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On Estimation for Reliability of Stress-Strength Model Based on Topp-Leone Distribution

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Abstract: The Paper points at an important point, which is estimate the reliability of system used for stress-strength model when the stress and strength follows Topp-Leone distribution. In this context, this work including two models of stress –strength, the first one when the system has one component with strength subject to one stress, while the another model concern with system has one component which has strength subject for two bounded stresses .The expressions of system reliability of two considered models were derived and estimate using different methods. Comparisons between the considered estimators were made depending on simulation technique based on statistical criterion namely mean squared error.

Keywords: Topp-Leone Distributions; Stress-Strength Model; Monte Carlo simulation, *Mean Squared Error*

حول تقدير معولية نموذج الاجهاد – المتانة بالاعتماد على توزيع Topp-Leone

الملخص

يشير البحث إلى نقطة مهمة تتعلق بتقدير معولية نظام في نموذج الاجهاد – المتانة عندما تتبع كل من الاجهاد والمتانة توزيع Topp-Leone .في هذا السياق يتضمن العمل في هذا البحث نموذجين من الاجهاد – المتانة , يتضمن الاول عندما يحتوي النظام على مركبة واحدة لها قوة (متانة) وتتعرض الى اجهاد واحد فقط بينما يتضمن النموذج الثاني مركبة واحدة في النظام ولها متانة وتتعرض الى اجهادين مقيدين اثنين من الاعلى والاسفل .تم اشتقاق معولية النظام للنموذجين وايضا تم تقديرها بطرائق تقدير مختلفة وايضا تمت المقارنة بين هذه المقدرات باستخدام محاكاة مونتي كارلو اعتمادا على معيار احصائي وهو متوسط مربعات الخطأ.

I. Introduction

One of the most popular techniques for data analysis is the stress-strength system, which is employed in a wide range of disciplines like developed engineering, military presentations, healthiness, and useful skills.

The reliability of system in stress-strength model is an assessment of a module's dependability in terms of the random variable X, which stands in for the stress the module is exposed to, and Y, which stands in for the component's capacity to withstand the potential stress.

In stress–strength model of system, the strength X exposed the stress Y. Both random variables X and Y supposed to follows specific probability distribution with definite parameters. The reliability in stress–strength model refers to the probability that strength overdoes stress, i.e. $P(X > Y)$, roughly p. This topic consumes numerous presentations in several ranges. For example, if Y denotes the extreme heaviness produced by overflowing and X signifies the strength of the leg of a bridge on a stream, then p is the probability that the bridge will be hard. Additional instance, if Y and X are respectively symbolize the regulator and conduct groups, then P processes the treatment consequence. Then the estimation of P will be significant in creation interpretations. The system fails when the stress is too great for it to handle.

The stress-strength concept is particularly significant in the research on reliability. Most of the concerns in the statistical approach to the stress-strength model are predicated on the premise that the component strengths are randomly and uniformly distributed and are exposed to a single stress [1].

A system experiences a stress Y and strength X when demonstrated in a conventional reliability stress-strength analysis. Both random variables are considered to follows specific distribution with known or unknown parameters. The probability of strength exceeding stress, or P(X>Y), indicates the reliability of the system. This topic has many applications in many fields

The stress-strength models of the types *P(Y<X)*, *P(Y<X<Z)*, where *X*, *Y* and *Z* are independent random variables refers to strength X and two stress Y and Z, and its follows specific distribution. These two models have wide requests in several of engineering subareas, psychology, genetics, medical trials and others. Kotz [2].

Isaam, K. , Taha, A. and Abbas,N. They Estimated P(Y<X) using different estimation methods[3] .Chandra and Owen [4] derived maximum likelihood estimators (MLEs) and consequent uniform minimum variance unbiased estimators (UMVUEs) for *R= P(Y<X< Z)*.

Singh [5] offered the minimum variance unbiased, maximum likelihood and empirical estimators of *R= P(Y<X<Z)*, where *X*, *Y* and *Z* are independent random variables and follows the normal distribution. Dutta and Sriwastav [6] estimated *R* when *X*, *Y* and *Z* are exponentially distributed. Ivshin [7] studied the MLE and UMVUE of *R* when *X*, *Y* and *Z* are either uniform or exponential random variables with unknown location parameters. Wang et al. [8] make statistical inference for *P(X<Y<Z)* via two methods, the nonparametric normal approximation and the jackknife empirical likelihood.

The Topp-Leone (TL) distribution is therefore J-shaped through its support. Percentage data, rates, particle sizes and specific chemical procedure yield data that can be displayed by this distribution. The TL distribution has a finite support, and various data sets in reliability and life testing are showed using finite support distributions. [9].

Particularly, when the reliability is measured as the proportion of the quantity of effective trials to the amount of whole trials, the TL distribution can well be functional. In stress–strength model the distributions have uses in many spaces. For example, if *Y* denotes the maximum section elongation and *X* signifies the tensile strength of a piece of some material, then p processes the quality of the material. Another example, if *Y* refers the radius of the base of a small cup and *X* represents the radius of the circular depression in the center of a saucer then P represents the probability of holding the cup. Also, a consumer research organization may want to compare sales percentages of two products with a different advertisement policy each.

Topp and Leone [10] presented the TL distribution and display its properties and also showed its applications for some failure data. Nadarajah and Kotz [11] derive some properties of the TL distribution and provided an expression for its characteristic function. Kotz and Van Dorp [12] given a generalized TL distribution to model some economic facts and they also clear a reflected general TL distribution and studied its properties. Ghitany *et al.* [13] considered the related of the reliability function of this distribution such as the hazard rate; mean residual life, reversed hazard rate, expected inactivity time and their stochastic orderings.

The topic of the Topp-Leone (TL) distribution dealt with this research, and the probability density function (pdf) is

$$f(X) = 2\alpha(1-x)x^{\alpha-1}(2-x)^{\alpha-1} \tag{1}$$

where $0 < x < 1$ and $0 < \alpha < 1$.

The distribution function of the TL distribution is given by

$$F(X) = \begin{cases} 0\,, & x \le 0\,, \\ x^{\alpha}(2-x)^{\alpha} & 0 < x < 1 \\ 1 & x \ge 1 \end{cases} \tag{2}$$

So if u follows uniform distribution, then $X = 1 - \sqrt{1 - U^{1/\alpha}}$ has the TL (α) distribution.
Consequently, the hazard rate will be as below

$$h(x) = \frac{f(x)}{1-F(x)} = 2\alpha\frac{(1-x)[1-(1-x)^2]^{\alpha-1}}{1-[1-(1-x)^2]^{\alpha}} \quad (3)$$

The rest of this paper is structured as follows. In Section II, the expression of R_1= P(X > Y) and R_2= *P(Y<X< Z)* will be derived. Maximum likelihood estimator MLE, the Moment estimator MOM and the Pre-test single stage shrinkage estimator SH of *R* is obtained in Section III. Monte Carlo Simulation and Numerical Outcomes are laid out in Section IV. Finally, conclusions are presented in Section V.

II. Expression of R_1= P(X > Y) and R_2= *P(Y<X< Z)*

- **Derivation of R_1 = P(X > Y)**

This Section concentrates on estimating the reliability of when X and Y have independent Topp-Leone distributions. Let n be the number of observations distributed according to the Topp-Leone

Now let $X \sim$ TL *(α)* be independent of $Y \sim$ TL *(β)*. Then

$$R_1 = P(X > Y) = \int_0^1 F_Y(x) f_x(x)\, dx$$

$$R_1 = 2\alpha\int_0^1 x^{\alpha+\beta-1}(1-x)(2-x)^{\alpha+\beta-1}\, dx \quad (4)$$

$$R_1 = \frac{\alpha}{\alpha+\beta}$$

Where α and β are unknown.

- **Derivation of R2 = P(Y<X<Z)**

This Section concentrates on estimating the reliability of R_2 = P(Y<X<Z) when Z, Y and X have independent Topp-Leone distributions such that X, Y and Z are independent and they are distributed Topp-Leone with scale parameters$\alpha,\ \beta,\ \gamma$ respectively such that the p.d.f of the strength X is

$$f_x(x,\alpha) = 2\alpha(1-x)x^{\alpha-1}(2-x)^{\alpha-1} \quad x > 0\ ,\alpha > 0\,.$$

Consequently, the p.d.f of the stresses Y, Z are given respectively by

$$f_y(y,\alpha) = 2\beta(1-y)y^{\beta-1}(2-y)^{\beta-1} \quad y > 0\ ,\beta > 0\,.$$

$$f_Z(z,\gamma) = 2\gamma(1-z)z^{\gamma-1}(2-z)^{\gamma-1} \quad z > 0\ ,\gamma > 0\,.$$

The reliability system of this model P(Y<X<Z) given by

$$R_2 = P(Y < X < Z) = \int_0^1 P(Y < X, X < Z)\, f(x)dx$$

$$= \int_0^1 F_Y(x)\bar{F}_z(x)\, f(x)dx$$

$$= \int_0^1 F_Y(x)f(x) - \int_0^1 F_Y(x)F_z(x)\, f(x)dx$$

$$= \frac{\alpha}{\alpha+\beta} - \int_0^1 x^{\beta}(2-x)^{\beta} x^{\gamma}(2-x)^{\gamma} 2\alpha(1-x)x^{\alpha-1}(2-x)^{\alpha-1}\,dx$$

$$= \frac{\alpha}{\alpha+\beta} - \int_0^1 2\alpha x^{(\alpha+\beta+\gamma)}(2-x)^{(\alpha+\beta+\gamma)-1}(1-x)x^{\alpha-1}\,dx$$

$$= \frac{\alpha}{\alpha+\beta} - \frac{\alpha}{\alpha+\beta+\gamma}$$

$$R_2 = \frac{\alpha\gamma}{(\alpha+\beta)(\alpha+\beta+\gamma)} \tag{5}$$

Where , β and γ are unknown.

III. Estimation of $R_1 = P(X > Y)$ and $R_2 = P(Y<X<Z)$

Maximum Likelihood Estimation of R_1 , R_2

The Maximum likelihood estimator MLE technique is an important and commonly estimator, since its has a good property for estimate which is known as invariant property [14] .

This Section deals with MLE of reliability R_1=P(X>Y) and R_2 =P(Y < X < Z) when X, Y and Z are independent Topp-Leone distribution with scale parameters (α, β, γ) respectively.

Let $x_1, x_2, \ldots x_n$ be a random strength sample of size n with p.d.f. as in (eq.3) then let $y_1, y_2, \ldots y_m$. . and $z_1, z_2, \ldots z_w$ be the random samples with p.d.f . as in (eq.3) .The Maximum Likelihood function of the observed sample is:

$$L(\alpha,\beta,\gamma,x,y,z) = \prod_{i=1}^{n} f(x_i) \prod_{j=1}^{m} f(y_j) \prod_{k=1}^{w} f(z_k) \tag{6}$$

$$= \prod_{i=1}^{n} 2\alpha(1-x_i){x_i}^{\alpha-1}(2-x_i)^{\alpha-1} \prod_{j=1}^{m} 2\beta(1-y_j){y_j}^{\beta-1}(2-y_j)^{\beta-1} \prod_{k=1}^{w} 2\gamma(1 - z_k){z_k}^{\gamma-1}(2-z_k)^{\gamma-1}$$

Taking the logarithm for the above likelihood function eq (6) and the partial derivative for the log-likelihood function with respect to unknown parameters α β andγ, respectively and equating the partial derivative to zero to solve this equation:

$$\frac{\partial \mathrm{Ln}\, L(x_i)}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \log[x_i(2-x_i)] = 0$$

$$\frac{\partial \mathrm{Ln}\, L(y_j)}{\partial \beta} = \frac{m}{\beta} + \sum_{j=1}^{m} \log[y_j(2-y_j)] = 0$$

$$\frac{\partial \mathrm{Ln}\, L(z_k)}{\partial \gamma} = \frac{w}{\gamma} + \sum_{B=1}^{w} \log[z_B(2-z_B)] = 0$$

The results of the above equations give MLEs of the parameters:

$$\hat{\alpha}_{ml} = \frac{-n}{\sum_{i=1}^{n} \log[x_i(2-x_i)]} \tag{7}$$

$$\hat{\beta}_{ml} = \frac{-m}{\sum_{j=1}^{m} \log[y_j(2-y_j)]} \tag{8}$$

$$\hat{\gamma}_{ml} = \frac{-\mathrm{w}}{\sum_{B=1}^{w} \log[\mathrm{z_B}(2-\mathrm{z_B})]} \tag{9}$$

We obtain the MLE of R_1 and R_2 as

$$R_{(ML)1} = \frac{\hat{\alpha}_{ml}}{\hat{\alpha}_{ml}+\hat{\beta}_{ml}} \quad , R_{(ML)2} = \frac{\hat{\alpha}_{ml}\hat{\gamma}_{ml}}{(\hat{\alpha}_{ml}+\hat{\beta}_{ml})(\hat{\alpha}_{ml}+\hat{\beta}_{ml}+\hat{\gamma}_{ml})} \tag{10}$$

Moment Estimation Method of R_1 and R_2

This section concern with the moment estimator method MOM of R_1 and R_2.The moment estimators of the unknown parameters α,β and γ will be obtained by equating the population moments with the corresponding sample moments. The population means of random variables X,Y and Z are as below

$$\mu'_x = E(x) = 1 - \frac{\sqrt{\pi}}{2}\left(\frac{\Gamma(\alpha+1)}{\Gamma\left(\alpha+\frac{3}{2}\right)}\right) \tag{11}$$

$$\mu'_y = E(y) = 1 - \frac{\sqrt{\pi}}{2}\left(\frac{\Gamma(\beta+1)}{\Gamma\left(\beta+\frac{3}{2}\right)}\right) \tag{12}$$

$$\mu'_z = E(z) = 1 - \frac{\sqrt{\pi}}{2}\left(\frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\frac{3}{2})}\right) \tag{13}$$

Suppose that X= (x_1 , x_2 , …x_n) be a random sample of size n and Y=(y_1 , y_2 …y_m) be a random sample of size m and Z=(z_1 , z_2 …z_w) be a random sample of size w follows Topp-Leone distribution with unknown scales parameter α,β and γ .

Then the means of the first and sample moments are given by

$$\mu_x = \bar{x} = \frac{1}{n}\sum_{i=1}^{n}(x_i) \quad \mu_y = \bar{y} = \frac{1}{m}\sum_{j=1}^{m}(y_i) \quad \mu_z = \bar{z} = \frac{1}{w}\sum_{B=1}^{w}(z_i)$$

By equating the samples moments with the corresponding population moments, then $\mu'_x = \mu_x$, $\mu'_y = \mu_y$, $\mu'_z = \mu_z$

$$\bar{x} = 1 - \frac{\sqrt{\pi}}{2}\left(\frac{\Gamma(\alpha+1)}{\Gamma\left(\alpha+\frac{3}{2}\right)}\right), \bar{y} = 1 - \frac{\sqrt{\pi}}{2}\left(\frac{\Gamma(\beta+1)}{\Gamma\left(\beta+\frac{3}{2}\right)}\right), \bar{z} = 1 - \frac{\sqrt{\pi}}{2}\left(\frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\frac{3}{2})}\right) \tag{14}$$

The moment estimator of α,β and γ denoted by $\hat{\alpha}_{mo}$, $\hat{\beta}_{mo}$ and $\hat{\gamma}_{mo}$ can be obtained from (14), respectively as,

$$\hat{\alpha}_{mo} = \left(\frac{\Gamma\left(\alpha_\circ+\frac{3}{2}\right)}{\Gamma(\alpha_\circ)}\right).\frac{2}{\sqrt{\pi}}(1-\bar{x}) \tag{15}$$

$$\hat{\beta}_{mo} = \left(\frac{\Gamma\left(\beta_\circ+\frac{3}{2}\right)}{\Gamma(\beta_\circ)}\right).\frac{2}{\sqrt{\pi}}(1-\bar{y}) \tag{16}$$

$$\hat{\gamma}_{mo} = \left(\frac{\Gamma\left(\gamma_\circ+\frac{3}{2}\right)}{\Gamma(\gamma_\circ)}\right).\frac{2}{\sqrt{\pi}}(1-\bar{z}) \tag{17}$$

Consequently ,we obtain the Moment estimators of R_1 and R_2 as

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$$R_{(MO)1} = \frac{\widehat{\alpha}_{mo}}{\widehat{\alpha}_{mo} + \hat{\beta}_{mo}} \quad , \quad R_{(MO)2} = \frac{\widehat{\alpha}_{mo}\hat{\gamma}_{mo}}{(\widehat{\alpha}_{mo} + \hat{\beta}_{mo})(\widehat{\alpha}_{mo} + \hat{\beta}_{mo} + \hat{\gamma}_{mo})} \tag{18}$$

Pre-test single stage shrinkage estimator (SH) of R_1 and R_2

Some time may we have a prior information value (point guess) of the parameter to be estimated. If this value is in the neighborhood of the accurate value, the shrinkage procedure is valuable to obtain an improved estimator. Thompson in [15], Isaam, K, Taha, A. and Abbas,N.[16] and others suggested shrunken estimators for different distributions when a prior estimate or guess point is available. They indicated that these estimators perform better in the term of mean squared error when a guess value θ_0 close to the true value θ. Pre- test estimator is considered for estimating the parameter θ when a guess point (prior estimate) θ_0 is available about θ due the past knowledge or similar cases. From the empirical studies it has been established that the shrinkage estimators performs better than the usual estimator when the guess point is very close to the true value of the parameter. Therefore to make sure whether θ is closed to θ_0 or not, we may test H_0:$\theta = \theta_0$ against H_1: $\theta \neq \theta_0$, so we denote by R to the critical region for above test.

Thompson in 1968 recommended shrinking the usual estimator $\hat{\theta}$ of θ towards the prior guess point θ_0 and suggested the estimator $\tilde{\theta} = K\hat{\theta} + (1 - K(\hat{\theta}))\theta_\circ$, where $(1-K)$ represents the experimenters belief in the guess point θ_0. He found the estimator $\tilde{\theta}$ which is more efficient than usual estimator $\hat{\theta}$ if the true value θ is close to θ_0 (H_0 accepted) but may be less efficient otherwise, therefore to resolve the uncertainty that a guess point value is approximately the true value or not, a pre- test of significance may be employed. So he take the usual estimator $\hat{\theta}$ when θ is far away from θ_0 (H_0 rejected) after he made the pre- test.

Thus, the pre-test shrunken estimator has the following form ; A.N.Salman [17]

$$\tilde{\theta} = \begin{cases} K\hat{\theta} + (1-K)\theta_\circ & if \hat{\theta} \in R \\ \hat{\theta} & if\ \hat{\theta} \notin R \end{cases} \tag{19}$$

Where R may be pre- test region for acceptance the null hypothesis H_0 as we mentioned above, $\hat{\theta}$ is the usual ML estimator of θ and K is a constant shrinkage weight factor such that

$0 \leq K \leq 1$.

In this research we may assume the region as follow:

$$R = \{(\theta_\circ - \theta)^2 \leq MSE\} \tag{20}$$

$$R = \left(\theta_\circ - \sqrt{MSE}\ ,\ \theta_\circ + \sqrt{MSE}\ \right) \tag{21}$$

In this research, we suppose

Case 1 Suppose that

$$K_n = \left|\frac{Sinn}{n}\right|^2 \quad K_m = \left|\frac{Sinm}{m}\right|^2 \quad K_w = \left|\frac{Sinw}{w}\right|^2 , 0 \leq K_n, K_m, K_w \leq 1 \text{ and } \theta = \theta_\circ .$$

Where, θ may be referred to α , β and γ .

Thus, the shrinkage estimator of the scale parameters α, β and γ of the random variables X,Y, Z that follows Topp-Leone distribution will be as follows:

$$\tilde{\alpha}_{sh1} = \begin{cases} K_n\hat{\alpha} + (1-K_n)\alpha_\circ & if \hat{\alpha} \in R \\ \hat{\alpha} & if\ \hat{\alpha} \notin R \end{cases} \tag{22}$$

$$\tilde{\beta}_{sh1} = \begin{cases} K_m\hat{\beta} + (1-K_m)\beta_\circ & if \hat{\beta} \in R \\ \hat{\beta} & if\ \hat{\beta} \notin R \end{cases} \tag{23}$$

$$\tilde{\gamma}_{sh1} = \begin{cases} K_w\hat{\gamma} + (1-K_w)\gamma_\circ & if \hat{\gamma} \in R \\ \hat{\gamma} & if\ \hat{\gamma} \notin R \end{cases} \tag{24}$$

Case 2 Suppose that

$$\mathbf{A_n} = \sqrt{\mathbf{e}^{\frac{-\mathbf{n}}{\mathbf{10}}}} \ , \mathbf{A_m} = \sqrt{\mathbf{e}^{\frac{-\mathbf{m}}{\mathbf{10}}}} \ , \mathbf{A_w} = \sqrt{\mathbf{e}^{\frac{-\mathbf{w}}{\mathbf{10}}}} \ , \qquad 0 \le \mathrm{A_n}, \mathrm{A_m}, \mathrm{A_w} \le 1 \ \text{and} \ \theta = \theta_\circ \ .$$

Where , θ may be referred to α , β $and \gamma$.

$$\tilde{\alpha}_{sh2} = \begin{cases} \mathrm{A_n}\hat{\alpha} + (1 - \mathrm{A_n})\alpha_\circ & if \hat{\alpha} \in R \\ \hat{\alpha} & if \ \hat{\alpha} \notin R \end{cases} \tag{25}$$

$$\tilde{\beta}_{sh2} = \begin{cases} \mathrm{A_m}\hat{\beta} + (1 - \mathrm{A_m})\beta_\circ & if \hat{\beta} \in R \\ \hat{\beta} & if \ \hat{\beta} \notin R \end{cases} \tag{26}$$

$$\tilde{\gamma}_{sh2} = \begin{cases} \mathrm{A_w}\hat{\gamma} + (1 - \mathrm{A_w})\gamma_\circ & if \hat{\gamma} \in R \\ \hat{\gamma} & if \ \hat{\gamma} \notin R \end{cases} \tag{27}$$

we obtain the Mom of R_1 and R_2 as

$$R_{(SH)1} = \frac{\widehat{\alpha}_{SH1}}{\widehat{\alpha}_{SH1} + \hat{\beta}_{SH1}} \quad , R_{(SH)2} = \frac{\widehat{\alpha}_{SH1}\hat{\gamma}_{SH1}}{(\widehat{\alpha}_{SH1} + \hat{\beta}_{SH1})(\widehat{\alpha}_{SH1} + \hat{\beta}_{SH1} + \hat{\gamma}_{SH1})} \tag{28}$$

Consequently , we obtain

$$R_{(SH2)1} = \frac{\widehat{\alpha}_{SH2}}{\widehat{\alpha}_{SH2} + \hat{\beta}_{SH2}} \quad , R_{(SH22} = \frac{\widehat{\alpha}_{SH2}\hat{\gamma}_{SH2}}{(\widehat{\alpha}_{SH2} + \hat{\beta}_{SH2})(\widehat{\alpha}_{SH2} + \hat{\beta}_{SH2} + \hat{\gamma}_{SH2})} \tag{29}$$

IV. Monte Carlo Simulation and Numerical Outcomes

An extensive numerical investigation will be conducted in this section to compare the performance of the various estimators for unlike sample sizes and parameter values for the Topp-Leone distribution. The properties investigated result in mean square errors (MSEs). Matlab 2018 statistical software was used for all computations. A simulation results are conducted to examine and compare the performance of the estimates for shape respecting to the MSE. The best estimator has the smallest value of MSE.

The steps for estimating the parameters, $R_1 = P(X > Y)$ and $R_2 = P(Y< X < Z)$ can be summarized as follows:

Generate 10000 random samples from Topp-Leone distribution with the sample sizes;(n ,m)= (25,50,75,100) and the parameter values are selected as α = (1.2,1.5,3) and β =(3 ,1.5,1.2) for R_1 .Also (n, m ,w)= (25,25,25),(50,50,50),(75,75,75), (100,100,100), (50,25,25), (75,25,25), (100,25,25), (25,50,25), (50,75,50), (75,100,75), (25,100,25), (25,25,50), (50,50,75), (75,75,100) and parameter values are selected as α = (1.5,1.5,1.5), β =(2.5 ,1.5,3.5) and γ =(3.5 ,1.5,2.5) for R_2.

One can conclude from the simulation results which is used to determine the best consequence of the proposed estimation methods (ML, MO, SH1, SH2) using different samples for the system reliability $R_1=p(Y<X)$ and $R_2 = P(Y< X < Z)$ grounded the Topp-Leone distribution (T-L). The simulation results of the proposed estimation methods are demonstrated in annexed tables and distinguish that Pre-test single stage shrinkage estimator (SH1) of system reliability R_1 , R_2 satisfied the smallest mean squared error in overall; this infers that $\hat{R}_{SH1}$ was the best than the others for two considered models.

V. Conclusion

From above results, it observed that in general the best performance of the consider estimators (ML, MO, SH1, SH2,) under the different sample sizes and for the different Parameters of this study is the pre-test single stage shrinkage estimator (SH1) of system reliability R_1 , R_2 for two considered models .This important method has proven its efficiency in estimation as prior estimate approach to real value which depends on classical estimation method and prior information (initial estimate) as a linear combination and make pretest region to know how close the initial value from the actual value .

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Appendix A

Table A1. Shown estimates of R_1, when $R_1= 0.28571$, $\alpha = 1.2$, $\beta = 3$.

n	m	$\hat{R}_{ML}$	$\hat{R}_{MO}$	$\hat{R}_{SH1}$	$\hat{R}_{SH2}$
25	25	0.22795	0.76549	0.26949	0.65166
	50	0.29000	0.69709	0.28616	0.59951
	75	0.25700	0.83259	0.27238	0.71323
	100	0.30891	0.70636	0.29865	0.60523
50	25	0.22768	0.72594	0.26577	0.60214
	50	0.25496	0.79200	0.28253	0.66948
	75	0.31010	0.66498	0.28581	0.59447
	100	0.38670	0.58104	0.29034	0.5384
75	25	0.38350	0.58687	0.30097	0.54219
	50	0.36616	0.64820	0.29003	0.58296
	75	0.27682	0.72916	0.28453	0.62165
	100	0.32255	0.65676	0.28536	0.57684
100	25	0.31769	0.69842	0.28650	0.60441
	50	0.30583	0.72137	0.28373	0.61462
	75	0.34595	0.62115	0.28556	0.55979
	100	0.24077	0.78389	0.28439	0.66114

Table A2. MSE of R_1, when $R_1= 0.28571$, $\alpha = 1.2$, $\beta = 3$.

n	m	ML	MO	SH1	SH2	Best
25	25	3.3362e-07	2.3018e-05	2.6327e-08	1.3392e-05	SH
	50	1.8372e-09	1.6923e-05	1.9773e-11	9.8467e-06	SH
	75	8.2468e-08	2.9907e-05	1.7781e-08	1.8277e-05	SH
	100	5.3794e-08	1.7695e-05	1.6745e-08	1.0209e-05	SH
50	25	3.3683e-07	1.938e-05	3.9771e-08	1.0012e-05	SH
	50	9.4604e-08	2.5633e-05	1.0121e-09	1.4727e-05	SH
	75	5.9453e-08	1.4384e-05	9.8482e-13	9.5327e-06	SH
	100	1.0197e-06	8.7219e-06	2.1393e-09	6.3848e-06	SH
75	25	9.5616e-07	9.5616e-07	2.3269e-08	6.5779e-06	SH
	50	6.4721e-07	1.314e-05	1.8666e-09	8.8355e-06	SH
	75	7.9146e-09	1.9664e-05	1.3949e-10	1.1286e-05	SH
	100	1.3571e-07	1.3767e-05	1.2546e-11	8.4754e-06	SH
100	25	1.0226e-07	1.7032e-05	6.1133e-11	1.0157e-05	SH
	50	4.0479e-08	1.898e-05	3.926e-10	1.0818e-05	SH
	75	3.6288e-07	1.1252e-05	2.3293e-12	7.5116e-06	SH
	100	2.0201e-07	2.4818e-05	1.7668e-10	1.4095e-05	SH

Table A3. Shown estimation when R1= 0.5, beta1 = 1.5, beta2= 1.5.

n	m	ML	MO	SH1	SH2
25	25	0.52585	0.44398	0.50746	0.47914
	50	0.52351	0.34866	0.50910	0.40764
	75	0.53547	0.52974	0.50933	0.52313
	100	0.52507	0.54220	0.50871	0.53144
50	25	0.47686	0.56972	0.56972	0.53694
	50	0.51562	0.53771	0.50141	0.52235
	75	0.50862	0.52169	0.50146	0.5146
	100	0.53958	0.40917	0.50059	0.44168
	25	0.51675	0.46870	0.50091	0.47168
	50	0.52847	0.53250	0.50018	0.52059

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75	75	0.53742	0.42200	0.50086	0.45271
	100	0.55972	0.43488	0.50177	0.46178
100	25	0.47113	0.61394	0.49444	0.55794
	50	0.49079	0.53128	0.50097	0.52011
	75	0.52326	0.53541	0.50074	0.53282
	100	0.53481	0.43170	0.50023	0.45482

Table A4. MSEs of R_1 when $\alpha = 1.5$, $\beta = 1.5$.

n	m	ML	LS	SH1	SH2	Best
25	25	6.6847e-08	3.138e-07	5.561e-09	4.3506e-08	SH1
	50	5.5253e-08	2.2904e-06	8.2734e-09	8.5312e-07	SH1
	75	1.2578e-07	8.8426e-08	8.6961e-09	5.3482e-08	SH1
	100	6.2857e-08	1.7812e-07	7.5812e-09	9.8874e-08	SH1
50	25	5.3536e-08	4.8611e-07	4.537e-10	1.3645e-07	SH1
	50	2.4407e-08	1.4221e-07	1.9898e-10	4.9937e-08	SH1
	75	7.4261e-09	4.7039e-08	2.1352e-10	2.1311e-08	SH1
	100	1.5667e-07	8.2494e-07	3.4689e-11	3.4007e-07	SH1
75	25	2.8048e-08	9.7946e-08	8.2667e-11	8.0202e-08	SH1
	50	8.1083e-08	1.0562e-07	3.4017e-12	4.2379e-08	SH1
	75	1.4002e-07	6.0841e-07	7.3392e-11	2.2367e-07	SH1
	100	3.5666e-07	4.2403e-07	3.1286e-10	1.4606e-07	SH1
100	25	8.332e-08	1.2983e-06	3.0867e-09	3.3573e-07	SH1
	50	8.4884e-09	9.7818e-08	9.4062e-11	4.0457e-08	SH1
	75	5.4121e-08	1.2542e-07	5.5441e-11	1.0773e-07	SH1
	100	1.2119e-07	4.6645e-07	5.3448e-12	2.0412e-07	SH1

Shown estimation when R= 0.71429, beta1 = 3, beta2= 1.2

n	m	ML	MO	SH1	SH2
25	25	0.61334	0.34578	0.69064	0.41593
	50	0.61151	0.43041	0.69210	0.46764
	75	0.72315	0.23630	0.72105	0.35348
	100	0.79167	0.21785	0.73058	0.34541
50	25	0.64462	0.22901	0.69405	0.33525
	50	0.73712	0.25847	0.71695	0.37117
	75	0.67832	0.28617	0.71318	0.38605
	100	0.66978	0.27653	0.71211	0.37392
75	25	0.67920	0.24415	0.71638	0.34904
	50	0.74174	0.26702	0.71454	0.38343
	75	0.74139	0.24329	0.71603	0.36362
	100	0.72348	0.27315	0.71518	0.38607
100	25	0.76171	0.42327	0.71933	0.48892
	50	0.68467	0.30166	0.71191	0.39779
	75	0.73210	0.24035	0.71535	0.35858
	100	0.68185	0.31757	0.71511	0.40896

mse

n	m	ML	MO	SH1	SH2	Best
25	25	1.0191e-06	1.358e-05	5.5891e-08	8.9014e-06	SH1
	50	1.0562e-06	8.0585e-06	4.9205e-08	6.0834e-06	SH1
	75	7.8616e-09	2.2847e-05	4.5744e-09	1.3018e-05	SH1
	100	5.9891e-07	2.4645e-05	2.6549e-08	1.3607e-05	SH1
50	25	4.853e-07	2.3549e-05	4.095e-08	1.4367e-05	SH1
	50	5.2132e-08	2.0777e-05	7.0912e-10	1.1773e-05	SH1
	75	1.2934e-07	1.8328e-05	1.2289e-10	1.0774e-05	SH1
	100	1.9804e-07	1.9163e-05	4.7435e-10	1.1585e-05	SH1
75	25	1.2307e-07	2.2103e-05	4.3956e-10	1.334e-05	SH1
	50	7.5363e-08	2.0004e-05	6.3547e-12	1.0946e-05	SH1

	75	7.349e-08	2.2184e-05	3.0541e-10	1.2297e-05	SH1
	100	8.4551e-09	1.946e-05	8.0755e-11	1.0773e-05	SH1
100	25	2.6294e-07	1.6149e-05	1.995e-10	9.6001e-06	SH1
	50	8.7697e-08	1.7026e-05	5.6266e-10	1.0017e-05	SH1
	75	3.174e-08	2.2462e-05	1.1351e-10	1.2653e-05	SH1
	100	1.0524e-07	1.5738e-05	6.7611e-11	9.3224e-06	SH1
R=0.175 , ζ_1=1.5, ζ_2=2.5, ζ_3=3.5and q=10000						
Method / **n,m,w**		**Mle**	**Mom**	**SH1**	**SH2**	**Best**
(25,25,25)	$\hat{R}$	0.21921	0.12232	0.18717	0.15602	SH1
	MSE	1.9542e-07	2.7753e-07	1.4807e-08	3.6021e-08	
(50,50,50)	$\hat{R}$	0.15329	0.15246	0.17317	0.17118	
	MSE	4.7154e-08	5.0796e-08	3.3671e-10	1.4578e-09	SH1
(75,75,75)	$\hat{R}$	0.16695	0.15469	0.17473	0.18071	
	MSE	6.486e-09	4.1266e-08	7.4067e-12	3.2602e-09	SH1
(100,100,100)	$\hat{R}$	0.20633	0.10786	0.17513	0.13863	
	MSE	9.8161e-08	4.5076e-07	1.5626e-12	1.3226e-07	SH1
(50,25,25)	$\hat{R}$	0.17236	0.10592	0.17598	0.13517	SH1
	MSE	6.9677e-10	4.7716e-07	9.5917e-11	1.5864e-07	
(75,25,25)	$\hat{R}$	0.19945	0.11034	0.17960	0.13528	
	MSE	5.9768e-08	4.1811e-07	2.119e-09	1.5773e-07	SH1
(100,25,25)	$\hat{R}$	0.19819	0.10900	0.18023	0.15131	
	MSE	5.3782e-08	4.3554e-07	2.733e-09	5.6115e-08	SH1
(25,50,25)	$\hat{R}$	0.17935	0.12105	0.17843	0.15127	
	MSE	1.8924e-09	2.9108e-07	1.1757e-09	5.6325e-08	SH1
(50,75,50)	$\hat{R}$	0.18495	0.11786	0.17604	0.15383	
	MSE	9.9087e-09	3.2649e-07	1.0817e-10	4.4799e-08	SH1
(75,100,75)	$\hat{R}$	0.17638	0.13235	0.1747	0.15697	SH1
	MSE	1.9162e-10	1.8189e-07	8.7569e-12	3.2494e-08	
(25,100,25)	$\hat{R}$	0.19069	0.11492	0.17653	0.1392	
	MSE	2.4633e-08	3.6099e-07	2.3519e-10	1.2816e-07	SH1
(25,25,50)	$\hat{R}$	0.23891	0.12393	0.1901	0.15131	
	MSE	4.0841e-07	2.6079e-07	2.2813e-08	5.611e-08	SH1
(50,50,75)	$\hat{R}$	0.16162	0.1557	0.17367	0.16523	
	MSE	1.7909e-08	3.7239e-08	1.7809e-10	9.5422e-09	SH1
(75,75,100)	$\hat{R}$	0.15119	0.13748	0.17433	0.16805	
	MSE	5.6692e-08	1.408e-07	4.4538e-11	4.8245e-09	SH1
R=0.16667 , ζ_1=1.5, ζ_2=1.5, ζ_3=1.5and q=10000						
Method / **n,m,w**		**Mle**	**Mom**	**SH1**	**SH2**	**Best**
(25,25,25)	$\hat{R}$	0.15676	0.18541	0.16392	0.17445	SH1
	MSE	9.82e-09	3.5124e-08	7.5502e-10	6.0599e-09	
(50,50,50)	$\hat{R}$	0.1404	0.21087	0.16415	0.18916	
	MSE	6.8985e-08	1.9543e-07	6.3217e-10	5.0602e-08	SH1
(75,75,75)	$\hat{R}$	0.1454	0.17792	0.16618	0.17245	
	MSE	4.3016e-08	1.2655e-08	2.3994e-11	3.3414e-09	SH1
(100,100,100)	$\hat{R}$	0.17169	0.16521	0.1667	0.16773	
	MSE	2.522e-09	2.1217e-10	1.3565e-13	1.1396e-10	SH1
(50,25,25)	$\hat{R}$	0.12897	0.29976	0.15687	0.23538	SH1
	MSE	1.4211e-07	1.7713e-06	9.5912e-09	4.7217e-07	
(75,25,25)	$\hat{R}$	0.1175	0.20579	0.14959	0.18288	
	MSE	2.4171e-07	1.5307e-07	2.9154e-08	2.6288e-08	SH1
(100,25,25)	$\hat{R}$	0.16667	0.14741	0.23725	0.21319	
	MSE	3.7084e-08	4.9826e-07	4.7221e-09	2.1646e-07	SH1
	$\hat{R}$	0.18239	0.12568	0.17596	0.14649	

(25,50,25)	MSE	2.4725e-08	1.6795e-07	8.6408e-09	4.072e-08	SH1
(50,75,50)	$\hat{R}$	0.16269	0.15295	0.1671	0.15776	
	MSE	1.5778e-09	1.8804e-08	1.907e-11	7.9395e-09	SH1
(75,100,75)	$\hat{R}$	0.13608	0.20373	0.1662	0.18712	SH1
	MSE	9.3539e-08	1.374e-07	2.1631e-11	4.1833e-08	
(25,100,25)	$\hat{R}$	0.16371	0.2123	0.1644	0.19272	
	MSE	8.7604e-10	2.0823e-07	5.1378e-10	6.7874e-08	SH1
(25,25,50)	$\hat{R}$	0.13021	0.22961	0.15341	0.2007	
	MSE	1.3291e-07	3.9616e-07	1.7581e-08	1.1581e-07	SH1
(50,50,75)	$\hat{R}$	0.1759	0.13769	0.16607	0.14916	
	MSE	8.5328e-09	8.3974e-08	3.5792e-11	3.0655e-08	SH1
(75,75,100)	$\hat{R}$	0.15083	0.18493	0.1664	0.17714	
	MSE	2.5079e-08	3.3366e-08	7.1447e-12	1.0978e-08	SH1

R=0.1 , ζ_1=1.5, ζ_2=3.5, ζ_3=2.5and q=10000

Method / n,m,w		Mle	Mom	SH1	SH2	Best
(25,25,25)	$\hat{R}$	0.1439	0.18133	0.11091	0.17023	SH1
	MSE	1.9272e-07	6.6146e-07	1.1897e-08	4.9322e-07	
(50,50,50)	$\hat{R}$	0.13639	0.14832	0.10279	0.1621	
	MSE	1.3241e-07	2.3344e-07	7.8026e-10	3.8563e-07	SH1
(75,75,75)	$\hat{R}$	0.10428	0.20421	0.099843	0.18633	
	MSE	1.8297e-09	1.0859e-06	2.4697e-12	7.4528e-07	SH1
(100,100,100)	$\hat{R}$	0.10724	0.18486	0.099781	0.18265	
	MSE	5.2398e-09	7.2017e-07	4.8049e-12	6.8303e-07	SH1
(50,25,25)	$\hat{R}$	0.11243	0.18481	0.10198	0.17709	SH1
	MSE	1.5447e-08	7.1931e-07	3.9077e-10	5.9426e-07	
(75,25,25)	$\hat{R}$	0.11521	0.17945	0.10323	0.17097	
	MSE	2.3139e-08	6.3123e-07	1.0435e-09	5.0368e-07	SH1
(100,25,25)	$\hat{R}$	0.10514	0.16091	0.10201	0.16608	
	MSE	2.6379e-09	3.7101e-07	4.033e-10	4.3671e-07	SH1
(25,50,25)	$\hat{R}$	0.10903	0.18471	0.10377	0.19206	
	MSE	8.1614e-09	7.1758e-07	1.42e-09	8.4754e-07	SH1
(50,75,50)	$\hat{R}$	0.1112	0.21963	0.10171	0.19207	
	MSE	1.2543e-08	1.4312e-06	2.9161e-10	8.4765e-07	SH1
(75,100,75)	$\hat{R}$	0.10183	0.21353	0.099982	0.18867	SH1
	MSE	3.3388e-10	1.289e-06	3.1663e-14	7.8616e-07	
(25,100,25)	$\hat{R}$	0.12711	0.19176	0.11079	0.17787	
	MSE	7.3519e-08	8.4195e-07	1.1645e-08	6.063e-07	SH1
(25,25,50)	$\hat{R}$	0.12573	0.19876	0.10639	0.18526	
	MSE	6.6195e-08	9.7531e-07	4.0889e-09	7.2684e-07	SH1
(50,50,75)	$\hat{R}$	0.11281	0.1895	0.10081	0.18458	
	MSE	1.6403e-08	8.0111e-07	6.6176e-11	7.1541e-07	SH1
(75,75,100)	$\hat{R}$	0.13862	0.15759	0.10038	0.16339	
	MSE	1.4917e-07	3.317e-07	1.4333e-11	4.0187e-07	SH1

R=0.11667 , ζ_1=3.5, ζ_2=2.5, ζ_3=1.5and q=10000

Method / n,m,w		Mle	Mom	SH1	SH2	Best
(25,25,25)	$\hat{R}$	0.11541	0.25489	0.11629	0.21104	SH1
	MSE	1.5718e-10	1.9104e-06	1.4074e-11	8.9072e-07	
(50,50,50)	$\hat{R}$	0.13119	0.22845	0.11754	0.20084	
	MSE	2.1082e-08	1.2496e-06	7.7031e-11	7.0857e-07	SH1
(75,75,75)	$\hat{R}$	0.10116	0.25097	0.116	0.21287	
	MSE	2.4034e-08	1.8038e-06	4.4505e-11	9.256e-07	SH1
(100,100,100)	$\hat{R}$	0.11467	0.21453	0.1164	0.19455	
	MSE	3.9876e-10	9.5781e-07	7.0629e-12	6.0656e-07	SH1
(50,25,25)	$\hat{R}$	0.12773	0.22401	0.11957	0.19822	SH1

	MSE	1.2229e-08	1.1522e-06	8.4428e-10	6.6501e-07	
(75,25,25)	$\hat{R}$	0.14116	0.18953	0.12366	0.18391	
	MSE	5.9999e-08	5.3089e-07	4.8937e-09	4.522e-07	SH1
(100,25,25)	$\hat{R}$	0.13117	0.20523	0.12032	0.19259	
	MSE	2.1028e-08	7.8439e-07	1.334e-09	5.7639e-07	SH1
(25,50,25)	$\hat{R}$	0.11238	0.18634	0.11584	0.17234	
	MSE	1.8361e-09	4.855e-07	6.8685e-11	3.0991e-07	SH1
(50,75,50)	$\hat{R}$	0.12615	0.2232	0.11812	0.19613	
	MSE	8.9875e-09	1.1349e-06	2.1166e-10	6.3138e-07	SH1
(75,100,75)	$\hat{R}$	0.11136	0.23389	0.11633	0.20431	SH1
	MSE	2.8167e-09	1.3742e-06	1.1273e-11	7.682e-07	
(25,100,25)	$\hat{R}$	0.13305	0.20444	0.1194	0.18279	
	MSE	2.6836e-08	7.7047e-07	7.4595e-10	4.3717e-07	SH1
(25,25,50)	$\hat{R}$	0.13803	0.15661	0.11943	0.16303	
	MSE	4.5657e-08	1.5957e-07	7.6543e-10	2.1493e-07	SH1
(50,50,75)	$\hat{R}$	0.12893	0.22632	0.117	0.19254	
	MSE	1.5035e-08	1.2024e-06	1.1364e-11	5.7566e-07	SH1
(75,75,100)	$\hat{R}$	0.13302	0.19233	0.11656	0.18769	
	MSE	2.6751e-08	5.7249e-07	1.0731e-12	5.0448e-07	SH1

R=0.23333 , ζ_1=3.5, ζ_2=1.5, ζ_3=2.5and q=10000

Method / n,m,w		Mle	Mom	SH1	SH2	Best
(25,25,25)	$\hat{R}$	0.21429	0.13464	0.22908	0.15108	SH1
	MSE	3.6267e-08	9.7408e-07	1.8092e-09	6.7651e-07	
(50,50,50)	$\hat{R}$	0.19785	0.11743	0.2306	0.13693	
	MSE	1.2591e-07	1.3433e-06	7.4572e-10	9.2927e-07	SH1
(75,75,75)	$\hat{R}$	0.21823	0.10601	0.23325	0.13887	
	MSE	2.2806e-08	1.6212e-06	6.592e-13	8.9235e-07	SH1
(100,100,100)	$\hat{R}$	0.21461	0.12208	0.23348	0.15091	
	MSE	3.5065e-08	1.2378e-06	2.1933e-12	6.7935e-07	SH1
(50,25,25)	$\hat{R}$	0.2452	0.11016	0.23601	0.14458	SH1
	MSE	1.4077e-08	1.5172e-06	7.1686e-10	7.8772e-07	
(75,25,25)	$\hat{R}$	0.17944	0.14782	0.2162	0.1579	
	MSE	2.9049e-07	7.3125e-07	2.9371e-08	5.6829e-07	SH1
(100,25,25)	$\hat{R}$	0.26793	0.10931	0.24479	0.14556	
	MSE	1.1967e-07	1.5382e-06	1.3124e-08	7.705e-07	SH1
(25,50,25)	$\hat{R}$	0.18239	0.10106	0.22039	0.13968	
	MSE	2.595e-07	1.7496e-06	1.6755e-08	8.7702e-07	SH1
(50,75,50)	$\hat{R}$	0.22786	0.10771	0.23371	0.13791	
	MSE	2.9949e-09	1.5781e-06	1.3946e-11	9.106e-07	SH1
(75,100,75)	$\hat{R}$	0.24163	0.10018	0.23389	0.13539	SH1
	MSE	6.878e-09	1.7731e-06	3.0849e-11	9.5927e-07	
(25,100,25)	$\hat{R}$	0.188	0.13646	0.22451	0.15624	
	MSE	2.0554e-07	9.3851e-07	7.7868e-09	5.943e-07	SH1
(25,25,50)	$\hat{R}$	0.29333	0.10743	0.24206	0.14321	
	MSE	3.6e-07	1.5852e-06	7.6197e-09	8.1225e-07	SH1
(50,50,75)	$\hat{R}$	0.18978	0.14026	0.23203	0.1577	
	MSE	1.8966e-07	8.6626e-07	1.6882e-10	5.7203e-07	SH1
(75,75,100)	$\hat{R}$	0.22703	0.10696	0.23361	0.13775	
	MSE	3.9738e-09	1.597e-06	7.6452e-12	9.1362e-07	SH1

Co-author Information

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