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Article

The Quadrature of the Circle in Electrotherapy

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Abstract

Electrotherapy and neurostimulation universally employ the rectangular (square) waveform as their standard stimulation signal. This article demonstrates that this choice constitutes a fundamental error of physical, mathematical, and neurophysiological nature, perpetuated since the mid-twentieth century through three converging factors: insufficient signal theory training in medical and paramedical curricula; technological drift toward ever-steeper wavefronts perceived as progress; and inadequate spectral disclosure by medical device manufacturers. We recall that the founders of electrical neurophysiology — Du Bois-Reymond (1843) and Helmholtz (1850) — stimulated with smooth-envelope signals, involuntarily close to membrane physiological requirements. We analyze the technological stratigraphy that progressively established the square wave as the unquestioned norm, and identify two erroneous assertions in the French foundational literature (Dumoulin & de Bisschop, 1987; Crépon, 1994) as crystallization points of the error in clinical practice. We present spectral and energetic calculations demonstrating the inadequacy of the rectangular signal relative to the biological bandwidth of the excitable membrane: for a 600 μ s rectangular pulse at 50 Hz, Fourier harmonics extend to 81,650 Hz, wavefront components exceed 5 MHz, and the calculated peak power reaches 7.75×10^8 W, against 6.1×10^5 W for the equivalent sinusoidal signal. We propose an optimal biomimetic signal described by a parametric Bézier curve whose inflection points correspond to the conformational time constants of voltage-gated ion channels as described by the Hodgkin-Huxley model (1952). This zero-mean signal respects the natural opening and inactivation kinetics of sodium and potassium channels, concentrating its energy within the physiologically relevant bandwidth. We discuss documented clinical consequences of the fundamental error: peri-electrode fibrosis in deep brain stimulation (DBS), progressive impedance drift, and the relative inefficacy of consumer TENS devices. This work is published open access under Creative Commons CC-BY 4.0. All parameters of the optimal signal are fully described herein, establishing permanent publication priority and excluding subsequent patent filing on this concept.

Keywords: neurostimulation; electrotherapy; rectangular waveform; sinusoid; Bézier curve; ion channels; Hodgkin-Huxley; biological bandwidth; TENS; DBS; quadrature of the circle

From the rectangular waveform to the biomimetic signal: analysis of a fundamental error and proposal for an optimal waveform

1. Opening: The Quadrature of the Circle

1.1. A Mathematical Problem Two Thousand Years Old

The *quadrature of the circle* is one of the founding problems of ancient geometry: to construct, using only a ruler and compass, a square of equal area to a given circle. For two millennia, mathematicians and philosophers attempted to solve it. In 1882, Ferdinand von Lindemann demonstrated its impossibility once and for all: π is a transcendental number, the root of no polynomial with rational coefficients [1]. No finite algebraic construction can bridge the circle from the square.

1.2. Vitruvian Man: The Square and the Circle as Geometries of Life

Vitruvius, in his *De architectura* (1st century BCE), describes man as the measure of all things, indscribable both in a square and in a circle [2]. Leonardo da Vinci illustrated this duality around

1490 in what remains the most celebrated image in the history of human anatomy. Two superimposed silhouettes: one inscribed in the circle and the other in the square. The circle is not an approximation of an improved square. It is its primary form, the form of movement, rotation, and life.

1.3. *The Missed Quadrature of Electrotherapy*

Electrotherapy has for 150 years pursued its own quadrature of the circle: substituting a rectangular signal for the curved, continuous, periodic form of the neuronal action potential.

Lindemann's result applies here by analogy: substitution is impossible without loss. One cannot replace the circle with the square without leaving something behind. That "something" is what we shall now quantify.

2. What the Founders already Knew

2.1. *Du Bois-Reymond and the Wave of Negativity (1843)*

Emil Heinrich Du Bois-Reymond (1818–1896), working in Berlin under Johannes Müller, demonstrated in 1843 that a stimulus applied to the surface of a nerve produces a local decrease in electrical potential — which he named the *wave of relative negativity* (*negative Schwankung*) — propagating along the fiber [3]. This discovery founded the field of scientific electrophysiology.

What is decisive for our argument: Du Bois-Reymond was stimulated using mechanical interruption circuits and induction coils. The signals produced had mechanically constrained rise times, rounded, damped, quasi-sinusoidal. Involuntarily, instruments of the era produced physiological signals close to the target. And stimulation worked, at modest voltages.

2.2. *Helmholtz and the Measurement of Conduction Velocity (1850)*

Hermann von Helmholtz (1821–1894) measured in 1850 the propagation velocity of the nerve impulse in frog nerve-muscle preparations, obtaining values between 25 and 43 m / s [4]. His apparatus used induction coils (Ruhmkorff spirals) for stimulation. Again, the stimulation signal was a quasi-sinusoidal impulse envelope — the natural product of an inductance discharge.

Helmholtz did not choose the sinusoid through physiological reasoning. He was constrained to it by the available technology. That constraint was a stroke of luck.

2.3. *The Historical Rupture*

The discoveries of Du Bois-Reymond and his contemporaries could have provided a rigorous physiological foundation for electrotherapeutic practice from the mid-nineteenth century onward. They were "coldly received by electrotherapists, who criticized them severely for several decades" [5]. The dissociation between fundamental electrophysiology and clinical practice is ancient. It is not the product of a lack of data. It is a refusal.

3. The Technological Stratigraphy of the Error

The fundamental error of electrotherapy was not born from a single bad decision. It was built up in successive layers, each technological advance perfecting the wrong target.

3.1. *Level 1: The Manual Switch and the Induction Coil (19th century)*

The first stimulators produced signals with mechanically constrained rise times, rounded, and damped. These instruments were effectively stimulated at modest energies. The error did not yet exist: technology was involuntarily correct.

3.2. *Level 2: Vacuum Tube Electronics (1920–1960)*

The first electronic generators allowed for the first time the *choice* of waveform. The square wave became *possible* and immediately desirable because it appeared to be precise, reproducible and parameterizable. Its duration could be read directly from an oscilloscope screen.

This is the bifurcation point. The rectangular signal was not adopted following a physiological demonstration. It was adopted because it was *convenient for the human operator*. The quality criterion became visual legibility, not biological adequacy.

3.3. Level 3: The Operational Amplifier (1960–1980)

The advent of $\mu A741$ and its successors provided slew rates of several $V / \mu s$. Rise times could now be quasi-vertical. The plateaus were perfectly flat. Technology reached the *perfection of the ideal square wave*.

The ideal square wave is, as we shall demonstrate, the worst possible neurophysiological signal.

3.4. Level 4: The Fast MOSFET and the Microprocessor (1980–2000)

The generators of the era of foundational publications (Dumoulin & de Bisschop, 1987; Crépon, 1994) produced wavefronts in a few hundred nanoseconds. The engineering was remarkable. The target was still the square wave.

3.5. Level 5: High-Voltage Digital Stimulation (2000 – present)

Deep brain stimulation (DBS), cochlear implants, and spinal neurostimulators operate with pulses whose fronts reach a few tens of nanoseconds, at voltages that can exceed 100 V. Jamali *et al.* [6] proposed in 2019 a stimulation method at 1 MHz with sub-microsecond pulses, reporting a reduction in charge per pulse from 60 nC to 0.3 nC. At these frequencies, energy is predominantly dissipated as heat. The device resembles a low-power electrosurgical unit more than a neurostimulator.

The law of this stratigraphy: at each technological generation, the quality criterion was *fidelity to the rectangle* — steeper rise, flatter plateau, sharper descent. Nobody asked whether the rectangle was the right target. This is the *error of perfection*: to optimize brilliantly in the wrong direction.

4. Codification of the Error in the Foundational Literature

4.1. Two Assertions, One Generation of Error

Assertion 1 — Dumoulin & de Bisschop (1987).

In *Électrothérapie* [7], p. 33: direct current is presented as the ideal waveform because “it is easy to establish and break a circuit.”

This claim confuses the electrical properties of a resistive load with the physiology of an excitable membrane. In terms of pure resistance, the establishment of a direct current does indeed produce sustained depolarization. On a neurological membrane, the same maneuver produces a single initial depolarization, followed by accommodation and then electrical silence — as Hodgkin and Huxley had demonstrated 35 years earlier [8]. The neuron *responds to the variation of potential* (dV/dt), not to its absolute value.

Assertion 2 — Crépon (1994).

In *Électrophysiothérapie* [9], p. 23: the advantage of the rectangular signal is that “it is possible to rise to the plateau value in zero time.”

A zero-time rise is a Heaviside discontinuity. Its Fourier series expansion is an infinite sum of sinusoids whose amplitudes never tend to zero. In real devices, a 100 ns front corresponds to a cutoff frequency of approximately 3.5 MHz — several orders of magnitude above the physiologically relevant bandwidth of the neuron.

4.2. A Matter of Training, not Negligence

Guy de Bisschop held positions in clinical neurophysiology at the hospitals of Marseille and Martigues, and lectured at Paris V. Francis Crépon authored major pedagogical references, still in print in 2014. These authors were not lacking in rigor — they lacked specific conceptual tools: Fourier spectral analysis applied to non-sinusoidal signals, the concept of biological bandwidth, and the

molecular biophysics of ion channels. These tools are taught in undergraduate physics and biomedical engineering. They were not and are still not systematically included in medical and paramedical curricula.

4.3. The Responsibility of Manufacturers

Neurostimulation device manufacturers employ engineers in electronics and signal processing. These engineers know the spectral content of their signals. They measure the impedance of the load. They calculate peak power delivered.

The technical manuals of neurostimulation devices do not to this day specify the spectral content of the delivered signal, the cutoff frequency –3 dB, the calculated maximum power, nor the physiological limits beyond which injected energy ceases to be neurophysiologically relevant.

This information was available. We will leave it to the reader to appreciate why it did not find its way into user manuals.

5. Physical and Mathematical Demonstration

5.1. Spectral Analysis of the Rectangular Signal

A rectangular signal of period T , amplitude A , and duty cycle $1/2$ decomposes into a Fourier series:

$$f(t) = \frac{4A}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin\left(\frac{2\pi nt}{T}\right) \tag{1}$$

For a pulse of duration $\tau = 600\ \mu\text{s}$ at frequency $f_0 = 50\ \text{Hz}$:

- Fundamental harmonic: $f_1 = 50\ \text{Hz}$
- Significant harmonics extending to $f_n \approx 81,650\ \text{Hz}$ (for $n \approx 1633$)
- Wavefront duration $\Delta t \approx 100\ \text{ns}$ (modern device): $f_{\text{max}} \approx 1/(2\Delta t) \approx 5\ \text{MHz}$

The physiologically relevant bandwidth of a neuron is determined by the duration of the action potential: 1–2 ms, corresponding to a cutoff frequency of approximately 500–1000 Hz. The vast majority of the spectral energy of the rectangular signal is deposited at frequencies to which the membrane is *blind* — it cannot respond to them through selective ionic mechanisms. This energy is dissipated as heat.

5.2. Comparative Peak Power Calculation

Table 1. Comparative energy parameters — rectangular vs. sinusoidal signal.

Parameter	Rectangular	Sinusoidal
Minimum impedance	31 Ω	187 Ω
Maximum impedance	2,000 Ω	187 Ω
Impedance variation	$\times 64$	constant
Peak power ($I_{\text{max}}^2 \times Z_{\text{min}}$)	$7.75 \times 10^8\ \text{W}$	$6.1 \times 10^5\ \text{W}$
Mean amplitude	3.92 mA	3.46 mA
Ratio $P_{c,\text{rect}}/P_{c,\text{sin}}$		≈ 1273

Two key observations:

- (1) The peak power of the rectangular signal is approximately 1273 times greater than that of the sinusoidal signal for comparable clinical parameters. The commercial claim that the rectangular signal “uses less energy” is doubly incorrect: the mean rectangular amplitude is 13% *higher* than the sinusoidal amplitude, *and* the peak power exceeds it by more than three orders of magnitude.
- (2) The impedance varies by a factor of 64: the actual charge delivered to the membrane varies proportionally across frequencies. The rectangular signal is intrinsically unstable against a biological load, unlike the sinusoidal signal whose impedance is constant.

5.3. The sinusoid: omnipresent and unrecognized

There is a profound irony in the situation described. The sinusoidal signal is:

- The form of every oscillatory motion (pendulum, spring, tide)
- The projection of uniform circular motion — Vitruvian Man's circle
- The form of sound waves, light, electromagnetic radiation
- The form of the alternating current that *powers* the stimulation devices themselves
- The approximate form of the action potential

And yet, inside the neurostimulation device itself, the mains sinusoid is rectified, chopped, and deformed — to produce a rectangular signal before it reaches the patient. The physiological signal is deliberately destroyed at the input of the device.

One of the authors verified experimentally, during a 50 Hz mains electrification incident (1985), that the pure mains sinusoid produces effective and reproducible muscular tetanization at modest voltage — exactly what Du Bois-Reymond observed with his induction coils.

We optimized for the human observer. We forgot to optimize for the neuron.

6. Membrane Neurophysiology: What the Neuron Requires

6.1. The Hodgkin-Huxley Model (1952)

The Hodgkin-Huxley model [8], Nobel Prize 1963, describes the dynamics of the action potential:

$$C_m \frac{dV}{dt} = -g_{Na}(V - E_{Na}) - g_K(V - E_K) - g_L(V - E_L) + I_{ext} \quad (2)$$

The conductances g_{Na} and g_K are governed by activation and inactivation variables (m , h , n) obeying first-order differential equations with voltage-dependent time constants $\tau_m(V)$, $\tau_h(V)$, $\tau_n(V)$ — the conformational time constants of the channel proteins.

6.2. What the Square Wave Imposes on the Membrane

Rise front ($\Delta t \rightarrow 0$): theoretically infinite dV/dt . Voltage-gated sodium channels are trans-membrane proteins — mechanical objects with finite conformational time constants. Their opening cannot be instantaneous. The imposed electrical constraint bears no relationship to any physiological stimulation.

Plateau ($dV/dt = 0$): the potential is held constant. Yet it is precisely dV/dt that is the relevant signal for sodium channels. A constant potential produces accommodation: channels progressively inactivate. Stimulation becomes ineffective.

Falling front: a second discontinuity, of opposite sign. A second mechanical constraint on the membrane proteins.

In chronic stimulation (DBS, prolonged TENS), these repeated mechanical constraints on channel proteins may induce conformational fatigue, local neuroinflammation, and peri-electrode fibrotic reaction.

6.3. Membrane Accommodation: A Neglected Phenomenon

Membrane accommodation — the progressive decrease of excitability under constant current — has been described in the electrophysiological literature since the 19th century. Pflüger's law (1859) [10] states that it is the *variation* of current, not its intensity, that determines the contractile response. This law underpins the concepts of chronaxie and rheobase — both of which, it should be noted, are measured using rectangular signals, introducing a systematic measurement bias that the literature has never corrected.

7. Documented Clinical Consequences

7.1. Peri-Electrode Fibrosis in DBS

Deep brain stimulation uses chronically implanted electrodes delivering high-frequency rectangular pulses. The literature documents a progressive impedance drift following implantation [11–13]:

- Impedance doubling within 12 days [11]
- Factor of 10 within 22 days under continuous stimulation
- Correlation between impedance increase ($> 4000 \Omega$) and loss of clinical efficacy [12]
- Post-mortem studies: peri-electrode fibrous sheath, reactive astrocytosis, neuronal loss, neuroinflammation

The cause of this fibrosis is described as *unknown* in recent meta-analyses [14]. Our hypothesis: chronic membrane aggression by an inadequate signal $\rightarrow$ local neuroinflammation $\rightarrow$ reactive fibrosis $\rightarrow$ impedance increase $\rightarrow$ compensatory voltage increase $\rightarrow$ aggravation of aggression. A vicious cycle not identified as such in the literature.

7.2. Cardiac Pacemakers and Implantable Defibrillators: The Weightiest Argument

Chronic cardiac pacing by implanted pacemakers may constitute the most widespread and clinically critical application of the rectangular waveform in medicine. More than five million new devices are implanted annually worldwide [15]. Tens of millions of patients carry, permanently implanted in direct contact with myocardial or nodal tissue, an electrode delivering continuous rectangular impulses — 24 hours a day, 365 days a year, for 8 to 15 years.

Myocardial tissue is excitable tissue in the full sense: it possesses voltage-gated ion channels, conformational time constants, a refractory period, and susceptibility to accommodation. The arguments developed in this article therefore apply to it in their entirety.

Peri-electrode impedance drift in chronic cardiac stimulation.

The formation of a fibrous capsule around the pacemaker electrode tip after implantation is a clinically well-documented phenomenon, universally known to cardiologists [16]. It manifests as an increase in stimulation impedance in the weeks following implantation, followed by stabilization at a level above the initial baseline. The practical consequences are:

- Increased stimulation threshold requiring reprogramming of delivered energy
- Increased battery consumption, reducing device longevity
- In extreme cases, loss of capture requiring surgical re-intervention

The official cause of this fibrosis: foreign body inflammatory reaction. This explanation, while partially accurate, does not account for why the inflammation is systematically localized to the active electrode-tissue interface, nor why it is more pronounced under active stimulation than in its absence [17].

Our hypothesis applied to cardiac pacing.

The mechanism proposed in this article — chronic membrane aggression by an inadequate signal $\rightarrow$ inflammation $\rightarrow$ fibrosis $\rightarrow$ impedance increase $\rightarrow$ compensatory energy increase $\rightarrow$ aggravation — applies to myocardial tissue with an additional dimension: the resting heart rate (60–80 bpm) implies chronic stimulation at relatively low pulse frequency, but the rectangular rise fronts remain identical. Each impulse imposes on myocardial tissue the same spectral discontinuities as on the neuronal membrane.

The energetic argument reversed.

Pacemaker manufacturers justify the use of ultra-short rectangular pulses (typically 400–500 μ s) by battery longevity optimization: a short pulse consumes less energy than a long one. This argument is valid in terms of charge per pulse.

However, if a significant fraction of each pulse’s energy is dissipated as heat in biologically out-of-band harmonics, the *useful* energetic efficiency — energy actually converted into myocardial depolarization — is lower than the charge calculation suggests. Furthermore, the peri-electrode fibrosis induced by spectral mismatch forces progressive energy increases over time, gradually negating the benefit of the short pulse.

The energetic logic of the rectangular signal is circular: it creates the very conditions that require its intensification.

Cochlear implants.

The same reasoning applies to cochlear implants, whose electrodes deliver biphasic rectangular pulses to auditory nerve fibers. The progressive loss of efficacy and peri-electrode histological changes observed after cochlear implantation [18] deserve to be re-examined through the lens of spectral mismatch.

In terms of public health, the question of the optimal waveform in chronic cardiac pacing exceeds by several orders of magnitude that of TENS or even DBS. This is the weightiest argument in this article — and, to our knowledge, one that has never been formulated in the literature.

7.3. Consumer TENS Devices

Tens of millions of TENS (Transcutaneous Electrical Nerve Stimulation) devices are sold annually worldwide, all using rectangular signals, all direct heirs of the chain of errors described in this article. Clinical evaluation of their efficacy in chronic pain remains controversial [19]. No rigorous comparative study has, to date, compared rectangular and biomimetic signals on identical physiological parameters.

8. The Optimal Biomimetic Signal: Complete Description

8.1. Design Principles

An optimal neurostimulation signal must satisfy the following constraints:

- 1. **Zero net charge** — to prevent tissue electrolysis and net ionic migration
- 2. **Spectrum concentrated within the biological bandwidth** — useful energy between ~1 Hz and ~2 kHz
- 3. **Rise time with respect to conformational time constants** of Na⁺ channels ($\tau_m \approx 0.1 - 0.5$ ms)
- 4. **Repolarization phase with respect to the kinetics of K⁺** ($\tau_n \approx 1-5$ ms)
- 5. **Post-potential hyperpolarization** reproducing the relative refractory period

8.2. The Bézier Curve as a Modeling Tool

A cubic Bézier curve is defined by four control points P_0, P_1, P_2, P_3 :

$$\mathbf{B}(t) = (1 - t)^3 P_0 + 3(1 - t)^2 t P_1 + 3(1 - t) t^2 P_2 + t^3 P_3, \quad t \in [0, 1]$$

(3)

The power of this representation for our application is twofold:

- (a) Each control point P_i corresponds to an identifiable ionic event: Na⁺ channel opening (P_1), depolarization peak (P_2), Na⁺ inactivation, and K⁺ activation (P_3). The coordinates are therefore *biologically interpretable* and measurable by electrophysiology.
- (b) The curve is C^∞ — infinitely differentiable. There are no discontinuities, no fronts, and no parasitic high-frequency harmonics. The spectrum is naturally bounded.

8.3. Parametric Description of the Optimal Signal

The complete signal is composed of three concatenated segments Bézier, ensuring continuity in value and first derivative at junction points.

Segment 1 — Depolarization (duration $T_1 \approx 0.5\text{--}1$ ms).

$P_0 = (0, 0)$	resting potential
$P_1 = (0.1 T_1, 0.05)$	onset of Na^+ channel opening
$P_2 = (0.4 T_1, 0.85)$	rapid Na^+ activation
$P_3 = (T_1, 1.0)$	depolarization peak

This segment reproduces the apparent exponential activation of sodium channels — rapid but not discontinuous.

Segment 2 — Rapid repolarization (duration $T_2 \approx 1\text{--}2$ ms).

$P_0 = (T_1, 1.0)$	
$P_1 = (T_1 + 0.2 T_2, 0.9)$	Na^+ inactivation
$P_2 = (T_1 + 0.6 T_2, -0.1)$	K^+ activation
$P_3 = (T_1 + T_2, -A_h)$	hyperpolarization trough

where $A_h \approx 0.15\text{--}0.25$ (post-potential hyperpolarization, 15–25% of the depolarization amplitude).

Segment 3 — Return to rest (duration $T_3 \approx 3\text{--}8$ ms).

$P_0 = (T_1 + T_2, -A_h)$	
$P_1 = (T_1 + T_2 + 0.3 T_3, -0.7 A_h)$	K^+ inactivation
$P_2 = (T_1 + T_2 + 0.7 T_3, -0.1 A_h)$	Na/K pump
$P_3 = (T_1 + T_2 + T_3, 0)$	return to rest

Zero net charge condition.

$$\int_0^{T_1+T_2+T_3} \mathbf{B}(t) \, dt = 0$$

(4)

satisfied by numerically adjusting A_h and T_3 .

8.4. Comparative Visualization

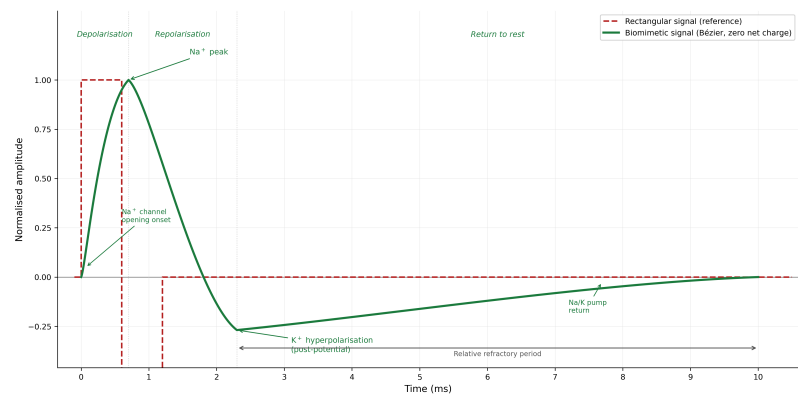


Figure 1. Comparison of the proposed biomimetic signal (green) and the standard rectangular signal (red, dashed). The biomimetic signal reproduces the phases of the action potential: progressive depolarization respecting Na^+ kinetics, peak, K^+ repolarization, post-potential hyperpolarization, and return to rest. Zero net charge is guaranteed. The rectangular signal presents non-physiological discontinuities at the rising and falling fronts.

8.5. Advantages of the Biomimetic Signal

1. **Concentrated spectrum:** absence of discontinuities $\Rightarrow$ energy naturally limited to the biological bandwidth
2. **Respect for ionic kinetics:** the signal *invites* channels to open according to their own natural dynamics, rather than forcing them through a discontinuity
3. **Stable impedance:** absence of high-frequency components $\Rightarrow$ predictable and constant load
4. **Reduced thermal energy:** minimization of energy deposited outside the biological band
5. **Biologically interpretable parameters:** each control point corresponds to a measurable and adjustable membrane time constant, tunable to the properties of the target tissue

9. Discussion

9.1. The Irony of the Unrecognized Sinusoid

The sinusoidal signal is the most universal mathematical form in nature. It is present in the main current that powers the stimulation device itself. And at the final step of the chain, it is deliberately destroyed to produce a rectangle.

The sinusoid is paradoxically foreign to us — not because it is rare, but because its “useful” duration cannot be read directly from an oscilloscope screen. We preferred the signal whose duration can be measured with a ruler on the display. The square wave has a visual advantage: the pulse duration is immediately apparent. The sinusoid does not offer this legibility.

We optimized for the human observer. We forgot to optimize for the neuron.

9.2. Toward Spectral Normalization of Neurostimulation Signals

This article argues for the introduction, in medical device safety standards for neurostimulation, of a minimum spectral specification:

- Mandatory declaration of the spectral content of the delivered signal (-3 dB cutoff frequency; energy fraction in the band > 10 kHz)
- Declaration of the calculated peak power
- Justification of spectral adequacy with respect to the biological bandwidth of the target tissue

These requirements are technically trivial for any manufacturer equipped with a spectrum analyzer. Their absence from current standards is the regulatory gap that has allowed the error to persist.

9.3. Limitations and Future Directions

The biomimetic signal proposed in this article is described on theoretical grounds and from available literature on ion channel time constants. Experimental validation requires:

1. In vitro electrophysiological studies comparing the efficiency of action potential triggering across waveforms (rectangular vs. Bézier biomimetic)
2. In vivo animal studies measuring stimulation threshold, fiber selectivity, and peri-electrode inflammatory response
3. Clinical studies comparing the analgesic efficacy and tolerability of biomimetic versus rectangular TENS
4. Systematic characterization of DBS peri-electrode fibrosis as a function of waveform

These studies are feasible with current technology. Arbitrary waveform generators can reproduce the described signal exactly. Digital signal processing enables real-time implementation Bézier at frequencies well above those required.

10. Open Publication and Priority

All parameters of the biomimetic signal described in this article are published under the Creative Commons Attribution 4.0 International License (CC-BY 4.0). This publication establishes permanent and opposable prior art. No patent application claiming this signal, its parameters, its calculation method, or its electronic implementation may be validly filed after the deposit date of this preprint.

Any manufacturer is free to implement this signal without royalties, provided the source is cited. Universal dissemination of the optimal signal is preferable to any industrial protection.

11. Conclusions

Electrotherapy has attempted for 150 years to achieve the quadrature of the circle: substituting a rectangular signal for the curved form of the neuronal action potential. Lindemann's 1882 proof of the mathematical impossibility of the quadrature finds its biophysical analog here: one cannot replace the curved signal of the neuron with a rectangular signal without energetic loss, membrane aggression, and clinical consequences.

This error is the product of a convergence: the visual convenience of the rectangular signal on an oscilloscope; technological drift toward ever-steeper wavefronts; insufficient signal theory training; and the absence of spectral requirements in medical device standards.

The proposed biomimetic signal, a Bézier curve whose control points correspond to the time constants of voltage-gated ion channels, is now implementable in real time with available technology. It is published freely.

The circle is not an approximation of an improved square. It is the form that the neuron has been waiting for all along.

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