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Article

Embedding EEG Trajectories in a Möbius-Like Manifold: An Exploratory Study

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Abstract

Time–frequency decompositions, statistical descriptors and nonlinear dynamical methods can be used to assess electroencephalographic (EEG) signals. These approaches treat signals as time series evolving in Euclidean state spaces, potentially overlooking global geometric constraints. We explore an alternative representation in which EEG dynamics are embedded in a Möbius-like manifold, a non-orientable geometric structure capable of encoding cyclic evolution and phase-dependent state inversion within a compact topological space. We combine normalized signal amplitude with instantaneous phase derived from the Hilbert transform to generate a three-dimensional trajectory representing the temporal evolution of neural electrical activity. Using EEG recordings from a healthy young adult, we reconstructed trajectories and computed path geometric descriptors, including torsion and cumulative winding number. These two quantities characterize local twisting and global rotations, providing information about the trajectories of oscillatory neural activity. We derive testable hypotheses from our proof-of-concept study, e.g., transitions between oscillatory regimes should appear as changes in torsion or winding structure. We conclude that embedding neural signals in non-orientable manifolds could provide a methodological framework for assessing physiological and pathological brain activity.

Keywords: topology; curvature; attractor; recurrence; dynamics

Introduction

Electroencephalographic (EEG) signal analysis employs time–frequency decompositions, spectral and statistical descriptors and nonlinear methods such as attractor reconstruction and entropy metrics (Zahra et al. 2017; Kwessi and Edwards 2021; Durka et al. 2024; Jiao et al. 2024; Pourdavood and Jacob 2024; Ren et al. 2024; Lu et al. 2024). Also, geometric and topological approaches like delay embeddings and topological data analysis have been used to capture global structure in neural dynamics beyond local signal features (Lainscsek and Sejnowski 2015; Xu, Drougard and Roy 2021; Yamanashi et al. 2021; Chiarion et al. 2023; Manjunath, Perea and Sathyanarayana 2023; Pourdavood and Jacob 2024; Hazarika et al. 2024; Xu, Drougard and Roy 2025). These approaches, representing signals as trajectories in Euclidean spaces or orientable manifolds like cylinders or tori derived from phase variables, may fail to capture intrinsic symmetries and phase-dependent inversions.

We explore an alternative geometric representation in which EEG trajectories are embedded in a Möbius-like manifold (Gorinov 2004; Hofmann and Martin 2015; Rieger 2026). Combining normalized signal amplitude with the instantaneous phase extracted through the Hilbert transform, we produce a representation in which each time point of the signal corresponds to a position along a three-dimensional trajectory. The phase variable introduces cyclic organization, whereas the amplitude component determines the transverse coordinate of the trajectory. By identifying states that differ by half an oscillatory cycle through an inversion of amplitude orientation, the resulting space acquires the topology of a non-orientable surface analogous to a Möbius strip. This means that

oscillatory evolution corresponds to continuous motion along a twisted manifold rather than along a simple cylindrical surface. This allows local twisting of the path to be quantified through torsion, whereas the cumulative number of rotations of the trajectory around the manifold to be captured by the winding number (Yin et al. 2018; Chang et al. 2025).

We will proceed as follows. First, we describe the methods used to reconstruct the Möbius-like embedding and compute geometric descriptors of the trajectory. Then, we present trajectory representations and derived measures from EEG recordings. Finally, we discuss the conceptual and practical implications for the analysis of EEG dynamics.

METHODS

We evaluated an EEG recording acquired from a healthy 20-year-old subject, analyzing the first 4000 ms of activity from channel *C3* sampled at 1 ms intervals. We used signal normalization, Hilbert-transform phase extraction, a Möbius-like geometric embedding and trajectory-based differential geometry to reconstruct the temporal evolution of the EEG trace as a path in a three-dimensional non-orientable manifold. Then, we computed topological and geometric descriptors, including cumulative winding number, curvature, torsion and recurrence structure. We aimed to identify discriminable trajectories' signatures, namely local twisting, global rotational accumulation and repeated visits to nearby regions of the embedded space.

Data acquisition and temporal discretization. The analyzed signal consisted of a univariate EEG time series from channel *C3* of a healthy 20-year-old subject. Let the raw trace be denoted by s_n , where $n = 0, 1, \dots, N - 1$ indexes discrete samples. Because each sample corresponded to 1ms, physical time was defined by

$$t_n = n\Delta t, \Delta t = 1 \text{ ms} = 10^{-3} \text{ s.}$$

The first portion of the recording used for the trajectory analysis contained $N = 4000$ samples, corresponding to a total duration

$$T = N\Delta t = 4000 \text{ ms} = 4 \text{ s.}$$

The signal was treated as a discrete realization of an underlying continuous function $s(t)$, with the approximation

$$s(t_n) \approx s_n.$$

All subsequent operations were performed on this finite sequence. The present section focuses on the single-channel analysis of *C3*. No temporal smoothing, band-pass filtering, artifact rejection or epoch averaging was applied before the Möbius-like embedding, so that the embedding procedure operated directly on the recorded sequence after normalization.

Amplitude standardization and analytic signal construction. The raw series s_n was transformed into a dimensionless standardized amplitude x_n to remove scale dependence and center the data. If

$$\mu = \frac{1}{N} \sum_{n=0}^{N-1} s_n$$

denotes the sample mean and

$$\sigma = \left(\frac{1}{N} \sum_{n=0}^{N-1} (s_n - \mu)^2 \right)^{1/2}$$

denotes the population-standard deviation, the normalized signal was defined as

$$x_n = \frac{s_n - \mu}{\sigma}.$$

This transformation preserved waveform shape while rescaling the signal to zero mean and unit variance. Instantaneous phase was then extracted from the analytic signal. For a continuous signal $x(t)$, the Hilbert transform is

$$H[x](t) = \frac{1}{\pi} \text{p. v.} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau,$$

where p. v. denotes the Cauchy principal value (Criscuolo and Scuderi 1998; Hasegawa 2004). The corresponding analytic signal is

$$z(t) = x(t) + iH[x](t).$$

In discrete form, numerical estimation of the analytic signal yielded

$$z_n = x_n + i \tilde{x}_n,$$

where \tilde{x}_n is the discrete Hilbert transform of x_n . From z_n , the instantaneous amplitude and phase were

$$a_n = |z_n| = \sqrt{x_n^2 + \tilde{x}_n^2}, \phi_n = \arg(z_n).$$

Because phase returned by the argument function is defined modulo 2π , an unwrapped phase sequence Φ_n was computed to eliminate artificial jumps of approximately 2π . The wrapped phase used in the embedding was then

$$\theta_n = \Phi_n \bmod 2\pi, \theta_n \in [0, 2\pi).$$

This step assigned to every EEG sample both a normalized scalar amplitude x_n and a cyclic phase coordinate θ_n . These two variables formed the basis of the subsequent manifold embedding. Their pairing created a phase-amplitude representation in which the same time series was mapped into a space with explicit cyclic organization.

Topological identification and Möbius-like state space. A phase shift of π transforms a sinusoidal waveform into its sign-reversed version, since

$$\sin(\theta + \pi) = -\sin(\theta), \cos(\theta + \pi) = -\cos(\theta).$$

For this reason, amplitude-phase states related by a half-cycle shift can be identified through

$$(x, \theta) \sim (-x, \theta + \pi).$$

Equivalently, if one starts from a cylindrical phase-amplitude space with periodic identification $(x, 0) \sim (x, 2\pi)$, the additional pairing

$$(x, \theta) \sim (-x, \theta + \pi)$$

produces a non-orientable quotient space topologically equivalent to a Möbius strip. This identification was not interpreted as an anatomical property of the brain or scalp, but as a mathematical rule imposed on the signal representation. The construction associates states of opposite amplitude sign that differ by half an oscillatory cycle, thereby preserving phase relations while reducing redundancy in the representation of cyclic motion.

To achieve three-dimensional coordinates, the phase-amplitude pair (x_n, θ_n) was mapped to a standard Möbius-strip parametrization. Let $R > 0$ denote the radius of the central circular axis and w_n denote a bounded transverse coordinate derived from amplitude. We used

$$w_n = \text{clip}\left(\frac{x_n}{3}, -0.8, 0.8\right),$$

where $\text{clip}(u, \alpha, \beta)$ truncates u to the interval $[\alpha, \beta]$. This bounded scaling prevented very large standardized amplitudes from generating excessive excursions away from the strip. With $R = 1.5$, the embedding coordinates were defined by

$$\begin{aligned} X_n &= \left(R + w_n \cos \frac{\theta_n}{2}\right) \cos \theta_n, \\ Y_n &= \left(R + w_n \cos \frac{\theta_n}{2}\right) \sin \theta_n, \\ Z_n &= w_n \sin \frac{\theta_n}{2}. \end{aligned}$$

The discrete EEG series therefore generated a trajectory

$$\mathbf{r}_n = (X_n, Y_n, Z_n) \in \mathbb{R}^3.$$

Because the parametrization contains half-angle terms, one full traversal in θ reverses local transverse orientation, matching the Möbius identification. Therefore, the embedded EEG evolved as an ordered set of points on a twisted surface rather than as a curve in an untwisted cylindrical phase space.

Trajectory reconstruction and differential-geometric observables. The ordered sequence \mathbf{r}_n was interpreted as a discrete trajectory approximating a differentiable curve $\mathbf{r}(t)$. Numerical derivatives were obtained using finite differences implemented through centered discrete gradients. Writing

$$\mathbf{r}_n = \begin{pmatrix} X_n \\ Y_n \\ Z_n \end{pmatrix},$$

the first derivative was approximated by

$$\mathbf{r}'_n \approx \frac{\mathbf{r}_{n+1} - \mathbf{r}_{n-1}}{2\Delta t},$$

the second derivative by

$$\mathbf{r}''_n \approx \frac{\mathbf{r}_{n+1} - 2\mathbf{r}_n + \mathbf{r}_{n-1}}{\Delta t^2},$$

and the third derivative by discrete differentiation of \mathbf{r}''_n . In practice, the numerical gradient operator provided consistent discrete estimates at all interior points, with one-sided approximations at boundaries. Curvature was computed from the classical Frenet formula

$$\kappa_n = \frac{\|\mathbf{r}'_n \times \mathbf{r}''_n\|}{\|\mathbf{r}'_n\|^3 + \varepsilon},$$

where \times denotes the vector cross product, $\|\cdot\|$ is the Euclidean norm and ε is a small regularization constant introduced numerically to avoid division by zero at points with very small speed. Curvature measures the instantaneous bending of the path. Torsion was computed as

$$\tau_n = \frac{(\mathbf{r}'_n \times \mathbf{r}''_n) \cdot \mathbf{r}'''_n}{\|\mathbf{r}'_n \times \mathbf{r}''_n\|^2 + \varepsilon},$$

where \cdot denotes the scalar product. Torsion measures the local deviation of the path from planarity, that is, the extent to which the trajectory twists in three dimensions. Since all derivatives were estimated with $\Delta t = 1\text{ms}$, the independent variable of κ_n and τ_n was directly physical time.

The embedded path was also inspected through its arclength increment,

$$\Delta \ell_n = \|\mathbf{r}_{n+1} - \mathbf{r}_n\|,$$

and cumulative arclength

$$L_m = \sum_{n=0}^{m-1} \Delta \ell_n.$$

Although arclength was not the primary descriptor shown in the figures, it served as an internal consistency check that the trajectory evolved continuously over the manifold. Together, \mathbf{r}_n , κ_n , τ_n and L_m provided a differential-geometric description of the EEG trace after the Möbius-like transformation.

Winding number estimation and oscillatory accumulation. Global rotation around the Möbius-like manifold was quantified through the cumulative winding number. In continuous form, if $\theta(t)$ is an unwrapped phase, the winding accumulated over an interval $[t_0, t_1]$ is

$$N_w(t_0, t_1) = \frac{1}{2\pi} \int_{t_0}^{t_1} \frac{d\Phi}{dt} dt = \frac{\Phi(t_1) - \Phi(t_0)}{2\pi}.$$

For the discrete EEG trajectory we used the unwrapped phase sequence Φ_n , so the cumulative winding up to sample m was

$$W_m = \frac{\Phi_m - \Phi_0}{2\pi}.$$

Equivalent numerical estimation can be written in incremental form as

$$W_m = \sum_{n=1}^m \frac{\Phi_n - \Phi_{n-1}}{2\pi}.$$

Because one full oscillatory cycle corresponds to an increase of approximately 2π in unwrapped phase, W_m measures the accumulated number of rotations executed by the trajectory around the strip's central loop. Its temporal slope,

$$\frac{dW}{dt} = \frac{1}{2\pi} \frac{d\Phi}{dt},$$

is directly related to instantaneous frequency. In discrete form,

$$f_n \approx \frac{\Phi_{n+1} - \Phi_{n-1}}{4\pi\Delta t},$$

with $\Delta t = 10^{-3}\text{s}$ when frequency is expressed in hertz. In our analysis, the winding curve was retained as a cumulative topological marker rather than reduced to a frequency estimate. This preserved its interpretation as a monotone measure of how the embedded EEG trajectory progresses around the non-orientable manifold. Since the trajectory was built from the first 4000ms, the winding number was evaluated for the same interval as torsion and curvature. Plotting W_m against physical

time yielded a direct visualization of rotational accumulation in the embedded state space. This quantity complemented torsion because it described a global rotational property rather than a local twisting property.

Recurrence structure and distance-based trajectory organization. To characterize repeated visits of the EEG trajectory to nearby regions of the Möbius-like manifold, pairwise Euclidean distances were computed between all embedded points. For samples i and j ,

$$D_{ij} = \|\mathbf{r}_i - \mathbf{r}_j\| = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2}.$$

The resulting matrix $D \in \mathbb{R}^{N \times N}$ is symmetric, nonnegative and has zeros on the diagonal. Visualization of D_{ij} as an image yields a recurrence-distance map in which darker values indicate closer proximity of trajectory states. A thresholded recurrence matrix may also be defined as

$$R_{ij} = \Theta(\epsilon - D_{ij}),$$

where Θ is the Heaviside step function and $\epsilon > 0$ is a chosen spatial threshold. The full distance matrix rather than a binarized recurrence map was used for visualization, because it retains graded information about state similarity without introducing threshold dependence. Diagonal continuity reflects temporal continuity of the curve; off-diagonal dark structures indicate returns to similar regions of the manifold at separated times.

The analysis was implemented in Python using the libraries pandas for spreadsheet import, numpy for numerical array operations, scipy.signal for analytic-signal estimation via the Hilbert transform, scikit-learn for principal component analysis when a multichannel summary was inspected, and matplotlib for visualizations.

RESULTS

We report the geometric and topological properties obtained after embedding the EEG signal from channel C3 into a Möbius-like manifold using amplitude-phase coordinates derived from the Hilbert transform. The analysis focused on trajectory organization, rotational accumulation, local twisting and recurrence patterns measured directly from the embedded path.

Geometric structure of the embedded EEG trajectory. The Möbius-like embedding transformed the discrete EEG sequence into an ordered three-dimensional trajectory $\mathbf{r}(t) = (X(t), Y(t), Z(t))$ whose evolution reflects the temporal progression of the signal (Figure 1). Over the analyzed interval of 4000 samples, the distribution of points along the surface revealed non-uniform occupancy patterns, with visually identifiable bands where successive trajectory segments accumulated more densely. The trajectory followed smooth loops around the central axis of the strip, reflecting the cyclic structure of oscillatory neural activity encoded by the phase variable. Visual inspection of the density representation confirmed that the trajectory revisited similar regions of the manifold repeatedly over time (Figure 2A). These regions correspond to clusters of nearby coordinates in the embedded space and indicate recurrent dynamical states of the signal. The recurrence distance matrix further revealed repeated patterns through symmetric off-diagonal structures that indicate temporal intervals where the trajectory approached previously visited states (Figure 2B). Because the distance matrix compares every pair of embedded points, the appearance of these structured patterns indicates that the reconstructed trajectory does not behave as a random walk in the manifold but instead evolves through geometrically organized paths.

Taken together, these observations establish that the Möbius embedding converts the EEG time series into a structured trajectory whose geometry reflects repeated oscillatory regimes.

Topological descriptors of the trajectory. Quantitative descriptors derived from the embedded path characterize both local geometric deformation and global rotational structure (Figure 3). Torsion measures the instantaneous twisting of the trajectory in three dimensions and was computed from the first three derivatives of $\mathbf{r}(t)$. The resulting torsion time series displays continuous fluctuations over the 4000 ms interval, indicating that the trajectory does not evolve within a single planar subspace but instead undergoes repeated twisting as it moves across the manifold (Figure 3A). These variations correspond to local changes in trajectory geometry occurring on the millisecond time scale of the EEG signal. The cumulative winding number captures the global rotational behavior of the trajectory by integrating the unwrapped phase variable. The winding curve increases monotonically

across the analyzed interval, indicating repeated rotations of the trajectory around the central loop of the Möbius surface (Figure 3B). Because each full oscillatory cycle corresponds approximately to one rotation in the embedded space, the slope of this curve reflects the temporal progression of oscillatory activity in the signal. The monotonic increase in winding number confirms that the trajectory advances continuously along the cyclic coordinate defined by the phase variable. Torsion reflects local twisting of the path, whereas the winding number quantifies the cumulative number of rotations executed by the trajectory.

The coexistence of these descriptors demonstrates that the EEG signal contains both local geometric variability and persistent global cyclic structure when represented in the Möbius-like manifold.

Overall, the embedding of the EEG signal in a Möbius-like manifold produced a continuous trajectory with organized geometric and topological patterns. The trajectory exhibited repeated visits to specific regions of the manifold, persistent rotational accumulation and time-dependent twisting captured by torsion. Therefore, representing EEG signals as trajectories in a non-orientable manifold could provide measurable geometric descriptors of neural dynamics.

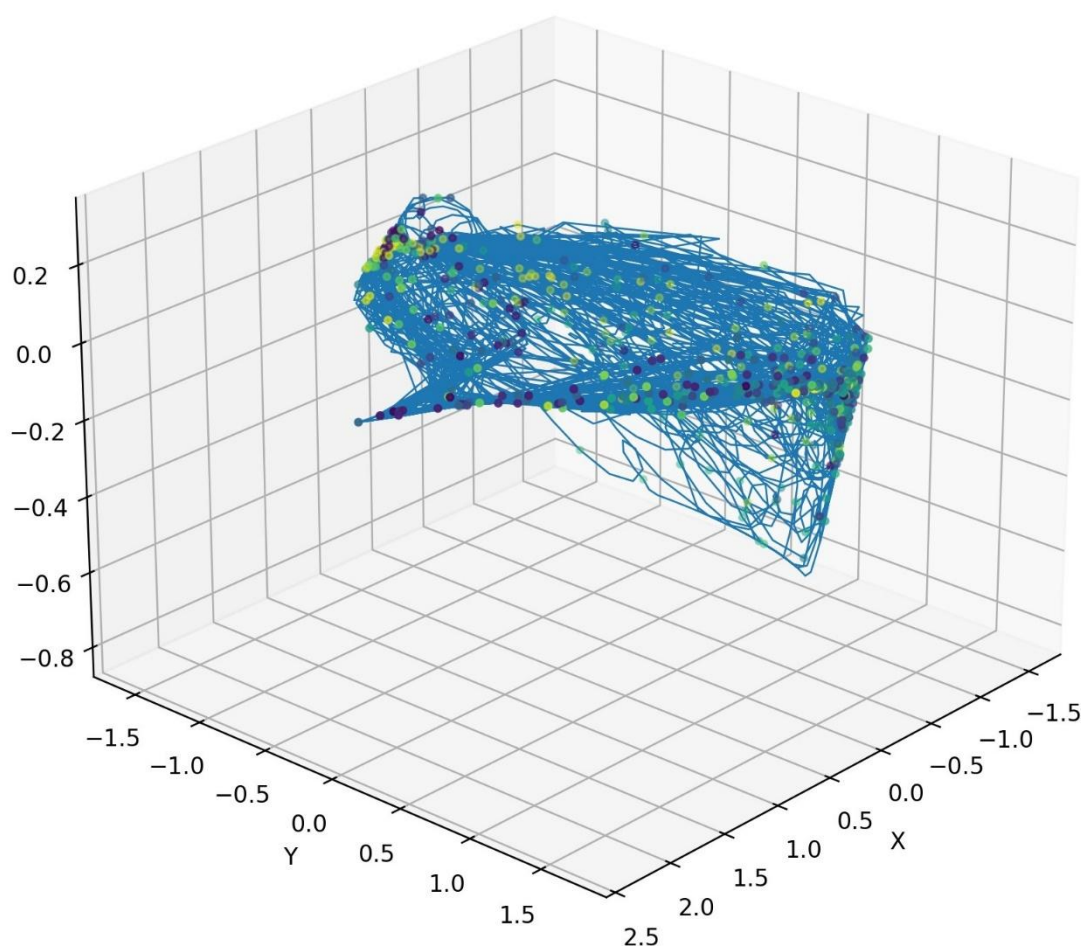


Figure 1. Möbius-like trajectory map of EEG activity. The electrical signal recorded from channel C3 of a healthy 20-year-old subject (sampling interval 1 ms, corresponding to 1 kHz) was embedded in a Möbius-like manifold constructed from the normalized signal amplitude and its instantaneous phase obtained via the Hilbert transform. The figure displays the first 4000 ms of the recording. Each EEG sample is mapped to coordinates (X, Y, Z) using a Möbius parametrization, producing a continuous trajectory that represents the temporal evolution of neural electrical activity. The blue curve shows the ordered trajectory of the signal on the manifold. The colored dots correspond to individual sampled points along the trajectory and are color-coded according to

time progression (milliseconds): darker colors indicate earlier samples and progressively lighter colors indicate later samples within the 0–4000 ms interval.

A B

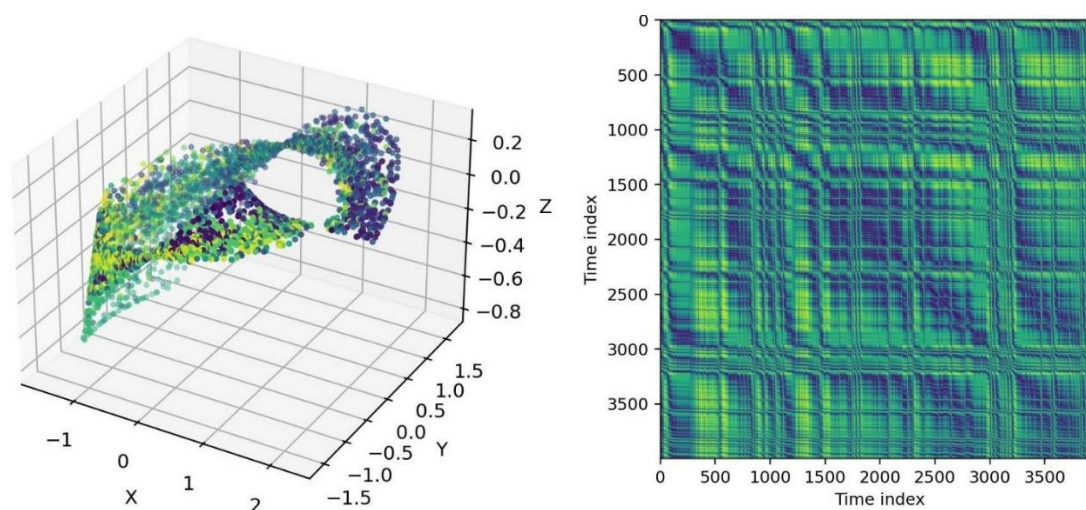


Figure 2. Topological features of the EEG trajectory embedded in a Möbius-like manifold (first 4000 ms). (A) Möbius trajectory density map. Each point represents one EEG sample mapped onto the Möbius-like manifold. Points are colored according to time progression during the first 4000 ms of the recording, with darker colors corresponding to earlier samples and progressively lighter colors indicating later samples. Regions where many points accumulate indicate preferred dynamical channels of the EEG trajectory, reflecting recurrent oscillatory states of neural electrical activity. (B) Recurrence distance map. Pairwise distances between trajectory points are represented in a two-dimensional matrix indexed by time. Darker regions indicate similar states of the trajectory, revealing recurrent patterns and repeated dynamical configurations of the EEG signal. The diagonal structures reflect the temporal continuity of the trajectory and the recurrence of oscillatory regimes.

A B

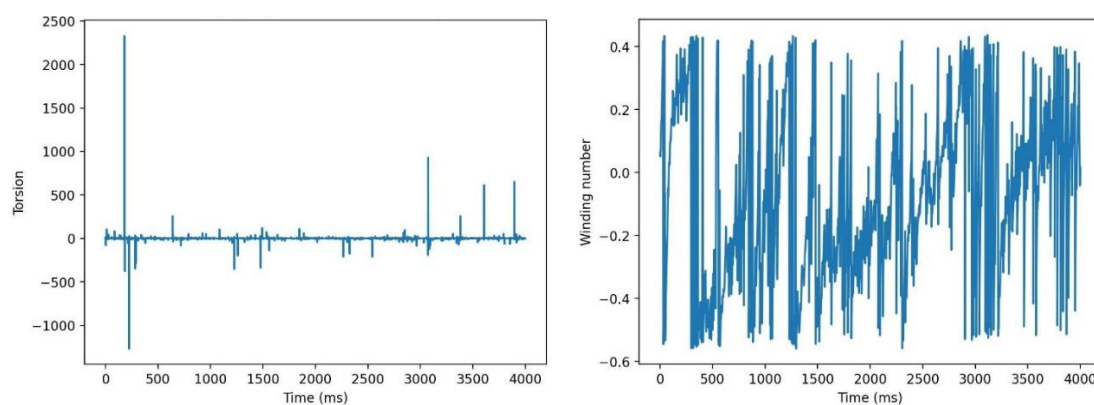


Figure 3. Two topological descriptors of the EEG trajectory embedded in a Möbius-like manifold. (A) Torsion of the EEG trajectory over the first 4000 ms. Torsion measures the degree to which the path twists in three-dimensional space and is computed from the derivatives of the trajectory vector $\mathbf{r}(t)$. Changes in torsion reflect variations in the local geometric structure of the trajectory and may correspond to transitions between oscillatory regimes or modifications in the temporal organization of the EEG signal. Because the data are sampled every 1 ms, the horizontal axis represents physical time in milliseconds. (B) Cumulative winding number illustrating the accumulated number of rotations of the trajectory around the Möbius-like manifold over the same time interval. The winding number is derived from the temporal evolution of the phase variable and indicates how many times the trajectory wraps around the embedded space. Since one oscillatory cycle corresponds approximately

to a full rotation around the manifold, the slope of the curve reflects the instantaneous oscillatory activity of the EEG signal.

Conclusions

We asked whether the temporal evolution of an EEG signal can be represented as a trajectory evolving on a non-orientable geometric manifold and whether this representation yields measurable descriptors of neural electrical activity. The EEG trace recorded from channel C3 of a healthy subject was transformed into amplitude–phase coordinates derived from the Hilbert transform and embedded into a Möbius-like manifold. Our simulations showed that the embedded signal forms a continuous path progressing around the manifold, while exhibiting local twisting and repeated returns to nearby regions of the space. The winding number captured the cumulative rotations associated with oscillatory cycles, whereas torsion quantified local three-dimensional deformation of the path. Recurrence and density representations indicated that the trajectory repeatedly occupied restricted regions of the manifold. These observations indicate that EEG dynamics can be represented as a geometric trajectory in a non-orientable state space whose properties can be measured through topological quantities.

Our study has limitations. The Möbius-like manifold is introduced as a representational device rather than as a structure derived from established electrophysiological principles. The identification of amplitude–phase states through the relation $(x, \theta) \sim (-x, \theta + \pi)$ follows from oscillatory symmetry arguments but has not been empirically demonstrated for electroencephalographic signals. Torsion, curvature and winding number were computed from the reconstructed trajectory, but their numerical values depend entirely on the adopted mapping and parameterization. The analysis was performed on a single EEG trace from one individual, which limits generalizability. It remains to be determined whether similar geometric structures appear consistently across different EEG channels, individuals and behavioral states and whether longer recordings produce stable trajectory patterns over extended time scales.

Experimentally testable hypotheses can be drawn. First, the cumulative winding number should scale linearly with time during periods of stable oscillatory rhythms. The slope dW/dt should approximate the instantaneous oscillatory frequency extracted from standard spectral analysis, providing a quantitative correspondence between rotational accumulation and classical frequency measures.

Second, transitions between oscillatory regimes should appear as localized increases in torsion magnitude. This prediction can be evaluated by comparing torsion peaks with independently detected changes in dominant frequency bands or phase resets, with the expectation that larger torsion amplitudes coincide with sharper transitions in the underlying EEG dynamics.

Third, recurrence maps derived from the embedded trajectory should reveal repeated visits to similar regions of the manifold during stable rhythmic activity. Quantitatively, the density of recurrence points within a fixed spatial threshold should increase during sustained oscillatory epochs and decrease during irregular or broadband activity.

Fourth, if the Möbius-like identification captures phase symmetry between half-cycle–shifted states, pairs of trajectory points separated by approximately π in phase should cluster spatially within the manifold, a prediction that can be tested by measuring pairwise distances between amplitude–phase states satisfying $\theta_j \approx \theta_i + \pi$.

Future research may also examine how trajectory geometry changes under different physiological conditions, whether alternative non-orientable embeddings produce comparable descriptors and how geometric measures interact with established nonlinear metrics in neural signal analysis.

Our approach introduces computational observables that may support new forms of signal characterization. Because our embedding converts a time series into a spatial trajectory, quantities like trajectory morphology, spatial recurrence and rotational progression can be incorporated into automated pipelines for signal segmentation, where different portions of a recording are classified

according to trajectory morphology rather than spectral content alone. Our approach may also support visualization of neural dynamics in compact geometric spaces that preserve temporal ordering while reducing dimensional redundancy in cyclic signals. In addition, trajectory-based descriptors can be integrated with existing signal-processing techniques to generate hybrid representations.

In conclusion, we examined whether transforming an EEG recording into a trajectory on a twisted geometric surface yields measurable descriptors of neural signal organization. Our continuous representation introduces a different analytical viewpoint in which temporal activity is interpreted through the geometry of its reconstructed path, providing a complementary perspective for investigating the organization of neural electrical recordings.

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

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Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.

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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 5.2 to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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