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Not peer-reviewed version

Failed Zero-Forcing Number in Neutrosophic Graphs

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Posted Date: 26 February 2022

doi: 10.20944/preprints202202.0343.v1

Keywords: failed zero-forcing number; maximal set; vertex

Failed Zero-Forcing Number in Neutrosophic Graphs

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Abstract

New setting is introduced to study failed zero-forcing number and failed zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex but now reverse approach is on demand which is finding biggest set which doesn't force. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then failed zero-forcing number $\mathcal{Z}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximal cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. Failed zero-forcing neutrosophic-number $\mathcal{Z}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximal neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex. Failed zero-forcing number and failed zero-forcing neutrosophic-number are about a set of vertices which are applied into the setting of neutrosophic graphs. The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, bipartite-neutrosophic graphs, and t-partite-neutrosophic graphs are investigated in the terms of maximal set which forms both of failed zero-forcing number and failed zero-forcing neutrosophic-number. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach facilitates identifying vertices which form failed zero-forcing number and failed zero-forcing neutrosophic-number. In path-neutrosophic graphs, the set of vertices such that every given two vertices in the set, have distance at least two, forms maximal set but with slightly differences, in cycle-neutrosophic graphs, the set of vertices such that every given two vertices in the set, have distance at least two, forms maximal set. Other classes have same approaches. In complete-neutrosophic graphs, a set of vertices excluding two vertices leads us to failed zero-forcing number and failed zero-forcing neutrosophic-number. In star-neutrosophic graphs, a set of vertices excluding only two vertices and containing center, makes maximal set. In complete-bipartite-neutrosophic graphs, a set of vertices excluding two vertices from same parts makes intended set but with slightly differences, in complete-t-partite-neutrosophic graphs, a set of vertices excluding two vertices from same parts makes intended set. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. Using basic set not to extend this set to set of all vertices has key role to have these notions in the form of failed zero-forcing number and failed zero-forcing neutrosophic-number. The cardinality of a set has eligibility to form failed zero-forcing number but the neutrosophic cardinality of a set has eligibility to call failed zero-forcing neutrosophic-number. Some results get more frameworks and perspective about these definitions. The way in that, two vertices don't have unique connection together, opens the way to do some approaches. A vertex could affect on other vertex but there's no usage of edges. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Failed Zero-Forcing Number, Maximal Set, Vertex
AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in **Ref.** [16], neutrosophic set in **Ref.** [2], related definitions of other sets in **Refs.** [2, 14, 15], graphs and new notions on them in **Refs.** [5–12], neutrosophic graphs in **Ref.** [3], studies on neutrosophic graphs in **Ref.** [1], relevant definitions of other graphs based on fuzzy graphs in **Ref.** [13], related definitions of other graphs based on neutrosophic graphs in **Ref.** [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.1. *Is it possible to use mixed versions of ideas concerning "Failed Zero-Forcing Number", "Failed Zero-Forcing Neutrosophic-Number" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Unique connections amid two vertices have key roles to assign failed zero-forcing number and failed zero-forcing neutrosophic-number. Thus they're used to define new ideas which conclude to the structure failed zero-forcing number and failed zero-forcing neutrosophic-number. The concept of not having unique edge inspires us to study the behavior of vertices in the way that, some types of numbers, failed zero-forcing number and failed zero-forcing neutrosophic-number are the cases of study in the setting of individuals. In both settings, a corresponded number concludes the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of failed zero-forcing number and failed zero-forcing neutrosophic-number are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", sets of vertices have the key role in this way. General results are obtained and also, the results

 doi:10.20944/preprints202202.0343.v1

about the basic notions of failed zero-forcing number and zero forcing neutrosophic-number are elicited. Some classes of neutrosophic graphs are studied in the terms of neutrosophic failed zero-forcing number, in section "Setting of Neutrosophic Failed Zero-Forcing Number," as individuals. In section "Setting of Neutrosophic Failed Zero-Forcing Number,", neutrosophic failed zero-forcing number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs and complete-t-partite-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Neutrosophic Failed Zero-Forcing Number," and "Setting of Failed Zero-Forcing Neutrosophic-Number," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for star-neutrosophic graphs and cycle-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.2 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.2. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.3. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\Sigma_{e\in E}\Sigma_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1, σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1, σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1, σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

The main definition is presented in next section. The notions of failed zero-forcing number and failed zero-forcing neutrosophic-number facilitate the ground to introduce new results. These notions will be applied on some classes of neutrosophic graphs in upcoming sections and they separate the results in two different sections based on introduced types. New setting is introduced to study failed zero-forcing number and failed zero-forcing neutrosophic-number. Leaf-like is a key term to have these notions. Forcing a vertex to change its color is a type of approach to force that vertex to be zero-like. Forcing a vertex which is only neighbor for zero-like vertex to be zero-like vertex but now reverse approach is on demand which is finding biggest set which doesn't force.

Definition 1.5. (Failed Zero-Forcing Number).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Failed zero-forcing number** $\mathcal{Z}'(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

(ii) **Failed zero-forcing neutrosophic-number** $\mathcal{Z}'_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is maximum neutrosophic cardinality of a set S of black vertices (whereas vertices in $V(G) \setminus S$ are colored white) such that $V(G)$ isn't turned black after finitely many applications of "the color-change rule": a white vertex is converted to a black vertex if it is the only white neighbor of a black vertex.

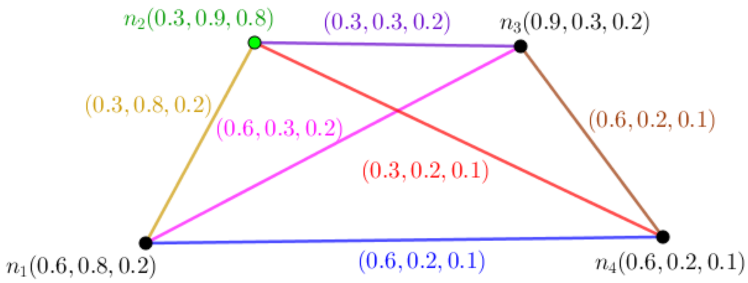

Figure 1. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.

In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.6. In Figure (1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex. Thus n_1 and n_2 aren't black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_3, n_4\}$ could form failed zero-forcing number;

(ii) if $S = \{n_2, n_3, n_4\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. Thus n_3 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iv) if $S = \{n_1, n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_2 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_3\}, \{n_2, n_4\}$, and $\{n_3, n_4\}$;

(vi) 3.6 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

2 Setting of Neutrosophic Failed Zero-Forcing Number

In this section, I provide some results in the setting of neutrosophic failed zero-forcing number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, and star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph are both of cases of study and classes which the results are about them.

In next result, a complete-neutrosophic graph is considered in the way that, its neutrosophic failed zero-forcing number and its failed zero-forcing neutrosophic-number this model are computed. A complete-neutrosophic graph has specific attribute which implies every vertex is neighbor to all other vertices in the way that, two given vertices have edge is incident to these endpoints.

Proposition 2.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. If S is a set of black vertices and $S < \mathcal{O}(NTG) - 1$, then there are x and y such that they've more than one neighbor in S. Thus the color-change rule doesn't imply these vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.2. In Figure (2), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex. Thus n_1 and n_2 aren't black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_3, n_4\}$ could form failed zero-forcing number;

(*ii*) if $S = \{n_2, n_3, n_4\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_1, n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. Thus n_3 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iv*) if $S = \{n_1, n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_2 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*v*) 2 is failed zero-forcing number and its corresponded sets are $\{n_1, n_2\}$, $\{n_1, n_3\}$, $\{n_1, n_4\}$, $\{n_2, n_3\}$,$\{n_2, n_4\}$, and $\{n_3, n_4\}$;

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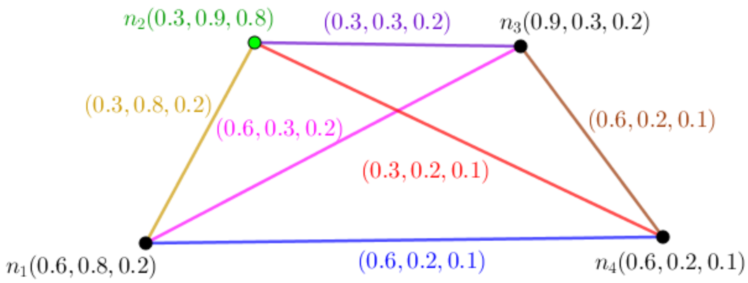

Figure 2. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

(vi) 3.6 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

In next result, a path-neutrosophic graph is considered and failed zero-forcing number and its failed zero-forcing neutrosophic-number of this model are computed for a leaf in specific case. In next result where being leaf-like and having its unique edge are key hypotheses, the set of black forms failed zero-forcing number and its failed zero-forcing neutrosophic-number.

Proposition 2.3. *Let* $NTG : (V, E, \sigma, \mu)$ *be a path-neutrosophic graph. Then*

$$\mathcal{Z}'(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Every non-leaf vertex is a neighbor for two vertices. Non-leaf vertices with distance two, are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

□

In next part, one odd-path-neutrosophic graph is depicted. Failed zero-forcing number and its failed zero-forcing neutrosophic-number are computed. In next part, one even-path-neutrosophic graph is applied to compute its failed zero-forcing number and its failed zero-forcing neutrosophic-number, too.

Example 2.4. There are two sections for clarifications.

(a) In Figure (3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

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(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2, n_4\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other side, n_5 and n_3 are only white neighbor of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4\}$ could form failed zero-forcing number;

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex and after that n_3 is only white neighbor of n_2. Thus the color-change rule implies n_3 is black vertex after that n_4 is only white neighbor of n_3. Thus the color-change rule implies n_4 is black vertex after that n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_2, n_3, n_4 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded set is $\{n_2, n_4\}$;

(vi) 3 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_2, n_4\}$.

(b) In Figure (4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2, n_4\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other side, n_5 and n_3 are only white neighbor of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4\}$ could form failed zero-forcing number;

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex and after that n_3 is only white neighbor of n_2. Thus the color-change rule implies n_3 is black

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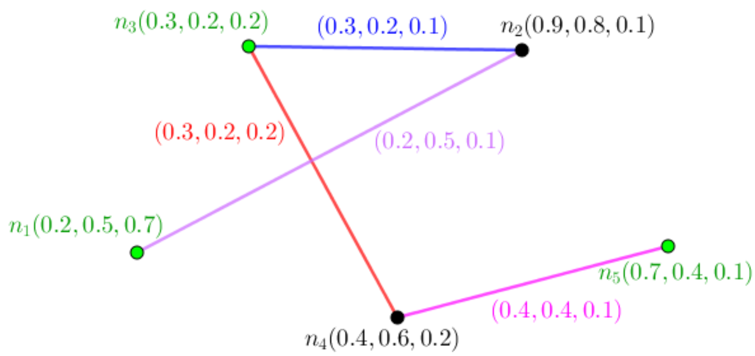

Figure 3. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

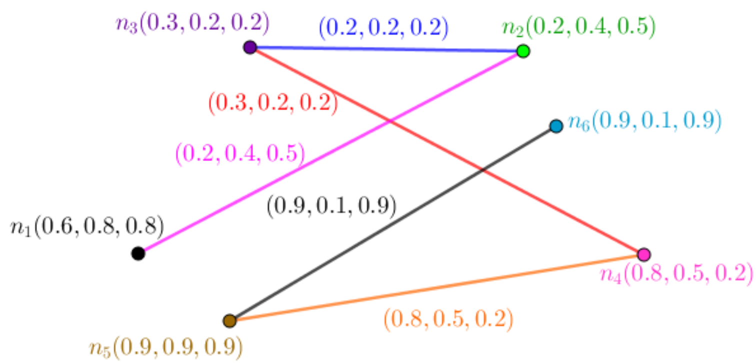

Figure 4. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

vertex after that n_4 is only white neighbor of n_3. Thus the color-change rule implies n_4 is black vertex after that n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_2, n_3, n_4, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4\}$, and $\{n_3, n_5\}$;

(vi) 3.4 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_3, n_5\}$.

The set of vertices forms failed zero-forcing number and its failed zero-forcing neutrosophic-number.

Proposition 2.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}'(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex is a neighbor for two vertices. Vertices with distance two, are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of “the color-change rule”. So

$$\mathcal{Z}'(NTG) = \lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6. There are two sections for clarifications.

(*a*) In Figure (5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of “the color-change rule”;

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of “the color-change rule”;

(*iii*) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of “the color-change rule”. Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of “the color-change rule”;

(*v*) 3 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(*vi*) 4.9 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(*b*) In Figure (6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

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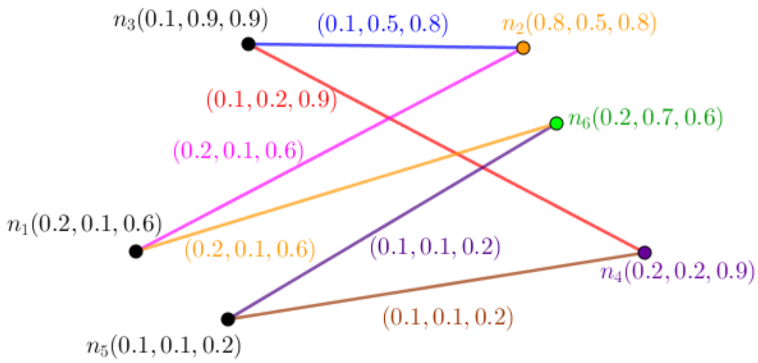

Figure 5. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4\}$,
$\{n_3, n_5\}$,
$\{n_2, n_5\}$, $\{n_4, n_1\}$,
and $\{n_1, n_3\}$;

(vi) 3.7 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3\}$.

Proposition 2.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center

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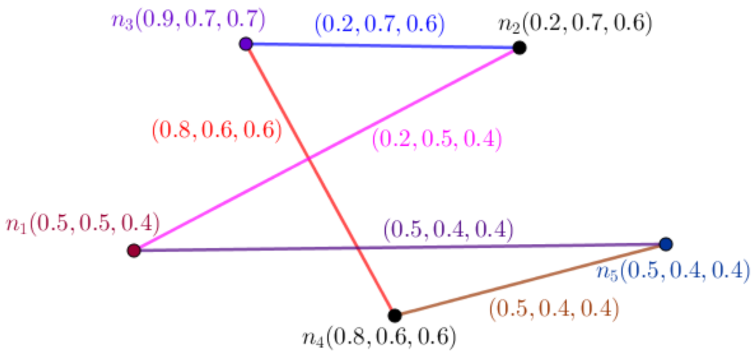

Figure 6. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

is only neighbor for all vertices. Hence all vertices excluding two vertices but containing center are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.8. There is one section for clarifications. In Figure (7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 are black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_3. Thus the color-change rule implies n_1 is black vertex and after that n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_4, n_5\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1. Thus the color-change rule doesn't imply n_2 and n_3 are black vertices. n_1 is only white neighbor of n_4 but $n_1 \in S$. Thus the color-change rule doesn't imply n_1 is black vertex. n_1 is only white neighbor of n_5 but $n_1 \in S$. Thus the color-change rule doesn't imply n_1 is black vertex. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_1, n_4, n_5\}$ could form failed zero-forcing number;

(iv) if $S = \{n_2, n_3, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_3. Thus the color-change rule implies n_1 is black vertex and after that n_4 is only white neighbor of n_1. Thus the color-change rule implies n_4 is black vertex. Thus n_1 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

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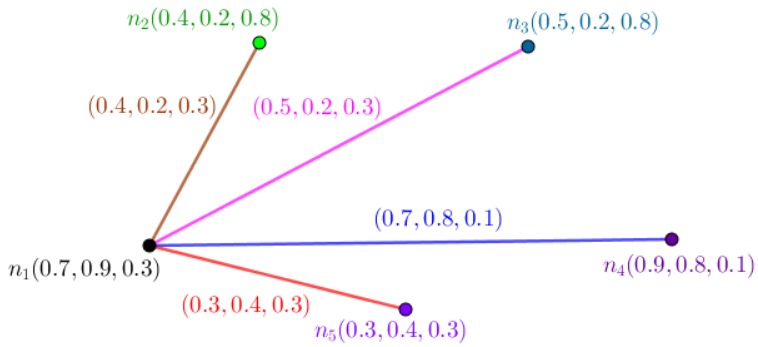

Figure 7. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

(*v*) 3 is failed zero-forcing number and its corresponded sets are $\{n_1, n_2, n_3\}$,$\{n_1, n_2, n_4\}$, $\{n_1, n_2, n_5\}$,$\{n_1, n_3, n_4\}$, $\{n_1, n_3, n_5\}$, and $\{n_1, n_4, n_5\}$;

(*vi*) 5.1 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3, n_4\}$.

Proposition 2.9. *Let $NTG : (V, E, \sigma, \mu)$ be a bipartite-neutrosophic graph. Then*

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence all vertices excluding two vertices from same parts are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

□

The clarifications about results are in progress as follows. A bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.10. There is one section for clarifications. In Figure (8), a bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_1, n_3\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. In other side, n_4 is only white neighbor of n_3. Thus the color-change rule implies n_4 is black vertex. Thus n_2 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_1, n_2\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. In other side, n_4 is only

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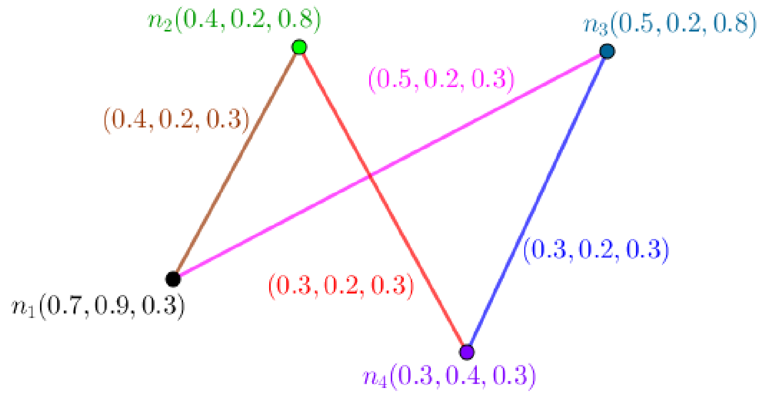

Figure 8. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

white neighbor of n_2. Thus the color-change rule implies n_4 is black vertex. Thus n_3 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_4\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1 and n_4, simultaneously. Thus the color-change rule doesn't imply n_2 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_1, n_4\}$ could form failed zero-forcing number;

(iv) if $S = \{n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_4. Thus the color-change rule implies n_3 is black vertex. In other side, n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_1, n_4\}$, and $\{n_2, n_3\}$;

(vi) 2.9 is failed zero-forcing neutrosophic-number and its corresponded set are $\{n_1, n_4\}$, and $\{n_2, n_3\}$.

Proposition 2.11. *Let $NTG : (V, E, \sigma, \mu)$ be an t-partite-neutrosophic graph such that $t \neq 2$. Then*

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence all vertices excluding two vertices in same part are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'(NTG) = \mathcal{O}(NTG) - 2.$$

□

The clarifications about results are in progress as follows. A t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

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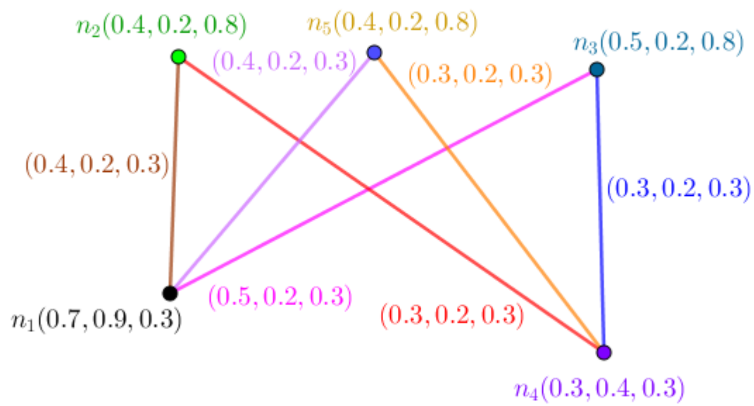

Figure 9. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

Example 2.12. There is one section for clarifications. In Figure (9), a t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_1, n_3, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. In other side, n_4 is only white neighbor of n_3. Thus the color-change rule implies n_4 is black vertex. Thus n_2 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_1, n_2, n_5\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. In other side, n_4 is only white neighbor of n_2. Thus the color-change rule implies n_4 is black vertex. Thus n_3 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_4, n_5\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1 and n_4, simultaneously. Thus the color-change rule doesn't imply n_2 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_1, n_4, n_5\}$ could form failed zero-forcing number;

(iv) if $S = \{n_2, n_4, n_5\}$ is a set of black vertices, then n_3 is only white neighbor of n_4. Thus the color-change rule implies n_3 is black vertex. In other side, n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 3 is failed zero-forcing number and its corresponded sets are $\{n_1, n_4, n_5\}$,$\{n_1, n_4, n_2\}$,
$\{n_1, n_4, n_3\}$, and $\{n_5, n_2, n_3\}$;

(vi) 4.4 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3, n_4\}$.

3 Setting of Failed Zero-Forcing Neutrosophic-Number

In this section, I provide some results in the setting of failed zero-forcing neutrosophic-number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, and star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph are both of cases of study and classes which the results are about them.

In next result, a complete-neutrosophic graph is considered in the way that, its neutrosophic failed zero-forcing number and its failed zero-forcing neutrosophic-number this model are computed. A complete-neutrosophic graph has specific attribute which implies every vertex is neighbor to all other vertices in the way that, two given vertices have edge is incident to these endpoints.

Proposition 3.1. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-neutrosophic graph. Then*

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y)\}_{x,y \in V}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. Every vertex is a neighbor for every given vertex. Assume $|S| > 2$. If S is a set of black vertices and $S < \mathcal{O}(NTG) - 1$, then there are x and y such that they've more than one neighbor in S. Thus the color-change rule doesn't imply these vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(y)\}_{x,y \in V}.$$

□

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.2. In Figure (10), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- *(i)* if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_2 is black vertex. n_1 is white neighbor of n_3 and n_4. Thus the color-change rule doesn't imply n_1 is black vertex. Thus n_1 and n_2 aren't black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_3, n_4\}$ could form failed zero-forcing number;
- *(ii)* if $S = \{n_2, n_3, n_4\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";
- *(iii)* if $S = \{n_1, n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. Thus n_3 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

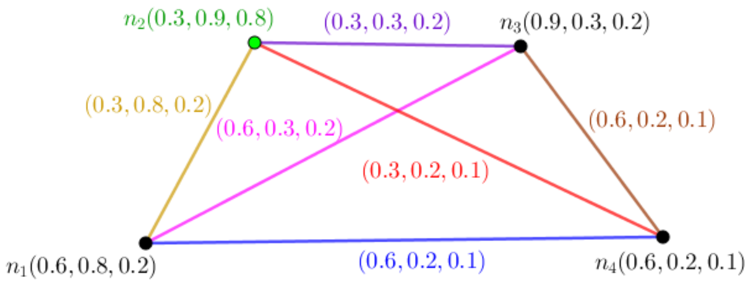

Figure 10. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

(iv) if $S = \{n_1, n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_2 is black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_1, n_2\}, \{n_1, n_3\}, \{n_1, n_4\}, \{n_2, n_3\}, \{n_2, n_4\}$, and $\{n_3, n_4\}$;

(vi) 3.6 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_2\}$.

Proposition 3.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{Z}'_n(NTG) = \max\{\Sigma_{i=1}^3 \sigma_i(x_j) + \Sigma_{i=1}^3 \sigma_i(x_{j+s}) + \cdots\}_{s \geq 2,\ x_z \text{ isn't a leaf}}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Every non-leaf vertex is a neighbor for two vertices. Non-leaf vertices with distance two, are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'_n(NTG) = \max\{\Sigma_{i=1}^3 \sigma_i(x_j) + \Sigma_{i=1}^3 \sigma_i(x_{j+s}) + \cdots\}_{s \geq 2,\ x_z \text{ isn't a leaf}}.$$

□

In next part, one odd-path-neutrosophic graph is depicted. Failed zero-forcing number and its failed zero-forcing neutrosophic-number are computed. In next part, one even-path-neutrosophic graph is applied to compute its failed zero-forcing number and its failed zero-forcing neutrosophic-number, too.

Example 3.4. There are two sections for clarifications.

(a) In Figure (11), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

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(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_2, n_4\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other side, n_5 and n_3 are only white neighbor of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_1\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex and after that n_3 is only white neighbor of n_2. Thus the color-change rule implies n_3 is black vertex after that n_4 is only white neighbor of n_3. Thus the color-change rule implies n_4 is black vertex after that n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_2, n_3, n_4 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*v*) 2 is failed zero-forcing number and its corresponded set is $\{n_2, n_4\}$;

(*vi*) 3 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_2, n_4\}$.

(*b*) In Figure (12), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_2, n_4\}$ is a set of black vertices, then n_1 and n_3 are only white neighbor of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other side, n_5 and n_3 are only white neighbor of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_1\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex and after that n_3 is only white neighbor of n_2. Thus the color-change rule implies n_3 is black

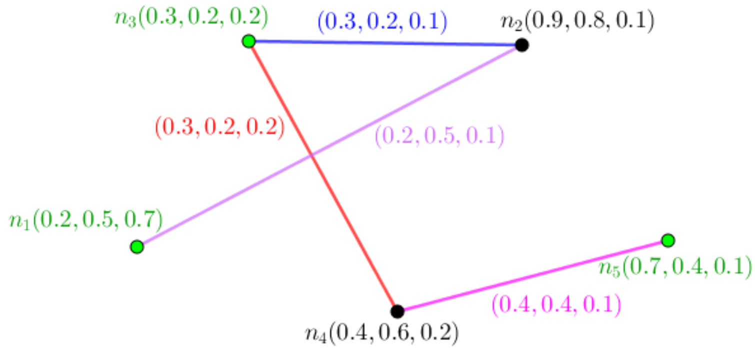

Figure 11. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

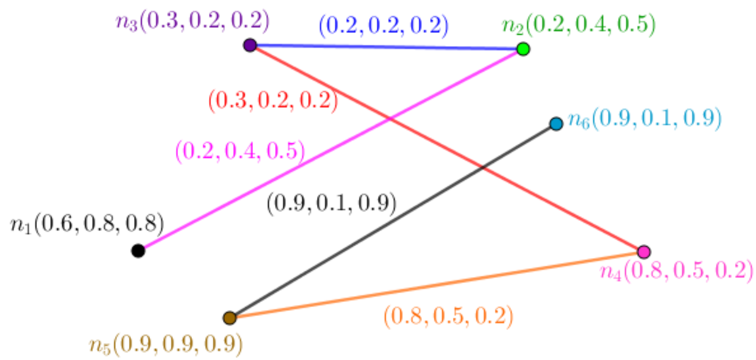

Figure 12. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

vertex after that n_4 is only white neighbor of n_3. Thus the color-change rule implies n_4 is black vertex after that n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_2, n_3, n_4, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4\}$, and $\{n_3, n_5\}$;

(vi) 3.4 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_3, n_5\}$.

The set of vertices forms failed zero-forcing number and its failed zero-forcing neutrosophic-number.

Proposition 3.5. *Let* $NTG : (V, E, \sigma, \mu)$ *be a cycle-neutrosophic graph. Then*

$$\mathcal{Z}'_n(NTG) = \max\{\Sigma_{i=1}^3 \sigma_i(x_j) + \Sigma_{i=1}^3 \sigma_i(x_{j+s}) + \cdots\}_{s \geq 2}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Every vertex is a neighbor for two vertices. Vertices with distance two, are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'_n(NTG) = \max\{\Sigma_{i=1}^3 \sigma_i(x_j) + \Sigma_{i=1}^3 \sigma_i(x_{j+s}) + \cdots\}_{s \geq 2}.$$

□

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6. There are two sections for clarifications.

(a) In Figure (13), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 3 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(vi) 4.9 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(b) In Figure (14), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white

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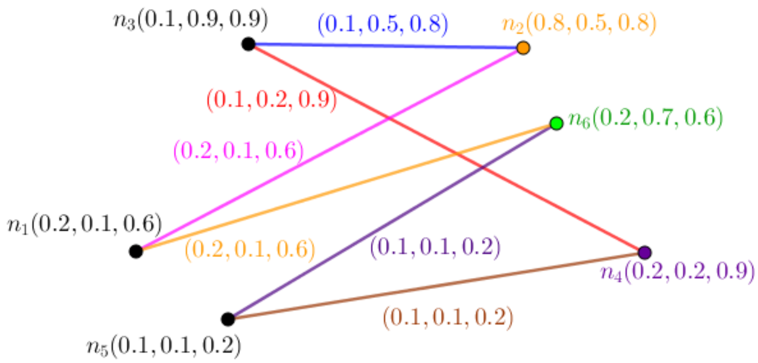

Figure 13. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4\}$,
$\{n_3, n_5\}$,
$\{n_2, n_5\}$, $\{n_4, n_1\}$,
and $\{n_1, n_3\}$;

(vi) 3.7 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3\}$.

Proposition 3.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y)\}_{x,y \in V, x,y \neq c}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. Hence all vertices excluding two vertices but containing center are only members of S is a maximal set of black vertices which doesn't force.

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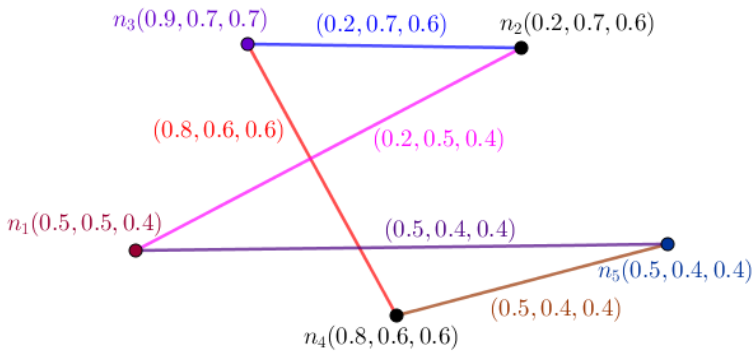

Figure 14. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^{3}\sigma_i(x) + \Sigma_{i=1}^{3}\sigma_i(y)\}_{x,y\in V,x,y\neq c}.$$

□

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.8. There is one section for clarifications. In Figure (15), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 are black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_3. Thus the color-change rule implies n_1 is black vertex and after that n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_1, n_4, n_5\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1. Thus the color-change rule doesn't imply n_2 and n_3 are black vertices. n_1 is only white neighbor of n_4 but $n_1 \in S$. Thus the color-change rule doesn't imply n_1 is black vertex. n_1 is only white neighbor of n_5 but $n_1 \in S$. Thus the color-change rule doesn't imply n_1 is black vertex. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_1, n_4, n_5\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_2, n_3, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_3. Thus the color-change rule implies n_1 is black vertex and after that n_4 is only white neighbor of n_1. Thus the color-change rule implies n_4 is black vertex. Thus n_1 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

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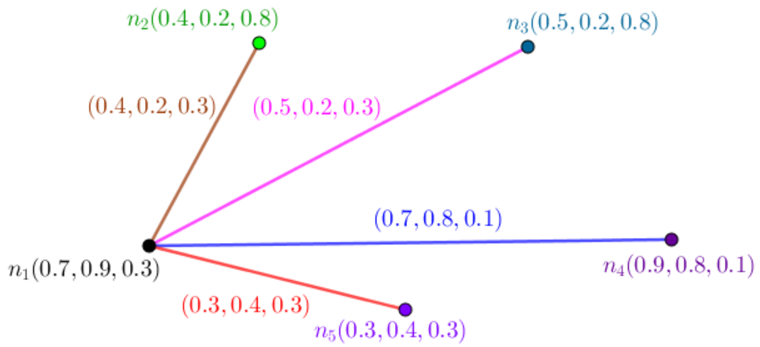

Figure 15. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

(v) 3 is failed zero-forcing number and its corresponded sets are $\{n_1, n_2, n_3\}$,$\{n_1, n_2, n_4\}$, $\{n_1, n_2, n_5\}$,$\{n_1, n_3, n_4\}$, $\{n_1, n_3, n_5\}$, and $\{n_1, n_4, n_5\}$;

(vi) 5.1 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3, n_4\}$.

Proposition 3.9. *Let $NTG : (V, E, \sigma, \mu)$ be a bipartite-neutrosophic graph. Then*

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x')\}_{x,x' \in V_i}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence all vertices excluding two vertices from same parts are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x')\}_{x,x' \in V_i}.$$

□

The clarifications about results are in progress as follows. A bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.10. There is one section for clarifications. In Figure (16), a bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_1, n_3\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. In other side, n_4 is only white neighbor of n_3. Thus the color-change rule implies n_4 is black vertex. Thus n_2 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_1, n_2\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. In other side, n_4 is only

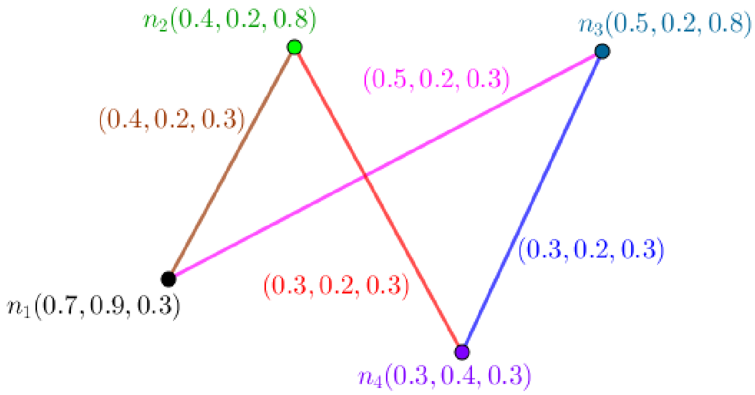

Figure 16. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

white neighbor of n_2. Thus the color-change rule implies n_4 is black vertex. Thus n_3 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_1, n_4\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1 and n_4, simultaneously. Thus the color-change rule doesn't imply n_2 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_1, n_4\}$ could form failed zero-forcing number;

(iv) if $S = \{n_2, n_4\}$ is a set of black vertices, then n_3 is only white neighbor of n_4. Thus the color-change rule implies n_3 is black vertex. In other side, n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_1, n_4\}$, and $\{n_2, n_3\}$;

(vi) 2.9 is failed zero-forcing neutrosophic-number and its corresponded set are $\{n_1, n_4\}$, and $\{n_2, n_3\}$.

Proposition 3.11. *Let* $NTG : (V, E, \sigma, \mu)$ *be a t-partite-neutrosophic graph such that* $t \neq 2$. *Then*

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x')\}_{x,x' \in V_i}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. Hence all vertices excluding two vertices in same part are only members of S is a maximal set of black vertices which doesn't force. Thus the color-change rule doesn't imply all vertices are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". So

$$\mathcal{Z}'_n(NTG) = \mathcal{O}_n(NTG) - \min\{\Sigma_{i=1}^3 \sigma_i(x) + \Sigma_{i=1}^3 \sigma_i(x')\}_{x,x' \in V_i}.$$

□

The clarifications about results are in progress as follows. A t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.12. There is one section for clarifications. In Figure (17), a t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_1, n_3, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. In other side, n_4 is only white neighbor of n_3. Thus the color-change rule implies n_4 is black vertex. Thus n_2 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_1, n_2, n_5\}$ is a set of black vertices, then n_3 is only white neighbor of n_1. Thus the color-change rule implies n_3 is black vertex. In other side, n_4 is only white neighbor of n_2. Thus the color-change rule implies n_4 is black vertex. Thus n_3 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_1, n_4, n_5\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1 and n_4, simultaneously. Thus the color-change rule doesn't imply n_2 and n_3 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_1, n_4, n_5\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_2, n_4, n_5\}$ is a set of black vertices, then n_3 is only white neighbor of n_4. Thus the color-change rule implies n_3 is black vertex. In other side, n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_3 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*v*) 3 is failed zero-forcing number and its corresponded sets are $\{n_1, n_4, n_5\}$,$\{n_1, n_4, n_2\}$, $\{n_1, n_4, n_3\}$, and $\{n_5, n_2, n_3\}$;

(*vi*) 4.4 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3, n_4\}$.

4 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models aren't complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

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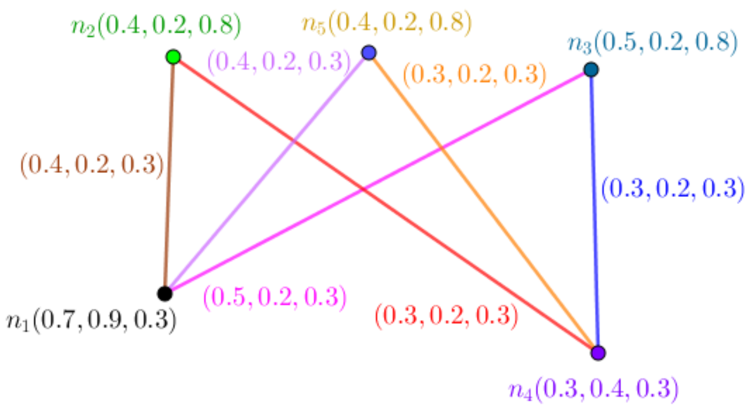

Figure 17. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

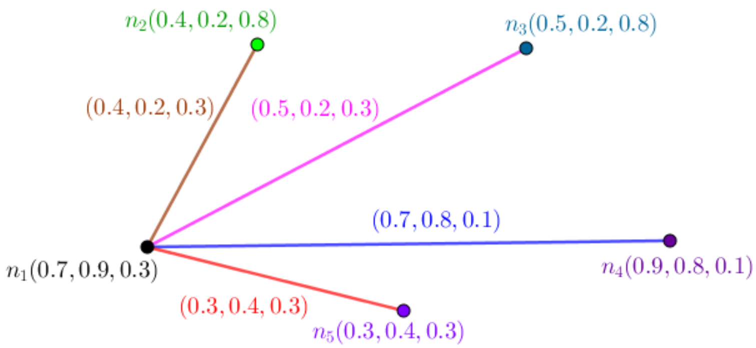

Figure 18. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1), clarifies about the assigned numbers to these situations.

Table 1. Scheduling concerns its Subjects and its Connections as a neutrosophic graph and its alliances in a Model.

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.3, 0.4, 0.3)$
Connections of NTG	E_1	$E_2 \cdots$	E_4
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.4, 0.3)$

4.1 Case 1: Star Model alongside its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number

Step 4. (Solution) The neutrosophic graph alongside its failed zero-forcing number and its failed zero-forcing neutrosophic-number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates some connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of failed zero-forcing number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (18). This model is strong. And the study proposes using specific number which is called failed zero-forcing number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (18). In Figure (18), an star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) if $S = \{n_2, n_3, n_4, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 are black vertex. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*ii*) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_3. Thus the color-change rule implies n_1 is black vertex and after that n_2 is only white neighbor of n_1. Thus the color-change rule implies n_2 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*iii*) if $S = \{n_1, n_4, n_5\}$ is a set of black vertices, then n_2 and n_3 are white neighbors of n_1. Thus the color-change rule doesn't imply n_2 and n_3 are black vertices. n_1 is only white neighbor of n_4 but $n_1 \in S$. Thus the color-change rule doesn't imply n_1 is black vertex. n_1 is only white neighbor of n_5 but $n_1 \in S$. Thus the color-change rule doesn't imply n_1 is black vertex. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_1, n_4, n_5\}$ could form failed zero-forcing number;

(*iv*) if $S = \{n_2, n_3, n_5\}$ is a set of black vertices, then n_1 is only white neighbor of n_3. Thus the color-change rule implies n_1 is black vertex and after that n_4 is only white neighbor of n_1. Thus the color-change rule implies n_4 is black vertex. Thus n_1 and n_4 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(*v*) 3 is failed zero-forcing number and its corresponded sets are $\{n_1, n_2, n_3\}$,$\{n_1, n_2, n_4\}$, $\{n_1, n_2, n_5\}$,$\{n_1, n_3, n_4\}$, $\{n_1, n_3, n_5\}$, and $\{n_1, n_4, n_5\}$;

(*vi*) 5.1 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3, n_4\}$.

4.2 Case 2: Cycle Model alongside its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number

Step 4. (Solution) The neutrosophic graph alongside its failed zero-forcing number and its failed zero-forcing neutrosophic-number as model, propose to use specific

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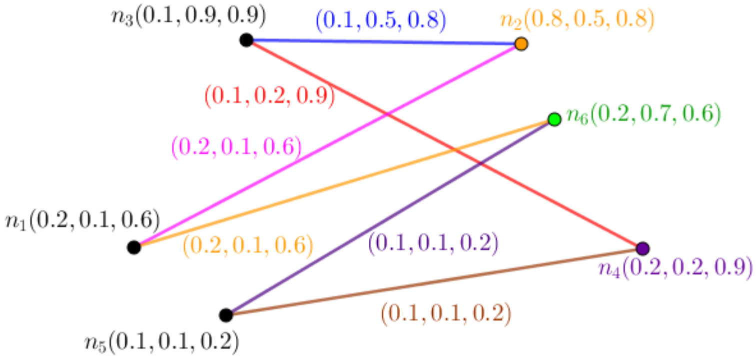

Figure 19. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

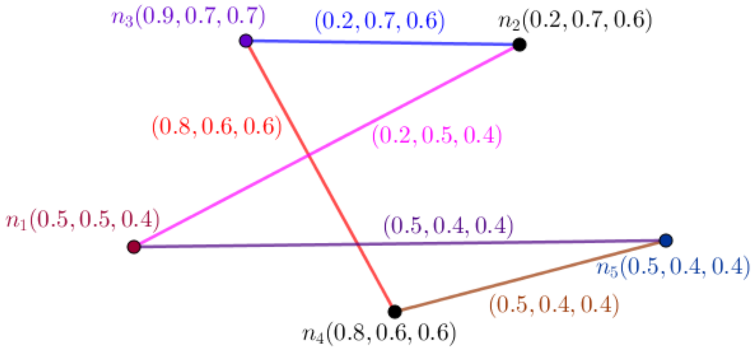

Figure 20. A Neutrosophic Graph in the Viewpoint of its Failed Zero-Forcing Number and its Failed Zero-Forcing Neutrosophic-Number.

number. Every subject has connection with every given subject in deemed way. Thus the connection is applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of failed zero-forcing number and failed zero-forcing neutrosophic-number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are six subjects and five subjects which are represented in the formation of two models as Figures (19), (20). These models are neutrosophic strong as individual. And the study proposes using specific number which is called failed zero-forcing number and failed zero-forcing neutrosophic-number for these models. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figures (19), (20). There is one section for clarifications.

(a) In Figure (19), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

- (i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex and after that n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2, n_5 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";
- (ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_6 is only white neighbor of n_5. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_6 is only white neighbor of n_5. Thus the color-change rule implies n_6 is black vertex. Thus n_1, n_2 and n_6 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";
- (iii) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;
- (iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are

black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 3 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4, n_6\}$ and $\{n_1, n_3, n_5\}$;

(vi) 4.9 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_2, n_4, n_6\}$.

(b) In Figure (20), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) if $S = \{n_3, n_4\}$ is a set of black vertices, then n_2 is only white neighbor of n_3 and n_5 is only white neighbor of n_4. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. In other side, n_5 is only white neighbor of n_4. Thus the color-change rule implies n_5 is black vertex. Thus n_1, n_2 and n_5 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(ii) if $S = \{n_3, n_4, n_5\}$ is a set of black vertices, then n_2 is only white neighbor of n_3. Thus the color-change rule implies n_2 is black vertex and after that n_1 is only white neighbor of n_2. Thus the color-change rule implies n_1 is black vertex. Thus n_1 and n_2 are black vertices. Hence $V(G)$ is turned black after finitely many applications of "the color-change rule";

(iii) if $S = \{n_2, n_4, n_6\}$ is a set of black vertices, then n_1 and n_3 are only white neighbors of n_2. Thus the color-change rule doesn't imply n_1 and n_3 are black vertices. In other view, n_5 and n_3 are only white neighbors of n_4. Thus the color-change rule doesn't imply n_5 and n_3 are black vertices. In last view, n_5 and n_4 are only white neighbors of n_6. Thus the color-change rule doesn't imply n_5 and n_4 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule". Thus $S = \{n_2, n_4, n_6\}$ could form failed zero-forcing number;

(iv) if $S = \{n_1\}$ is a set of black vertices, then n_2 and n_6 are only white neighbor of n_1. Thus the color-change rule doesn't imply n_2 and n_6 are black vertices. Hence $V(G)$ isn't turned black after finitely many applications of "the color-change rule";

(v) 2 is failed zero-forcing number and its corresponded sets are $\{n_2, n_4\}$, $\{n_3, n_5\}$,
$\{n_2, n_5\}$, $\{n_4, n_1\}$,
and $\{n_1, n_3\}$;

(vi) 3.7 is failed zero-forcing neutrosophic-number and its corresponded set is $\{n_1, n_3\}$.

5 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

Notion concerning failed zero-forcing number and failed zero-forcing neutrosophic-number are defined in neutrosophic graphs. Neutrosophic number is also reused. Thus,

Question 5.1. *Is it possible to use other types failed zero-forcing number and failed zero-forcing neutrosophic-number arising from operations of different vertices to define new failed zero-forcing number and failed zero-forcing neutrosophic-number?*

Question 5.2. *Are existed some connections amid different types of failed zero-forcing number and failed zero-forcing neutrosophic-number in neutrosophic graphs?*

Question 5.3. *Is it possible to construct some classes of which have "nice" behavior?*

Question 5.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 5.5. *Which parameters are related to this parameter?*

Problem 5.6. *Which approaches do work to construct applications to create independent study?*

Problem 5.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

6 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning failed zero-forcing number and failed zero-forcing neutrosophic-number arising operations of different vertices to study neutrosophic graphs. New neutrosophic number is reused which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. Comparisons amid number and edges are done by using neutrosophic tool. The connections of vertices which aren't clarified by unique edges differ them from each other and put them in different categories to represent a number

Table 2. A Brief Overview about Advantages and Limitations of this study

Advantages	Limitations
1. Defining Failed Zero-Forcing Number	1. Wheel-Neutrosophic Graphs
2. Defining Failed Zero-Forcing Neutrosophic-Number	
3. Study on Classes	2. Study on Families
4. Using Operations of Vertices	
5. Avoid Unique Edges	3. Same Models in Family

which its value is called either failed zero-forcing number or forms failed zero-forcing neutrosophic-number. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2), some limitations and advantages of this study are pointed out.

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