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Newly Proposed Matrix Reduction technique Under Mean Ranking Method for Solving Trapezoidal Fuzzy Transportation problems Under Fuzzy Environment

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Abstract

In this paper, improved matrix Reduction Method is proposed for the solution of fuzzy transportation problem in which all inputs are taken as fuzzy numbers. Since ranking fuzzy number is important tool in decision making, Fuzzy trapezoidal number is converting in to crisp set by using Mean techniques and solved by proposed method for fuzzy transportation problem. We give suitable numerical example for unbalanced and compare the optimal value with other techniques. The Result shows that the optimum profit of transportation problem using proposed technique under robust ranking method is better than the other method. Novelty: The numerical illustration demonstrates that the new projected method for managing the transportation problems on fuzzy algorithms.

Keywords: Mean; Mean of Trapezoidal Fuzzy Numbers; Trapezoidal Fuzzy Numbers; Transportation Problem; and Fuzzy Transportation Problem

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Introduction

In our daily activities, problems such as fixing cost of goods, profit for sellers, giving decisions for real life multi-objective functions, balancing the need of customer and supply, etc. are seeking a solution by Transportation problem. But there are diverse situation due to the uncertainty, measurement inaccuracy, lack of confidence, computational errors, high information cost, whether condition. Due to this Zadeh, (1978) had introduced in the notion of fuzziness. A method of solving fuzzy transportation problem based on assumption of uncertainity about transportation cost was introduced by (Kaur & Kumar, 2012). A parametric plan for solving transportation problem under fuzziness is also proposed (Saad & Abass, 2003). (Hitchcock, 1941) initiated the fundamental transportation problem on the distribution of a product from several sources to numerous localities.

Transportation problems can be solved by using zero point method and zero suffix method to get the minimum cost. Mohideen & Kumar (2010) were used the zero point method of multiplication operation and concluded that it is better than both Vogel's Approximation method and the modified distribution method.

(Sharma et al., 2012) explained that the zero point method as the symmetric procedure for transportation problems is easy to be applied for any type of transportation problem with an objective function (maximum or minimum). This method provides the decisions for an optimal solution of the logistic problems in the transportation problem. In solving transportation problem, Pandian & Natarajan(2010) used zero point method with trapezoidal fuzzy number to find the optimal value of the objective function. This method is the systematic procedure that can be used to make the decisions, when we solve the various types of logistics problems involving fuzzy parameters. Concerning with the fuzzy transportation problem, Samuel (2012) also studied the method of the increasing of zero point, and concluded that this method was simpler and more efficient compared to VAM, SVAM, GVAM, RVAM, BVAM. He stated that this method was easy to be understood that provided the optimal solution.

In the currently, researchers are investigating regarding to transportation solution using zero suffix method. (Fegade et al, 2012) are applied zero suffix method under Robust Ranking method and fuzzy cost reaches optimal. (Christi, & Kumari, 2015) applied the robust ranking with the zero suffix method and they got the optimal solution of fuzzy problems accurately and effectively.

Pandian & Natarajan,(2010) proposed a new algorithm namely zero point method for finding optimal solution for a fuzzy transportation, the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers. Ngastiti & Surarso, (2018) investigate the number of iterations, allocations, optimized solutions of fully fuzzy transportation problems by using Zero Suffix methods and Zero Point methods with robust ranking technique and get the zero suffix method uses less iteration than Zero pint method, whereas the allocation and minimum optimum solution are the same.

Dinagar & Keerthivasan, (2020).are solving transportation problem with modern zero suffix method under fuzzy environment. Kalyani & Nagarani, (2020) are solved a fully fuzzy transportation problem with hexagonal fuzzy number. Geetha & Selvakumari (2020) are also proposed Recommended Range method for solving fuzzy transportation problem using pentagonal fuzzy numbers.

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The present study was proposed a new way of solving transportation problem, named Improved Matrix Reduction Method Using Mean Method to solve the fuzzy transportation problem ((Ngastiti & Surarso, 2018) and (Geetha & Selvakumari, 2020)).

Preliminaries

Definition

Fuzzy Transportation problem (FTP) is a linear programming problem with the specific structure. If in a transportation issue, all parameters and variables are fuzzy, then it includes fuzzy transportation issues. The fuzzy transportation issue (FTP) is formulated as follows:

$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij,}$$

Subject to constraint

$$\sum_{j=1}^{n} x_{ij,} = a_i \, for \; i = 1,2,3,\dots,m.$$

$$\sum_{i=1}^{n} x_{ij,} = b_j \, for \; j = 1,2,3,\dots,n.$$

$$\sum_{j=1}^{n} a_i = a_i \approx \sum_{j=1}^{n} b_i = b_j \quad for x_{ij,} \geq 0 \; for \; all \; i \; and \; j.$$

Definition for fuzzy set membership function

A fuzzy set A is characterized by a membership function, mapping elements of a domain, space or universe of discourse X to the unit closed interval [0, 1]. That is,$A = \{\langle x, \mu_A(x) | x \epsilon X\rangle\}$ it is mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$is called membership value of $x\epsilon X$, in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1]

Definition for Normal Fuzzy Set

A fuzzy set A of the universe of discourse X it is called normal fuzzy set if there exist at least one $x\epsilon X$ such that $\mu_A(x) = 1$

Definition for interval number

Let R, it is the set of real numbers and then a closed interval [a, b], it is said to be an interval number, where $a, b \,\epsilon \mathbb{R} \; with \; a \leq b$

Convex normalized fuzzy set

A fuzzy number μ, it is a convex normalized fuzzy set of the crisp set such that for only one $x\epsilon X$ and $\mu_A(x) = 1 \; and \; \mu_A(x)$, it is piecewise continuous

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Support of Fuzzy number

Let u, it is a fuzzy number and then the support of μ is defined by$\sup(u) = \overline{\{x|u(x) > 0\}}$ where $\overline{\{x|u(x) > 0\}}$, represents the closure of $\{x|u(x) > 0\}$

Fuzzy Number Concepts

A fuzzy set A, of the real R, with a membership function $\mu_A: \mathbb{R} \Rightarrow [0, 1], it\ is\ called$ fuzzy number if

a. A, it must be convex and normal fuzzy set
b. The support of A, it must be bounded
c. α_A, it must be closed interval for every $\alpha \epsilon [0,1]$

Definition for Trapezoidal Fuzzy Number

A fuzzy number$A = (a, b, c, d)$,it is said to be Hexagonal Fuzzy number if it its membership function,$\mu_A(x)$ it is given by

$$\mu_A(x) = \begin{cases} 0, if\ x \leq a \\ \frac{x-a}{b-a}, if\ \ b \leq x \leq a \\ 1, if\ b \leq y \leq c \\ \frac{d-x}{d-c}, if\ c \leq y \leq d \\ 0,\ \ if\ x \geq a \end{cases}$$

Arithmetic on Trapezoidal Fuzzy Numbers

Let us take two Hexagonal Fuzzy numbers are

$$A_H = (a_1, a_2, a_3, a_4)\ and\ \ B_H = (b_1, b_2, b_3, b_4)$$

and then

a. $A_H + B_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
b. $A_H - B_H = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
c. $A_H \times B_H = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4)$

Mean Technique

What Is Mean?
A mean is the simple mathematical average of a set of two or more numbers. The mean for a given set of numbers can be computed in more than one way, including the arithmetic mean method, which uses the sum of the numbers in the series, and the geometric mean method, which is the average of a set of products. However, all of the primary methods of computing a simple average produce the same approximate result most of the time.

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The arithmetic mean is the simplest and most widely used measure of a mean, or average. It simply involves taking the sum of a group of numbers, then dividing that sum by the count of the numbers used in the series. For example, take the numbers 34, 44, 56, and 78. The sum is 212. The arithmetic mean is 212 divided by four, or 53.

Mathematical Formulation

Proposed Algorithm

Step1. Construct the transportation table and examine whether total demand equals total supply then go to step 2. If it is not balanced add the dummy source then go to step 2.

Step2. By using Mean technique, convert the fuzzy cost into crisp values to the given transportation problem

Step3. For the row-wise difference between maximum and minimum of each row, and it is divided by the number of columns of the cost matrix.

Step4. For the column-wise difference between maximum and minimum of each column, and it is divided by the number of rows of the cost matrix.

Step5. We find the maximum of the resultant values and find the corresponding minimum cost value and do the allocation of that particular cell of the given matrix. Select any one if more than one maximum consequent value are there.

Step6. Repeated procedures 1 to 5 until all the allocations are completed.

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Result and Discussion

Numerical Example

Consider the Fuzzy Transportation Problem that is a company has three sources they are A, B, C and three destinations, they areD, E, F and the fuzzy transportation cost for unit quantity of the product from i^{th} source and j^{th}destination, it is given by

From	D	E	F	SUPPLY
A	(35000,5000,60000,75000)	(45000,60000,70000,90000)	(30000,45000,55000,70000)	(2,3,5,7)
B	(55000,65000,80,000,100000)	(55000,70000,85000,115000)	(45,0000,60000,75000,95000)	(0,1,3,6)
C	(40000,,45000,55000,80000)	(45000,55000,650000,85000)	(40000,50000,55000,70000)	(1,3,4,5)
Demand	(1,2,4,6)	(2,4,5,8)	(1,3,5,7)	

Table1: Fully Fuzzy Transportation Table

We use Mean techniques for the defuzzification process. Next, we give comparison of results of the implication of zero point method and zero suffix method.

By Mean Method with new proposed Method

$$M(\tilde{a}) = \frac{a+b+c+d+e+f}{6}, for\ \tilde{a}\ is\ fuzzy\ Number$$

$$M(35000{,}5000{,}60000{,}75000) = \frac{35000+5000+60000+75000}{4} = 30250$$

Similarly

$M(45000{,}60000{,}70000{,}90000) = 66250, M(30000{,}45000{,}55000{,}70000) = 50{,}000,$

$M(55000{,}65000{,}80{,}000{,}100000) = 75000, M(55000{,}70000{,}85000{,}115000) = 81250,$ M$(45{,}0000{,}60000{,}75000{,}95000) = 68750$

Rank of all Supply

$$M(2,3,5,7) = 4.25, M(0,1,3,6) = 2.5\ \ M(1,3,4,5) = 3.25$$

Rank of all Demand

$$M(1,2,4,6) = 3.25, M(2,4,5,8) = 4.75, M(1,3,5,7) = 4$$

From	D	E	F	SUPPLY
A	30250	66250	50,000	4.25
B	75000	81250	68750	2.5

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C	55000	62500	53750	3.25
demand	3.25	4.75	4	

Table2: Crisp Transportation Table

From table 2, we get the total supply and total demand as follows:

$$\sum_{j=1}^{n} a_i = 4.25 + 2.5 + 3.25 = 10$$

$$\sum_{j=1}^{n} b_i = 3.25 + 4.75 + 4 = 12$$

It was found that $\sum_{j=1}^{n} a_i < \sum_{j=1}^{n} b_i$ so the transportation problem was not balanced. Next, to be balanced,

$\sum_{j=1}^{n} a_i = \sum_{j=1}^{n} b_i$, then we have the following table 3.

From	D	E	F	SUPPLY
A	55000	66250	50,000	4.25
B	75000	81250	68750	2.5
C	55000	62500	53750	3.25
dummy	0	0	0	2
demand	3.25	4.75	4	

Table3. Balanced Full Fuzzy Transportation Table

Implementation of Proposed Method

In this section we apply proposed method to get optimum solution of cost transportation problem.

from	D	E	F	SUPPLY	$\frac{max - min}{3}$
A	55000	66250	50,000	4.25	5416.67
B	75000	81250	68750	2.5	4166.67
C	55000	62500	53750	3.25	2916.67
dummy	0	0	0	2	
demand	3.25	4.75	4		
$\frac{max - min}{3}$	6666.67	6250	6250		

Table 4: row-wise difference and column-wise difference between maximum and minimum of each row (column), and it is divided by the number of columns (rows) of the cost matrix

Again we find the maximum of the resultant values and find the corresponding minimum cost value and allocate the particular cost cell of the given matrix. If we have more than one maximum resultant values, we can select anyone.

From	D	E	F	SUPPLY	$\frac{max - min}{3}$
A	$55000^{3.25}$	66250	50,000	4.25	5416.67
B	75000	81250	68750	2.5	4166.67
C	55000	62500	53750	3.25	2916.67
Dummy	0	0	0	2	
demand	3.25	4.75	4		
$\frac{max - min}{2}$	10,000	9375	9375		

Table 5: the first allocation cost cell of the given matrix

The same procedure will be followed again and again until we reach the final allocation.

From	D	E	F	SUPPLY
A	$55000^{3.25}$	66250	$50,000^{1}$	4.25
B	75000	$81250^{1.5}$	68750^{1}	2.5
C	55000	62500	53750	3.25
Dummy	0	0	0^{2}	2
Demand	3.25	4.75	4	

Table6. Optimal Solution with Matrix Reduction Method

Table 6 is a transportation table with an optimal value using the newly proposed Matrix Reduction method.

By solving newly method we get the following allocation as follows:

$$x_{11} = 3.25, x_{13} = 1, x_{22} = 1.5, x_{23} = 1, x_{32} = 3.25$$

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We get the minimum transportation costas follows:

$$Z = 55000 * 3.25 + 3.25 * 62500 + 1 \times 50{,}000 + 1.5 \times 81250 = 553750$$

Comparison to the existing method

The comparison of the proposed method with the existing plan is tabulated below, in which it is clearly shown that the proposed method provides the optimal results

Name of method	Optimum solution
Zero point method under robust ranking method (Ngastiti & Surarso, (2018))	597500
Zero suffix method under robust ranking method (Ngastiti & Surarso, (2018))	597500
Proposed method	553750

Table 7: Comparison Table of the Result

Conclusion

In this paper, the transportation costs are considered as imprecise fuzzy numbers. Here, Mean Method had been transformed fuzzy transportation problem into crisp transportation. We can have the optimal solution from crisp and fuzzy optimal total cost of given example. The comparison result shows that the optimal value of proposed method is 553750, which is better than zero point method and zero suffix method 597.500.

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