

QED and Higgs-like Domain Wall Dynamics in Einstein-Cartan Gravity

By L.C. Garcia de Andrade

departamento de fis teorica IF Rio de janeiro Brasil

Abstract

Spacetime torsion is known to be highly suppressed at the end of inflation, called preheating. Of course this fact does not avoid that metric could affect the cosmic dynamos at preheating phase as was showed by Bassett et al, in the general relativity (GR) case. In this paper, is to show that a torsionful magnetogenesis in QED effective lagrangean, drives either a torsion damping or an amplification mechanism or dynamo effect, depending upon torsion chirality. The present extension of Bassett et al theory yields a new magnetic field expression $B \sim a^{(1 \pm \beta)}$, obtained from dynamo equation in QED. Here a cosmic factor $\beta = g_T \eta k$ is present. In a second example we find Higgs inflationary fields in Einstein-Cartan gravity thick domain walls (DWs). Recently a modified Einstein-Cartan gravity was given by M Shaposhnikov et al [PRL (2020)] to obtain higgs like inflatons as a portal to dark energy. In the case of thick DW we assume that there is a torsion squared influence, since we are in the early universe where torsion is not suppressed and may have an appreciable magnitude as show by Paul and SenGupta (EPJ C (2019)). A static DW solution is obtained when inflationary potential vanishes, and higgs potential is a helical function. The number of e-folds is also computed. Recently without inflation domain wall dynamos were obtained in Einstein-Cartan gravity (EC) [Garcia de Andrade, Ann Phys 432 (2021)] where the spins of the nucleons were orthogonal to the wall.

Key-words: Modified gravity theories; cosmic dynamo; QED and preheating; domain walls; higgs inflation

¹Departamento de Física Teórica - IF - UERJ - Rua São Francisco Xavier 524, Rio de Janeiro, RJ, Maracanã, CEP:20550 and Institute for Cosmology and Philosophy of Nature, Trg, Florjana 16, Krizvic, Croatia. e-mail: luizandra795@gmail.com

1 Introduction

Recently Shaposhnikov et al [1] have investigate a metric-affine theory alternative of general relativity which is a generalization of Einstein-Cartan theory of gravity (EC) [2], which contains other invariants besides the Ricci scalar in Riemann-Cartan spacetime. This is due to the fact that in EC gravity the Riemann-Cartan curvature tensor is totally asymmetric and violates parity. Earlier the author has showed how to obtain cosmic dynamos from the electromagnetic gravity field Lagrangean where the term $\epsilon^{ijkl}R_{ijkl}$ is added to the action. In the Shaposhnikov et al Einstein-Catan gravity this term is also added to the action and called Holst term. Their action they claim is a more complete for EC, gives rise to Higgs inflation. Of course the simple presence of a scalar gravity sector is not enough to guarantee that the scalar field is a Higgs field [3]. Therefore here we shall investigate non-minimal coupling of EC gravity to QED and scalar gravity sectors, via non-minimal coupling and investigate static domain walls (DWs) in the Riemann-flat curvature plus torsionful spacetime and the QED gravity-sector also with non-minimal coupled with torsion. Previously B Bassett et al [4] investigated the variation of the magnetic energy density compared to electromagnetic radiation in a photon fluid. This has been done in GR magnetogenesis framework. They found a cosmic dynamo as a metric perturbation at preheating or by the end of inflation. Since torsion is highly suppressed at that time epoch of universe, one is led to think that torsion might not contribute to magnetogenesis process [5]. Nevertheless, in this paper we show that in EC [6] magnetogenesis, torsion damping effect affects the cosmic dynamo equation with non-adiabatic helical magnetic fields, by decreasing the magnetic energy density to values compatible to astronomical observations [7]. Note since the EC gravity and GR are equivalent in the absence of matter sector [8], we must use in this QED the fermionic matter sector or the scalar gravity as source of torsion. In GR case, a similar result with helical fields has been established by Schober et al [9]. Present results are accomplished by considering a semi-minimal coupling quantum electrodynamics (QED) on a torsionful inflationary case. We assume that torsion does not couple to the electromagnetic field minimally, in the QED case, and no massive photon to break gauge invariance is produced [10]. Recently Khotari et al [11] considered modifications of the minimal coupling of electromagnetism with torsion [12] by considering spin-one fields of non-abelian nature, driven by torsionful magnetogenesis. This paper contains the derivation of a torsionful cosmic dynamo equation, obtained by variation of the torsionful QED effective Lagrangean. The value of torsion in terms of cosmic time dependent factor, is substituted into this cosmic dynamo equation and solved. Its solution is shown to depend upon a parameter β written in terms of torsion coupling, electric resistivity and the wave vector k . This results in a magnetic field damping, driven by torsion coupling of the magnetic field energy density. Results agree with the observations of modern astronomy. Several types of dynamos have been presented

in the literature [13]. For example chiral dynamos instabilities [14] endowed with torsion. To our knowledge, this is the first time a cosmic dynamo in a torsionful helical magnetogenesis is found to be compatible with observations with the aid of torsion and QED. Non-adiabatic and superadiabatic magnetic field can be found from the present solution of dynamo equations. Besides dynamos in QED sector we also find dynamos from the presence of magnetic fields external to a static DW with torsion. Since torsion at early universe where DWs are present is of the order of $1MeV$ [15] we find that the torsion depends upon the DW equations from Euler-Lagrange equations. The paper is organised as follows: In section 2 we present the effective QED lagrangean with semi-minimal coupling and found an differential equation for the magnetic vector potential. In section 3 we obtain galactic dynamo seeds and compute values of $r = \frac{\rho_B}{\rho_\gamma}$ from dynamo equation. In section 4 we present the derivation of the cosmic dynamo equation in the background of torsionful QED cosmology, and solutions are found depending upon the torsion coupling electric resistivity coupling. In section 5 we investigate the role of a torsion on a static DW with scalar inflation in EC gravity without Nieh-Yan inflation or Holst invariant terms in the action as considered by Shaposhnikov et al [16]. In section 6 we derive the dynamo equations in comoving coordinates of DW static metric. Section 7 is left to discussions and conclusions.

2 QED effective Lagrangean and semi-minimal torsion-photon coupling RF^2

Though torsion effects are highly suppressed in comparison with curvature ones of Einstein gravity sector, we do not consider here Minkowski space since as can be easily shown here from the field equations that torsion vanishes in Minkowski space. From Mazzitelli et al [17] the QED effective Lagrangean is

$$S = \frac{1}{m^2} \int d^4x (-g)^{\frac{1}{2}} \left(-\frac{1}{4} F^2 + (m^2 + \epsilon R) \phi \bar{\phi} - D_j \phi D^j \bar{\phi} \right) \quad (1)$$

Here $D_i = \partial_i - ieA_i$ is the covariant derivative for the scalar fields. Spedalieri et al [17] have computed an effective Lagrangean for the e.m field by integrating the quantum scalar field. Via dimensional regularisation they obtained the effective Lagrangean [17]

$$\mathcal{L}_{eff} = -\frac{1}{4} F^2 + \frac{1}{2} \frac{1}{4\pi^{\frac{d}{2}}} \left(\frac{m}{\mu} \right)^{d-4} \sum a_j(x) m^{4-2j} \Gamma(j - \frac{d}{2}) \quad (2)$$

The first Schwinger-De Witt (SDW) coefficients from Spedalieri et al work are

$$a_0 = 1 \quad (3)$$

$$a_1 = -(\epsilon - \frac{1}{6})R \quad (4)$$

$$a_2 = \frac{1}{180} (R_{ijkl} R^{ijkl} - R_{ij} R^{ij}) + \frac{1}{2} (\epsilon - \frac{1}{6})^2 R^2 + \frac{1}{6} (\epsilon - \frac{1}{5}) R - \frac{e^2}{12} F^2 \quad (5)$$

$$a_3 = \dots + \frac{e^2}{60} R_{ijkl} F^{ij} F^{kl} - \frac{e^2}{90} R_{ij} F^{ik} F^{jl} + \left(\frac{1}{6} - \epsilon\right) R F^2 + \dots \quad (6)$$

Here we note that due to the use of semi-minimal coupling where torsion, which is also our gravitational field, appears only in a_2 as first term, since in the semi-minimal coupling torsion does not appear in the covariant derivative and consequently not in the electromagnetic field. Accordingly torsion appears only in the curvatures for the first time in a_2 . Moreover, from semi-minimal coupling I shall consider the following effective Lagrangean in Riemann-Cartan spacetime

$$\mathcal{L}_{eff} = -\frac{1}{4} F^2 \left(1 + \frac{b}{m^2} R\right) \quad (7)$$

where we have taken $n = 1$ such as in Widrow and Turner [9]. From this effective Lagrangean we obtain the field equations for the Friedmann spatially flat metric

$$ds^2 = a^2(-d\eta^2 + dx^2) \quad (8)$$

as

$$\partial^i (F_{ij} (1 + \frac{bR}{m^2})) = 0 \quad (9)$$

With appropriated approximations we obtain

$$[\ddot{A}_k + k^2 A_k] \left(1 + \frac{bR}{m^2}\right) + \frac{b\dot{R}}{m^2 R} \dot{A}_k = 0 \quad (10)$$

these equations may yet be approximated for high coherence scales where $k^2 \ll 1$. We also assume here that in Riemannian case inflation-ary epoch $R \gg \gg m^2$ so this would reduce the last equation to

$$[\ddot{A}_k + \frac{\dot{R}}{R} \dot{A}_k] = 0 \quad (11)$$

where R is the Ricci scalar. This shows that although there is no inflation here we consider that torsion has a similar behaviour so actually $\dot{K} \gg \gg m^2$.

3 Galactic dynamo seeds in RF^2 semi-minimal coupling

In this section we shall solve equation (11) in the case of curved spacetime with torsion, and performing the semi-minimal coupling where the Ricci scalar is approximated taken as $2\dot{K}$, where K is the time component K^0 , which to simplify matters is the only homogeneous component of contortion, an algebraic combination of torsion. Here we assume linearisation of the Ricci-Cartan scalar

$$R = g_{ij} R^{ij} = R^* + 2\nabla_i K^i - K^2 \quad (12)$$

where $K^j = K^{rj}_{r}$, represents the trace of torsion tensor, R^* is the Riemannian Ricci scalar that shall be taken as constant like in de Sitter or Einstein space. Let us now perform the variation of the Lagrangean density $\sqrt{g}\mathcal{L}$ with respect to the scale cosmological factor a , and contortion K . This would complete the Einstein-Cartan-Maxwell equations systems

with propagating torsion. This can be done easily by computing the Euler Lagrange equations

$$\frac{d}{dt} \frac{\partial \sqrt{g} \mathcal{L}}{\partial \dot{a}} - \frac{\partial \sqrt{g} \mathcal{L}}{\partial a} = 0 \quad (13)$$

$$\frac{d}{dt} \frac{\partial \sqrt{g} \mathcal{L}}{\partial \dot{K}} - \frac{\partial \sqrt{g} \mathcal{L}}{\partial K} = 0 \quad (14)$$

The last equation determines contortion K in terms of the scale factor a. This yields

$$K = -\frac{3\dot{a}}{a} \quad (15)$$

Before applying this result to the expression for the Ricci-Cartan scalar, let us express this scalar in terms of the scalar a and torsion K. Then, we are left with the following expression

$$R = g_{ij} R^{ij} = R^* + 2\dot{K} - K^2 + \partial_t \ln \sqrt{g} K \quad (16)$$

or

$$R = g_{ij} R^{ij} = -6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] + 2\dot{K} - K^2 + (\partial_t \ln a^3) K \quad (17)$$

which yields

$$\dot{R} = \dot{R}^* + \ddot{K} + \left(\frac{\dot{a}}{a} + 2K\right)\dot{K} + 3\left[\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right]K \quad (18)$$

The expression for \ddot{K} is

$$\ddot{K} = -3\left[\frac{\ddot{a}}{a} - 3\frac{\ddot{a}\dot{a}}{a^2} + \left(\frac{\dot{a}}{a}\right)^3\right] \quad (19)$$

The expression for Ricci-Cartan scalar $\sqrt{g}R$ is

$$a^3 R = -3[3\ddot{a}a^2 + 7\dot{a}^2 a] \quad (20)$$

Substitution of this expression into the Euler-Lagrange equation above leads to

$$\ddot{a}a - 4\dot{a}\dot{a} = 0 \quad (21)$$

By making use of the ansatz $a \sim t^n$, where n is a real number, one obtains the following algebraic equation

$$n(n-2) - 4n^2 = 0 \quad (22)$$

which yields immediatly $n = -\frac{2}{3}$, and $a \sim t^{-\frac{2}{3}}$, which represents a contracting phase of the cosmological model with torsion. Therefore from the above expression for K, one obtains $K \sim a^{\frac{3}{2}}$. In terms of cosmic time, the contortion and cosmic factor is $K \sim -\frac{2}{3}t^{-1}$ and $a \sim t^{-\frac{2}{3}}$. This shows that in the contracting phase of the cosmological bouncing model the contraction of the universe goes faster than torsion, whereas in the expansion inflationary factor the torsion decays faster than cosmic expansion, showing that torsion is really highly suppressed by inflation. Now to investigate how magnetic field can be highly compressed in the contracting phase giving rise to a kind of dynamo action, one simply compute the ratio $\frac{\dot{R}}{R}$ as

$$\frac{\dot{R}}{R} = \frac{[3\ddot{K} - \frac{2}{3}(K^2)]}{[3\dot{K} - \frac{2}{3}K^2]} \quad (23)$$

Since the torsion is a very weak field at preheating phase, this can be approximated to

$$\frac{\dot{R}}{R} \approx \frac{\ddot{K}}{\dot{K}} \quad (24)$$

This expression then yields

$$\frac{\dot{R}}{R} \approx 2t^{-1} \quad (25)$$

Therefore, substitution of this value into the Fourier transformed equation for A_k above one obtains

$$\ddot{A}_k + 2t^{-1}\dot{A}_k = 0 \quad (26)$$

solution of this differential equation yields

$$A_k \sim A_{k(0)} t^{-2} \quad (27)$$

Since the magnetic field $B_{seed} = ikA$ and $B_{seed} \sim B_G t^{-2}$. Since $B_G = 10^{-6} G$, $B_{seed} \sim 10^{-40} Gauss$ which is able to seed the galactic dynamo.

4 Torsion damping and chirality in QED magnetogenesis compatible with observations

In this section derivation of dynamo equation from the electromagnetic equations is given, where torsion is introduced by the coupling of the new torsional covariant derivative: $\nabla_i = \partial_i + igK_i$ where K^i is the contortion vector assumed in previous sections. Here g is the torsion coupling. One notices that the partial differentiation substitutes the usual Riemann covariant operator, since we have assumed here that torsion coupling is not present in principle in the definition of electromagnetic field 2-tensor F_{ij} . Hence, the Maxwell equations in Friedmann torsionful universe are given by

$$\nabla \times (a^2 \mathbf{E}) = \partial_t (a^2 \mathbf{B}) \quad (28)$$

which is the modified Faraday equation in curved Riemann Friedmann spacetime, whereas the Ampere modified equation is

$$\nabla \times (a^2 \mathbf{B}) = \mathbf{J} \quad (29)$$

whereas the absence of monopole equation is given by

$$\nabla \cdot (a^2 \mathbf{B}) = 0 \quad (30)$$

Since the a cosmic factor does depend only upon cosmic time t , this expression is equivalent to the regular Maxwell one $div B = 0$ given in Riemann-flat spacetime without torsion. By taking the coupling derivative above we obtain

$$\nabla \times (a^2 \mathbf{B}) + ig[a^2 \mathbf{K} \times \mathbf{B}] = \mathbf{J} \quad (31)$$

and

$$\nabla \times [\mathbf{v} \times (a^2 \mathbf{B})] = \partial_t (a^2 \mathbf{B}) \quad (32)$$

Along with Ohm's law

$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{v} \times \mathbf{B} \quad (33)$$

From a straightforward algebra leads to the cosmic dynamo equation in QED torsionful magnetogenesis as

$$\nabla \times (a^2 \mathbf{B}) + ig[a^2 \mathbf{K} \times \mathbf{B}] = \mathbf{J} \quad (34)$$

and

$$\partial_t(a^2 \mathbf{B}) - \eta[\lambda^2 - g[\pm k.K]](a^2 \mathbf{B}) = 0 \quad (35)$$

where $\mathbf{k} \cdot \mathbf{K} = \pm k.K$ means that if one considers the spin vector proportional to torsion vector, a positive or negative chirality associated with a plus or minus sign respectively, implies that either spin or torsion vector flips and are parallel or antiparallel. Now from expression (15) for the cosmic factor dependence of contortion one obtains a complete dynamo equation in curved space with torsion as

$$\partial_t(a^2 \mathbf{B}) - \eta[\lambda^2 \mp (3g[k.\frac{\dot{a}}{a}])](a^2 \mathbf{B}) = 0 \quad (36)$$

where the electric resistivity is $\eta = \sigma^{-1}$. Here factor lambda appears in the equation due to the assumed helicity of magnetic fields and equation

$$\nabla \times \mathbf{B} = \lambda \mathbf{B} \quad (37)$$

A simple solution of the equation (36) is

$$B \sim e^{-\eta \lambda^2 t} a^{(1 \pm \beta)} \quad (38)$$

where $\beta := g_T \eta k$, with $g_T = -g$ g being positive is the new torsion coupling constant. Therefore, if the magnetic field is enhanced or damping by torsional effects depends only upon torsion chirality. Now, with this solution at hand, we may perform the astrophysical analysis of this solution. The first astrophysical point one can make is the one which concerns exponential Ohmic resistivity effect which depends on magnetic helicity factor lambda squared and resistivity factor. Furthermore, one notes that if the β is positive and greater than one the magnetic field is further reduced due to torsion damping and magnetic field energy density is $\rho_B \sim B^2 \sim a^{-(\beta-1)}$ which shows that torsion may really damp the magnetic field via β factor. Moreover, this allows one to place a bound on torsion coupling g_T . Then, since $\beta \geq 1$ one obtains $g_T \geq \frac{1}{3} \sigma \lambda$. Here λ is the inverse of wave vector k . This wave vector was introduced into dynamo equation by using the operator $ik = \nabla$, which turns the dynamo into a real equation. We also note that when β vanishes our solution reduces to $\rho_B \sim a^{-2}$ which is the non-adiabatic magnetic field solution found by Bassett et al. Nevertheless differently from their solution, ours contains an exponential decaying factor that does appear due to magnetic helicity. Then, if the magnetic helicity vanishes the torsionless solution really reduces to Bassett et al one. On the other hand, $\beta \leq 1$ implies that the magnetic energy density does not agree with astronomical observations despite than in our case we have the advantage to have a decaying exponential with time. This fact implies that even in the absence of torsion a strong decaying in magnetic field energy density is found due the coupling between Ohmic resistivity and helicity factor squared, being always positive. We also note that, in the long wavelength limit k^{-1} implies that k is small, which masquerades the torsion effect. In the early universe stage cosmic time $t \rightarrow 0$, and the effect of the exponential decay disappears. Therefore the case of Bassett et al, torsionless magnetogenesis at preheating is recovered. Note as well, that the growth in the magnetic energy density is slower now.

5 Scalar gravity-torsion sector in static DW

In this section, however, we assume that there is a distinct source for torsion, namely a scalar field which is given by a DW with torsion. In the example of a fermionic sector we have already obtained [18] a DW in EC gravity where the spin of nucleons are fermionic sources torsion and are polarised along orthogonal directions of the planar DW. In this case we assume that the present domain wall is static and a solution is naturally obtained from the lagrangean In general EC gravity is sourced by fermionic sector and its energy momentum tensor gives rise naturally to torsion. The scalar torsionless sector DW lagrangean is given by

$$S = \int d^4x (-g)^{\frac{1}{2}} [g^{ij} \partial_i \phi \partial_j \phi - \frac{\lambda}{2} (\phi^2 - \eta^2)^2] \quad (39)$$

variation of the action above yields the following wave equation for the scalar field

$$[\partial_t^2 - \nabla^2] \phi + 2\lambda(\phi^2 - \eta^2)\phi = 0 \quad (40)$$

In the case of static DW this wave equation reduces to

$$\frac{d^2 \phi}{dz^2} - 2\lambda(\phi^2 - \eta^2)\phi = 0 \quad (41)$$

which yields the following classical solution for the Minkowski DW as

$$\phi(z) = \eta \tanh\left(\frac{z}{\delta_0}\right) \quad (42)$$

where $\delta_0 = \frac{1}{\eta\sqrt{\lambda}}$ is the thickness of the wall in flat spacetime. Since the denominator is essentially the mass of the Higgs-like boson, as observed by Dolgov et al this δ_0 is microscopically small, otherwise the if it is cosmologically large, this boson would have a tiny , and would generate long range forces which would contradict LHC experiments. Moreover, Higgs boson would not be so long and so hard to be discovered. Now let us use a non-minimal coupling between torsion and the Higgs-like field to obtain the action

$$S = \int d^4x \left(-\frac{\lambda}{2} (\phi^2 - \eta^2)^2 + D_j \phi D^j \phi \right) \quad (43)$$

Here $D_i = \partial_i - iS_i$ is the covariant derivative for the scalar fields containing the minimal coupling this time, between torsion axial vector of components S_i and the Higgs-like field. Now let us express the last action explicitly in terms of the torsion by minimal coupling. This yields

$$\mathcal{L}_{S^2\phi} = -\frac{1}{2} g^{ij} [\partial_i \phi \partial_j \phi + 2iS_i \phi \partial_j \phi - S^2 \phi^2 - \frac{\lambda}{2} (\phi^2 - \eta^2)^2] \quad (44)$$

By applying the Euler-Lagrange equation to this lagrangean w.r.t higgs-like scalar field and torsion vector respectively yields

$$\partial_k \left[\frac{\partial \mathcal{L}}{\partial \partial_k \phi} \right] - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \quad (45)$$

$$\partial_k \left[\frac{\partial \mathcal{L}}{\partial \partial_k S_i} \right] - \frac{\partial \mathcal{L}}{\partial S_i} = 0 \quad (46)$$

Now from the lagrangean for the interaction between torsion and Higgs-like fields in DW one obtains the following field equations: First the one which gives the torsion vector sourced by Higgs-like field as

$$S_k = i \partial_k \ln \phi \quad (47)$$

which shows that torsion vector sourced by Higgs-like field is a complex torsion. This kind of affine connection appears in loop quantum gravity. Let us now solve the remaining equation (45). This yields the following equation

$$\partial_k \partial^k \phi - \left[\frac{\lambda}{2} (\phi^2 - \eta^2)^2 + S^2 \right] \phi = 0 \quad (48)$$

Since $S^2 = S_k S^k$ we shall be able to plugg equation (47) into this last wave equation for Higgs like field in DWs, to obtain

$$\partial_k \partial^k \phi - \left[\frac{\lambda}{2} (\phi^2 - \eta^2)^2 + \partial_k \ln \phi \partial^k \ln \phi \right] \phi = 0 \quad (49)$$

which can approximated by

$$\partial_k \partial^k \phi - \left[\frac{\lambda}{2} (\eta^2)^2 - \frac{\partial_k \phi \partial^k \phi}{\phi^2} \right] \phi = 0 \quad (50)$$

As been shown recently, [19] since by the end of inflation torsion is highly suppressed in scalar-torsion gravity sector, one may approximate this equation in order we obtain the same solution as before for flat DW which shows from expression (47) that the z-component of torsion is given by

$$S_z = i S_0 \operatorname{sech}^2 \left(\frac{z}{\delta_0} \right) \quad (51)$$

Now let us examine this solution for torsion more closely from the physical point of view. Since hyperbolic secant has an acute Gaussian format so is squared secant and so is torsion vector component in the direction of z-coordinate orthogonal to DW. This seems to indicate that both torsion sourced either by fermionic sector or scalar higgs-like sector, has a similar behaviour orthogonally to flat DW with torsion. If δ_0 is small microscopically but finite, then when we approach the wall as $z \rightarrow \text{infy}$, the Gaussian function tends to be a delta Dirac distribution. This could for example represent a string crossing a DW. Primordial magnetic DW with Debye screening has been recently investigated which gives also motivation to obtain the dynamo equation in comoving coordinates in the next section as we did for QED sector dynamo.

6 Chiral dynamos in scalar higgs-like sector Einstein-Cartan static DWs

In this section we shall obtain the magnetic dynamo equation in the background of a planar thick static wall in comoving coordinates, given by the metric line element [19]

$$ds^2 = e^{az} (dt^2 - dz^2) - e^{-az} (dx^2 + dy^2) \quad (52)$$

From this metric by assuming just one non-trivial component of the magnetic field orthogonal to the wall as B_z we have from the chiral dynamo equation without the convection term the following equation

$$\partial_t B^z - \eta \partial_z^2 B^z = \mu_5 B^z \quad (53)$$

Now by considering the metric (52) and the comoving coordinates for the contravariant component of the magnetic field as $B^z = e^{-az} B_z$, where a is an integration constant, one obtains the chiral dynamo equation in the explicitly format as

$$\partial_t (e^{-az} B_z) - \eta \partial_z^2 (e^{-az} B_z) = \mu_5 (e^{-az} B_z) \quad (54)$$

Here μ_5 is the chiral chemical potential. Furthermore, we note that we are not considering the convective term here that depends on the velocity of the chiral plasma for example. As usual the z -component of the magnetic field depends upon the z and t -cosmic coordinates. Then the solution of this equation is given by

$$B_z = B_{(0)} e^{a^2(\eta + \mu_5)t} \quad (55)$$

therefore we note that the factor γ of the dynamo amplification of the magnetic field is given by $\gamma = a^2(\eta + \mu_5)$, which is always positive. To complete the solution one simply multiply this magnetic field component by the comoving component to obtain

$$B^z = B_{(0)} e^{a^2(\eta + \mu_5)t - az} \quad (56)$$

Therefore one notice that the domain wall sourced by the Higgs-like field is compatible with the one we have obtained previously in the fermionic sector where the nucleons spins were polarised along the orthogonal direction to the DW.

7 Conclusions and Outlook

In this paper we showed that torsional Ohmic effects also affects amplification of magnetic fields during preheating, though the effects seems to be minimal due to the fact that the magnetic field at decoupling is $B_{dec} \sim 10^{-25} G$, whereas respective torsion effects are $B_{torsion} \sim 10^{-37} G$. However, despite of the weakness of this field, this value is still able to seed galactic dynamo. This result which is our main goal in the paper has been obtained by finding a solution of QED equations with torsion semi-minimally coupled with electromagnetic field. This does not introduce a massive photon as in some previously investigation like in Proca magnetogenesis recently investigated by Garcia de Andrade [12]. Effective Lagrangean can be used to determine the torsion which can be used to seed galactic dynamos. From a cosmic dynamo torsionful QED background equation we show that torsion contributes as a damping couple with Ohmic resistivity to decrease the values of the magnetic densities in several situations of astrophysically interest to provide results compatible with astronomical observations. The motivation from this study came from some work by Campanelli et al [20] where investigation of similar issues were undertaken in general relativistic backgrounds of Riemannian geometry, and by the work of Salim et al [21] on the amplification of the

magnetic field in bouncing cosmological models. Another aspects of magnetogenesis can be found in Pandey [14]. Besides, in the long wave length limit $\eta k \rightarrow 0$ the factor $\beta \sim k\eta K$, can be neglected and really destroys contortion action by the vanishing of β . Therefore, from this limit GR magnetogenesis preheating expressions are recovered. The extension of this paper to chiral dynamos may appear in near future. We also showed that the torsion decouples from the expansion of magnetic field. Recently R. Banerjee [22] has informed us that values of B – field as low as $10^{-32} Gauss$ may also seed galactic dynamos. Probably one of the most interesting features of introducing torsion here, is that since dark energy is repulsive gravity [23] and dark matter attractive, the slow down in the magnetic field growing discussed above may be due to torsion contributions to dark energy. A more detailed investigation on that matter must appear in a forthcoming paper. Actually previously, Gasperini [23] that is possible to regularize the curvature singularity in a radiation dominated universe by the repulsive effect of spin-spin interaction in Einstein-Cartan gravity. Other type of dynamos in topological defects such as domain walls can be found recently in literature [7]. Magnetogenesis is such an important subject that appeared very recently on a scientific magazine for popular science [24]. **Following the evolution of a double thick domain wall, one containing matter the other antimatter, given by Dolgov et al [25] in the context of inflationary universe, one may further generalise the ideas discussed here in section 5. This is work under progress.**

8 Acknowledgements

We would like to express my gratitude to R Banerjee for helpful discussions on primordial magnetism. Support from my wife Ana Paula Teixeira de Araujo. Financial support from University of State of Rio de Janeiro (UERJ) is grateful acknowledged.

References

- [1] M Shaposhnikov, A Shkerin, I Timiryasov, S Zell, JCAP 02 (2021) 08; [Erratum: JCAP 10(E01)].
- [2] V de Sabbata and C Sivaram, Spin and Torsion Gravitation (1995) world scientific, Singapore and New York. L Widrow, Rev Mod Phys **74**: 775 (2001). M Turner and L Widrow, Phys Rev **D** (1988). L C Garcia de Andrade, Nuclear Phys **B** (2011). L Garcia de Andrade, Phys Lett **B** 468, 28 (2011).
- [3] G Karananas, M Shaposhnikov, A Shkerin, S Zell, Scale and Weyl Invariance in Einstein-Cartan Gravity, arXiv: 2108. 05897v2 [hep-ph] (2021).
- [4] B A Bassett, G Polfrone, S Tsujikawa and F Viniegra, Phys Rev D **63** (2001) 023506.
- [5] F W Hehl and Yu N Obukhov, Foundations of Electrodynamics: Charge flux and Metric.
- [6] L Garcia de Andrade, Topology in Einstein-Cartan magnetogenesis and dynamo effects, Lambert Academic Publishers, Moldavia (2021).

- [7] L Garcia de Andrade, Annals of Physics 433, 24 (2021). L Garcia de Andrade, Chiral spinning cosmic string dynamo in Einstein-Cartan gravity, submitted to Ann of Physics (2021). I. T. Drummond and S. J. Hathrell, Phys. Rev. D **22**, 343 (1980).
- [8] G Karananas, M Shaposhnikov, I Timiryasov, S Zell, Phys Rev Lett **120**, 161301 (2021).
- [9] J Schober, I Rogachevskii and A Brandenburg, Production of a chiral magnetic anomaly with emerging turbulence and mean-field dynamo action, ArXiv: 2107.12945v1 [Phys-Plasm-ph] (2021). J Schober, I Rogachevskii, Axel Brandenburg, Dynamo instabilities in plasmas with inhomogeneous chiral chemical potential, arXiv: 2107.13028v1 [physics.plasmas](2021).
- [10] L Garcia de Andrade, Int J Geom Math in Phys (2021).
- [11] Khotari et al, Torsion driven magnetogenesis at inflationary universe, Phys Rev D (2020). C Tsagas, Phys Rev D **72**: 123509 (2005). J D Barrow, C Tsagas, Phys Rev **D 77**, 107302 (2008). J D Barrow, C Tsagas and K Yamamoto, Phys Rev D **85**, 047503 (2012).
- [12] L Garcia de Andrade, Cosmic Magnetism in modified gravity theories (2017), Editions europeennes universitaires editors.
- [13] V Arnold and B Khesin, Topological methods in Hydrodynamics, (1980) Springer verlag. S Childress and A D Gilbert, Stretch, twist and Fold: The Fast Dynamo, springer, (1996).
- [14] K.L. Pandey and S.K. Sethi, Astrophysical J., **762**: 15, (2013).
- [15] R Banerjee, private communication (2013).
- [16] M Gasperini, GRG J **30**, (1998) 1703-1709.
- [17] D. Mazzitelli and F. Spedalieri, Phys. Rev. D (1988). M Turner and M Widrow, Phys Rev **D** (1988).
- [18] L. Garcia de Andrade, Ann Phys **432** (2021).
- [19] L. Garcia de Andrade, Class Quantum Gravity **6**, (1999).
- [20] L Campanelli, P Cea, G L Fogli, Phys Lett **B 680**, 125 (2009).
- [21] J. Salim, N Souza, S P Bergliaffa and T Prokopec, JCAP **0704**: 011, (2007).
- [22] Garcia de Andrade, Ann Phys **432** (2021).
- [23] M Gasperini, GRG J **30**, (1998) 1703-1709. R Banerjee, private communication (2013).
- [24] D Garrison, Magnetogenesis: The first 10 seconds, Sky and Telescope, printed in Canada.
- [25] A Dolgov, S I Godunov, A S Rudenko, Evolution of Thick domain walls in inflationary and $p = \omega\rho$ universe, arXiv: 1711. 04704v2 [gr-qc] (2018).