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A Simple Equation for the Proton Charge Radius

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Abstract: In this short paper we will propose a new equation for the proton charge radius which is inversely proportional to the Rydberg constant and directly proportional to the running value of the fine structure constant on the protonic scale. This will define the proton charge radius only by the effective value of the fine structure constant and the Rydberg constant. This new equation also disproves the old value of the proton charge radius 0.877(50) which was measured by using electron-proton scattering and further solidifies the newer and smaller values 0.8414(19) provided by NIST.

Keywords: proton charge radius; particle physics; theoretical physics; atomic physics

Introduction

The current state of the proton charge radius essentially boils down to having two different experimental values. The first one, which will be the focus of this paper, is the more recent muonic hydrogen Lamb shift experiment [1] by R. Pohl et al from 2010, which has a measured value of the proton charge radius of $r_p = 0.84184(67)\ \mathrm{fm}$ which is in disagreement with the mean-square charge radius derived from electron-proton scattering experiments [2,3] from older experiments where they measured a value $r_p = 0.897(18)\ \mathrm{fm}$. Other older experiments [4,5] that used electron-proton scattering measured a value $r_p = 0.877(50)\ \mathrm{fm}$.

We will solve this "proton radius puzzle" by proposing a new empirical equation for the proton charge radius.

The Proton Charge Radius Equation

The new equation for the proton charge radius is based on my previous work from ref. [6]. We will however add the strong coupling $\alpha_s(Q)$ on the Q scale:

$$r_p = \frac{N}{R_\infty} \cdot \int_0^{\alpha(Q)} x^N dx \cdot \frac{(N+1)}{\alpha_s(Q) \cdot \pi} \tag{1}$$

where $\alpha(Q) = \alpha(m_p)$ is the running value of the fine structure constant on the protonic scale, $\alpha_s(m_p)$ is the running value of the strong coupling on the proton scale and N is the number of colors which refers to the number of different "types" or "charges" of the strong interaction that quarks can possess. After we solve the integral, we get:

$$r_p = \frac{N \cdot \alpha^{N+1}(m_p)}{\alpha_s(m_p) \cdot \pi \cdot R_\infty} \tag{2}$$

and since $N = 3$:

$$r_p = \frac{3 \cdot \alpha^4(m_p)}{\alpha_s(m_p) \cdot \pi \cdot R_\infty} \tag{3}$$

The running or effective value of the fine structure constant $\alpha(Q)$ is usually calculated by using renormalization. In the $\overline{\mathrm{MS}}$ renormalization scheme the effective value of the fine structure constant is obtained by using the equation:

$$\alpha(Q) = \frac{\alpha}{1 - \hat{\Pi}(Q)} \tag{5}$$

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where $\widehat{\Pi}(Q)$ is the vacuum polarization function which can be written as:

$$\widehat{\Pi}(Q) = \sum_{i=1}^{\infty} \widehat{\Pi}^{i}(Q) \tag{6}$$

where each term receives contributions from all fermion flavors. In the $\overline{MS}$ renormalization scheme the counter terms are chosen so that they only contain divergent pieces with the addition of certain constants. One-loop counter terms, two loop corrections, muonic, tauonic and hadronic contributions are:

$$\widehat{\Pi}(Q) = \frac{\alpha_0}{3\pi} \cdot \ln\left(\frac{m_p^2}{m_e^2}\right) + \frac{\alpha_0^2}{4\pi^2} \cdot \ln\left(\frac{m_p^2}{m_e^2}\right) + \Delta\alpha_{muon} + \Delta\alpha_{tau} + \Delta\alpha_{had}(m_p^2) \tag{7}$$

where α_0 is the fine structure constant at low energy, name on the electron scale, m_p is the proton mass and m_e is the electron mass. $\Delta\alpha_{muon}$ accounts for the vacuum polarization from muons, $+\Delta\alpha_{tau}$ accounts for tau contributions and $\Delta\alpha_{had}(m_p^2)$ are the hadronic contribution at the proton mass scale. We get:

$$\alpha^{-1}(m_p) = 134.265(34) \tag{8}$$

Although the calculation from renormalization provides a far less accurate value $\alpha^{-1}(m_p) = 134(01)$. To calculate $\alpha_s(Q)$ we use:

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \cdot \ln\left(\frac{Q}{\Lambda}\right)} \tag{9}$$

where the beta function $\beta_0 = (11N - 2N_f)/3$ is the first coefficient of the beta function, N_f is the number of active flavors (quark types) at the scale Q, $N_f = 3$ in the range of energy scales relevant to the proton when $Q = m_p$, and $\Lambda \approx 200$ MeV is the s the QCD scale parameter. This provides us with a value $\alpha_s(m_p) = 0.32(09)$.

We obtain a value of the proton charge radius $r_p = 0.8412(42)$ fm that is in great agreement with the newer and smaller experimental value [1] and the NIST value 0.84075(64) fm.

Conclusions

The proton charge radius is proportional to the running value of the fine structure constant on the proton scale and it's inversely proportional to the running value of the strong coupling on the proton scale and the Rydberg constant. We can easily conclude that the older and larger value of the proton charge radius is incorrect by using the equation from eq.(2) by pointing out that if $r_p = 0.877(50)$ fm then $\alpha^{-1}(m_p) = 132.875(47)$ which is impossible because the running value of the fine structure constant on the tauonic scale is [7] $\alpha^{-1}(m_\tau) = 133.557(37)$. Having in mind that the tau lepton is almost twice as massive as the proton, it's impossible for $\alpha(m_p)$ to have a larger value than $\alpha(m_\tau)$.

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