Technical Note

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The CMB Temperature Is Simply the Geometric Mean: $T_{cmb} = \sqrt{T_{min} T_{max}}$ of the Minimum and Maximum Temperature in the Hubble Sphere

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The CMB Temperature Is Simply the Geometric Mean: $T_{cmb} = \sqrt{T_{min}T_{max}}$ of the Minimum and Maximum Temperature in the Hubble Sphere

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Abstract: In the Hubble sphere, we assume that the wavelength of pure energy spreads out in all directions. The maximum wavelength in the Hubble sphere is then the circumference of the Hubble sphere. We assume the minimum wavelength occurs in a Planck mass black hole, which is given by $4\pi R_{s,p} = 8\pi l_p$. Here, we build further on the geometric mean CMB approach by Haug and Tatum and conclude that the CMB temperature is simply given as: $T_{cmb} = \sqrt{T_{min}T_{max}}$, which is the geometric mean of the minimum and maximum physically possible temperatures in the Hubble sphere. This is again means the CMB temperature simply is the geometric mean of the Hawking temperature of the Hubble sphere (in black hole cosmology) and the Hawking temperature of the Planck mass black hole, se we have also $T_{cmb} = \sqrt{T_{Haw,H}T_{Haw,p}}$.

Keywords: CMB temperature; geometric mean temperature; minimum temperature; maximum temperature; Hubble sphere; cosmology

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1. Introduction

The Λ-CDM model, despite its success in many areas, is not able to predict the CMB temperature today. See, for example, Narlikar and Padmanabhan [1], which states:

> *"The present theory is, however, unable to predict the value of T at $t = t_0$. It is therefore a free parameter in SC (Standard Cosmology)."*

The CMB temperature is likely the most precisely measured cosmological parameter [2–5], but it is clearly not fully understood within Λ-CDM cosmology. In recent years, however, there has been a breakthrough in understanding the CMB temperature and its connection to the Hubble parameter, which we will soon revisit.

We will be operating within a black hole $R_{H_t} = ct$ cosmology. Although black hole cosmology is much less well-known than Λ-CDM, it is not new; it dates back at least to 1972 with a paper by Pathria (1972) [6]. The topic continues to be actively discussed by various researchers to this day [7–16].

There are also multiple variations of $R_{H_t} = ct$ cosmologies, all of which share the common feature that the universe has expanded—or is at least related to—the speed of light; see [17–22]. The Melia $R_H = ct$ model is the best known among these, and he has done a tremendous job demonstrating that $R_H = ct$ cosmology can perform at least as well as, and often better than, the Λ-CDM model.

However, in this work, we will focus specifically on black hole $R_{H_t} = ct$ cosmology, as described by Haug and Tatum [23], which is a subcategory within $R_{H_t} = ct$ cosmologies.

2. The CMB Temperature as a Geometric Mean of the Minimum and Maximum Temperature in the Hubble Sphere

The geometric mean plays an important role in thermodynamics and in other areas of physics [24–27]. For example, the optimal reheating pressure is given as the geometric mean of the maximum

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and minimum pressure: $P_{reheating} = \sqrt{P_{max}P_{min}}$, and the optimal intercooling in an ideal two-stage compressor is also given by the geometric mean pressure, $P_{intercooling} = \sqrt{P_{max}P_{min}}$, see [28]. The geometric mean temperature: $\sqrt{T_{hot}T_{cold}}$ play a central role in Carnot engines where it defines a type of equilibrium, see [29,30]. That geometric means could also potentially play an important role in the thermodynamics of cosmic temperatures should not come as a surprise.

Haug and Tatum [31] have recently shown that the CMB temperature, at a deeper physical level, is likely just linked to the geometric mean of the shortest and longest possible wavelengths in the Hubble sphere. They presented their formula as:

$$T_{cmb} = \hbar \frac{c}{\sqrt{\bar{\lambda}_{min}\bar{\lambda}_{max}}} \frac{1}{4\pi k_b} = \hbar \frac{c}{\bar{\lambda}_{gm}} \frac{1}{4\pi k_b} \tag{1}$$

where they assumed the shortest wavelength $\bar{\lambda}_{min} = l_p = \sqrt{\frac{G\hbar}{c^3}}$ was the Planck [32,33] length and the maximum wavelength was the diameter of the Hubble sphere $\bar{\lambda}_{max} = 2R_H$, they mention also the circumference could be the limiting factor. This, again, they demonstrate to be consistent with the CMB formula heuristically first suggested by Tatum et al. [34]. Haug and Wojnow [35] have further demonstrated the CMB formula fully consistent with this can be derived from the Stefan-Boltzmann law. The Stefan-Boltzmann law holds for a perfect black body and the CMB is the closest we likely get to a perfect black body in the real world as stated by for example Muller et al. [36] :

> *"Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body characterized by a temperature T_0 at $z = 0$, which is measured with very high accuracy, $T_0 = 2.72548 \pm 0.00057k$."*

Haug [37] has recently expanded on the geometric mean approach of Haug and Tatum and shown that the CMB formula can even be written directly in the form:

$$T_{cmb} = \frac{\sqrt{T_{max}T_{min}}}{4\pi} = \frac{\sqrt{T_p T_{min}}}{4\pi} \tag{2}$$

Where he suggested $T_{max} = T_p = \frac{1}{k_b} = \sqrt{\frac{\hbar c^5}{G}} = \frac{m_p c^2}{k_b} = \frac{E_p}{k_b}$, which is the Planck [32,33] temperature, and $T_{min} = \hbar \frac{c}{2R_H} \frac{1}{k_b}$. Furthermore k_b is the Boltzmann constant.

The question is: why does the geometric mean temperature have to be multiplied by the constant $\frac{1}{4\pi}$? We now think we have an answer to that. Electromagnetic waves (such as CMB radiation), as well as matter waves, tend to spread out in all directions—like throwing a stone into a lake, where the waves propagate in all directions. If a Planck-mass Schwarzschild black hole is the smallest black hole and the Hubble sphere is a cosmic black hole, then the maximum wavelength is the circumference of the Hubble sphere, $\bar{\lambda}_{max} = 4\pi R_H$, and the minimum wavelength is the circumference of a Schwarzschild Planck-mass black hole, which is $\bar{\lambda}_{min} = 4\pi R_{s,p} = 8\pi l_p$.

The minimum and maximum energies and temperatures in the Hubble sphere are then:

$$E_{min} = \hbar \frac{c}{4\pi R_{H_t}}, \qquad T_{min} = \frac{E_{min}}{k_b} \tag{3}$$

and the maximum energy and maximum temperature is then

$$E_{max} = \hbar \frac{c}{4\pi R_{s,p}} = \hbar \frac{c}{8\pi l_p}, \qquad T_{max} = \frac{E_{max}}{k_b} \tag{4}$$

The CMB temperature is then always given by

$$T_{cmb} = \sqrt{T_{max}T_{min}} \approx 2.725K \tag{5}$$

While the maximum temperature is always constant, $T_{max} = \hbar \frac{c}{8\pi l_p} \frac{1}{k_b}$, the minimum temperature varies as we travel along the cosmic epoch as we assume $R_{H_t} = ct$. It is worth mention that the minimum temperature now always is equal to the Hawking [38] temperature $T_{min} = \frac{\hbar c}{4\pi R_H} = T_{Haw} =$

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$\frac{\hbar c}{4\pi R_s}$ when the Hubble radius is equal to the Schwarzschild radius $R_H = R_s$ as it will be in a black hole Hubble universe where the equivalent mass is the critical Friedmann mass.

This means the CMB temperature also can be seen as simply the geometric mean of the Hawking temperature of the Hubble sphere and a Planck mass black hole. We call the Hawking temperature of the Hubble sphere for the Hawking Hubble temperature:

$$T_{min} = T_{Haw,H} = \frac{\hbar c^3}{k_b 8\pi G M_{BH}} = \hbar \frac{c}{4\pi R_s}\frac{1}{k_b} = \hbar \frac{c}{4\pi R_H}\frac{1}{k_b} \tag{6}$$

We here assume that the mass of the Hubble sphere is the critical Friedmann [39] mass, $M_{cr} = \frac{c^2 R_H}{2G}$. If we solve for the Hubble radius in terms of the critical Friedmann mass, we get $R_H = \frac{2GM_{cr}}{c^2}$, and we can see that it must be identical to the Schwarzschild radius of a black hole with mass equal to the critical Friedmann mass: $R_s = \frac{2GM}{c^2}$. This is not a new result, but it is important for understanding why we can apply the Hawking temperature to a black hole Hubble sphere universe.

This represents the minimum temperature within the Hubble sphere. In addition, we have the Hawking–Planck temperature, which is the Hawking temperature of a Schwarzschild black hole, given by:

$$T_{max} = T_{Haw,p} = \hbar \frac{c}{4\pi R_{s,p}}\frac{1}{k_b} = \hbar \frac{c}{4\pi 2 l_p}\frac{1}{k_b} = \hbar \frac{c}{8\pi l_p}\frac{1}{k_b} \tag{7}$$

The CMB temperature is then given by:

$$T_{cmb} = \sqrt{T_{max} T_{min}} = \sqrt{T_{Haw,H} T_{Haw,p}} \approx 2.725K \tag{8}$$

Based on a $H_0 \approx 66.9\ km/s/Mpc$ as reported by Haug and Tatum [22]. We further assume it follows the $R_{H_t} = ct$ cosmology, where the circumference of the black hole Hubble sphere was smaller in the past. The geometric mean formula is consistent with the observed relation $T_t = T_0(1+z)$, see [4,40–42].

Alternatively we can express the CMB temperature from energies:

$$T_{cmb} = \sqrt{E_{max} E_{min}} \frac{1}{k_b} \tag{9}$$

The maximum energy is naturally much smaller than the energy in the Hubble sphere, this can be seen as the maximum possible energy from a single particle or photon (or perhaps even graviton) that we conjecture is linke to the Planck scale and actually a Schwarzschild Planck mass black hole. It is common for researchers working on quantum gravity to assume the Planck scale will play an important role, see for example [43–46]. So it should not be a big surprise the Planck scale also play an important role for the CMB.

It is naturally remarkable that, based on recent years of research on the CMB temperature, we can now accurately predict the CMB temperature today—something the Λ-CDM model has not been able to do and still cannot, as it is not compatible with $R_H = ct$ black hole cosmology. Even more important than predicting the CMB temperature is the fact that the approach developed in recent years has found the exact mathematical relationship between the CMB temperature and the Hubble parameter. Tatum et al. [47], as well as Haug and Tatum [22], have recently demonstrated that one can predict the Hubble parameter much more precisely than with other methods. This is possible because the CMB temperature can be used to determine the Hubble constant due to these new exact mathematical relationships.

Based on the geometric mean approach above we get:

$$H_0 = \frac{T_{cmb,0}^2}{T_{max}}\frac{k_b 4\pi}{\hbar} = \frac{T_{cmb,0}^2}{T_{Haw,p}}\frac{k_b 4\pi}{\hbar} = 66.8943 \pm 0.0287\ km/s/Mpc \tag{10}$$

we have used the Fixsen [3] measured CMB temperature now (at $z = 0$): $T_0 = 2.72548 \pm 0.00057K$. This is as expected in line with the research just mentioned above.

3. Conclusion

Based on years of research on the CMB temperature by several authors, we can now conclude that the CMB temperature, in its simplest and most understandable form, is simply the geometric mean of the minimum and maximum temperatures possible in the Hubble sphere. The CMB temperature is given by $T_{cmb} = \sqrt{T_{max}T_{min}}$. This also means that the CMB temperature is the geometric mean temperature of the Hawking Hubble temperature and the Hawking Planck temperature: $T_{cmb} = \sqrt{T_{max}T_{min}} = \sqrt{T_{Haw,H}T_{Haw,p}} \approx 2.725K$.

This has important implications, as it provides a precise mathematical relationship between the CMB temperature and the Hubble parameters, as well as a deeper physical understanding of the CMB temperature. Unlike in the Λ-CDM model, we can now accurately predict the CMB temperature in black hole $R_{H_t} = ct$ cosmology. In addition, we can predict the Hubble parameter much more precisely, as recentyl demonstrated by Tatum et al. [47] and Haug and Tatum [22].

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