Review

An Extremely Low Mass Ratio Binary at the Key Stage of Evolution

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Abstract: This study presents multi-band photometric observations and detailed period analysis of a totally eclipsing binary system exhibiting low photometric amplitude. The system demonstrates characteristic W Ursae Majoris (EW)-type light curves with complete eclipses, enabling precise determination of fundamental parameters through light curve modeling. Photometric analysis reveals that ASASSN-V J171815.10+450432.9 (hereafter J171815) represents a marginal contact binary system with an extreme mass ratio ($q < 0.09$; the more massive component is designated as the primary star), approaching the theoretical lower limit for stable contact configurations. Furthermore, our investigation of orbital period variations uncovers a long-term period increase at a rate of $\frac{dP}{dt} = (1.08 \pm 0.05) \times 10^{-6} \, \text{day yr}^{-1}$, which is likely attributable to ongoing mass transfer between components. This interpretation aligns with the system's geometric configuration and observed light curve asymmetries. The unique characteristics presented by this binary system serve as a rare opportunity for in-depth research on the mass ratio theory, and also provide an important opportunity for testing the Thermal Relaxation Oscillation (TRO) theory.

Keywords: close binaries; mass ratio; near-contact binaries

1. Introduction

As defined by Shaw [1], near-contact binaries (NCBs) represent a distinctive subtype within the category of close binaries, where at least one component approaches but does not fully fill its Roche lobe. These systems exhibit diverse morphological configurations, encompassing marginal contact, semi-detached, and near-detached states [2], serving as crucial transitional objects bridging the evolutionary gap between detached and fully contact binaries. According to TRO theory, these binaries undergo oscillations between shallow contact states and semi-detached configurations. In shallow contact binary systems, the component stars have just started to share a common envelope. This distinctive feature makes them ideal subjects for validating the relevant theory and for exploring in depth the complex processes involved in the formation and evolution of over-contact binary systems. These NCBs have relatively short evolutionary timescales [3,4], which results in their relative rarity in the universe. Given their scarcity, there is a pressing need to gather more observational data, as such data are crucial for understanding their fundamental physical characteristics.

Prior theoretical investigations (e.g., [5,6]) established that tidal instability arises in close binaries when the system's orbital angular momentum J_{orb} falls below three times the rotational angular momentum of the primary component (J_{rot}). Beyond this threshold, the breakdown of synchronized rotation initiates rapid orbital inspiral, ultimately leading to coalescence into a single object. This instability criterion for contact binaries aligns with the theoretically predicted minimum mass ratio q_{min}, below which systems are predicted to undergo catastrophic orbital decay. However, because different studies employ different research methods, the proposed values for q_{min} are different. For instance, Yang and Qian [7] statistically deduced that q_{min} could be as low as 0.044. In contrast, Pešta and Pejcha [8] demonstrated that the value of q_{min} is contingent upon the type of contact binary. Specifically, for late - type systems with orbital periods exceeding 0.3 days, q_{min} is approximately 0.087;

systems with shorter periods, conversely, exhibit a higher q_{min} of around 0.246. Early - type contact binaries, on the other hand, display a lower q_{min}, approximately 0.030. Recent comprehensive reviews by Arbutina and Wadhwa [9] and Wadhwa et al. [10] have thoroughly examined the critical mass ratio and orbital stability of low-q contact binaries. Detection of contact binary systems with observed q values below theoretical predictions poses a significant challenge to existing astrophysical models. Until now, only a few systems with $q < 0.1$ have been identified, including TIC 55007847 ($q \approx 0.08$, [11]), TYC 4002 - 2628 - 1 ($q \approx 0.048$, [12]) and TYC 3801 - 1529 - 1 ($q \approx 0.036$, [13]). These ultra-low-q systems provide a unique perspective for observing stellar merger processes.

Based on the International Variable Star Index [1], CSS-J171815.1+450432 (other names: ASASSN-V J171815.10+450432.9) was a newly discovered contact binary with a low amplitude (0.24 CV band) and a short period (0.317907d) by Drake et al. [14]. In this paper, BVR_c photometric observations and their corresponding photometric solutions for the totally eclipsing binary J171815 are presented. Meanwhile, orbital period variations are also investigated, revealing a continuous period increase.

2. Observations and Data Reduction

2.1. Ground Based Observation

Multi-band photometric observations for the totally eclipsing binary J171815 were conducted using the newly installed 50 cm telescope located at Wendeng Biblical Mountain in Weihai (WH50, 37°10.9′ N, 121°47.6′ E). The observational system consists of a PlaneWave CDK20 telescope (f/6.8) coupled with a Dhyana 4040BSI sCMOS detector featuring 4096×4096 square pixels. With an image scale of $\sim 0''.54$ pixel^{-1}, the system provides a $36.7' \times 36.7'$ field of view. Image acquisition was done with MaxIm DL. During the photometric observation, the standard Cousins filters (BVR_c) were employed. Bias, dark and flat frame were obtained during the observations, which were used to calibrate the images of the target. A completed BVR_c band light curves (LC_1) were obtained on June 25, 26 and 30, 2024. The typical exposure times for B, V and R_c bands are 140 s, 90 s and 60 s, respectively. Differential photometry was performed using aperture techniques with carefully selected reference stars: the comparison star (α_{2000} 17^{h}18^{m}21^s.3 ; δ_{2000} 44°56′15″.8) and check star (α_{2000} 17^{h}18^{m}21^s.2 ; δ_{2000} 45°09′49″.1). The phase-folded light curves (Figure 1) were computed using the ephemeris: $Min.I = HJD2460487.10958 + 0.317907E$.
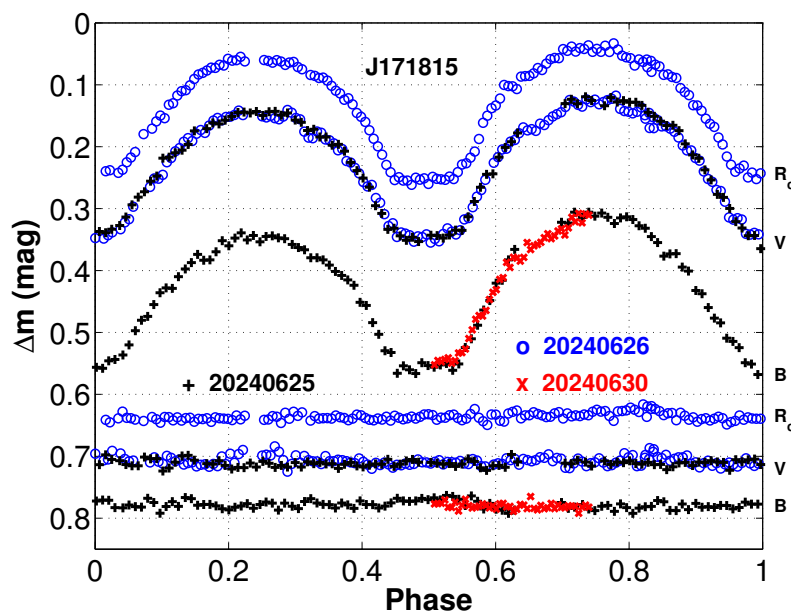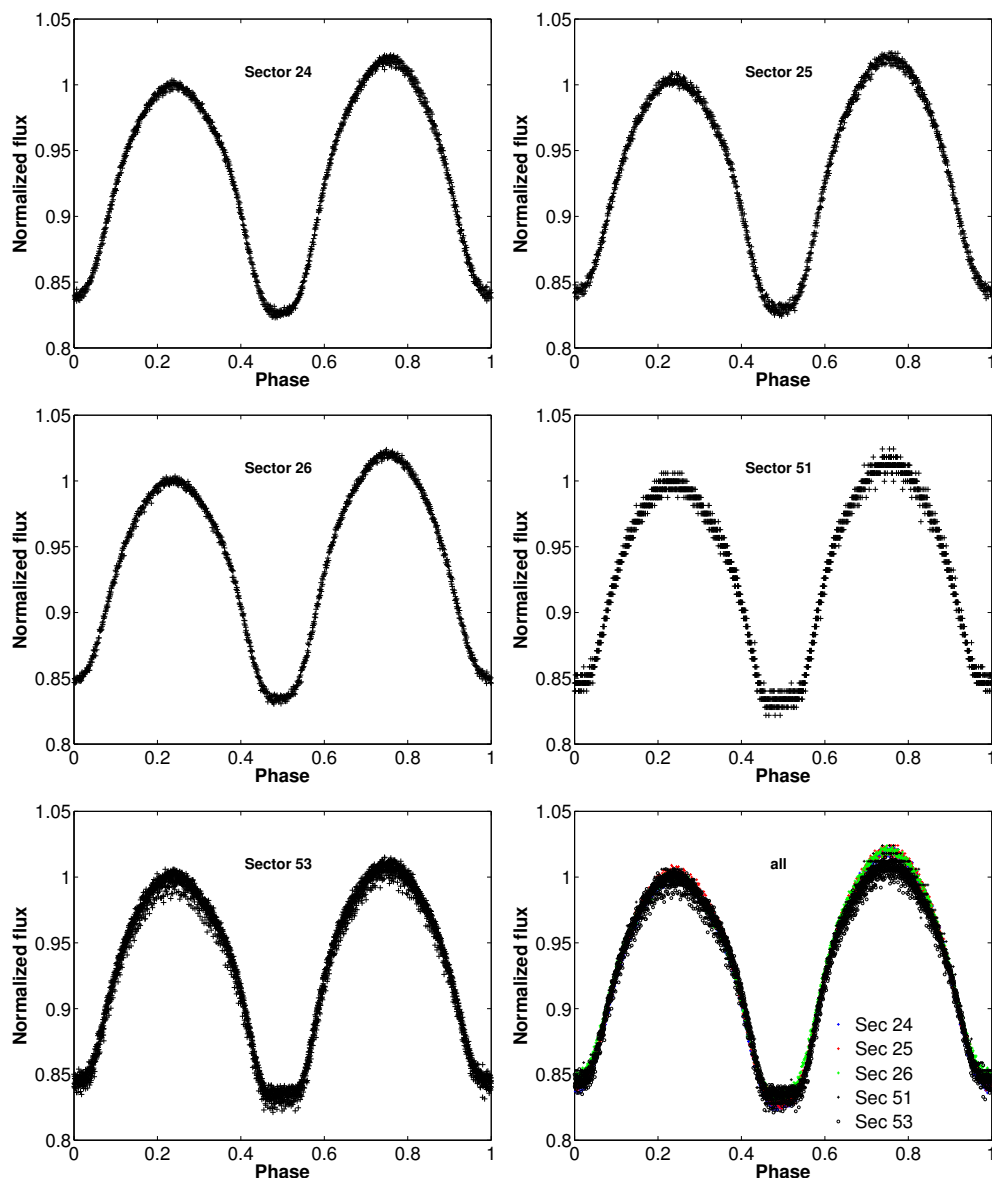

Figure 1. BVR_c light curves of J171815 observed by 50 cm telescope at Weideng station of Weihai observatory.

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[1]   https://www.aavso.org/vsx/index.php

2.2. TESS Observation

NASA's Transiting Exoplanet Survey Satellite (TESS; [15]) provides continuous photometric observations, making it possible to conduct a detailed analysis of the light curve shapes of eclipsing binary systems. We found that J171815 was observed by TESS in Sectors 24, 25, 26 at 30-minute cadence, while in sectors 51, and 53 at 10-minute cadence. Then we download the relevant data from the Mikulski Archive for Space Telescopes (MAST)[2]. After acquiring the local minimum times and applying them to phase the light curves, we excluded data points with evident discrepancies in our analysis. Then, we normalized the light curves according to the values at phase 0.25. All phased light curves from TESS are illustrated in Figure 2. Moreover, for the sake of comparison, we included all these light curves in the last panel of the same figure. As can be seen from the figure, the secondary minimum distinctly shows a flat bottom characteristic.

Figure 2. The normalized TESS light curves observed of J171815 at different sectors. For comparison, all the light curves were shown in the last panel.

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[2]   https://archive.stsci.edu/

3. Orbital Period Investigation

For the purpose of examining the orbital period's variation, we adhered to the procedures described in [16–18] to calculate the minimum times of TESS data. As a result, 20 minimum times were obtained. Since the TESS data use the Barycentric Julian Date (BJD), we transformed the minimum times determined by our observations to BJD using online tools[3] [19]. Through these efforts, we identified 24 eclipsing minima, which are listed in Table 1. Using the following equation:

$$Min.I(BJD) = 2459698.69977 + 0.317907E \tag{1}$$

the diagrams of $O - C$ are obtained and displayed in Figure 3. The diagram of $O - C$ illustrates an upward parabolic change. By using the least square method to fit the curve, we derived the following ephemeris formula:

$$Min.I(BJD) = 2459698.698950(\pm0.000712) + 0.3179064$$
$$(\pm0.0000003)E + 4.693(\pm1.640) \times 10^{-10} \times E^2. \tag{2}$$

Table 1. CCD times of light minimum for J171815.

BJD +2450000	Error (days)	Min.	E	O-C(day)	BJD +2450000	Error (days)	Min.	E	O-C(day)
8961.15962[a]	±.00011	I	-2320.0	0.00409	9698.69977[a]	±.00012	I	0.0	0.00000
8961.31635[a]	±.00009	II	-2319.5	0.00187	9698.85715[a]	±.00014	II	0.5	-0.00157
8974.51138[a]	±.00008	I	-2278.0	0.00376	9712.36981[a]	±.00013	I	43.0	0.00004
8974.66855[a]	±.00009	II	-2277.5	0.00197	9712.52697[a]	±.00016	II	43.5	-0.00175
8988.81727[a]	±.00013	I	-2233.0	0.00383	9745.74946[a]	±.00009	I	148.0	-0.00055
8988.97430[a]	±.00016	II	-2232.5	0.00191	9745.90766[a]	±.00012	II	148.5	-0.00130
9002.48734[a]	±.00016	I	-2190.0	0.00390	9759.41945[a]	±.00008	I	191.0	-0.00056
9002.64418[a]	±.00014	II	-2189.5	0.00179	9759.57765[a]	±.00010	II	191.5	-0.00131
8988.81728[a]	±.00013	I	-2233.0	0.00384	10487.11039[b]	±.00022	I	2480.0	0.00127
8988.97429[a]	±.00015	II	-2232.5	0.00190	10487.26761[b]	±.00042	II	2480.5	-0.00047
9002.48734[a]	±.00016	I	-2190.0	0.00390	10488.06468[b]	±.00032	I	2483.0	0.00183
9002.64416[a]	±.00013	II	-2189.5	0.00177	10488.22153[b]	±.00036	II	2483.5	-0.00027

Note (a) TESS, (b) WH50cm.
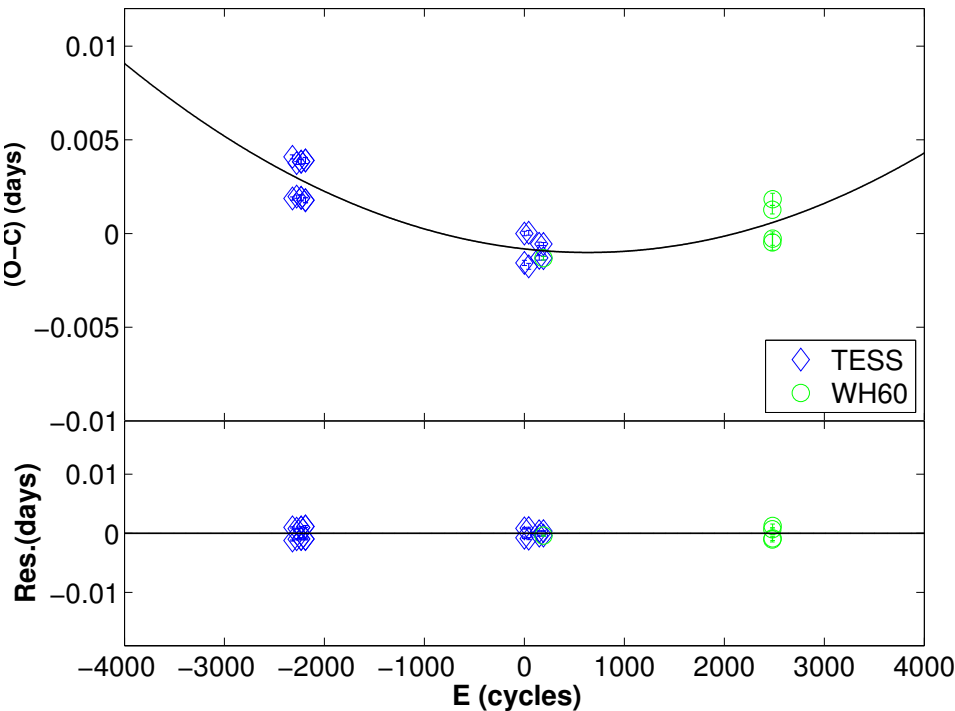
Figure 3. O-C diagram of J171815. Different symbols refer to different observations.

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[3] https://astroutils.astronomy.osu.edu/time/hjd2bjd.html

According to Equation (2), the residuals are calculated and displayed at the bottom of Figure 3. The increase in the secular period of Equation (2) is calculated to be $dP/dt = 1.08 \times 10^{-6} day \cdot yr^{-1}$. Considering the short timescale and the limited observational data available for J171815, the observed long-term orbital trend may represent only a portion of the actual orbital period variation. Therefore, additional observations are required to confirm and further understand these findings.

4. Photometric Solutions

We employed the Wilson-Devinney (W-D) code [20–25] to obtain photometric solutions for the binary system. A pivotal step in deriving the photometric parameters of the binary system was determining the effective temperatures, as accurate temperature values are essential for reliable modeling. Using data from the Gaia mission [26,27], we calculated the effective temperature to be 5495 K. As reported by Andrae et al. [28], Gaia-derived effective temperatures within the 3000–10,000 K range are typically associated with uncertainties of approximately 324 K.

Notably, the light curves of J171815 display extremely low amplitudes, suggesting that the primary component predominantly contributes to the system's overall luminosity. Consequently, we adopted the effective temperatures obtained from the Gaia mission as the effective temperature for the primary component. In accordance with the guidelines proposed by Lucy [29] and Ruciński [30], we set the bolometric albedo coefficients and gravity darkening coefficients for both components. The bolometric albedo coefficients were assigned as $A_1 = A_2 = 0.5$, while the gravity darkening coefficients were set to $g_1 = g_2 = 0.32$.

The light curves of the target exhibit small amplitudes, suggesting an extremely low mass ratio. This presents a considerable challenge in directly measuring the mass ratio from radial velocity (RV) observations. However, the presence of total eclipses in the system supports the reliability of photometric results obtained using the Wilson–Devinney (W–D) code, as corroborated by earlier studies [31,32]. Given that the target displays typical EW-type light curves, we initially adopted mode 3 for the photometric analysis, which is suitable for overcontact binary systems. To estimate the initial mass ratio, we applied the q-search method, beginning with a sequence of trial mass ratios starting at 0.06. For each tested mass ratio, parameters including the surface potential (Ω), orbital inclination (i), secondary component temperature (T_2), and the luminosity of the primary component (L_1) were allowed to vary. Light curve models were generated for each q and compared to the observed data. The optimal model, selected by minimizing a statistical indicator such as the chi-squared (χ^2) value, yielded an estimate for the initial mass ratio.

During the q-search process, we observed that certain trial mass ratios did not converge even after 200 iterations for the target J171815. To address this, we expanded the q-search approach to incorporate modes 4 and 5. Mode 4 is intended for semi-detached binary systems in which the primary star fills its critical Roche lobe, whereas mode 5 applies to semi-detached systems where the less massive component reaches its Roche lobe. Eventually, convergent solutions for J171815 were obtained under modes 3, 4, and 5. The plots illustrating the correlation between the tested mass ratios q and the sums of weighted squared deviations Σ are displayed in the left panel of Figure 4.

As previously stated, the TESS mission observed the binary system. It monitored sectors 24, 25, and 26 at a 30 - minute sampling rate, while sectors 51 and 53 were observed at a more frequent 10 - minute cadence, as depicted in Figure 2. The figure clearly reveals that the 30 minute cadence data suffered substantially from phase - smearing effects. These effects can distort the true shape of the light curve and lead to inaccuracies in the photometric analysis. Furthermore, the light curves obtained from different sectors exhibit remarkable similarity in their profiles. This similarity strongly suggests that there were no significant intrinsic changes in the system's brightness during the observation period. Given these factors, the light curves from sector 53 were selected for deriving the photometric solutions. Sector 53's data stood out because of its higher data density and lower scatter. For comparison, the q-search method was applied to the TESS data set. The plots in the right panels of Figure 4 illustrate the relationships between the tested mass ratios q and Σ.
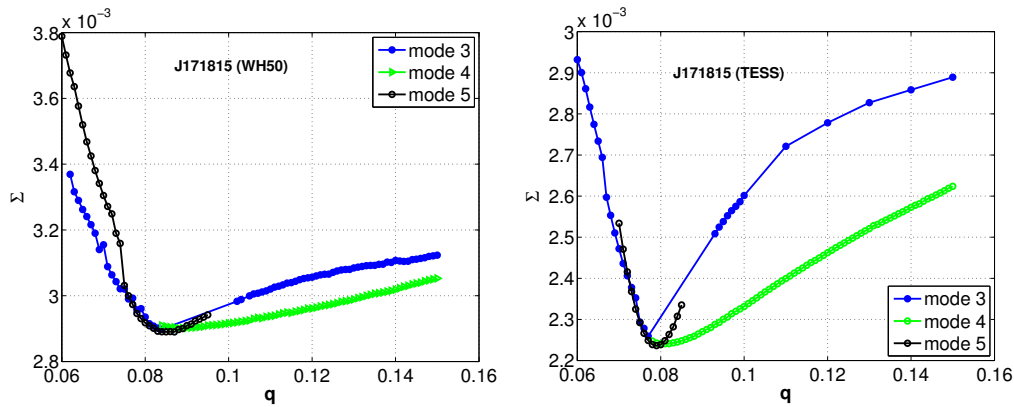
Figure 4. The left panel displays the q-search results derived from data obtained using the WH50 telescope, while the right panel presents the q-search results utilizing data from TESS.

Based on the left panel of Figure 4, the minimum values of Σ derived from modes 3, 4, and 5 are 0.084, 0.088, and 0.087, respectively. They are quite similar to the results derived from TESS, which yielded a minimum value of Σ at $q = 0.077$, $q = 0.080$, and $q = 0.079$ for modes 3, 4, and 5, respectively. Given the long - term increasing trend in the orbital period, which indicates mass transfer from the less massive component to the more massive one, modes 3 and 5 are considered more plausible than mode 4. Therefore, modes 3 and 5 were selected for subsequent photometric solutions. The corresponding mass ratios from our data and TESS data were used as initial mass ratios. These values were then treated as adjustable parameters in the following analysis, together with other key parameters such as the luminosity of the primary (L_1), phase shift, secondary temperature (T_2), potential (Ω), and inclination (i). From Figures 1 and 2, we can see that all the light curves show asymmetry with brightness enhancement at the secondary light maximum. Just as the asymmetry in some NCBs [33–35] has been explained by a hot spot on the binaries, we added a hot spot to the primary component. Since the light curve exhibits total eclipses, the derived photometric solutions should be reliable [31,36]. The final photometric solutions from the BVR_c data and the TESS data are summarized in Table 2, labeled $Mode3(BVR_c)$, $Mode3(\text{TESS})$, $Mode5(BVR_c)$ and $Mode5(\text{TESS})$, respectively. The theoretical light curves obtained for mode 3 and mode 5 were shown in Figure 5.
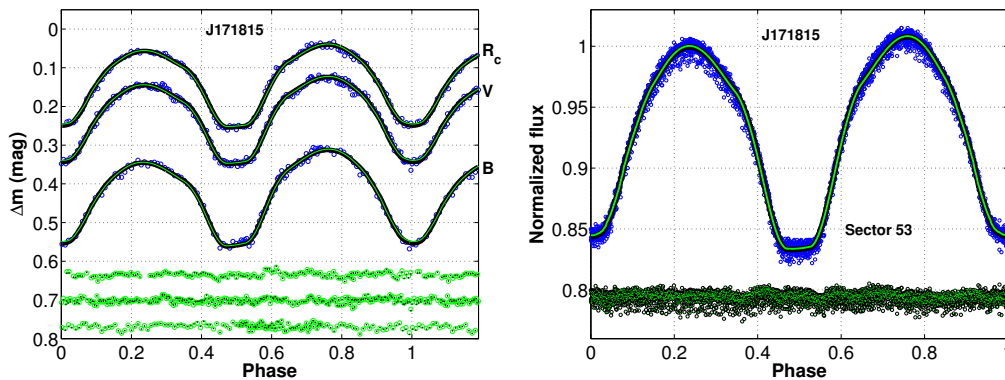
Figure 5. The left panel, the fitted light curves for BVR_c band are exhibited, whereas in the right panel, the fitted light curves for TESS are provided. The green solid line stands for mode 3, while the black solid line represent mode 5.

In order to investigate whether J171815 is contaminated by the visible star in its surroundings, we consulted the Gaia database for nearby stars: the closest star is located 27 arcseconds away, with a *g*-band magnitude of 19.72. This star is significantly dimmer than J171815, which has a *g*-band magnitude of 13.10. Given these findings, the contribution of third light from nearby stars is negligible. Since most W UMa systems contain a third component, the mass ratio we obtained should be regarded as the lower limit of the binary's mass ratio.

Table 2. Photometric solutions for J171815 derived from the BVR_c observations and TESS data.

Parameters	$Mode3(BVR_c)$	$Mode3(TESS)$	$Mode5(BVR_c)$	$Mode5(TESS)$
$T_1(K)$	5495	5495	5495	5495
$T_2(K)$	5632(9)	5754(4)	5612(9)	5738(4)
$q(M_2/M_1)$	0.081(1)	0.077(1)	0.084(1)	0.079(1)
$i(°)$	69.7(2)	69.8(1)	70.3(3)	70.0(1)
Ω_{in}	1.897	1.885	1.908	1.891
$\Omega_{out})$	1.844	1.834	1.853	1.839
Ω_1	1.896(1)	1.885(1)	1.919(3)	1.895(1)
Ω_2	1.896	1.885	1.908	1.891
f	1(2)%	1(1)%		
$f1$			99(1)%	100(1)%
$f2$			100%	100%
r_1	0.598(1)	0.601(1)	0.589(1)	0.597(1)
r_2	0.192(1)	0.189(1)	0.194(1)	0.191(1)
$\theta(\text{rad})$	0.3466	0.4532	0.3466	0.3384
$\lambda(\text{rad})$	1.6796	1.7660	1.7190	1.7597
$r_s(\text{rad})$	0.2096	0.2077	0.2089	0.2073
T_s/T	1.1673	1.0891	1.1673	1.1164
$\Sigma\omega(O-C)^2$	0.001769	0.001899	0.001605	0.001738

5. Discussions and Conclusions

The photometric solutions and detailed orbital period analysis for J171815 are carried out for the first time. Photometric solutions derived from multi-band observations and TESS data are listed in Table 2. From Figure 4, it is evident that the light curve can be well reproduced by both mode 3 and mode 4. Given that no significant abrupt changes in the slope of the observed light curves were detected around the orbital phases of approximately 0.36 and 0.64 (see Figure 1 and 2), mode 3 appears to be a more suitable choice. Photometric solutions derived from mode 3 suggest that J171815 is a W-type contact binary system, where the more massive star is cooler than the less massive one. The system has an ultra low mass ratio and a shallow contact degree, where f is defined as $f = \frac{\Omega_{in}-\Omega}{\Omega_{in}-\Omega_{out}}$, with Ω_{in} being the Roche lobe potential at the L_1 Lagrangian point, Ω_{out} the potential at the L_2 Lagrangian point and Ω the actual potential of the star.

In an effort to determine the global parameters of J171815, we utilized the final results from its light-curve solutions (mode3 of TESS)). Assuming that the more massive primary star in the system is on the main sequence, we estimated its mass using data from an online table [37]. This led to a mass estimate of $M_1 = 0.94 M_\odot$. Applying Kepler's third law ($M = 0.0134a^3/P^2$), we calculated the semi-major axes (the separations between the components) to be approximately $1.97 R_\odot$. Based on the photometric solutions provided in Table 2, the radii of the primary and secondary components were calculated. These values are $R_1 = 1.17 R_\odot$ (primary) and $R_2 = 0.38 R_\odot$ (secondary). The luminosities of the binaries were then computed based on the Stefan - Boltzmann law ($L = 4\pi\sigma T^4 R^2$), which resulted in $L_1 = 1.13 L_\odot$ and $L_2 = 0.14 L_\odot$.

For the first time, we analyzed the minimum times calculated for constructing and plotting the $O-C$ diagrams for J171815, which are shown in Figure 3, which reveal an upward parabolic trend. The quadratic term in the orbital period ephemeris (Equation (2)) reveals a secular increase in the orbital period at a rate of ($\frac{dP}{dt} = 1.08 \times 10^{-6}$ day yr^{-1}), which indicates mass transfer from the less massive primary component to the more massive secondary star. Then, using Equation (3):

$$\frac{\dot{P}}{P} = -3\dot{M}_1\left(\frac{1}{M_1} - \frac{1}{M_2}\right) \tag{3}$$

we calculate the mass accretion rate of the primary component as $\frac{dM_1}{dt} = 9.11 \times 10^{-8} M_\odot$ yr^{-1}.

In summary, our analysis based on photometric solutions and orbital period studies indicates that J171815 has an ultra-low mass ratio. The observed positive ($\frac{dP}{dt} = 1.08 \times 10^{-6}$ day yr^{-1}) supports this scenario, suggesting mass transfer from the less massive star to the more massive one through Roche

lobe overflow, which may cause brightness enhancement around the secondary light maximum. Given its very small mass ratios and marginal critical states, J171815 is an important candidate for continuous monitoring. Such observations are crucial for testing theories of binary mergers and thermal relaxation oscillations, providing valuable insights into the evolution of these systems.

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