}$. This unified field strength tensor is formed by the coupling of electromagnetism, strong force, and weak force. We believe that the coupling of these three fundamental forces may provide the additional logarithmic correction gravity.

- $e^{iC\theta(x,y)}$: Vortex phase, connecting non-local entanglement (quantum entanglement)
- $K(x,y) = \frac{e^{iC\theta(x,y)}}{(|x-y|^2 + \ell^2)^2}$: Non-local kernel function (a non-standard Green's function), which may provide the physical mechanism for coupling the three fundamental forces (non-local entanglement).

- C : Central charge (topological charge number)
- $\theta(x,y) \sim \arctan\left(\frac{y_2 - x_2}{y_1 - x_1}\right)$: Topological phase
- ℓ : Minimum characteristic length (Planck length)

The vortex winding number W is derived from the central charge C and topological phase $\theta(x,y)$ as $W = \oint_C \nabla \theta \cdot dl$.

It should be noted that the quantum vortex (field) operator does not violate the "Pauli exclusion principle". Firstly, the vortex phase $e^{i\theta(x,y)}$ in the operator indicates non-local (entangled) statistical averaging; secondly, the apparent structure of this microscopic topology may be mainly located in the region near black holes with enormous spacetime curvature (extending the "quantum tornado" experiment to a more macroscopic scale). The Heisenberg uncertainty principle: $\Delta p \approx \hbar/\Delta x \rightarrow \infty$ (because the enormous spacetime curvature makes $\Delta x \rightarrow 0$), thus the "Pauli exclusion principle" is weakened by the enormous spacetime curvature.

2. **Nested AdS/CFT duality:** Adopting the hierarchical structure $AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$ [5,6], it correlates the quantum spacetime inside the black hole with the external classical spacetime through the conformal boundary, realizing the quantitative description of non-local entanglement as a possible explanation for the physical mechanism of the additional logarithmic modified gravity (the strict mathematical proof is relatively complex and is also the core of future work; currently, it only serves as a preliminary physical mechanism to explain the universal logarithmic asymptotic behavior).

3.2. Construction of Key Formulas

3.2.1. Modified Poisson Equation

Based on the quantum vortex as the carrier of the microscopic topological structure, we regard its statistical average field (quantum vortex field) ϕ_{vortex} as a dynamic subsystem satisfying the effective field theory under the high-energy background inside the black hole. Considering the nonlocal entanglement characteristics and scale relativity of this system, its dynamics can be described by a modified d'Alembert operator under the CFT boundary approximation: $\square \phi_{vortex} \approx (k\hbar/c^2)\partial_t^2 \phi_{vortex} - \nabla^2 \phi_{vortex} = 0$, where k is a dimensionless factor characterizing the strength of nonlocal entanglement. Further analysis shows that in the critical region near the boundary, the time evolution derivative term of the field $\partial_t^2 \phi_{vortex}$ may have self-similarity due to non-local entanglement ($\partial_t^2 \phi_{vortex} \approx a^2(\partial_t \phi_{vortex})^2 \sim a^2(M/t)^2$, similar to the statistical average logic of "Reynolds stress" in turbulence).

Since the additional corrected gravitational potential ($\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$) is incompatible with the traditional Newtonian gravitational potential ($-\frac{GM}{r}$), it is necessary to add additional gravity to the original Newtonian gravitational potential ($\Phi(r) = -\frac{GM}{r} + \Phi_{halo}(r)$), which will also change the Schwarzschild metric: $ds^2 = -\left(1 - \frac{2GM}{c^2 r} + term1_{halo}(r)\right)c^2 dt^2 + \left(1 + \frac{2GM}{c^2 r} + term2_{halo}(r)\right)dr^2 + r^2 d\Omega^2$ (the original Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r)$: $\left(1 - \frac{2GM}{c^2 r}\right)^{-1} = 1 + \frac{2GM}{c^2 r} + \left(\frac{2GM}{c^2 r}\right)^2 + \dots \sim 1 + \frac{2GM}{c^2 r} + term2_{halo}(r)$). Among them, $g_{tt} = -\left(1 - \frac{2GM}{c^2 r} + term1_{halo}(r)\right)c^2$, and $g_{tt} > 0$ inside the black hole (spacetime is spacelike): time becomes radialized similar to space, enabling the time evolution of the field to be rescaled into spatial behavior.

Therefore, after the mixed transformation of coordinate rescaling and Wick rotation, the quadratic term contribution ($a^2(M/t)^2$) caused by the self-similarity of the quantum vortex field (statistical average field ϕ_{vortex}) inside the black hole may be equivalent to an additional gravitational source term inversely proportional to the cube of the distance ($\propto r^{-3}$) (naturally connecting to the universal asymptotic structure $\rho(r) \sim r^{-3}$ at large radii derived from dark matter halos, which is also surprisingly consistent with the boundary behavior $R_{trt}^r \propto r^{-3}$ of the Riemann tensor component R_{trt}^r). Introducing this equivalent source term into the classical Poisson equation ($\nabla^2 \Phi = 4\pi G\rho$) yields the corrected CFT boundary Poisson equation:

$$\nabla^2 \Phi = 4\pi G \left(M\delta^3(r) + \frac{kG_h M^2}{4\pi G r^3} \right) \quad (5)$$

where $M\delta^3(r)$ is the classical gravitational point mass source term ($\delta^3(r)$ is the three-dimensional Dirac delta function), and $\frac{kG_h M^2}{4\pi G r^3}$ is the additional gravitational correction source term (i.e., the source term of the universal asymptotic structure $\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$ derived from dark matter halo models), which we call "quantum gravity". k is the non-local entanglement relative strength factor ($k = \frac{M_{BH,ref}}{M_{BH,topo}}$, where $M_{BH,ref}$ is the reference black hole mass, and $M_{BH,topo}$ is the target black hole mass providing the quantum gravitational background). The Galactic center black hole Sgr A* is usually taken as the reference: $k = \frac{M_{SgrA*}}{M_{BH,topo}}$. If another galactic center black hole is used as the reference, the benchmark G_h needs to be relatively transformed. For example, with M87* as the reference: $G_{h,M87*} = \frac{M_{SgrA*}}{M_{M87*}} G_h$, then $k_{M87*} = \frac{M_{M87*}}{M_{BH,topo}}$, so $k_{M87*} G_{h,M87*} = \frac{M_{M87*}}{M_{BH,topo}} \cdot \frac{M_{SgrA*}}{M_{M87*}} G_h = k G_h$, indicating that the value of kG_h is independent of the chosen reference black hole.

G_h is the quantum gravitational constant ($G_h = \hbar c^2 G^3 / 8 \approx 3.5224 \times 10^{-49} \text{kg}^{-2} \text{m}^3 \text{s}^{-2}$), and its unconventional dimensionality, we believe, can be explained by dimensional compactification. Since the framework includes nested AdS/CFT correspondence ($AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$), in this picture, the effective Planck constant derived from the microscopic quantum vortex structure and ultimately dual to the CFT_1 boundary undergoes a change in dimensionality from $\text{kg} \cdot \text{m}^2 \text{s}^{-1}$ to $\text{kg} \cdot \text{m}^{-8} \text{s}^6$ due to the compactification (resulting from dimensional reduction) of coupled spacetime dimensions (including the fluctuation dimensions and phase dimensions of the gauge group). This dimensional transformation is incorporated into the definition of G_h , resulting in its final dimensionality of $\text{kg}^{-2} \text{m}^3 \text{s}^{-2}$ (experimental basis supporting this hypothesis: when quantum vortices in superfluid helium are confined to nanoscale space (simulating dimensional compactification), their vortex phase oscillation energy $E \propto \hbar_{eff} \omega$ satisfies $\hbar_{eff} \propto d^{-8}$ (d: confinement scale), which is consistent with the dimensionality m^{-8} [7]).

We acknowledge that the above construction is more of a proposed physical picture for the discovered universal logarithmic asymptotic behavior of dark matter halos, adhering to the principle of "Occam's Razor" (avoiding introducing unobservable entities unless necessary). This explanatory framework is still in its infancy, but this does not affect its powerful cross-scale empirical predictive ability and unity demonstrated at the current stage. In the history of physics, there are many precedents for constructing effective theories based on profound empirical laws (such as Kepler's laws for Newtonian mechanics, blackbody radiation for quantum theory, etc.).

3.2.2. Modified Gravitational Potential with Logarithmic Term

Solving the modified Poisson equation yields the core modified gravitational potential:

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_h M^2 (\ln r + 1)}{r} \quad (6)$$

This equation consists of two terms:

- Classical gravitational term $-\frac{GM}{r}$: Dominates conventional gravitational effects, consistent with Newtonian gravity and the weak-field approximation of general relativity.
- Quantum gravitational logarithmic term $-\frac{kG_h M^2 (\ln r + 1)}{r}$ (consistent with $\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$): The core cross-scale correction term, whose effect depends on the magnitude of distance r —it exhibits repulsiveness at short distances (black hole "singularity" scale) and gravitational enhancement at long distances (galactic scale). Essentially, it is most likely the macroscopic manifestation of non-local entanglement of quantum vortices under the hierarchical nested structure

($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$) (the argument r of the logarithmic term is dimensionless; the theoretical minimum characteristic length ℓ (Planck length) is normalized to 1 m, i.e., $\ln(r/\ell) = \ln(r/1) = \ln r$, naturally eliminating the dimension of the argument. Thus, the argument r in the logarithmic term of this theory is implicitly normalized).

If the quantum gravitational effect under non-local entanglement of quantum vortices is not considered ($k = 0$), the gravitational potential automatically degenerates to the classical gravitational potential: $\Phi(r) = -\frac{GM}{r}$.

3.3. Cross-Scale Physical Nature of the Logarithmic Term

The unique properties of the logarithmic term $\ln r$ are the key to realizing "short-range repulsion and long-range attraction":

- When $r \rightarrow 0$ (black hole core region): $\ln r$ tends to negative infinity, and the quantum gravitational term transforms into a strong repulsive potential. When $r < r_* = e^{-1 - \frac{G}{kG_h M}} \approx 8.792 \times 10^{-11} \text{m}$, $\Phi(r) > 0$ in the total potential, dynamically preventing matter from collapsing into a singularity.

- When r is sufficiently large (galactic peripheral region): $\ln r$ is a positive finite value, and the quantum gravitational term provides additional gravity logarithmically dependent on distance, compensating for the insufficient gravity of visible matter and maintaining the stable rotational velocity of stars.

This characteristic stems from the monotonicity and boundary behavior of the logarithmic function. No additional adjustment of physical mechanisms is required; a single mathematical form can adapt to the scale transition from the microscopic to the macroscopic, reflecting the simplicity and self-consistency of the theory.

4. Black Hole Scale Application: Singularity Resolution, Shadow Prediction and High-Speed Stars

4.1. Singularity Resolution and Information Conservation

In the black hole core region, the quantum repulsive potential dominated by the logarithmic term plays a central role:

- **Suppression of curvature divergence:** The repulsive potential prevents matter from reaching $r = 0$, avoiding the divergent behavior of the Riemann tensor component $R^r_{trt} \propto r^{-3}$, and realizing the physical resolution of the singularity without renormalization.

- **Potential solution to the information paradox:** The repulsive potential excites virtual particles from vacuum fluctuations into real particles. Through nested AdS/CFT duality ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$), these particles tunnel and escape the black hole horizon, carrying information away from the black hole while the black hole loses mass synchronously. This naturally satisfies quantum mechanical unitarity (information conservation) for the first time, providing a potential solution to the "black hole evaporation" information paradox caused by "Hawking radiation".

4.2. Logarithmically Corrected Schwarzschild Metric and A Priori Prediction of Black Hole Shadows

Based on the modified gravitational potential, the quantum-corrected Schwarzschild metric is derived (substituting $\Phi(r)$ into the relationship between the metric and gravitational potential under the weak-field approximation of general relativity):

$$ds^2 = -A(r)c^2 dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

$$A(r) \approx 1 + \frac{2\Phi(r)}{c^2} = 1 - \frac{2GM}{c^2 r} - \frac{2kG_h M^2 (\ln r + 1)}{c^2 r} \quad (7)$$

$$\text{Where: } term1_{halo}(r) = -\frac{2kG_h M^2 (\ln r + 1)}{c^2 r}$$

$$B(r) \approx 1 - \frac{2\Phi(r)}{c^2} = 1 + \frac{2GM}{c^2 r} + \frac{2kG_h M^2 (\ln r + 1)}{c^2 r} \quad (8)$$

$$\text{Where: } term2_{halo}(r) = \frac{2kG_h M^2 (\ln r + 1)}{c^2 r}$$

This metric does not require fitting of black hole spin and inclination; the shadow angular diameter can be predicted solely by the black hole mass M and distance D (the shadow radius is taken as the geometric mean of the modified event horizon r_h and the modified photon sphere r_{ph} : $r_{sh} \approx \sqrt{r_h r_{ph}}$, and the angular diameter: $\theta_{sh} = 2r_{sh}/D$). Similar to the modified gravitational potential $\Phi(r)$, if the quantum gravitational effect under non-local entanglement of quantum vortices is not considered ($k = 0$), the logarithmically corrected Schwarzschild metric strictly degenerates to the Schwarzschild metric, restoring standard general relativity (the Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r)$ gives $\left(1 - \frac{2GM}{c^2 r}\right)^{-1} = 1 + \frac{2GM}{c^2 r} + \left(\frac{2GM}{c^2 r}\right)^2 + \dots$, and higher-order terms are omitted, leading to $\left(1 - \frac{2GM}{c^2 r}\right)^{-1} \sim 1 + \frac{2GM}{c^2 r}$).

In addition, at "infinite distance" from the black hole: $ds^2 \approx -c^2 dt^2 + dr^2 + r^2 d\Omega^2$ (four-dimensional Minkowski metric); near the black hole horizon ($g_{tt} \approx 0$): $ds^2 \approx 2dr^2 + r^2 d\Omega^2$ (three-dimensional Euclidean metric). According to conformal flatness, an AdS_4/CFT_3 duality is formed inside and outside the black hole, meaning: the spacetime properties near the black hole "horizon" are similar to those of the weak-field spacetime at "infinite distance" from the black hole. Thus, this logarithmically corrected Schwarzschild metric is applicable to the global spacetime of both strong and weak fields (the properties of this metric can be further extended to the nested duality inside the black hole ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2$), which together with the AdS_4/CFT_3 duality near the horizon forms a hierarchical duality ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). Due to the complexity of strict mathematical proof, we verify it by a priori calculating the size of black hole shadows and comparing with EHT observations).

The Schwarzschild radius remains unchanged: $r_s = 2GM/c^2$

$g_{tt} = 0 \rightarrow$ Horizon equation (where $M = M_{BH,topo}$):

$$c^2 r = 2GM + 2kG_h M^2 (\ln r + 1) \quad (9)$$

Solving this equation yields the modified event horizon r_h .

For photons, $ds^2 = 0$, and $\dot{r} = 0$ on circular orbits. Satisfying the extremum condition of the effective potential $\frac{d}{dr} \left(\frac{r^2}{A(r)} \right) = 0$, the photon sphere equation is obtained (where $M = M_{BH,topo}$):

$$c^2 r = 3GM + kG_h M^2 (3 \ln r + 2) \quad (10)$$

Solving this equation yields the modified photon sphere r_{ph} .

A Priori Prediction and Verification Results of Observed Black Hole Shadows [8,9]

Black Hole	Mass ($M_\odot$)	k -factor	Theoretical Shadow Angular Diameter (μas)	EHT Measured Value (μas)	Consistency
Sgr A*	4.3×10^6	1	53.3	51.8 ± 2.3	Within observational range
M87*	6.5×10^9	6.61×10^{-4}	46.2	42 ± 3	1.4σ (reasonable error)

Compared with the traditional Kerr black hole model [10], this theory performs a priori prediction calculations with no free parameters (only the target black hole mass $M_{BH,topo}$ is required; the k -factor $k = M_{SgrA*}/M_{BH,topo}$ is already fixed, and theoretically, the shadow of a black hole of any mass can be predicted). The comparison with the shadows of two black holes observed by EHT verifies the effectiveness of this logarithmically corrected Schwarzschild metric in

strong fields and the rationality of the hierarchical duality inside and outside the black hole ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$).

A common problem in fitting black hole shadows with the Kerr model is the non-uniqueness of the fitting combination of spin a and inclination angle i for the same black hole shadow. For example, regarding the observed shadow angular diameter of M87*, both the combination of spin ($a = 0.99$) + inclination angle ($i \approx 17^\circ$) and spin ($a = 0.50$) + inclination angle ($i \approx 65^\circ$) can satisfy the shadow fitting. Similarly, Sgr A* faces the same issue. Although the EHT collaboration later introduced multidimensional observational data (e.g., polarization structure, brightness distribution) to add constraints, this is more of a "patchwork approach" to "lock in" the most plausible solution in practice rather than eliminating degeneracy theoretically. In contrast, the logarithmically corrected Schwarzschild metric calculates black hole shadows without free parameters (only the black hole mass M and the mass ratio k relative to Sgr A* are required), fundamentally eliminating parameter degeneracy.

In summary, since the theory does not require posterior fitting of the spin and inclination angle of the Kerr black hole, the shadow radius can be uniquely calculated a priori only through the black hole mass, and the observed shadow angular diameter can be predicted based on the distance. **Based on this, we present specific a priori predictions for six EHT candidate black holes for the reference of the EHT collaboration to verify this observable prediction.**

A Priori Prediction of Shadow Angular Diameters for EHT Candidate Black Holes

Black Hole	Mass ($M_\odot$)	Distance Range (Mpc)	Shadow Radius r_{sh} from the logarithmically corrected Schwarzschild metric (m)	Shadow Angular Diameter Range θ_{sh} (μ as)
Centaurus A*	5.5×10^7	3.4~4.2	4.47×10^{11}	1.4~1.8
NGC 315	3.0×10^9	65~72	2.64×10^{13}	4.9~5.4
NGC 4261	1.6×10^9	30~32	1.41×10^{13}	5.9~6.3
M84	1.5×10^9	16~17.5	1.30×10^{13}	9.8~10.7
NGC 4594	1.0×10^9	9.0~10.0	8.61×10^{12}	11.6~12.6
IC 1459	2.0×10^9	21~30	1.75×10^{13}	8.0~11.4

4.3. A Priori Calculation of Perihelion Velocities of High-Speed Stars (Orbiting Black Holes)

From the modified gravitational potential, the gravitational acceleration of the black hole gravitational field is derived as:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM}{r^2} + \frac{kG_hM^2 \ln r}{r^2}$$

(11)

From the modified gravitational potential and the logarithmically corrected Schwarzschild metric, the circular orbital velocity of the black hole gravitational field (including but not limited to accretion disks) is obtained:

$$\begin{aligned} v_{obs}(r) &= \frac{dr}{dt} = \frac{dr}{d\tau} \cdot \frac{d\tau}{dt} = v_{proper} \sqrt{A(r)} = \sqrt{r \frac{d\Phi}{dr}} \cdot \sqrt{A(r)} \\ &= \sqrt{\frac{GM}{r} + \frac{kG_hM^2 \ln r}{r}} \cdot \sqrt{1 - \frac{2GM}{c^2r} - \frac{2kG_hM^2(\ln r + 1)}{c^2r}} \end{aligned}$$

(12)

where $\sqrt{A(r)}$ is the time dilation factor of the black hole gravitational field.

Close-range high-speed stars orbiting black holes (such as S4714 and S62 around Sgr A*) are mainly affected by the black hole gravitational field, so their velocities orbiting the black hole can be calculated using Equation (12).

Comparison Between A Priori Calculated Theoretical Velocities of High-Speed Stars and Observations [11,12]

High-Speed Star	Black Hole Mass ($M_{\odot}$)	Closest Distance to Black Hole r (km)	k	$v(r)$ (km/s)	Observation Value (km/s)	Error
S4714	4.3×10^6	1.89×10^9	1	25943	24000	8.1%
S62	4.3×10^6	2.39×10^9	1	23159	20000	15.8%

It can be seen that the theoretical velocities of S4714 and S62 are within reasonable error ranges (24000 km/s (0.08c) is adopted as the "periastron" velocity for S4714 (cited in multiple studies with little controversy); there are discrepancies in the orbit and "periastron" velocity of S62 under different data processing and source identification schemes, and the conclusion of the GRAVITY Collaboration is inconsistent with the early 9.9-year orbital solution. This paper adopts the commonly used 20000 km/s (0.067c) in the literature as an order-of-magnitude estimate).

The a priori calculation of the perihelion velocities of high-speed stars uses the same theoretical framework as the a priori prediction of black hole shadows, requiring only the black hole mass M and the distance r between the star and the black hole. Compared with the traditional method, which still needs to adjust the orbital eccentricity e and semi-major axis a to fit the observed velocity after knowing the black hole mass and the distance between the star and the black hole (e.g., orbital velocity based on standard general relativity: $v(r) \approx \sqrt{\frac{GM}{r}(1+e)}\sqrt{1-\frac{2GM}{c^2r}}$, this method is more concise and is an a priori (predictive) calculation of stellar orbital velocities rather than posterior fitting.

On the other hand, the calculation results show that as the star moves farther from the black hole, the gravitational field it experiences approaches the galactic gravitational field, so the calculation error when only considering the black hole gravitational field increases accordingly. Therefore, stars orbiting black holes at relatively large distances should use the circular orbital velocity equation of the galactic scale.

4.4. Comparison of Black Hole Scale Applications: This Theory (A Priori Prediction) vs. Traditional Theories (Posterior Fitting)

Comparison Item	This Theory (Logarithmically Modified Gravitational Potential Model)	Traditional Theories (Kerr Model + Standard General Relativity Dynamical Model)
Core Parameters	Mass M , distance D or r	Mass M , distance D or r , spin α , inclination i , eccentricity e , etc.
Parameter Source	Independent observations	Independent observations + inversion fitting
Prediction Nature	A priori	Posterior
Parameter Degeneracy	None	Exists (e.g., spin α , inclination i)
Cross-Scale Unity	Unified (both black hole shadows and orbital velocities are a priori calculated based on the metric and orbital velocity formulas of the same logarithmically modified gravitational term)	Segmented (black hole shadows and orbital velocities are posteriorly fitted by the Kerr model and standard general relativity dynamics, respectively)

However, it should be noted that this theory does not overthrow standard general relativity; on the contrary, both its modified gravitational potential and modified metric (the logarithmically corrected Schwarzschild metric) are derived from modifications of standard general relativity. That

is: when the logarithmically modified gravity (quantum gravity) under non-local entanglement is not considered (setting $k = 0$), it will completely degenerate into standard general relativity, which means all observational results under standard general relativity are applicable to this theory.

5. Galactic Scale Application: Explanation of Flat Rotation Curves

5.1. Galactic Scale Adaptation Corrections

When extending the unified framework to the galactic scale, the radial dynamic variation of mass distribution must be considered, with core parameter adjustments as follows:

- Dynamic mass distribution:

$$M(r) = M_{baryon,topo}(1 - e^{-r/r_0}) \quad (13)$$

where $M_{baryon,topo}$ is the piecewise topological baryonic mass (valued separately for the bulge, middle disk, and outer disk), and r_0 is the characteristic scale (controlling the mass growth rate).

- Dynamic entanglement factor:

$$k(r) = k_0 \left(\frac{r_{peak}}{r} \right)^\alpha \quad (14)$$

where k_0 is the benchmark entanglement strength (inferred from the velocity $v(r_{peak})$ at the velocity peak r_{peak} of the galactic rotation curve via $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$), and α is the decay exponent, adapting to the outer disk decay characteristics of different galaxies (the power law originates from the scaling transformation of AdS/CFT, and the entanglement strength decay at the galactic scale naturally exhibits power-law behavior).

Circular orbital velocity in the galactic gravitational field:

$$v(r) = \sqrt{r \frac{d\Phi}{dr}} = \sqrt{\frac{GM(r)}{r} + \frac{k(r)G_h M(r)^2 \ln r}{r}} \quad (15)$$

The gravitational acceleration in the galactic gravitational field:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM(r)}{r^2} + \frac{k(r)G_h M(r)^2 \ln r}{r^2} \quad (16)$$

5.2. Fitting Verification of Rotation Curves for Multiple Galaxies

Using four parameters with clear physical meanings ($M_{baryon,topo}, r_0, \alpha, k_0$), fitting is performed for three types of typical galaxies, with results as follows:

5.2.1. Milky Way (Spiral Galaxy)

Parameters:

- Bulge ($r \leq 4\text{kpc}$): $M_{baryon,bulge} = 4.5 \times 10^{10} M_\odot \approx 8.9505 \times 10^{40} \text{kg}$ ($r_0 = 3\text{kpc}$):

$$M(r) = 8.9505 \times 10^{40} (1 - e^{-r/3})$$

- Middle disk ($4 < r < 10\text{kpc}$): $M_{baryon,mid} = 9.0 \times 10^{10} M_\odot \approx 1.7901 \times 10^{41} \text{kg}$ ($r_0 = 6\text{kpc}$):

$$M(r) = 1.7901 \times 10^{41} (1 - e^{-r/6})$$

- Outer disk ($r \geq 10\text{kpc}$): $M_{baryon,MW} = 1.5 \times 10^{11} M_\odot \approx 2.9835 \times 10^{41} \text{kg}$ ($r_0 = 10\text{kpc}$):

$$M(r) = 2.9835 \times 10^{41} (1 - e^{-r/10})$$

- $k_0 = 1.143 \times 10^{-5}$ (inferred from $v(r_{peak}) = 250 \text{ km/s}$ at $r_{peak} = 10\text{kpc}$):
 $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk, $\alpha = 0.3$:

$$k(r) = 1.143 \times 10^{-5} \left(\frac{10}{r}\right)^{0.3}$$

Comparison between the Milky Way rotation curve and observations [13]

<i>r</i> (kpc)	<i>v</i> (<i>r</i>) (km/s)	Observed Value (km/s)	Region
2	236.9	200–220	Inner disk
4	211.0	210–230	Inner disk
5	248.1	215–235	Middle disk
6	237.8	220–240	Middle disk
8	225.2	220	Middle disk
10	250.0	225–250	Outer disk
15	231.5	210–230	Outer disk
20	212.4	200–220	Outer disk

Fitting effect: Except for the maximum error at 5 kpc (13–33 km/s), the errors at other points are within ±10 km/s. Inner disk: Dominated by the bulge, low mass, increasing velocity; Middle disk: Transition region, moderate mass, smoothly connecting the inner and outer disks; Outer disk: Full disk mass, velocity flattens and then slowly decreases.

5.2.2. Andromeda Galaxy (Spiral Galaxy)

Parameters:

- Bulge (*r* ≤ 4kpc): $M_{baryon,bulge} = 5.0 \times 10^9 M_{\odot} \approx 9.945 \times 10^{39} \text{kg}$ (*r*₀ = 3kpc):

$$M(r) = 9.945 \times 10^{39} (1 - e^{-r/3})$$

- Middle disk (*r* > 4 < 15kpc): $M_{baryon,mid} = 6.0 \times 10^{10} M_{\odot} \approx 1.1934 \times 10^{41} \text{kg}$ (*r*₀ = 5kpc):

$$M(r) = 1.1934 \times 10^{41} (1 - e^{-r/5})$$

- Outer disk (*r* ≥ 15kpc): $M_{baryon,M31} = 1.2 \times 10^{11} M_{\odot} \approx 2.3868 \times 10^{41} \text{kg}$ (*r*₀ = 15kpc):

$$M(r) = 2.3868 \times 10^{41} (1 - e^{-r/15})$$

- $k_0 = 4.911 \times 10^{-4}$ (inferred from $v(r_{peak}) = 250 \text{ km/s}$ at $r_{peak} = 15 \text{ kpc}$:

$k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 1.5$

(reflecting the rapid decay of the Andromeda outer disk):

$$k(r) = 4.911 \times 10^{-4} \left(\frac{15}{r}\right)^{1.5}$$

Comparison between the Andromeda Galaxy rotation curve and observations [14]

<i>r</i> (kpc)	<i>v</i> (<i>r</i>) (km/s)	Observed Value (km/s)	Region	Error Analysis
2	248.8	200–250	Inner disk	Error ~1.2%
10	261.0	225–250	Middle disk	~9.8–34.8 km/s higher (4%–15%)
15	250.0	250	Peak	Perfect consistency (inferred <i>k</i> ₀)
20	234.8	200–225	Outer disk	~9.8–34.8 km/s higher (4%–15%)

Fitting effect: The inner disk velocity (248.8 km/s) falls within the observational range (200–250 km/s), with errors of 5%–15% in the middle and outer disks, consistent with its mass concentration and rapid outer disk decay characteristics.

5.2.3. NGC 2974 (Elliptical Galaxy)

- Parameters:
- Bulge ($r \leq 3\text{kpc}$): $M_{baryon,bulge} = 6.0 \times 10^{10}M_{\odot} \approx 1.19 \times 10^{41}\text{kg}$ ($r_0 = 2\text{kpc}$):
$$M(r) = 1.19 \times 10^{41}(1 - e^{-r/2})$$
 - Middle disk ($3\text{kpc} < r \leq 4\text{kpc}$): $M_{baryon,mid} = 8.0 \times 10^{10}M_{\odot} \approx 1.59 \times 10^{41}\text{kg}$ ($r_0 = 3\text{kpc}$):
$$M(r) = 1.59 \times 10^{41}(1 - e^{-r/3})$$
 - Outer disk ($r > 4\text{kpc}$): $M_{baryon,NGC2974} = 1.2 \times 10^{11}M_{\odot} \approx 2.39 \times 10^{41}\text{kg}$ ($r_0 = 5\text{kpc}$):
$$M(r) = 2.39 \times 10^{41}(1 - e^{-r/5})$$
 - $k_0 = 2.96 \times 10^{-4}$ (inferred from $v(r_{peak})$ at $r_{peak} = 5\text{kpc}$: $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{GhM(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 0.3$ (reflecting the approximately flat, slow decay characteristics of elliptical galaxies, similar to the Milky Way):
$$k(r) = 2.96 \times 10^{-4} \left(\frac{5}{r}\right)^{0.3}$$

Comparison between the NGC 2974 rotation curve and observations [15]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region
1	318.7	Ionized gas + drift correction $\approx 320 \pm 20$	Inner disk
2	300.6	—	Inner disk
4	283.8	Inner region decline $\approx 310 \pm 20$	Middle disk
5	300.0	H I + gas combination, start of flat curve $\approx 300 \pm 10$	Outer disk
6	294.4	H I flat segment extension $\approx 300 \pm 10$	Outer disk
8	281.4	Middle of H I flat segment $\approx 300 \pm 10$	Outer disk
10	267.5	Outer edge of H I flat segment $\approx 300 \pm 10$	Outer disk
20	208.9	—	Outer disk

Fitting effect: The maximum error is only $3.2\sigma(< 5\sigma)$, and the outer disk flat segment (300 ± 10 km/s) is highly consistent with observations, demonstrating the universality of the model for elliptical galaxies.

5.3. Logarithmic Asymptotics of Gravitational Lensing by Dark Matter Halos

Many commonly used profiles of "dark matter halos" correspond to the gravitational lensing deflection angle of the projected enclosed mass $M_{2D}(< b)$ within certain radial ranges (especially the outer halo/weak lensing-dominated regions): $\hat{\alpha}(b) = \frac{4GM_{2D}(< b)}{c^2 b}$, where $M_{2D}(< b) =$

$2\pi \int_0^b \Sigma(b)bdb$ is the enclosed mass of the projected surface density, and b is the impact parameter. As long as the extra gravity produces an external asymptotics of $g_{extra}(r) \sim \frac{\ln r}{r^2}$ in 3D, the projected form naturally emerges: $\hat{\alpha}_{extra}(b) \propto \frac{\ln b}{b}$.

The gravitational lensing deflection angle under the weak-field approximation of general relativity (in the scalar potential form) is: $\hat{\alpha}(b) = \frac{2}{c^2} \int_{-\infty}^{+\infty} \nabla_{\perp} \Phi(\sqrt{b^2 + z^2})dz$. Substitute $\Phi(r)$ solved from the modified Poisson equation, combined with the 1/r-order expansion of the logarithmically corrected Schwarzschild metric ($\frac{2GM}{c^2r}, \frac{2kG_hM^2(\ln r+1)}{c^2r} \ll 1$), and adopt the "thin-lens" paraxial approximation:

$$\hat{\alpha}(b) \approx \frac{4GM(r)}{c^2b} + \frac{4k(r)G_hM(r)^2 \ln b}{c^2b}$$

(17)

It is evident that $\hat{\alpha}_{extra}(b) \approx \frac{4k(r)G_hM(r)^2 \ln b}{c^2b} \propto \frac{\ln b}{b}$, which is consistent with the logarithmic term appearing after the projection of the aforementioned "dark matter halo". When quantum gravitational effects (dark matter halo) are not considered ($k(r) = 0$), the gravitational lensing formula naturally reduces to the general relativity form: $\hat{\alpha}(b) \approx \frac{4GM}{c^2b}$.

5.4. Role of the Logarithmic Term at the Galactic Scale

In the peripheral regions of galaxies, the positive contribution of the logarithmic term $\ln r$ enables the quantum gravitational term to provide stable additional gravity, which is equivalent to the gravitational effect of the traditional dark matter halo but without the need to hypothesize unknown particles:

- Physical nature: The statistical average effect of non-local entanglement of quantum vortices at the galactic scale, transmitted as macroscopic gravity enhancement through AdS/CFT duality.
- Advantage: All parameters are correlatable with observations (e.g., $M_{baryon,topo}$ corresponds to stellar luminosity and gas distribution), avoiding the theoretical flaw of dark matter being "undetectable".
- Final summary: All standard halo models can be interpreted as effective parameterizations of this logarithmic term ($\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$), and their apparent diversity is essentially a reflection of different regularizations of the same asymptotic behavior

6. Cross-Scale Consistency and Theoretical Advantages

6.1. Consistency of Dual-Scale Mechanisms

Although the effects of the logarithmic term at the black hole and galactic scales seem opposite, they originate from the same physical nature:

- Scale correlation: Both the repulsive potential at the black hole scale and the additional gravity at the galactic (black hole gravitational field) scale are macroscopic manifestations of the topological structure and non-local entanglement of quantum vortices, with only changes in the sign and magnitude of $\ln r$ caused by distance r .
- Parameter unification: Core parameters such as the k -factor and G_h have consistent definitions across dual scales; only dynamic adjustments of $M(r)$ and $k(r)$ are made to adapt to scale differences, with no additional hypotheses.

6.2. Comparative Advantages over Traditional Theories

Comparison Dimension	This Theory (Quantum Gravitational Correction with Logarithmic Term)	Traditional Theories (Kerr Black Hole + Dark Matter)
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Singularity problem	Physically resolved, satisfying information conservation	Unresolved, with curvature divergence
Free parameters	None (black holes) / 4 physical parameters (galaxies)	Black holes require fitting of spin and inclination; galaxies rely on dark matter distribution hypotheses
Cross-scale unification	Covers microscopic to macroscopic scales under a single framework	Black hole and galactic dynamics are fragmented
Observational verification	Multiple verifications including black hole shadows, high-speed stars, galaxy rotation curves, and mathematical asymptotic behavior of dark matter halos	Dark matter particles not directly detected; black hole spin lacks independent verification
Physical picture	Clear image of quantum vortices + AdS/CFT duality	Dark matter nature unknown; Kerr black hole lacks microscopic physical support

7. Conclusions and Outlook

Through a unified non-perturbative quantum gravity framework, this paper reveals the cross-scale universality of the quantum gravity correction term containing a logarithmic term—it's extremely simple mathematical form not only resolves the singularity through a repulsive potential at the black hole core and may solve the information paradox but also maintains high stellar velocities and the flatness of rotation curves through additional gravity in black hole gravitational fields and galactic outskirts, eliminating the need for traditional assumptions such as dark matter and black hole spin fitting. Starting from the analysis of the mathematical asymptotic behavior of dark matter halo dynamics, this framework conducts preliminary cross-scale multiple verifications of the logarithmic-term-containing quantum gravity by combining black hole shadows (EHT observations), high-speed stars, rotation curves of multiple galaxies (astronomical measurements), and the mathematical asymptotics of gravitational lensing. It achieves, for the first time, a unified description of gravity from the microscopic to the macroscopic scale, providing an observable and reproducible empirical framework for quantum gravity theory that differs from current mainstream paths (such as string theory and loop quantum gravity). Furthermore, the prediction table of shadow angular diameters for six black holes presented in this paper makes this theory the only theoretical framework that can provide specific and falsifiable (unable to adjust spin α and inclination i) predictions for the next-generation EHT observations.

Future research can focus on: 1) Whether the logarithmically modified gravitational potential can be strictly derived from first principles based on the modified Einstein-Hilbert action or effective quantum field theory, combined with observations and experiments, which is the core task of future work (may require joint efforts of the academic community); 2) Attempting to apply the theoretical framework to the a priori prediction of multi-messenger astronomical research such as black hole thermodynamics, radio bursts, and astroparticle physics, and using next-generation high-precision telescopes such as EHT, JWST, CTA, Fermi-LAT, and IceCube to test the universal boundary of its application; 3) Studying the impact of this correction term on the Friedmann equation to explore possible insights into cosmological puzzles such as dark energy dynamics and Hubble tension; 4) Directly verifying the non-local entanglement (quantum entanglement) effect corresponding to the logarithmic term through laboratory simulations (such as superfluid helium quantum vortex systems), providing a more solid microscopic experimental foundation for the theory.

This study indicates that the gravitational behavior of the universe, from black holes to galaxies, may be governed by the same quantum gravitational mechanism, with the logarithmic term serving as the core carrier of this mechanism. It also strongly suggests that black holes and galaxies may share a common topological origin, which we interpret as follows: the overall dynamics of galaxy disks

may be the holographic manifestation of the quantum topological structure of their central black holes on the macrocosmic scale through hierarchical nesting ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). This idea resonates with multiple cutting-edge physical concepts such as quantum fluid cosmology, fractal cosmology, and recursive structures. With its simplicity and powerful cross-scale adaptability, this model may pave a brand-new path for the unified description of gravity in astrophysics.

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