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Mean—Variance—Entropy Model to Portfolio Optimization

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Abstract: The financial market is inherently a complex and dynamic environment, where investors must balance the trade-off between expected return and associated risk. While the classical Markowitz mean-variance model has served as a cornerstone for modern portfolio theory, it may not fully capture the informational uncertainty present in asset returns. Recent studies suggest that entropy-based measures can enhance the robustness of portfolio optimization by accounting for the uncertainty embedded in return distributions. This paper proposes an extended mean-variance-entropy framework for portfolio selection, aiming to maximize a return-to-risk adjusted with entropy ratio. A mathematical formulation of the optimization problem is developed, and an analytical solution for the optimal asset weights is derived. Furthermore, a practical application is conducted using two assets, highlighting the potential benefits of incorporating entropy in portfolio decision-making.

Keywords: Entropy; Portfolio Optimization; mean - variance – entropy model

MSC: 91B28

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1. Introduction

Portfolio optimization represents a cornerstone of modern financial theory, addressing the fundamental problem of allocating capital among a set of risky assets to achieve an optimal balance between expected return and associated risk. The classical framework for solving this problem was pioneered by Markowitz (1952) [11], whose mean-variance model formalized portfolio selection as a mathematical optimization problem. Within this paradigm, investors are assumed to be rational and risk-averse, constructing portfolios that either maximize expected return for a given level of risk or, conversely, minimize risk for a specified expected return. The model's formulation hinges on two critical parameters: the vector of expected returns and the covariance matrix of asset returns [11,12]. Despite its theoretical elegance, the practical application of the mean- variance model encounters significant limitations. Chief among these is the difficulty of estimating inputs with sufficient accuracy. Expected returns and covariances are typically inferred from historical data, yet such estimates are often unstable and may not reliably reflect future market conditions. As a result, optimized portfolios tend to exhibit high sensitivity to estimation errors [2,3], frequently leading to over-concentration in assets with seemingly high returns [7,8,14,19]. This issue has spurred extensive academic discourse and the development of numerous extensions to the original model. Over the decades, alternative models have emerged to mitigate these shortcomings, including the minimum-variance model, the mean-variance-skewness model, and the mean–Value-at-Risk (VaR) model [6,10,15,18].

Nevertheless, empirical findings—such as those presented by DeMiguel et al. [5] have demonstrated that many of these sophisticated approaches do not consistently outperform naive diversification strategies, such as equally weighted portfolios, particularly in out-of-sample

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performance. A key criticism of the mean-variance framework lies in its insufficient treatment of uncertainty beyond variance. In real-world financial contexts, investors evaluate risk and return concurrently, and the performance of a portfolio is intrinsically linked to its level of diversification. Entropy, a concept derived from information theory, offers a promising alternative for capturing the degree of uncertainty and diversification within a portfolio [1,4,9,16]. When incorporated into the optimization objective, entropy can enhance model robustness by penalizing portfolios with concentrated or imbalanced allocations.

In light of these considerations, the present study proposes an extension to the traditional mean-variance model by integrating an entropy-based measure into the optimization framework. The proposed model seeks to determine the set of asset weights that maximizes the expected return per unit of risk, as measured by the standard deviation, while simultaneously accounting for portfolio entropy as a diversification criterion}.

To validate the approach, an empirical case study is conducted using a portfolio composed of two assets, over the time period spanning January to March 2025. The performance of the entropy-augmented portfolio is then benchmarked against that of a conventionally optimized portfolio, providing insights into the practical benefits of the proposed methodology. The remainder of the paper is organized as follows: Section 2 presents the theoretical formulation of the proposed model, including its mathematical derivation and implementation. Section 3 discusses the empirical results and comparative analysis. Finally, Section 4 concludes the paper and outlines potential avenues for future research.

2. Materials and Methods

2.1. The Optimization Problem

Let us consider a portfolio consisting of n risky assets, with individual returns denoted as the random variables r_i, $i=\overline{1,n}$.

The means and covariances of returns are: $\mu_i = E(r_i) = E_i, \sigma_{ij} = cov(r_i, r_j)$, $i=\overline{1,n}$, $j=\overline{1,m}$

The portfolio is given by: $\omega = (\omega_1, \omega_2, \dots, \omega_n)$, cu $\sum_{i=1}^{n} \omega_i = 1$

The return of the portfolio ω is $R\omega = \sum_{i=1}^{n} \omega_i r_i$

The mean and variance of the portfolio return are:

$\mu_\omega = E(r_\omega) = E\left(\sum_{i=1}^{n} \omega_i r_i\right) = \sum_{i=1}^{n} \omega_i \mu_i$

$\sigma_\omega^2 = Var\left(\sum_{i=1}^{n} \omega_i r_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i\omega_j \sigma_{ij} = \sum_{i=1}^{n} \omega_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i\omega_j \sigma_{ij}$

Traditionally, portfolio optimization has been carried out by either maximizing expected return, minimizing variance, or achieving a trade-off between the two. Recently, however, entropy has been introduced as an additional objective criterion, capturing the degree of diversification and uncertainty in the portfolio structure {17} .

The portfolio optimization problem can be considered with respect to three utility functions: mean (maximization of the objective function), variance (minimization of the objective function), entropy (maximization of the objective function)

Case of Two Assets: Classical Mean-Variance Model

Assuming a portfolio composed of two assets $\omega = (\omega_1, \omega_2)$, cu $\omega_2 = 1 - \omega_1$

The variance minimization problem can be expressed a

min $g(\omega_1, \omega_2) = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_{12}$, where $\omega_1 + \omega_2 = 1$. Let $\omega = \omega_1$.

We obtain min $g(\varpi) = \varpi^2(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) - 2\varpi(\sigma_2^2 - \sigma_{12}) + \sigma_2^2$

$g'(\varpi) = 2\varpi(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) - 2(\sigma_2^2 - \sigma_{12})$

Because $g'(w) = 0$ we obtain $\omega^0 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 - 2\sigma_{12} + \sigma_2^2}$

Optimum portfolio is $\left(\frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 - 2\sigma_{12} + \sigma_2^2}, 1 - \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 - 2\sigma_{12} + \sigma_2^2}\right)$

Portfolio Optimization using Entropy

Entropy provides a rigorous mathematical framework for quantifying disorder and randomness within financial systems. Over time, researchers have recognized its potential as a more robust

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alternative to traditional risk metrics, such as variance or standard deviation, in the construction and optimization of investment portfolios. By leveraging entropy as a diversification metric, investors can systematically evaluate the degree of uncertainty in asset allocations, leading to more resilient portfolio structures [13,17,18,20]. The integration of entropy into portfolio theory allows for the development of sophisticated optimization models that transcend the limitations of mean-variance frameworks.

We will use in our study second order entropy.

Tsallis Entropy: $H(x) = -\sum_{i=1}^{n} x_i \frac{x_i^{\propto}-1}{\propto}, \propto \neq 0$: where $x = (x_1, x_2, \ldots, x_n)$, $\sum_{i=1}^{n} x_i = 1$

Second order entropoy (Tsallis with $\propto = 1$): $H(x) = 1 - \sum_{i=1}^{n} x_i^2$

Remarks:

1. For a given portfolio, this kind of entropy measures the correlation degree of the assets ($\sum_{i=1}^{n} x_i^2 = (\sum_{i=1}^{n} x_i)^2 - \sum_{j\neq i} x_i x_j = 1 - \sum_{j\neq i} x_i x_j$, *thus* $H(x) = 1 - \sum_{i=1}^{n} x_i^2 = \sum_{j\neq i} x_i x_j$)

2. A lower entropy implies greater concentration (lower diversification), whereas a higher entropy reflects greater diversification, which may contribute to portfolio liquidity

Case of Two Assets: Mean-Variance-Entropy Optimization

A more robust approach considers the joint maximization of return and entropy, while minimizing variance. Inspired by Hatemi and El-Khatib [7], we define the objective function as:

$F(\omega_1, \omega_2) = \frac{\varpi_1 E(r_1) + \varpi_2 E(r_2)}{\sqrt{(\varpi_1{}^2\sigma_1{}^2 + \varpi_2{}^2\sigma_2{}^2 + 2\varpi_1\varpi_2\sigma_{12}}}$. Optimum portfolio is:

$$\left(\frac{E_1\sigma_2^2 - E_2\sigma_{12}}{E_1(\sigma_2^2 - \sigma_{12}) + E_2(\sigma_1{}^2 - \sigma_{12})}, 1 - \frac{E_1\sigma_2^2 - E_2\sigma_{12}}{E_1(\sigma_2^2 - \sigma_{12}) + E_2(\sigma_1{}^2 - \sigma_{12})}\right)$$

A more complex approach is obtained when all three criteria are considered in the utility function, and the objective function includes both a profitability component (the numerator) and a variance component adjusted with the portfolio entropy (the denominator). We will present the results obtained for the case of two assets:

$Q(\omega_1, \omega_2) = \frac{E(\omega_1,\omega_2)}{Var(\omega_1,\omega_2) - \alpha H(\omega_1,\omega_2)}$, where: $E(\omega_1, \omega_2)$ represents the portfolio return, α is the importance given by the investor to diversification, $Var(\omega_1, \omega_2)$ is the portfolio variance, $H(\omega_1, \omega_2)$ is the portfolio entropy.

We use second-order entropy, defined as: $H(\omega_1, \omega_2) = 1 - (\omega_1^2 + \omega_2^2)$

Thus, the function to be maximized is:

$Q(\omega_1, \omega_2) = \frac{\varpi_1 E(r_1) + \varpi_2 E(r_2)}{\sqrt{\varpi_1{}^2\sigma_1{}^2 + \varpi_2{}^2\sigma_2{}^2 + 2\sigma_{12}\varpi_1\varpi_2 - \alpha[1-(\omega_1^2+\omega_2^2)]}}$ or equivalently,

$Q(\omega_1, \omega_2) = \frac{\varpi_1 E(r_1) + \varpi_2 E(r_2)}{\sqrt{\varpi_1{}^2\sigma_1{}^2 + \varpi_2{}^2\sigma_2{}^2 + 2\sigma_{12}\varpi_1\varpi_2 - 2\alpha\varpi_1\varpi}}$, or

$Q(\omega_1, \omega_2) = \frac{\varpi_1 E(r_1) + \varpi_2 E(r_2)}{\sqrt{\varpi_1{}^2\sigma_1{}^2 + \varpi_2{}^2\sigma_2{}^2 + 2(\sigma_{12}-\alpha)\varpi_1\varpi_2}}$, or

$Q(\omega_1, \omega_2) = \frac{\varpi_1 E_1 + \varpi_2 Er_2}{\sqrt{\varpi_1\sigma_1{}^2 + \varpi_2{}^2\sigma_2{}^2 + 2\sigma_\alpha\varpi_1\varpi_2}}$, where $\sigma_\alpha = \sigma_{12} - \alpha$

So, $Q(w) = \frac{\varpi E_1 + (1-\varpi)E_2}{\sqrt{\varpi^2\sigma_1{}^2 + (1-\varpi)^2\sigma_2{}^2 + 2\sigma_\alpha\omega(1-\varpi)}}$, Q : [0,1] → R continuous function

Or, $Q(w) = \frac{\varpi(E_1 - E_2) + E_2}{\sqrt{\varpi^2(\sigma_1{}^2 + \sigma_2{}^2 - 2\sigma_\alpha) - 2\varpi(\sigma_2{}^2 - \sigma_\alpha) + \sigma_2{}^2}} = \frac{f(\varpi)}{\sqrt{g(\varpi)}}$

We have that: $Q'(w) = \frac{f'(w)g(w) - \frac{1}{2}f(w)g'(w)}{g(w)^{\frac{3}{2}}}$. We impose the condition: $Q'(w) = 0$ and obtain: $f'(w)g(w) = \frac{1}{2}f(w)g'(w)$.Performing the calculations, we get: $\omega^0 = \frac{E_1\sigma_2^2 - E_2\sigma_\alpha}{E_1(\sigma_2^2 - \sigma_\alpha) + E_2(\sigma_1{}^2 - \sigma_\alpha)}$, $Q(\omega^0) = \sqrt{\frac{E_1^2\sigma_2^2 + E_2^2\sigma_1^2 - 2E_1E_2\sigma_\alpha}{\sigma_1{}^2\sigma_2^2 - \sigma_\alpha^2}}$

Additionally, we have: $Q(0) = \frac{E_2}{\sigma_2}$, $Q(1) = \frac{E_1}{\sigma_1}$ and Q: [0,1] → R has ω^0 as a stationary point.

We have: Q $(\omega^0) \geq$ Q(0) because $[Q(\omega^0)]^2 \geq [Q(0)]^2$ or $\frac{E_1^2\sigma_2^2+E_2^2\sigma_1^2-2E_1E_2\sigma_\alpha}{\sigma_1^2\sigma_2^2-\sigma_\alpha^2} \geq \frac{E_2^2}{\sigma_2^2}$ or $E_1^2\sigma_2^4 + E_2^2\sigma_\alpha^2 - 2E_1E_2\sigma_2^2\sigma_\alpha \geq 0$ or $(E_1\sigma_2^2 - E_2\sigma_\alpha)^2 \geq 0$

Similarly, Q $(\omega^0) \geq$ Q(1) because $[Q(\omega^0)]^2 \geq [Q(1)]^2$ or $\frac{E_1^2\sigma_2^2+E_2^2\sigma_1^2-2E_1E_2\sigma_\alpha}{\sigma_1^2\sigma_2^2-\sigma_\alpha^2} \geq \frac{E_1^2}{\sigma_1^2}$ or $E_2^2\sigma_1^4 + E_1^2\sigma_\alpha^2 - 2E_1E_2\sigma_1^2\sigma_\alpha \geq 0$ or $(E_2\sigma_1^2 - E_1\sigma_\alpha)^2 \geq 0$

Since ω^0 is a stationary point for Q: [0,1] → R and Q : [0,1] → R satisfies

Q $(\omega^0) \geq$ Q(0) , Q $(\omega^0) \geq$ Q(1), we obtain that $\omega^0 = \frac{E_1\sigma_2^2 - E_2\sigma_\alpha}{E_1(\sigma_2^2-\sigma_\alpha)+E_2(\sigma_1^2-\sigma_\alpha)}$ is a maximum point.

Thus, the optimal portfolio is $(\omega_1, \omega_2) = (\omega^0, 1-\omega^0) =$

$$\left(\frac{E_1\sigma_2^2 - E_2\sigma_\alpha}{E_1(\sigma_2^2-\sigma_\alpha)+E_2(\sigma_1^2-\sigma_\alpha)}, 1 - \frac{E_1\sigma_2^2 - E_2\sigma_\alpha}{E_1(\sigma_2^2-\sigma_\alpha)+E_2(\sigma_1^2-\sigma_\alpha)}\right)$$

2.2. Case Studies: Bitcoin and Ethereum Portfolio Optimization

In this case study, we consider a portfolio composed of Bitcoin (BTC) and Ethereum (ETH), two of the most prominent cryptocurrencies in the digital asset market. The data cover a historical period from January to March 2025, capturing relevant market fluctuations during that time. Using historical price data, we calculate statistics:

$E_1 = 0.074$, $\sigma_1^2 = 0.46$; $E_2 = 0.04$, $\sigma_2^2 = 1$; $\sigma_{12} = 0.203$. Based on these inputs, we implement and compare three portfolio optimization strategies: Minimum Variance, Mean-Variance, and Mean-Variance-Entropy (MVE).

Minimizing Variance Approach:

- Optimal Portfolio: $\left(\frac{\sigma_2^2-\sigma_{12}}{\sigma_1^2-2\sigma_{12}+\sigma_2^2}, 1-\frac{\sigma_2^2-\sigma_{12}}{\sigma_1^2-2\sigma_{12}+\sigma_2^2}\right)$ = *(0.76, 0.24)*
- Expected Return: $E(w) = \varpi_1E(r_1) + \varpi_2E(r_2)$ = *0.063*
- Variance: $\omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_{12}$ = *0.397*
- Risk (Standard Deviation): $\sqrt{\omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_{12}}$ = *0.63*
- Risk-Adjusted Return: $\frac{E(\omega)}{\sqrt{\omega_1^2\sigma_1^2+\omega_2^2\sigma_2^2+2\omega_1\omega_2\sigma_{12}}}$ = *0.16*

Mean-Variance Approach:

- Optimal Portfolio: $\left(\frac{E_1\sigma_2^2-E_2\sigma_{12}}{E_1(\sigma_2^2-\sigma_{12})+E_2(\sigma_1^2-\sigma_{12})}, 1-\frac{E_1\sigma_2^2-E_2\sigma_{12}}{E_1(\sigma_2^2-\sigma_{12})+E_2(\sigma_1^2-\sigma_{12})}\right)$ = *(0.97, 0.03)*
- Expected Return: $E(w) = \varpi_1E(r_1) + \varpi_2E(r_2)$ = *0.072*
- Variance: $\omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_{12}$ = *0.45*
- Risk: $\sqrt{\omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_{12}}$ = *0.67*
- Risk-Adjusted Return: $\frac{E(\omega)}{\sqrt{\omega_1^2\sigma_1^2+\omega_2^2\sigma_2^2+2\omega_1\omega_2\sigma_{12}}}$ = *0.1*

Mean-Variance-Entropy Approach:

- Optimal Portfolio: $\left(\frac{E_1\sigma_2^2-E_2\sigma_\alpha}{E_1(\sigma_2^2-\sigma_\alpha)+E_2(\sigma_1^2-\sigma_\alpha)}, 1-\frac{E_1\sigma_2^2-E_2\sigma_\alpha}{E_1(\sigma_2^2-\sigma_\alpha)+E_2(\sigma_1^2-\sigma_\alpha)}\right)$ = *(0.85, 0.15)*
- Expected Return: $E(w) = \varpi_1E(r_1) + \varpi_2E(r_2)$ = *0.069*
- Entropy-Adjusted Variance: $\sqrt{\varpi_1^2\sigma_1^2 + \varpi_2^2\sigma_2^2 + 2\sigma_{12}\varpi_1\varpi_2 - \alpha[1-(\omega_1^2+\omega_2^2)} = 0.4$
- Risk: $\sqrt{\varpi_1^2\sigma_1^2 + \varpi_2^2\sigma_2^2 + 2(\sigma_{12}-\alpha)\omega_1\omega_2}$ = *0.63*
- Risk-Adjusted Return: $\frac{\varpi E_1 + (1-\varpi)E_2}{\sqrt{\varpi^2\sigma_1^2+(1-\varpi)^2\sigma_2^2+2\sigma_\alpha\omega(1-\varpi)}}$ = *0.1.*

3. Results and Discussions

In this section, we analyze the empirical results of applying the Mean-Variance-Entropy (MVE) optimization model to our selected dataset of assets and we compare portfolio performance under three approaches: the Minimum Variance Portfolio, the classical Mean-Variance model, and the Mean-Variance-Entropy framework. The results highlight the risk-return balance and advantages of entropy-based diversification.

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3.1. Summary of Optimization Results

The optimization results highlight distinct characteristics for each portfolio type. The computed optimal weights, expected returns, and risk-adjusted returns are summarized in Table 1.

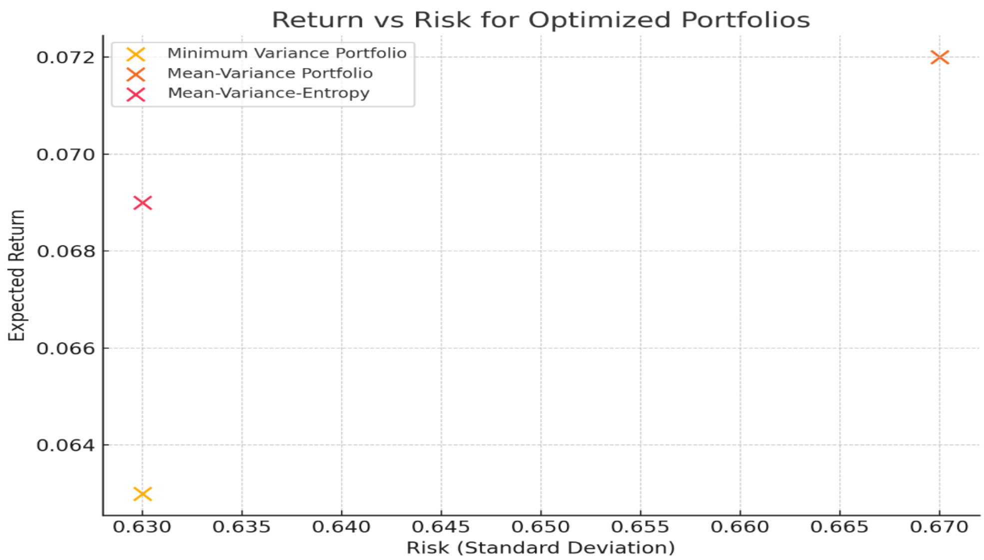
Portfolio Type	*Weights (BTC,ETH)*	*Return*	*Variance*	*Standard Deviation*	*Risk-Adjusted Return*
Minimum Variance	(0.76, 0.24)	0.063	0.397	0.63	0.16
Mean-Variance	(0.97, 0.03)	0.072	0.45	0.67	0.1
Mean-Variance-Entropy	(0.85, 0.15)	0.069	0.4	0.63	0.1

The minimum variance portfolio achieves the lowest standard deviation but sacrifices expected return. The classical mean-variance portfolio prioritizes return maximization but results in extreme allocation (97% in BTC). The MVE portfolio balances diversification while maintaining competitive returns.

3.2. Comparative Analysis

Risk-Return Trade-offs

The Mean-Variance portfolio achieves the highest expected return but introduces significant concentration risk, with 97% allocated to BTC. The Minimum Variance portfolio achieves the lowest risk but at the cost of reduced return. The Mean-Variance-Entropy (MVE) model offers a balanced alternative, optimizing the trade-off between return, risk, and diversification.



Impact of Entropy on Portfolio Allocation

By penalizing extreme weight distributions, entropy plays a crucial role in enhancing portfolio robustness. In our case, the MVE portfolio avoids excessive exposure to BTC and maintains a more stable structure through moderate ETH allocation. This mitigates potential tail risks associated with over-concentration.

Stability and Diversification

The entropy-based optimization yields a portfolio with improved diversification and reduced sensitivity to market shocks. Although the risk-adjusted return is similar to that of the Mean-Variance model, the stability offered by entropy integration provides added value, especially in volatile or emerging markets such as the crypto space

Implications and Future Directions

The empirical findings demonstrate that entropy-based portfolio optimization offers a compelling alternative to traditional mean-variance approaches. By mitigating concentration risk and enhancing diversification, the MVE model provides a more stable and balanced portfolio allocation. This is particularly relevant in emerging markets like BVB, where asset concentration can be a critical concern.

Future research could explore the application of entropy-driven optimization in multi-asset portfolios, alternative asset classes, and dynamic investment strategies. Additionally, integrating other risk measures, such as Conditional Value at Risk (CVaR), with entropy-based models could further enhance portfolio resilience.

4. Conclusions

This study proposes and validates an extended portfolio optimization model that integrates second-order entropy into the classical mean-variance framework. The proposed Mean-Variance-Entropy (MVE) model accounts not only for return and risk but also for diversification, measured through entropy.

The analytical formulation and empirical application on a BTC–ETH portfolio show that entropy-based optimization can effectively address concentration risk while maintaining competitive performance metrics. The comparative results demonstrate that, while the Mean-Variance model maximizes returns, it does so by introducing instability through concentrated asset weights. In contrast, the MVE approach achieves a better balance between return and risk diversification.

The originality of our approach lies in incorporating second-order entropy alongside classical risk-return parameters, allowing for a more flexible and realistic decision-making framework.

Future research could extend this model to multi-asset portfolios including alternative investments (e.g., stablecoins, DeFi assets), or explore dynamic strategies under regime-switching market conditions. Moreover, integrating other risk measures such as Conditional Value-at-Risk (CVaR) could enhance the practical applicability of entropy-based optimization, particularly in the context of volatile financial ecosystems.

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