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2D Behavior of Gravity at Large Distances

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Abstract: We show that the entropy or information flow of a gravitational system constrains the dependence of gravitational force on radial distance r which for very large r is equivalent to gravity in two dimensions. This explains the "origin" of dark matter and provides a natural solution to the problem of flat rotation curves of galaxies at very large distances and a larger deflection angle for a gravitational lens.

Keywords: dark matter; modified Newtonian gravity; holographic screen; galactic rotation curve; gravitational lensing

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1. Introduction

The holographic principle [1,2] states that a 3D system is actually 2D at the fundamental level. This conclusion came from the black hole physics [3–5]. AdS/CFT [6] presents another strong support for this idea. The idea of the holographic principle basically concerns itself with the storage of information of a black hole on a $2D$ surface. However, in [7], the authors showed that if we consider the information "flow" then a black hole behaves essentially as a $1D$ system. Therefore, if the storage of information is concerned, a black hole behaves as a 2D system, and if information flow is concerned a black hole behaves as a 1D system. In [8], treating a Schwarzschild black hole as a 1D system, its correct Bekenstein-Hawking(BH) entropy was derived while in [9], the correct form of BH entropy was derived by treating them as a 2D system. In the first paper, the information flow of black holes was the basis while the second paper claimed that since the phase space is modified which basically concerns the storage of states of the system, information storage should be the basis thereby treating the phase space as 2D. Some works on the 1D idea have been carried out [10–12]. In this paper, we show that the information "flow" of a gravitational system constrains the dependency of the force of gravity on radial distance r at very large distances. The force of gravity is shown to be modified as

$$F(r >> r_0) \propto \frac{1}{r} \tag{1}$$

where r_0 is a cutoff length scale that marks the departure from Newton's law of gravity. This modified dynamics (Equation (1)) solves the problem of observed flat rotation curves of galaxies at very large distances which is in conflict with Newton's law of gravity. This observation led to the advent of dark matter which is hypothesized to provide the "extra" mass needed to fit with observation. This is the basis of the standard cosmology ΛCDM. While another paradigm is Modified Gravity theories(see [13] for a nice discussion) that try to modify the Lagrangian of standard GR to account for this observation without any dark matter particles. This began with the proposal of Milgrom [14] who argued that Newton's law is modified at large distances or equivalently Newton's law of gravity behaves differently at these scales. Its relativistic generalization is Scalar-Vector-Tensor(STV) theory [15] which in the weak field limit reduces to Newtonian potential plus a repulsive Yukawa potential that accounts for the flat rotation curve. Theoretically, we can only account for the explanation of dark matter but we can not explain its "origin" per se. In this paper, we will show how dark matter naturally arises in a gravitational system. This is based on the idea that as far as information flow is concerned, black holes behave as a $1D$ system.

The paper is organized as follows: In section II, we briefly review the idea of entropy flow needed for the discussion of this paper. We elucidate our idea in section III and end the paper with some discussion.

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2. Entropy Flow: A Brief Review

The entropy $s(p)$ of any boson mode of momentum p in a thermal state at temperature T is [16]

$$s(p) = \frac{\epsilon(p)}{e^{\epsilon(p)}/T - 1} - \ln(1 - e^{-\epsilon(p)/T}) \tag{2}$$

The entropy current in one direction is given as

$$\dot{S} = \int_0^\infty s(p)v(p)\frac{dp}{2\pi\hbar} \tag{3}$$

$dp/2\pi\hbar$ being the number of modes per unit length in the interval dp which propagate in one direction. After integration by parts on the second term coming from Equation (2), we can write it as

$$\dot{S} = \frac{2}{T}\int_0^\infty \frac{\epsilon(p)}{e^{\epsilon(p)/T} - 1}\frac{d\epsilon(p)}{dp}\frac{dp}{2\pi\hbar} \tag{4}$$

The integral represents the unidirectional power P in the channel and gives

$$\dot{S} = 2P/T \tag{5}$$

The result of power is

$$P = \frac{\pi T^2}{12\hbar} \tag{6}$$

On eliminating T, we get Pendry's maximum entropy rate for power P [17]

$$\dot{S} = \left(\frac{\pi P}{3\hbar}\right)^{1/2} \tag{7}$$

This formula characterizes the one-dimensional transmission of entropy or information. For a hot closed black body with temperature T and area A in $3D$ space, we similarly obtain

$$P = \frac{\pi^2 T^4 A}{120\hbar^3} \tag{8}$$

$$\dot{S} = \frac{4P}{3T} \tag{9}$$

For a black hole system, $\dot{S}$ becomes [7]

$$\dot{S} = \left(\frac{\nu^2\Gamma\pi P}{480\hbar}\right)^{1/2} \tag{10}$$

similar to Pendry's limit for one-channel flow. This led the authors of [7] to conclude that Black holes are a $1D$ system in their entropy flow.

3. Behavior of Gravity at Large Distances

Let us reflect on the idea that black holes are $1D$ in their entropy or information flow. What was needed to discuss this idea was a boundary so that it has a temperature T. Using this line of thought, we can try to apply this idea to holographic screens which also have a finite temperature T. In our approach, we choose a system of a mass M with a holographic screen at a distance r (the screen stores all the data of the bulk). We further assume the screen to be spherical. We now treat this system as equivalent to the black hole system and claim that this behaves as a 1D system in its information flow. In what follows, it is clear that this claim that the system of mass M with a holographic screen follows the equation of a hot black body similar to a black hole is valid only if the distance of the holographic

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screen r is very large since as we show in this paper, this idea directly constrains the dependency of gravitational force on distance which is only possible to be modified at large distances only, in agreement with at least our experience so far[1]. A technical explanation shall be given later on. For now, let us verify whether or not Newton's law of gravity satisfies our claim. The temperature T of the screen is given as [19]

$$T = -\frac{1}{2\pi}e^{\phi}n^{\mu}a_{\mu} = \frac{1}{2\pi}e^{\phi}\sqrt{\nabla^{\mu}\phi\nabla_{\mu}\phi} \tag{11}$$

Here $n^{\mu} = \nabla^{\mu}\phi/\sqrt{\nabla^{\nu}\phi\nabla_{\nu}\phi}$ is the normal vector on the screen. $a^{\mu} = -\nabla^{\mu}\phi$ is the acceleration of a particle close to the holographic screen and ϕ is the Newtonian potential defined as

$$\phi = \frac{1}{2}\ln(-\xi^2) \tag{12}$$

where ξ is a global time-like Killing vector field. For a spherically symmetric spacetime, the metric reads

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 + r^2d\Omega^2 \tag{13}$$

where $d\Omega$ is the line element of a unit 2-sphere. Thus, the timeline killing vector and Newtonian potential is

$$\xi_{\mu} = (f(r), 0, 0, 0) \tag{14}$$

$$\phi = \frac{1}{2}\ln f(r) \tag{15}$$

Therefore, the temperature becomes[2][21]

$$T = \frac{1}{4\pi}|f'(r)| = \frac{M}{2\pi r^2} \tag{16}$$

This gives the power as

$$P = \frac{\pi^2 M T^3}{60} \tag{17}$$

This value leads to the system as effectively 2D (using Equation (9)) and not 1D as required. Let us see what happens if we modify the metric $f(r)$ such that it has now a logarithmic dependence on r as $f(r) = kr$ with k being a constant to be determined. This metric ansatz will give Newtonian gravity as k/r in the weak-field limit. It is important to note that the metric solution spelled above does not come from the Einstein-Hilbert action but rather from some modified theory of gravity that respects diffeomorphism invariance. In this case, the temperature becomes

$$T = \frac{k}{4\pi r} \tag{18}$$

This gives the power as

$$P = \frac{\pi k^2 T^2}{480} \tag{19}$$

and we obtain

$$\dot{S} = \left(\frac{\pi k^2 P}{270}\right)^{1/2} \tag{20}$$

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1 On solar system scales, Newton's law of gravity holds perfectly well while chances of modifications in the law of gravity are only possible at large distances in view of the peculiar behavior of rotation curves of galaxies. So far, this modification has only come to our knowledge via experiment. But in this paper, we present a possible "origin" of this modification for the first time.

2 In this section, we use units such that $G = c = k_B = \hbar = 1$

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Now, this expression is similar to Pendry's limit (Equation (7)), and the system effectively behaves as 1D. So, this expression for Newtonian gravity satisfies our requirement. It is easy to check that no other dependency of r will satisfy our requirement; thus, $f(r) \propto \ln r$ is the unique dependency. Before we evaluate the value of constant k, we return to how far the screen is placed. In general, the holographic screen can be placed at any distance. However, it is clear that our system cannot be blindly treated as a black body. But it turns out that in certain limits, it can be made to act as a black body (at least in the way that it approximately follows the Stefan-Boltzmann law). We, therefore, drop the assumption that the energy spectrum tends to infinity and keep the upper limit finite (yet large). The equation of power P is therefore given by

$$P = \frac{2\pi hA}{c^2} \int_0^{\epsilon(p)} \frac{\epsilon(p)^3}{e^{\frac{\epsilon(p)}{k_B T}} - 1} d\epsilon(p) \tag{21}$$

Let us perform the substitution $u = \frac{\epsilon(p)}{k_B T}$ to obtain

$$P = \frac{2\pi hA}{c^2} \left(\frac{k_B T}{h}\right)^4 \int_0^{\frac{\epsilon(p)}{k_B T}} \frac{u^3}{e^u - 1} du \tag{22}$$

When T is very small which happens when r is very large (since the Unrhuh temperature gives T, $T \propto a \propto 1/r^2$) and since $\epsilon(p)$ is finite (yet large), the upper limit of the integral can be approximated as approaching infinity ($\frac{\epsilon(p)}{k_B T} \to \infty$) and Equation (22) takes the standard form of power emitted by a black body (Equation (8)). If T is large (for relatively small r), the upper limit remains finite (and small), there is some dependency on T as well as $\epsilon(p)$ coming from the finite integral, and therefore it is not of the form of a black body and cannot be treated equivalently as a black hole system (which is taken to be a black body) and therefore as a 1D system. This brings us to an important conclusion, a system of mass M with a holographic screen at a distance r can be treated as a 1D system similar to a black hole only when the screen is at a very large distance. The immediate question that comes up in the mind is, how large r should be or what can be the minimum value of r. For this, we turn to the expression for a at a large distance, which varies as $1/r$. Let us apply this modified law for a spherical galaxy of mass M at large r which becomes $mv^2/r = ma = mk/r$. This gives

$$v^2 = k \tag{23}$$

This shows that the rotation curves are flat at large distances since k is a constant. From this, we infer that this dependence solves the problem of dark matter. We can find the value of k using the Tully-Fischer relation and MOND theory of Milgrom, where $v^4 = GMa_0$, which gives

$$k = \sqrt{GMa_0} \tag{24}$$

We, therefore, obtain that at large distance $r >> r_0$ with r_0 being a cutoff length scale, the law of gravity is modified as

$$F(r >> r_0) = \frac{\sqrt{GMa_0}}{r} \tag{25}$$

Thus, we find that at very large distances, the behavior of gravity is similar to gravity in two dimensions and obtained its exact expression as Equation (25).

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3.1. Gravitational Lensing

The deflection angle of light due to a gravitational lens of mass M in this case is[3] (following [22])

$$\delta = \frac{2}{c^2}\int \vec{\nabla}_{\perp}(V_N + V)ds \tag{26}$$

$$= \delta_1 + \frac{2b\sqrt{GMa_0}}{c^2}\int_{-\infty}^{\infty}\frac{dx}{(b^2+x^2)} \tag{27}$$

$$= \frac{4GM}{bc^2} + \frac{2\pi\sqrt{GMa_0}}{c^2} \tag{28}$$

$$\equiv \frac{4GM'}{bc^2} \tag{29}$$

where $M' = M\left(1 + \frac{b\pi}{2}\sqrt{\frac{a_0}{GM}}\right)$. Let us analyze this equation with an example. For this, we take $M = 10^{42}kg$, $b = 100kpc \approx 3.1 \times 10^{21}$, then we find that $M' \approx 7.5M$. Thus, we find that the lensing is 7.5 times larger than the standard case. This can solve the problem of a large deflection angle of light around a lens than the standard case given simply by δ_1.

4. Conclusions and Discussion

In this paper, we showed that a gravitational system with a very large radius can be treated as a 1D system in its entropy or information flow similar to a black hole. This idea led to the modification of Newton's law of gravity as $F(r >> r_0) \propto 1/r$. This modification solves the problem of flat rotation curves of galaxies at large distances as well as the large deflection angle of light around a gravitational lens: phenomena that are generally attributed to the presence of dark matter. Viewed in this way, this approach explains the "origin" of dark matter on theoretical grounds for the first time. A direct consequence of this approach is that it is not compatible with Verlinde's idea of entropic gravity [19]. In the entropic case, a holographic screen encloses a volume of space in which space is to emerge. Using the formula for entropic force given by $F\Delta x = T\Delta S$, he obtains Newton's law of gravity. But, as shown in this paper, at large distances, Newton's law of gravity is modified. Therefore, this conflicts with the idea that gravity is an entropic force. Another aspect is that the presence of a holographic screen violates the Bekenstein bound [20]. As pointed out by Verlinde [19], this could be due to two reasons. Firstly, the screen is not in a thermal state and second, the holographic screen does not follow the Bekenstein bound. However, our approach requires the screen to be in a thermal state(at temperature T) so that maximal entropy current occurs. We, therefore, take the second statement to be true that the Bekenstein bound is valid for bulk and not the screen(see [21]). We are hopeful that further studies can be done on this idea.

Conflicts of Interest: The author declares no conflicts of interest.

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