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Article

Thermodynamic Cosmology from the CMB Temperature and the Planck Temperature and The Relation to Quantum Cosmology

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Abstract: In this paper, we demonstrate that, in $R_H = ct$ cosmology, a wide range of cosmological parameters and properties can be expressed simply as functions of the CMB temperature relative to the Planck temperature, as well as the Hawking temperature relative to the CMB temperature. This approach offers a novel framework for studying the thermodynamics of the cosmos within black hole cosmology. Notably, it enhances the precision of predictions and introduces a minimum acceleration associated with a mass gap, which may provide a plausible explanation for galaxy rotation curves.

Keywords: Hubble sphere; Hubble geometry; Hubble sphere properties; CMB temperature; Hawking temperature; Planck temperature

1. CMB Temperature from the Stefan-Boltzman Law

Haug and Wojnow [1,2] have demonstrated that, from the Stefan-Boltzmann [3,4] law, when assuming $R_H = ct$ cosmology, one can derive the following equation for the CMB temperature:

$$T_0 = \frac{T_p}{8\pi} \sqrt{\frac{2l_p}{R_h}} \quad (1)$$

where $R_h = \frac{c}{H_0}$ is the Hubble radius. This can also be written as $T_0 = \frac{T_p}{8\pi} \sqrt{\frac{2l_p H_0}{c}} = \frac{T_p}{8\pi} \sqrt{2t_p H_0}$, where l_p is the Planck [5,6] length, t_p is the Planck time, and $T_p = \frac{1}{k_b} \sqrt{\frac{\hbar c^5}{G}} = \frac{m_p c^2}{k_b} = \frac{E_p}{k_b}$ is the Planck temperature, with k_b being the Boltzmann constant.

The importance of deriving the CMB temperature from the Stefan-Boltzmann law becomes clear when one understands that the CMB is an almost perfect black body and that the Stefan-Boltzmann law is valid only for systems close to perfect black bodies. Muller et al. [7] point out:

“Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body characterized by a temperature T_0 at $z = 0$, which is measured with very high accuracy, $T_0 = 2.72548 \pm 0.00057\text{K}$.”

The equation derived by Haug and Wojnow from the Stefan-Boltzmann law has also been shown to be fully consistent with a CMB formula suggested heuristically by Tatum et al. [8]. Additionally, the same CMB formula has recently been demonstrated to be derivable using a geometric mean approach by Haug and Tatum [9], as well as an approach involving quantized bending of light [10].

Equation (1) is valid for any previous cosmic epoch in an $R_H = ct$ universe and is expressed in its more general form as:

$$T_0 = \frac{T_p}{8\pi} \sqrt{\frac{2l_p}{R_t}} = \frac{T_p}{8\pi} \sqrt{2t_p H_t} \quad (2)$$

where the Hubble radius $R_H = R_t = \frac{c}{H_t}$ or the time dependent Hubble parameter: H_t is the only time-dependent parameter that follows the $R_t = ct$ principle (see [11–16]). This implies that the universe either expanded only at the speed of light or that we could be living in a steady-state black

hole universe, where light signals inside the sphere relative to us adhere to the $R_t = ct$ principle. Simply put, photons cannot travel faster than the well-known speed of light, c . The idea that the Hubble sphere could be a gigantic black hole is not new and have been suggested at least since 1972 by Pathria [17] and the idea of a black hole universe is actively discussed to this day (see [18–22]). The hypothesis of black hole universe is clearly much less popular than the Λ -CDM model, but it in our view deserved further investigation. It is first when the different hypothesis of the universe is extensively explored one can truly compare the different models in great detail.

2. Thermodynamic Cosmology from the CMB Temperature and the Planck Temperature

The Friedmann [23] equation is given by:

$$H_0^2 = \frac{8\pi G\rho_c + \Lambda c^2}{3} - \frac{k^2 c^2}{3} \quad (3)$$

In the case of the critical Friedmann equation under flat space ($k = 0$), we have:

$$H_0^2 = \frac{8\pi G\rho_c}{3} \quad (4)$$

where $\rho_c = \frac{M_c}{\frac{4}{3}\pi R_H^3} = \frac{3H_0^2}{8\pi G}$ is the critical Friedmann density.

As $H_0 = 32\pi^2 f_p \frac{T_0^2}{T_p^2}$, where $f_p = \frac{c}{l_p}$ is the Planck frequency, we can replace H_0 in the Friedmann equation, resulting in:

$$\begin{aligned} T_0^4 &= \frac{G\rho_c t_p^2 T_p^4}{384\pi^3} \\ T_0^4 &= \frac{l_p^4 c \rho_c T_p^4}{\hbar 384\pi^3} \\ T_0^4 &= \frac{\rho_c T_p^4}{\frac{m_p}{\frac{4}{3}\pi l_p^3} 512\pi^4} \\ T_0^4 &= T_p^4 \frac{\rho_c}{512\pi^4 \rho_p} \end{aligned} \quad (5)$$

where $\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$ is the Planck mass density for a Planck sphere. Often the Planck density is for simplicity defined as a Planck cube $\frac{m_p}{l_p^3}$ see Unnikrishnan and Gillies [24], but since we are working with black holes and as we will see the spherical Schwarzschild metric sphere density is more relevant.

This provides a way to express the Friedmann equation in thermodynamic form. This approach is rooted in what has been presented by Haug and Tatum [25], yielding the same output, but here the Friedmann equation itself written on a somewhat different form, with an emphasis on expressing it in terms of the Planck temperature in addition to the CMB temperature, thereby adhering more strictly to thermodynamics. Solving for the critical mass, we obtain:

$$\rho_c = 512\pi^4 \rho_p \frac{T_0^4}{T_p^4} \quad (6)$$

For the case of the critical density in past cosmic epochs, we have:

$$\rho_{c,t} = 512\pi^4 \rho_p \frac{T_0^4 (1+z)^4}{T_p^4} = 512\pi^4 \rho_p \frac{T_t^4}{T_p^4} \quad (7)$$

and further:

$$T_0 = \frac{T_p}{4\pi} \left(\frac{\rho_c}{2\rho_p} \right)^{1/4} \quad (8)$$

Table 1 shows a series of cosmological properties described as functions of the CMB temperature and the Planck temperature. In any of these formulas, T_0 can be replaced with T_t if one wants predictions for earlier cosmic epochs. This has major implications, as the CMB temperature has been measured much more precisely than H_0 . Additionally, it provides new insights into how the CMB temperature and Planck temperature appear to be closely connected to the cosmos.

Table 1. This table summarizes a series of properties for the Hubble sphere in an $R_h = ct$ universe, expressed in terms of the CMB temperature and the Planck temperature.

Property :	Expression	Comment
CMB temperature	$T_0 = T_p \frac{\sqrt{2t_p H_0}}{8\pi}$	See [1,2,8].
Hubble constant (Hubble frequency):	$H_0 = 32\pi^2 f_p \frac{T_0^2}{T_p^2}$	$f_p = \frac{c}{l_p}$. See [1,2,8].
Hubble radius	$R_H = \frac{c}{H_0} = \frac{l_p}{32\pi^2} \frac{T_p^2}{T_0^2}$	See [1].
Hubble time	$t_H = \frac{1}{H_0} = \frac{t_p}{32\pi^2} \frac{T_p^2}{T_0^2}$	see [1].
Hubble diameter	$D_H = 2R_H = \frac{l_p}{16\pi^2} \frac{T_p^2}{T_0^2}$	
Hubble circumference	$C_H = 2\pi R_H = \frac{l_p}{16\pi} \frac{T_p^2}{T_0^2}$	
Hubble surface	$A_H = 4\pi R_H^2 = \frac{l_p^2}{256\pi^3} \frac{T_p^4}{T_0^4}$	
Hubble volume	$V_H = \frac{4}{3}\pi R_H^3 = \frac{l_p^3}{24576\pi^5} \frac{T_p^6}{T_0^6}$	
Gaussian curvature	$k_H = \frac{1}{R_H^2} = \frac{1024\pi^4}{l_p^2} \frac{T_0^4}{T_p^4}$	
Critical mass	$M_c = \frac{c^3}{2GH_0} = \frac{m_p}{64\pi^2} \frac{T_p^2}{T_0^2}$	See [1].
Critical energy	$E_c = \frac{c^5}{2GH_0} = \frac{E_p}{64\pi^2} \frac{T_p^2}{T_0^2}$	See [1].
Critical mass density	$\rho_c = \frac{3H_0^2}{8\pi G} = 512\pi^4 \rho_p \frac{T_0^4}{T_p^4}$	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$.
Cosmological constant	$\Lambda = \left(\frac{H_0}{c} \right)^2 \Omega_\Lambda = \frac{1024\pi^4}{l_p^2} \frac{T_0^4}{T_p^4} \Omega_\Lambda$	
CMB and z	$z = \frac{T_0}{T_t} - 1$	Well known [27,28].
Hubble entropy	$S_{BH} = \frac{A}{4l_p^2} = \frac{1}{1024\pi^3} \frac{T_p^4}{T_0^4}$	See [1].
Mass gap	$m_{g,r} = \frac{m_p^2}{2M_c} = m_p 32\pi^2 \frac{T_0^2}{T_p^2}$	See [38].
Energy gap	$E_{g,r} = \frac{m_p^2 c^2}{2M_c} = E_p 32\pi^2 \frac{T_0^2}{T_p^2}$	$E_p = m_p c^2$
Minimum acceleration (radius)	$g_{min} = \frac{Gm_{g,r}}{l_p^2} = a_p 32\pi^2 \frac{T_0^2}{T_p^2}$	$a_p = \frac{Gm_p}{l_p^2}$. See [38].
Minimum acceleration (circumference)	$g_{min} = \frac{Gm_{g,c}}{l_p^2} = a_p 16\pi \frac{T_0^2}{T_p^2}$	$a_p = \frac{Gm_p}{l_p^2}$. See [38].

The mass gap is likely new to most readers. This is based on the assumption that the maximum reduced Compton wavelength is limited by the radius or alternatively the circumference of the Hubble sphere. The mass gap is if the Hubble radius is the limitation of this wavelength equal to $m_{g,r} = \frac{m_p^2}{2M_c}$, where $m_p = \sqrt{\frac{\hbar c}{G}}$ is the Planck mass and $M_c = \frac{c^3}{2GH_0}$ is the critical Friedmann mass, for derivation of the mass gap see [38]. Interestingly the mass gap leads to a gravitational gap, or we can call it a minimum acceleration equal to:

$$g_{min} = \frac{Gm_{g,r}}{l_p^2} = \frac{G}{l_p^2} \frac{m_p^2}{2M_c} = a_p 32\pi^2 \frac{T_0^2}{T_p^2} \approx 6.5 \times 10^{-10} \text{ m/s}^2 \quad (9)$$

where a_p is the gravitational acceleration. Or if alternatively the circumference of the Hubble sphere is the limitation factor for the maximum wavelength we get a minimum acceleration (gravitational acceleration gap) of:

$$g_{min} = \frac{Gm_{g,c}}{l_p^2} = a_p 16\pi \frac{T_0^2}{T_p^2} \approx 1.04 \times 10^{-10} \text{ m/s}^2 \quad (10)$$

This is very close to the minimum acceleration one operates with in the MOND model of Milgrom [26] that is one of the alternative models to explain galaxy rotation curves. The advantage of the minimum acceleration that comes out from the model here is that it also give a simple explanation for the cause of observed minimum gravitational acceleration. it is simply related to that all matter can never have a smaller gravitational acceleration than what is related to the mass gap inside the Hubble sphere. Again this seems to lead to no need for dark matter. That said this should naturally be carefully studied before one makes firm conclusions, but it is a new hypothesis worth investigating, but is the main topic of the paper just referred to and not of this paper which is more an overview paper of how one can write a series of cosmological properties on what we can call thermodynamic forms.

As the CMB temperature [29–31] can be much more precisely measured than H_0 (see [32–35]), this leads to a dramatic improvement in predictions of all these cosmological properties inside $R_H = ct$ cosmology; see [36]. This is naturally true even after taking into account uncertainty in the Planck units used in the formulas above. It even leads to a much more precise Hubble constant, as first demonstrated by Tatum et al. [37].

3. The Hawking Temperature Based Friedmann Equation

Haug [1] has demonstrated that:

$$H_0 = \frac{1}{2} f_p \frac{T_{Haw}^2}{T_0^2} \quad (11)$$

where T_{Haw} is the Hawking [39] temperature:

$$T_{Haw} = \frac{\hbar c}{k_b 4\pi R_s} \quad (12)$$

Here, R_s is the Schwarzschild radius, which we have shown in the previous section to be identical to the Hubble radius in a critical Friedmann universe. Thus, we have:

$$T_{Haw} = \frac{\hbar c H_0}{k_b 4\pi c} \quad (13)$$

Unlike the Planck temperature, determining the Hawking temperature still depends on knowing the Hubble constant. However, this provides valuable insight into how many cosmological parameters of the universe relate to the ratio between the CMB temperature and the Hawking temperature in a black hole universe.

This means we can re-write the critical Friedmann equation simply by replacing H_0 with this expression, resulting in:

$$\begin{aligned}
H_0^2 &= \frac{8\pi G\rho_c}{3} \\
\frac{1}{4}f_p^2 \frac{T_{Haw}^4}{T_0^4} &= \frac{8\pi G\rho_c}{3} \\
T_0^4 &= \frac{T_{Haw}^4 \rho_p}{8 \rho_c} \\
T_0 &= T_{Haw} \left(\frac{\rho_p}{8\rho_c} \right)^{1/4}
\end{aligned} \tag{14}$$

Solved for the critical density we get:

$$\rho_c = \frac{1}{8}\rho_p \frac{T_{Haw}^4}{T_0^4} \tag{15}$$

where $\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$ is the Planck mass density. Table 2 shows a series of cosmological properties expressed from the Hawking temperature and the CMB temperature consistent with $R_H = ct$ cosmology.

Table 2. This table summarizes a series of properties for the Hubble sphere in an $R_h = ct$ universe, expressed in terms of the CMB temperature and the Hawking temperature.

Property :	Expression	Comment
Hubble constant (Hubble frequency):	$H_0 = \frac{1}{2}f_p \frac{T_{Haw}^2}{T_0^2}$	$f_p = \frac{c}{l_p}$. See [1,2,8].
Hubble radius	$R_H = \frac{c}{H_0} = 2l_p \frac{T_0^2}{T_{Haw}^2}$	See [1].
Hubble time	$t_H = \frac{1}{H_0} = 2t_p \frac{T_0^2}{T_{Haw}^2}$	see [1].
Hubble diameter	$D_H = 2R_H = 4l_p \frac{T_0^2}{T_{Haw}^2}$	
Hubble circumference	$C_H = 2\pi R_H = 4\pi l_p \frac{T_0^2}{T_{Haw}^2}$	
Hubble surface	$A_H = 4\pi R_H^2 = 16\pi l_p^2 \frac{T_0^4}{T_{Haw}^4}$	
Hubble volume	$V_H = \frac{4}{3}\pi R_H^3 = \frac{32}{3}\pi l_p^3 \frac{T_0^6}{T_{Haw}^6}$	
Gaussian curvature	$k_H = \frac{1}{R_H^2} = \frac{1}{4l_p^2} \frac{T_{Haw}^4}{T_0^4}$	
Critical mass	$M_c = \frac{c^3}{2GH_0} = m_p \frac{T_0^2}{T_{Haw}^2}$	See [1].
Critical energy	$E_c = \frac{c^5}{2GH_0} = E_p \frac{T_0^2}{T_{Haw}^2}$	See [1].
Critical mass density	$\rho_c = \frac{3H_0^2}{8\pi G} = \frac{1}{8}\rho_p \frac{T_{Haw}^4}{T_0^4}$	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$.
Cosmological constant	$\Lambda = \left(\frac{H_0}{c}\right)^2 \Omega_\Lambda = \frac{1}{4l_p^2} \frac{T_{Haw}^4}{T_0^4} \Omega_\Lambda$	
CMB and z	$z = \frac{T_0}{T_i} - 1$	Well known [27,28].
Hubble entropy	$S_{BH} = \frac{A}{4l_p^2} = 4\pi \frac{T_0^4}{T_{Haw}^4}$	See [1].
Mass gap	$m_{p,r} = \frac{m_p^2}{2M_c} = \frac{1}{2}m_p \frac{T_{Haw}^2}{T_0^2}$	See [38].
Mass gap	$E_{p,r} = \frac{m_p^2 c^2}{2M_c} = \frac{1}{2}E_p \frac{T_{Haw}^2}{T_0^2}$.
Minimum acceleration (radius)	$g_{min} = \frac{Gm_{g,r}}{l_p^2} = a_p \frac{1}{2} \frac{T_{Haw}^2}{T_0^2}$	$a_p = \frac{Gm_p}{l_p^2}$. See [38].
Minimum acceleration (circumference)	$g_{min} = \frac{Gm_{g,c}}{l_p^2} = a_p \frac{1}{4\pi} \frac{T_{Haw}^2}{T_0^2}$	$a_p = \frac{Gm_p}{l_p^2}$. See [38].

4. Quantum Cosmology

Table 2 in the last section shows a series of cosmological properties and parameters expressed in terms of the ratio between the Hawking temperature and the CMB temperature. Haug [1] has demonstrated that:

$$\frac{T_0^2}{T_{Haw}^2} = \frac{l_p}{\bar{\lambda}_c} \quad (16)$$

where $\bar{\lambda}_c$ is the reduced Compton wavelength of the critical mass in the Hubble sphere: $\bar{\lambda}_c = \frac{\hbar}{M_c c}$. The reduced Compton frequency over the Planck time is given by $f = \frac{c}{\bar{\lambda}_c} t_p = \frac{l_p}{\bar{\lambda}_c}$. Haug [1,40] has discussed how this can be interpreted as the quantization of gravity—specifically, that the reduced Compton frequency over the Planck time can be incorporated into a quantized version of general relativity theory. While quantizing general relativity alone is not sufficient to unify it with quantum mechanics, it is a step toward better understanding gravity.

In Table 1, the ratio between the Planck temperature and the CMB temperature was used. At a deeper level, this ratio is equal to:

$$\frac{T_p^2}{T_0^2} = 32\pi^2 \frac{l_p}{\bar{\lambda}_c} \quad (17)$$

So again, we arrive at the reduced Compton frequency over the Planck time, multiplied by the constant $32\pi^2$. When we replace $\frac{T_0^2}{T_{Haw}^2}$ with its deeper constituent, $\frac{l_p}{\bar{\lambda}_c}$, and $\frac{T_p^2}{T_0^2}$ with its deeper constituent, $32\pi^2 \frac{l_p}{\bar{\lambda}_c}$, we find that both Tables 1 and 2 converge into Table 3. This essentially represents quantized cosmology, where $\frac{l_p}{\bar{\lambda}_c}$ again signifies the reduced Compton frequency over the Planck time, offering a new perspective on the quantization of gravity within general relativity theory [1].

Table 3. This table summarizes a series of properties for the Hubble sphere in an $R_h = ct$ universe, expressed in its most fundamental form involving the reduced Compton frequency over the Planck time.

Property :	Expression	Comment
Hubble constant (Hubble frequency):	$H_0 = \frac{f_p}{2\frac{l_p}{\bar{\lambda}_c}} = \frac{c\bar{\lambda}_c}{2l_p^2}$	$f_p = \frac{c}{l_p}$. See [1].
Hubble radius	$R_H = \frac{c}{H_0} = 2l_p \frac{l_p}{\bar{\lambda}_c}$	See [1].
Hubble time	$t_H = \frac{1}{H_0} = 2t_p \frac{l_p}{\bar{\lambda}_c}$	see [1].
Hubble diameter	$D_H = 2R_H = 4l_p \frac{l_p}{\bar{\lambda}_c}$	
Hubble circumference	$C_H = 2\pi R_H = 4\pi l_p \frac{l_p}{\bar{\lambda}_c}$	[41].
Hubble surface	$A_H = 4\pi R_H^2 = 16\pi l_p^2 \frac{l_p^2}{\bar{\lambda}_c^2}$	
Hubble volume	$V_H = \frac{4}{3}\pi R_H^3 = \frac{32}{3}\pi l_p^3 \frac{l_p^3}{\bar{\lambda}_c^3}$	[41].
Gaussian curvature	$k_H = \frac{1}{R_H^2} = \frac{1}{4l_p^2} \frac{l_p^4}{\bar{\lambda}_c^4}$	
Critical mass	$M_c = \frac{c^3}{2GH_0} = m_p \frac{l_p}{\bar{\lambda}_c}$	See [1].
Critical energy	$E_c = \frac{c^5}{2GH_0} = E_p \frac{l_p}{\bar{\lambda}_c}$	See [1].
Critical mass density	$\rho_c = \frac{3H_0^2}{8\pi G} = \frac{1}{8}\rho_p \frac{l_p^2}{\bar{\lambda}_c^2}$	$\rho_p = \frac{m_p}{\frac{4}{3}\pi l_p^3}$.
Cosmological constant	$\Lambda = \left(\frac{H_0}{c}\right)^2 \Omega_\Lambda = \frac{1}{4l_p^2} \frac{\bar{\lambda}_c^2}{l_p^2} \Omega_\Lambda$	
CMB and z	$z = \frac{T_0}{T_i} - 1 = \sqrt{\frac{\lambda_{cd}}{\bar{\lambda}_c}} - 1$	λ_c : Compton wavelength of critical mass [41].
Hubble entropy	$S_{BH} = \frac{A}{4l_p^2} = 4\pi \frac{l_p^2}{\bar{\lambda}_c^2}$	See [1].
Mass gap	$m_{p,r} = \frac{m_p^2}{2M_c} = \frac{1}{2}m_p \frac{\bar{\lambda}_c}{l_p}$	See [38].
Minimum acceleration (radius):	$g_{min} = \frac{Gm_{s,r}}{l_p^2} = a_p \frac{\bar{\lambda}_c}{2l_p} = \frac{c^2}{l_p^2} \frac{\bar{\lambda}_c}{2l_p}$	$a_p = \frac{Gm_p}{l_p^2}$. See [38].
Minimum acceleration (circumference):	$g_{min} = \frac{Gm_{s,c}}{l_p^2} = a_p \frac{\bar{\lambda}_c}{4\pi l_p} = \frac{c^2}{l_p^2} \frac{\bar{\lambda}_c}{4\pi l_p}$	$a_p = \frac{Gm_p}{l_p^2}$. See [38].

This is all consistent with a new approach to expressing the Schwarzschild metric. Specifically [1,42], we have:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$ds^2 = -\left(1 - \frac{2l_p}{r} \frac{l_p}{\bar{\lambda}}\right) c^2 dt^2 + \left(1 - \frac{2l_p}{r} \frac{l_p}{\bar{\lambda}}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (18)$$

where $\bar{\lambda}$ is the reduced Compton wavelength of the mass M . So, we see the same reduced Compton frequency over the Planck time here: $\frac{l_p}{\bar{\lambda}}$ in the Schwarzschild metric itself, which can again be linked to the ratio of the Planck temperature to the CMB temperature or the Hawking temperature to the CMB temperature, under the assumption of black hole $R_h = ct$ cosmology.

5. Conclusion

All cosmological properties that are typically expressed through the Hubble parameter can just as effectively be expressed through the observed CMB temperature and the Planck temperature. This is fully consistent with a new thermodynamic version of the Friedmann equation. Within the framework of $R_H = ct$ cosmology, this means we are free to make predictions based on either the Hubble parameter or the CMB temperature. Since the CMB temperature has been measured with far greater precision, this leads to a dramatic improvement in prediction accuracy, as demonstrated in a series of recent papers.

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