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Article

Conformal and geodesic mappings onto some special spaces

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Abstract: In the paper, we consider conformal mappings of Riemannian spaces onto Ricci-2-symmetric Riemannian spaces and geodesic mappings of spaces with affine connections onto Ricci-2-symmetric spaces. The main equations for the mappings are obtained as a closed system of differential equations of Cauchy-type in covariant derivatives. We have found the number of essential parameters which the solution of the system depends on. A similar approach was applied for the case of conformal mappings of Riemannian spaces onto Ricci-m-symmetric Riemannian spaces and geodesic mappings of spaces with affine connections onto Ricci-m-symmetric spaces.

Keywords: Space with affine connection; Riemannian space; Ricci-m-symmetric space; conformal mapping; geodesic mapping.

MSC: 53B05, 53B50, 35M10

1. Introduction

Conformal mappings were considered in many monographs, surveys, and papers. The theory of conformal mappings gives very important applications to the general relativity (e. g. [4,5,13–15]).

The question of whether a Riemannian space admits a conformal mapping onto some Einstein space was reduced by Brinkmann [3] to the question whether solutions of some nonlinear system of PDE's of Cauchy type with respect to unknown functions exists. This subject was set out in a monograph written by A. Z. Petrov [13].

In papers [6,11] the main equations of the above said mappings are reduced to a linear system of differential equations in covariant derivatives. Also, the mobility degree with respect to conformal mappings onto Einstein spaces was found.

In [11] authors have found an estimation of the first lacuna in a distribution of degree of mobility groups of Riemannian spaces with respect to conformal mappings onto Einstein spaces. It was proved in [11] that the maximal degree of mobility with respect to the conformal mappings is admitted by conformal flat spaces, and only by them. The paper presents a criterion in tensor form for Riemannian spaces, different from conformally Euclidean ones, for which the maximal degree of mobility $r = n - 1$, where n is the dimension of the spaces $(n > 2)$. Hence, the estimation of the first lacuna in a distribution of degree of mobility was obtained and the spaces with the maximal degree of mobility, different from conformally Euclidean ones, was distinguished.

In the above said explorations, it's supposed that all geometric objects under consideration belong to the sufficiently high smoothness class.

A paper [6] presents the minimal conditions on the differentiability of objects under consideration to be satisfied by conformal mappings of Riemannian spaces onto Einstein spaces. The main equations

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for the mappings are obtained as a closed system of differential equations of Cauchy-type in covariant derivatives taking into account the minimal requirements on the differentiability of metrics of spaces which are conformally equivalent.

A paper [1] is dedicated to conformal mappings of Riemannian spaces onto Ricci-symmetric spaces. The main equations for the mappings were reduced to a closed system of differential equations of Cauchy-type in covariant derivatives. Also, the authors have found the number of essential parameters which the solution of the system depends on. It is worth noting that the system is nonlinear.

The theory goes back to the paper [9] of T. Levi-Civita, in which the problem on the search for Riemannian spaces with common geodesics was stated and solved in a special coordinate system. We note a remarkable fact that this problem is related to the study of equations of dynamics of mechanical systems.

The theory of geodesic mappings has been developed by T. Thomas, H. Weyl, P. A. Shirokov, A. S. Solodovnikov, N. S. Sinyukov, J. Mikeš, and others [4,10,12,13,16].

The best known equations are the Levi-Civita equations obtained by him for the case of Riemannian spaces. Later, H. Weyl have obtained the same equations for geodesic mappings between spaces with affine connections.

N. S. Sinyukov [16] has proved that the main equations for geodesic mappings of (pseudo-)Riemannian spaces are equivalent to some linear system of differential equations of Cauchy-type in covariant derivatives.

A paper [17] extends the results to the case of geodesic mappings of geodesic mappings of equiaffine spaces with affine connections onto (pseudo-)Riemannian spaces.

In a paper [2], authors proved that the main equations of geodesic mappings of spaces with affine connections onto Rici-symmetric spaces are equivalent to some system of differential equations of Cauchy-type in covariant derivatives. In this paper, the main equations for conformal mappings of Riemannian spaces onto Ricci-2-symmetric Riemannian spaces and geodesic mappings of spaces with affine connections onto Ricci-2-symmetric spaces are obtained as a closed system of differential equations of Cauchy-type in covariant derivatives. We have found the number of essential parameters which the solution of the system depends on. The obtained results were extended for the case of conformal mappings of Riemannian spaces onto Ricci-m-symmetric Riemannian spaces and geodesic mappings of spaces with affine connections onto Ricci-m-symmetric spaces. We suppose throughout the paper, that all geometric objects under considerations are continuous and sufficiently smooth.

2. Basic conceptions of conformal and geodesic mappings

Let us consider conformal mapping f of a Riemannian space V_n with the metric tensor g onto a Riemannian space $\overline{V}_n$ with the metric tensor $\overline{g}$.

Let us suppose that the Riemannian spaces V_n and $\overline{V}_n$ are referred to a common coordinate system $x^1, x^2, \ldots, x^n$ relative to a mapping.

A diffeomorphism $f: \ V_n \longrightarrow \overline{V}_n$ is called a *conformal mapping* if in a common coordinate system $x^1, x^2, \ldots, x^n$ relative to the mapping their metric tensors g and $\overline{g}$ are proportional and the components of the tensors are in the relation

$$\overline{g}_{ij}(x) = e^{2\psi(x)} \cdot g_{ij}(x), \tag{1}$$

where $\psi(x)$ is a function of the x's.

From (1) it follows that conformal mappings preserve angles between tangent vectors of any pair of curves. Conformal mappings are completely characterized by that property.

From (1) it follows also that relations between the Christoffel symbols formed with respect to the two metric tensors are given by

$$\overline{\Gamma}^h_{ij}(x) = \Gamma^h_{ij}(x) + \delta^h_i \psi_j(x) + \delta^h_j \psi_i(x) - \psi^h(x) \cdot g_{ij}(x), \tag{2}$$

where $\psi_i = \dfrac{\partial \psi}{\partial x^i}$ is a gradient vector, $\psi^h(x) = g^{h\alpha}\psi_\alpha$, g^{ij} are components of the inverse matrix to g_{ij}, and δ^h_i is the Kronecker delta.

A conformal mapping is called *homothetic* if the function $\psi(x)$ is a constant i. e. $\overline{g}_{ij}(x) = c \cdot g_{ij}(x)$. The condition is equivalent to $\psi_i(x) = 0$, hence the mapping is also an affine one.

Let us recall that in Riemannian space V_n with a metric tensor $g_{ij}(x)$, the Riemann tensor, the Ricci tensor, and the scalar curvature are defined by the metric tensor as follows:

$$R^h_{ijk} = \frac{\partial \Gamma^h_{ik}}{\partial x^j} - \frac{\partial \Gamma^h_{ij}}{\partial x^k} + \Gamma^\alpha_{ik}\Gamma^h_{\alpha j} - \Gamma^\alpha_{ij}\Gamma^h_{\alpha k}, \qquad R_{ij} = R^\alpha_{ij\alpha}, \qquad R = R_{\alpha\beta}g^{\alpha\beta}. \tag{3}$$

It is known [4,10,13,16] that under conformal mappings a relationship between the Riemann tensors is presented by the formulas:

$$\overline{R}^h_{ijk} = R^h_{ijk} + \delta^h_k \psi_{ij} - \delta^h_j \psi_{ik} + g_{ij}\psi^h_k - g_{ik}\psi^h_j + (\delta^h_k g_{ij} - \delta^h_j g_{ik}) \triangle_1 \psi, \tag{4}$$

where $\psi_{ij} = \psi_{i,j} - \psi_i \psi_j$, $\psi^h_k = g^{h\alpha}\psi_{\alpha k}$, $\triangle_1 \psi = g^{\alpha\beta}\psi_\alpha\psi_\beta$, and symbol "," denotes the covariant derivative with respect to the metric tensor of a space V_n.

Contracting the equations (4) for h and k in the reduction, we get

$$\psi_{i,j} = \frac{\mu}{n-2}\, g_{ij} + \psi_i\psi_j - \frac{1}{n-2}\left(\overline{R}_{ij} - R_{ij}\right), \tag{5}$$

where μ is a certain invariant.

A curve is called a *geodesic* if the tangent vector field along the curve is parallel along the curve.

We say that a diffeomorphism $f\colon A_n \to \overline{A}_n$ is an *geodesic mapping* if any geodesic curve of A_n is mapped under f onto a geodesic curve in $\overline{A}_n$.

According to [10,12,13,16], a necessary and sufficient condition for the mapping f of a space A_n onto a space $\overline{A}_n$ to be geodesic is that in the common coordinate system $x^1, x^2, \ldots, x^n$ the *deformation tensor* $P^h_{ij}(x)$ of the mapping f

$$P^h_{ij}(x) = \overline{\Gamma}^{(}_{ij}x) - \Gamma^h_{ij}(x), \tag{6}$$

has to satisfy the condition

$$P^h_{ij}(x) = \psi_i(x)\delta^h_j + \psi_j(x)\delta^h_i. \tag{7}$$

The symbols $\Gamma^h_{ij}(x)$ and $\overline{\Gamma}^h_{ij}(x)$ are components of affine connections of the spaces A_n and $\overline{A}_n$ respectively, $\psi_i(x)$ is a covariant vector.

A geodesic mapping is called *nontrivial* if $\psi_i(x) \neq 0$. It is obvious, that any space A_n with affine connection admits a nontrivial geodesic mapping onto some space $\overline{A}_n$ with affine connection. Generally speaking, similar idea concerning geodesic mappings of Riemannian spaces onto Riemannian spaces is wrong. In particular, there are Riemannian spaces that do not admit nontrivial geodesic mappings onto Riemannian spaces.

3. Conformal mappings of Riemannian spaces onto Ricci-2-symmetric Riemannian spaces

A space $\overline{A}_n$ with affine connection (Riemannian space $\overline{V}_n$) is called Ricci-m-symmetric if its Ricci tensor $\overline{R}_{ij}$ satisfies the condition

$$\overline{R}_{ij|k_1k_2\ldots k_m} = 0, \tag{8}$$

where the symbol " | " denotes a covariant derivative with respect to the connection of the space $\overline{A}_n$. In particular, for the case of Ricci-2-symmetric spaces (8) is written as follows:

$$\overline{R}_{ij|km} = 0. \tag{9}$$

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Let us consider conformal mappings of a Riemannian space V_n with the metric tensor g onto another Riemannian space $\overline{V}_n$ with the metric tensor $\overline{g}$. If the spaces V_n and $\overline{V}_n$ are referred to the common coordinate system $x = (x^1, x^2, \ldots, x^n)$, then we get

$$\overline{R}_{ij|k} = \overline{R}_{ij,k} - P^{\alpha}_{ki}\overline{R}_{\alpha j} - P^{\alpha}_{kj}\overline{R}_{i\alpha}. \tag{10}$$

Taking account of (2), it follows from (10) that

$$\overline{R}_{ij|k} = \overline{R}_{ij,k} - \psi_i\overline{R}_{kj} - \psi_j\overline{R}_{ik} - 2\psi_k\overline{R}_{ij} + \psi^{\alpha}g_{ik}\overline{R}_{\alpha j} + \psi^{\alpha}g_{jk}\overline{R}_{i\alpha}. \tag{11}$$

Differentiating (11) with respect to x^m in the space V_n and taking into account $\psi^{\alpha}_{,m} = g^{\alpha\beta}\psi_{\beta,m}$, we obtain

$$\begin{aligned}\left(\overline{R}_{ij|k}\right)_{,m} &= \overline{R}_{ij,km} - \psi_{i,m}\overline{R}_{kj} - \psi_i\overline{R}_{kj,m} - \psi_{j,m}\overline{R}_{ik} - \psi_j\overline{R}_{ik,m} - 2\psi_{k,m}\overline{R}_{ij} - 2\psi_k\overline{R}_{ij,m} + \\ &+ g^{\alpha\beta}g_{ik}\overline{R}_{\alpha j}\psi_{\beta,m} + \psi^{\alpha}g_{ik}\overline{R}_{\alpha j,m} + g^{\alpha\beta}g_{jk}\overline{R}_{i\alpha}\psi_{\beta,m} + \psi^{\alpha}g_{jk}\overline{R}_{i\alpha,m}\,.\end{aligned} \tag{12}$$

According to the definition of covariant derivative we get

$$\left(\overline{R}_{ij|k}\right)_{,m} = \overline{R}_{ij|km} + P^{\alpha}_{mi}\overline{R}_{\alpha j|k} + P^{\alpha}_{mj}\overline{R}_{i\alpha|k} + P^{\alpha}_{mk}\overline{R}_{ij|\alpha}\,. \tag{13}$$

Taking account of (12) and (13), we have

$$\begin{aligned}\overline{R}_{ij|km} &= \overline{R}_{ij,km} - \psi_{i,m}\overline{R}_{kj} - \psi_i\overline{R}_{kj,m} - \psi_{j,m}\overline{R}_{ik} - \psi_j\overline{R}_{ik,m} - 2\psi_{k,m}\overline{R}_{ij} - 2\psi_k\overline{R}_{ij,m} + g^{\alpha\beta}g_{ik}\overline{R}_{\alpha j}\psi_{\beta,m} + \\ &+ \psi^{\alpha}g_{ik}\overline{R}_{\alpha j,m} + g^{\alpha\beta}g_{jk}\overline{R}_{i\alpha}\psi_{\beta,m} + \psi^{\alpha}g_{jk}\overline{R}_{i\alpha,m} - P^{\alpha}_{mi}\overline{R}_{\alpha j|k} - P^{\alpha}_{mj}\overline{R}_{i\alpha|k} - P^{\alpha}_{mk}\overline{R}_{ij|\alpha}\,.\end{aligned} \tag{14}$$

We introduce the tensor $\overline{R}_{ijk}$ defined by

$$\overline{R}_{ij,k} = \overline{R}_{ijk}\,. \tag{15}$$

Since the space $\overline{V}_n$ is Ricci-2-symmetric, it follows from (14) that

$$\begin{aligned}\overline{R}_{ijk,m} &= \theta_{im}\overline{R}_{kj} + \psi_i\overline{R}_{kjm} + \theta_{jm}\overline{R}_{ik} + \psi_j\overline{R}_{ikm} + 2\theta_{km}\overline{R}_{ij} + 2\psi_k\overline{R}_{ijm} - g^{\alpha\beta}\theta_{\beta m}(g_{ik}\overline{R}_{\alpha j} + g_{jk}\overline{R}_{i\alpha}) - \\ &- \psi^{\alpha}(g_{ik}\overline{R}_{\alpha jm} + g_{jk}\overline{R}_{i\alpha m}) + \theta^{\alpha}_{mi}\theta_{\alpha jk} + \theta^{\alpha}_{mj}\theta_{i\alpha k} + \theta^{\alpha}_{mk}\theta_{ij\alpha}\,,\end{aligned} \tag{16}$$

where

$$\begin{aligned}\theta_{ij} &= \frac{\mu}{n-2}\,g_{ij} + \psi_i\psi_j - \frac{1}{n-2}\left(\overline{R}_{ij} - R_{ij}\right),\\ \theta_{ijk} &= \overline{R}_{ijk} - \psi_i\overline{R}_{kj} - \psi_j\overline{R}_{ik} - 2\psi_k\overline{R}_{ij} + \psi^{\alpha}g_{ik}\overline{R}_{\alpha j} + \psi^{\alpha}g_{jk}\overline{R}_{i\alpha},\\ \theta^h_{ij} &= \delta^h_i\psi_j + \delta^h_j\psi_i - \psi^h g_{ij}.\end{aligned}$$

Let us differentiate (5) with respect to x^k in the space V_n and alternate the obtained result in j and k. In view of the Ricci identity and the fact that the Ricci tensor is symmetric, we get

$$\begin{aligned}(n-2)\psi_{\alpha}R^{\alpha}_{ijk} = &- g_{ij}\mu_{,k} + g_{ik}\mu_{,j} - g^{\alpha\beta}\psi_{\alpha}\left(g_{ik}\overline{R}_{\beta j} - g_{ij}\overline{R}_{\beta k}\right) + \\ &+ R_{ik,j} - R_{ij,k} + R_{ij}\psi_k - R_{ik}\psi_j + \mu(g_{ij}\psi_k - g_{ik}\psi_j).\end{aligned} \tag{17}$$

Let us multiply (17) by g^{ij} and contract for l and j. According to the Voss-Weyl formula $R_{ij,k}g^{jk} = \frac{1}{2}R_{,i}$, we obtain

$$(n-1)\mu_{,k} = g^{\alpha\beta}\left[(n-2)\psi_{\gamma}R^{\gamma}_{\beta k\alpha} - (n-1)\psi_{\beta}\overline{R}_{\alpha k} - \psi_{\beta}R_{\alpha k}\right] + [R + (n-1)\mu]\psi_k - \frac{1}{2}R_{,k}\,. \tag{18}$$

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Also, we have the notation
$$\psi_i = \psi_{,i} \,. \tag{19}$$
Obviously, in the space $\overline{V}_n$ the equations (5), (15), (16), (18) and (19) form a closed mixed system of PDE's of Cauchy type with respect to functions $\psi(x)$, $\psi_i(x)$, $\mu(x)$, $\overline{R}_{ij}(x)$ and $\overline{R}_{ijk}(x)$ and the functions $\overline{R}_{ij}(x)$ must satisfy the algebraic conditions $\overline{R}_{ij}(x) = \overline{R}_{ij}(x)$. Hense we have proved

Theorem 1. *In order that a Riemannian space V_n admits a conformal mapping onto a Ricci-2-symmetric space $\overline{V}_n$, it is necessary and sufficient that the mixed system of differential equations of Cauchy type in covariant derivatives* (5), (15), (16), (18) *and* (19) *has a solution with respect to functions $\psi(x)$, $\psi_i(x)$, $\mu(x)$, $\overline{R}_{ijk}(x)$ and $\overline{R}_{ij}(x)$ $(= \overline{R}_{ij}(x))$.*

It is obvious, that the general solution of the mixed system of Cauchy type depends on no more than $\frac{1}{2}n \cdot (n+1)^2 + n + 2$ essential parameters.

It is easy to see that if we differentiate covariantly with respect to x^ρ in the space V_n and taking into account the definition of covariant derivative writing expression for the tensor $(\overline{R}_{ij|km})_{,\rho}$, then we have obtained the expression for the covariant derivative of $\overline{R}_{ij|km\rho}$ through $\overline{R}_{ij,km\rho}$.

Hence in the case when the space $\overline{V}_n$ is Ricci-3-symmetric, the main equations for the mapping can be written in the form of a closed system of equations of Cauchy type in covariant derivatives.

Obviously, continuing this way, it is readily shown that the main equations for conformal mappings of Riemannian spaces onto Ricci-m-symmetric spaces also can be presented as a closed system of equations of Cauchy type in covariant derivatives.

4. Geodesic mappings of spaces with affine connections onto Ricci-2-symmetric spaces

Let us consider a geodesic mapping of a space A_n with affine connections onto a Ricci-2-symmetric space $\overline{A}_n$. Suppose that the spaces A_n and $\overline{A}_n$ are referred to a coordinate system common to the mapping.

One knows [10,16] that under conformal mappings a relationship between the Riemann tensors R^h_{ijk} and $\overline{R}^h_{ijk}$ of the spaces A_n and $\overline{A}_n$ respectively is presented by the formulas:

$$\overline{R}^h_{ijk} = R^h_{ijk} + P^h_{ik,j} - P^h_{ij,k} + P^\alpha_{ik} P^h_{\alpha j} - P^\alpha_{ij} P^h_{\alpha k} \,. \tag{20}$$

Taking into account that a deformation tensor P^h_{ij} of the connections is defined by (7), it follows from (20) that
$$\overline{R}^h_{ijk} = R^h_{ijk} - \delta^h_j \psi_{i,k} + \delta^h_k \psi_{i,j} - \delta^h_i \psi_{j,k} + \delta^h_i \psi_{k,j} + \delta^h_j \psi_i \psi_k - \delta^h_k \psi_i \psi_j \,. \tag{21}$$
Contracting the equations (21) for h and k we get

$$\overline{R}_{ij} = R_{ij} + n\psi_{i,j} - \psi_{j,i} + (1-n)\psi_i \psi_j \,. \tag{22}$$

Alternating (22) with respect to the indices i and j, we obtain

$$\overline{R}_{[ij]} = R_{[ij]} + (n+1)\psi_{i,j} - (n+1)\psi_{j,i} \,. \tag{23}$$

Here we denote by the brackets $[ij]$ an operation called antisymmetrization (or, alternation) without division with respect to the indices i and j. Taking account of (22), from (23) it follows

$$\psi_{i,j} = \frac{1}{n^2-1} \left[n\overline{R}_{ij} + \overline{R}_{ji} - (nR_{ij} + R_{ji}) \right] + \psi_i \psi_j \,. \tag{24}$$

Using the relation (10) and taking ito account that the deformation tensor is defined by (7), we find

$$\overline{R}_{ij|k} = \overline{R}_{ij,k} - 2\psi_k \overline{R}_{ij} - \psi_i \overline{R}_{kj} - \psi_j \overline{R}_{ik} \,. \tag{25}$$

Differentiating (25) with respect to x^m in the space A_n, we obtain

$$(\overline{R}_{ij|k})_{,m} = \overline{R}_{ij,km} - 2\psi_{k,m}\overline{R}_{ij} - 2\psi_k\overline{R}_{ij,m} - \psi_{i,m}\overline{R}_{kj} - \psi_i\overline{R}_{kj,m} - \psi_{j,m}\overline{R}_{ik} - \psi_j\overline{R}_{ik,m}\,. \tag{26}$$

Taking account of the formalas (13) and (7), from (26) it follows

$$\begin{aligned}\overline{R}_{ij|km} = \overline{R}_{ij,km} - 2\psi_{k,m}\overline{R}_{ij} - 2\psi_k\overline{R}_{ij,m} - \psi_{i,m}\overline{R}_{kj} - \psi_i\overline{R}_{kj,m} - \psi_{j,m}\overline{R}_{ik} - \psi_j\overline{R}_{ik,m} - \psi_i\overline{R}_{mj|k} -\\ - 3\psi_m\overline{R}_{ij|k} - \psi_j\overline{R}_{im|k} - \psi_k\overline{R}_{ij|m}\,.\end{aligned} \tag{27}$$

Suppose that the space $\overline{A}_n$ is Ricci-2-symmetric. Then, taking account of (15), (24) and (25) we have from (27),

$$\begin{aligned}\overline{R}_{ijk,m} = 2\rho_{km}\overline{R}_{ij} + 2\psi_k\overline{R}_{ijm} + \rho_{im}\overline{R}_{kj} + \psi_i\overline{R}_{kjm} + \rho_{jm}\overline{R}_{ik} + \psi_j\overline{R}_{ikm} +\\ + \psi_i\rho_{mjk} + 3\psi_m\rho_{ijk} + \psi_j\rho_{imk} + \psi_k\rho_{ijm}\,,\end{aligned} \tag{28}$$

where

$$\begin{aligned}\rho_{ij} =& \frac{1}{n^2-1}\left(n\overline{R}_{ij} + \overline{R}_{ji} - (R_{ij} + R_{ji})\right) + \psi_i\psi_j,\\ \rho_{ijk} =& \overline{R}_{ijk} - 2\psi_k\overline{R}_{ij} - \psi_i\overline{R}_{kj} - \psi_j\overline{R}_{ik}.\end{aligned}$$

Obviously, in the space $\overline{A}_n$ the equations (15), (28), and (24) form a closed system of differential equations of Cauchy type in covariant derivatives with respect to functions $\psi_i(x)$, $\overline{R}_{ij}(x)$ and $\overline{R}_{ijk}(x)$. Hence we have proved

Theorem 2. *In order that a space A_n with affine connection admits a geodesic mapping onto a Ricci-2-symmetric space $\overline{A}_n$, it is necessary and sufficient that the closed system of differential equations of Cauchy type in covariant derivatives (15), (28) and (24) has a solution with respect to functions $\psi_i(x)$, $\overline{R}_{ij}(x)$ and $\overline{R}_{ijk}(x)$.*

The general solution of the closed system of differential equations of Cauchy type in covariant derivatives (15), (28), and (24) depends on no more than $\frac{1}{2}n\cdot(n+1)^2+n$ essential parameters.

It is obvious that similarly to the case of conformal mappings, the main equations for geodesic mappings of spaces with affine connections onto Ricci-m-symmetric space could be obtained in a form of a closed system of equations of Cauchy type in covariant derivatives.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviation is used in this manuscript:

PDE Partial Differential Equation

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