Article

Not peer-reviewed version

A Simple Solution for the Proton Radius Puzzle

Nikola Perkovic *

Posted Date: 13 November 2023

doi: 10.20944/preprints202311.0785.v1

Keywords: partcle physics; atomic physics; proton radius doi:10.20944/preprints202311.0785.v1

Article

A Simple Solution for the Proton Radius Puzzle

Nikola Perkovic

Institute of Physics and Mathematics, Faculty of Science, University of Novi Sad, Serbia; percestyler@gmail.com

Abstract: In this brief paper we will propose a new equation for the proton charge radius. We will base our new equation on the Trinhammer-Bohr formula which states that the proton charge radius is directly proportional to the proton Compton wavelength. We will then propose a new equation for the proton Compton wavelength which is inversely proportional to the Rydberg constant and directly proportional to the running value of the fine structure constant on the protonic scale. This will define the proton charge radius only by the effective value of the fine structure constant and the Rydberg constant. This new equation also disproves the old value of the proton charge radius 0.877(50) which was measured by using electron-proton scattering and further solidifies the newer and smaller values 0.8414(19) provided by NIST.

Introduction

The current state of the proton charge radius essentially boils down to having two different experimental values. The first one, which will be the focus of this paper, is the more recent muonic hydrogen Lamb shift experiment [1] by R. Pohl et al from 2010, which has a measured value of the proton charge radius of $r_p = 0.84184(67)$ fm which is in disagreement with the mean-square charge radius derived from electron-proton scattering experiments [2,3] from older experiments where they measured a value $r_p = 0.897(18)$ fm. Other experiments [4,5] that used electron-proton scattering measured a value $r_p = 0.877(50)$ fm.

In 2019 two Danish physicists [6] Ole Trinhammer and Henrik Bohr, the grandson of the legendary physicist Niels Bohr, discovered a new equation for the proton charge radius where they concluded that the proton charge radius is directly proportional to:

$$r_p = \frac{2 \cdot \lambda_{c,p}}{\pi} \tag{1}$$

where $\lambda_{c,p}$ is the proton Compton wavelength. They measured the value $r_p = 0.841235641(10)$ fm, that agrees with the smaller value from the muonic Hydrogen Lamb shift experiment. In future reference, we will refer to their equation from eq. (1) above as the Trinhammer-Bohr equation.

A new equation for the proton charge radius

Even though the Trinhammer-Bohr equation from eq. (1) above does agree with the smaller experimental results that seem to be more persuasive, we still need a new equation that can safely and surely determine which results are true.

We will derive a new equation for the proton charge radius by proposing a new equation for the proton Compton wavelength. First we should explain that even though Compton wavelengths of all particles are expressed with the equation $\lambda_c = h/mc$ where h is the Pnack constant, m is the rest mass of the particle and c is, of course, the speed of light, there are alternative expressions.

For example, the electron Compton wavelength can also be expressed with the equation $\lambda_{c,e} = \alpha^2/2R_\infty$ where α is the fine structure constant and R_∞ is the Rydberg constant. Likewise there is also an alternative equation for the proton Compton wavelength that's inversely proportional to the Rydberg constant.

The new equation for the proton Compton wavelength that we propose in this paper is:

$$\lambda_{c,p} = \frac{N\,\pi}{2\,R_\infty} \cdot \int_0^{\alpha(m_p)} x^N dx \cdot (N+1) \tag{2}$$

where $\alpha(m_p)$ is the running value of the fine structure constant on the protonic scale and $N = 3$ is the number of constituent quarks of the proton. In the most basic sense a proton consists of three quarks, namely two up quarks and one down quark. After we solve the integral we get:

$$\lambda_{c,p} = \frac{N\,\pi \cdot \alpha^{N+1}(m_p)}{2\,R_\infty} \tag{3}$$

and since $N = 3$:

$$\lambda_{c,p} = \frac{3\pi \cdot \alpha^4(m_p)}{2\,R_\infty} \tag{4}$$

The running or effective value of the fine structure constant $\alpha(Q)$ is usually calculated by using renormalization. In the $\overline{\mathrm{MS}}$ renormalization scheme the effective value of the fine structure constant is obtained by using the equation:

$$\alpha(Q) = \frac{\alpha}{1 - \hat{\Pi}(Q)} \tag{5}$$

where $\hat{\Pi}(Q)$ is the photon vacuum polarization function which can be written as:

$$\hat{\Pi}(Q) = \sum_{i=1}^{\infty} \hat{\Pi}^i(Q) \tag{6}$$

where each term receives contributions from all fermion flavors. . In the $\overline{\mathrm{MS}}$ renormalization scheme the counter terms are chosen so that they only contain divergent pieces with the addition of certain constants. One-loop counter terms are proportional to:

$$\Delta = \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) + O(\varepsilon) \tag{7}$$

where γ_E is the Euler-Mascheroni constant. An appropriate choice for the 't Hooft mass is $\mu = m_p$. However, the process of calculating $\alpha(m_p)$ can be simplified because we know the experimental values of $\lambda_{c,p}$ and R_∞ so we can easily calculate from eq.(4) that:

$$\alpha^{-1}(m_p) = 134.264845(34) \tag{8}$$

After we introduce eq.(4) to the Trinhammer-Bohr equation from eq. (1) we get:

$$r_p = \frac{3 \cdot \alpha^4(m_p)}{R_\infty} \tag{9}$$

therefore the proton charge radius is defined in the terms of the running value of the fine structure constant on the protonic scale and the Rydberg constant.

Conclusions

We can easily conclude that the older and larger value of the proton charge radius is incorrect by using the equation from eq. (2) by pointing out that if $r_p = 0.877(50)$ fm then $\alpha^{-1}(m_p) = 132.875(47)$ which is impossible because the running value of the fine structure constant on the tauonic scale is [7] $\alpha^{-1}(m_\tau) = 133.557(37)$. Having in mind that the tau lepton is almost twice as massive as the proton, it's impossible for $\alpha(m_p)$ to have a larger value than $\alpha(m_\tau)$.

References

1. Pohl, R. et al., The Size of the Proton, Nature, 466 (2010).
2. Sick, I. On the rms-radius of the proton. Phys. Lett. B 576 (2003).

 doi:10.20944/preprints202311.0785.v1

3. Blunden, P. G. & Sick, I. Proton radii and two-photon exchange. Phys. Rev. C 72 (2005).
4. A1 Collaboration, Bernauer, J. et al., Phys. Rev. Lett., 105 (2010).
5. A1 Collaboration, Bernauer, J. et al., Phys. Rev. C, 90 (2014).
6. Trinhammer, O. & Bohr, H., On proton charge radius definition, EPL, 128 (2019).
7. Pivovarov, A., Phys. Atom. Nuclei 65 (2002).