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Article

The Multifunctional Modulation Micromechanical Gyro

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Abstract: An actual problem of inertial navigation related to the creation of a multifunctional device that performs the functions of a meter for both angular velocities and linear accelerations is considered. The possibility of constructing such a device based on a hybrid-type modulation micromechanical gyroscope has shown. The ability to measure linear accelerations has provided by the presence of a small symmetrical distance between the axes of the elastic suspension relative to the center of mass of the sensor element. A mathematical model of the device ("heavy" gyroscope) as a high-quality three-dimensional oscillatory system is constructed. It has shown numerically that the reaction of the system to the movement of an object has, along with precession, the observed nutation. This allows you to develop an algorithm for separating information about the angular and linear motion of the base and implement a two-component angular velocity meter and a two-component linear acceleration meter in one device.

Keywords: gyro; micromechanical system; newtonmeter

1. Introduction

Despite the progress made in the development of micromechanical gyroscopes (MMG), problems remain that limit the accuracy of their measurements ([1] – [12]). The most important of them is the presence of a small mass of the sensor element, which is not able to create the necessary value of the gyroscopic moment when measuring small values of the portable angular velocity, which requires an ultra-high sensitivity of the device's information search system. Another problematic factor that worsens the metrological properties of the device is a significant level of "zero bias" associated with its manufacturing technology [13].

To solve the first problem, in recent years there has been a tendency to switch to hybrid MMG that combine MEMS technologies with electromechanical elements. On the basis of a hybrid MMG, it was possible to create a two-axis angular velocity sensor with slightly increased dimensions, which made it possible to increase the sensitivity of the device [14]. However, this does not solve the second problem.

This article is a continuation of a series of papers devoted to the development of theoretical foundations for improving gyroscopic devices ([15] – [18]). To solve the problem of "zero displacement" of the signal, it is proposed to use the "modulation principle" of receiving and processing primary information in a mechanical circuit, which has proven itself well in vibration gyroscopes [19].

Russian scientists Golovan A.A., Sukhanov B.N., Belugin V.B. [20], Brozgul L.I. and other made a significant contribution to the development of the modulation direction in the theory of vibration gyroscopes.

A distinctive feature of MMG modulation is that the angular rotation of the base of the device is registering not by measuring this rotation relative to the main axis of the gyroscope fixed in inertial space, but by measuring the amplitude and phase of its rotor vibrations in the rotating coordinate system. In this case, information about the angular displacement of the base is contained in the specified parameters of the AC signal, and the

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presence of a constant component caused by the so-called "zero offset" does not affect the accuracy of taking MMG readings.

2. Problem statement

The two-coordinate hybrid MMG is able to measure the value of the angular velocity vector of rotation of the base lying in the plane of its sensitivity. When the device is tuning to resonance with respect to the measured angular velocity, it functions as an integrating gyroscope, since the deflection of its rotor (under low attenuation conditions) is proportional to the angle of rotation of the base.

The purpose of this study is to develop a multifunctional device that, along with the ability to measure angular deviations in two orthogonal directions that coincide with its sensitivity axes, allows you to measure the magnitude of the inertia force vector caused by rapid movement of the base. That is, the development of an angular velocity sensor that performs the functions of a newtonmeter.

Figure 1 presents a kinematic diagram of the proposed modulation micromechanical gyroscope – newtonmeter (MMG-N) of a hybrid type, including a movable base 1, a valve drive 2, a rotor 3, an elastic suspension 4, rotor mounting elements 5, an angle sensor 6 connected to the drive shaft.

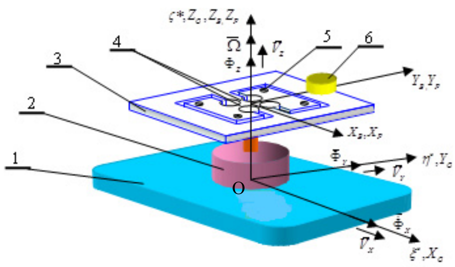

Figure 1. Kinematic scheme of modulation hybrid MMG-N.

We will show that linear acceleration can be measured due to a slight symmetrical separation of the elastic suspension axes relative to the center of mass of the sensor element.

3. Mathematical model

Figure 2 presents a diagram of the design of the modulation MMG-N rotor, a distinctive feature of which is the presence of separation (non-intersection) of elastic supports 1,2, which creates a "pendulum" of the rotor 3, due to the displacement of its center of mass relative to the suspension axes [21].

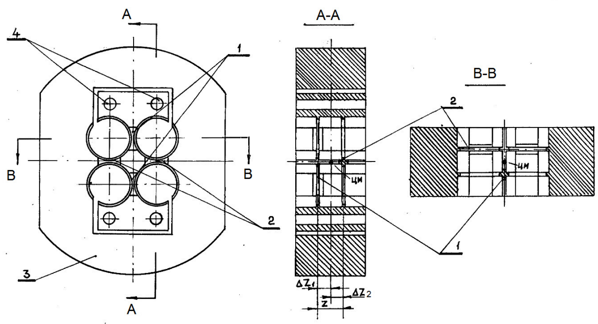

Figure 2. A diagram of the general view of the modulation MMG-N rotor.

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To describe the motion of a modulation gyroscope-newtonometer (MMG-N) mounted on a movable base, we introduce a coordinate system with the reference point at the center of mass of the gyroscope rotor and the axes directed at fixed stars, i.e. moving translationally in inertial space (Figure 1).

We connect with the movable base the coordinate system $X_0Y_0Z_0$, the axis OZ_0 of which coincides with the axis of rotation of the MMG-N rotor (Figure 3).

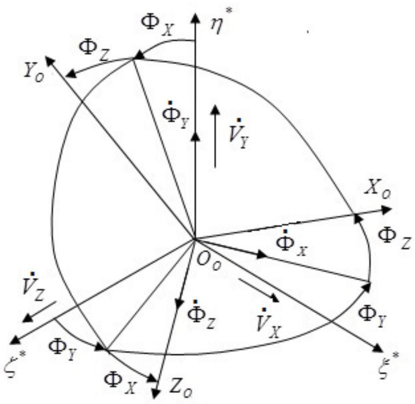

Figure 3. The position of the object in the reference coordinate system.

The motion of the base of the device will be considered known, i.e. in each instant of time the orientation of the coordinate system $X_0Y_0Z_0$ relative to the inertial system $\xi^*\eta^*\zeta^*$, and the projections $\dot{\Phi}_x, \dot{\Phi}_y, \dot{\Phi}_z$ of the vector of absolute angular velocity of the base on the axis of the system $\xi^*\eta^*\zeta^*$ are given functions of time.

In addition to the above-mentioned systems, four more axis systems $X_BY_BZ_B$, $X_pY_pZ_p$, $X_1Y_1Z_1$, $X_2Y_2Z_2$ are required, connected respectively to the drive shaft, with the axes of the main moments of inertia of the rotor and the corresponding axes of elastic supports (torsion bars), as shown in Figure 4.

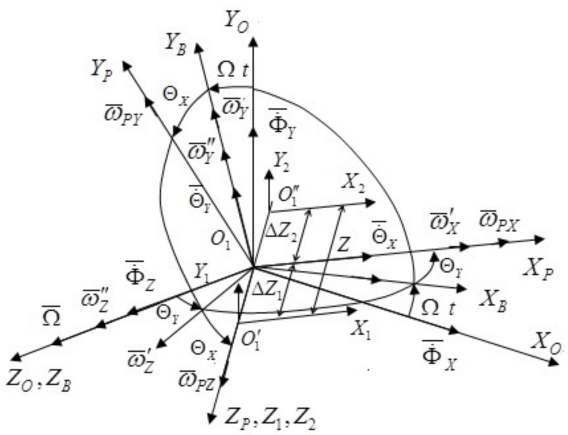

Figure 4. The position of the MMG-N sensing element relative to a moving object.

The origin of the axis systems $X_BY_BZ_B$ and $X_pY_pZ_p$, lies in the center of mass of MMG-N, and their position relative to the system $X_0Y_0Z_0$ set successive turns in the positive direction at angle Ωt to $X_BY_BZ_B$ and angles Θ_x, Θ_y for $X_pY_pZ_p$ (Ω – angular velocity of rotation of the rotor).

The presence of a separation of the torsion axes due to the Z value, as well as the displacement of the center of mass of the rotor relative to the corresponding axes of elastic supports by the value ΔZ_1 and ΔZ_2 (Figure 4), creates a static unbalance (pendulum) of the rotor, which makes it possible to classify such a device as a "heavy" gyroscope.

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Projections of the angular velocity of the base movement on the axis of the system $X_B Y_B Z_B$ after turning an angle Ωt have the form:

$$\omega_x'' = \dot{\Phi}_x \cos\Omega t + \dot{\Phi}_y \sin\Omega t,$$

$$\omega_y'' = \dot{\Phi}_y \cos\Omega t - \dot{\Phi}_x \sin\Omega t,$$

$$\omega_z'' = \dot{\Phi}_z + \Omega.$$

Projections of angular velocities on the axis of the system $X_p Y_p Z_p$ after turning an angle Θ_y we can write as:

$$\omega_x' = \omega_x'' \cos\Theta_y - \omega_z'' \sin\Theta_y,$$

$$\omega_y' = \omega_y'' + \dot{\Theta}_y,$$

$$\omega_z' = \omega_z'' \cos\Theta_y + \omega_x'' \sin\Theta_y,$$

and after turning an angle Θ_x , respectively:

$$\omega_{px} = \omega_x' + \dot{\Theta}_x,$$

$$\omega_{py} = \omega_y' \cos\Theta_x + \omega_z' \sin\Theta_x,$$

$$\omega_{pz} = \omega_z' \cos\Theta_x - \omega_y' \sin\Theta_x.$$

Since the MMG-N rotor has a "pendulum", as a result of its movement with angular velocities ω_{px},ω_{py} the center of mass moves translationally with a linear velocity, the projections of which on the axis of the system $X_p Y_p Z_p$ (4) have the form:

$$V_{px} = -\omega_{py} \Delta Z_1,$$

$$V_{py} = -\omega_{px} \Delta Z_2,$$

$$V_{pz} = 0.$$

To obtain a mathematical model of the gyroscopic system under consideration, we use the Lagrange method [22], choosing as generalized coordinates the rotation angles of the rotor Θ_x and Θ_y, that uniquely determine its position, and as generalized forces – the damping moments, torsion elastic moments, and moments caused by inertia forces. The latter, if the angles Θ_x and Θ_y are small, are defining by the following expressions:

$$M_{ix} = m\Delta Z_2 (\dot{V}_y \cos\Omega t - \dot{V}_x \sin\Omega t),$$

$$M_{iy} = m\Delta Z_1 (\dot{V}_x \cos\Omega t + \dot{V}_y \sin\Omega t).$$

The linearized mathematical model of MMG-N after factorization and accounting for generalized forces up to the first order of smallness has the form:

$$\begin{aligned}
&(J+\Delta J)\ddot{\Theta}_x + \mu\dot{\Theta}_x + (k_x + (C - J + \Delta J)\Omega^2)\Theta_x - (2J - C)\dot{\Theta}_y = \\
&-(J+\Delta J)(\ddot{\Phi}_x \cos\Omega t + \ddot{\Phi}_y \sin\Omega t) - \Omega(C + 2\Delta J)(\dot{\Phi}_y \cos\Omega t - \dot{\Phi}_x \sin\Omega t) \\
&+ (M + \Delta M)(\dot{V}_y \cos\Omega t - \dot{V}_x \sin\Omega t) + M_x \cos\Omega t + M_y \sin\Omega t, \\
&(J-\Delta J)\ddot{\Theta}_y + \mu\dot{\Theta}_y + (k_y + (C - J - \Delta J)\Omega^2)\Theta_y + (2J - C)\Omega\dot{\Theta}_x = \\
&(J-\Delta J)(-\ddot{\Phi}_y \cos\Omega t + \ddot{\Phi}_x \sin\Omega t) + \Omega(C - 2\Delta J)(\dot{\Phi}_x \cos\Omega t + \dot{\Phi}_y \sin\Omega t) \\
&-(M - \Delta M)(\dot{V}_x \cos\Omega t + \dot{V}_y \sin\Omega t) + M_y \cos\Omega t - M_x \sin\Omega t.
\end{aligned} \tag{1}$$

Differential equations (1) are obtained for the case when the MMG-N drive shaft rotates at a constant angular Ω, the torsional stiffness of torsion bars k_x, k_y are small

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compared to their bending stiffness, and the base performs both angular displacement with speed $\dot{\Phi}_x$ $\dot{\Phi}_y$,$\dot{\Phi}_z$ and translational motion with acceleration $\dot{V}_x$, $\dot{V}_y$,$\dot{V}_z$.

The following notation has entered here:

$$J = \frac{A + B + m(\Delta Z_1^2 + \Delta Z_2^2)}{2}, \; \Delta J = \frac{A - B + m(\Delta Z_1^2 - \Delta Z_2^2)}{2},$$

$$M = \frac{m(\Delta Z_1 + \Delta Z_2)}{2}, \; \Delta M = \frac{m(\Delta Z_1 - \Delta Z_2)}{2},$$

$\mu_x = \mu_y = \mu$ – the coefficient of viscous friction, $k_x = k_y = k$ – the torsional stiffness of the torsion bars, A, B, C – respectively, the equatorial and polar moments of inertia of the rotor, m – the weight of the rotor, M_x, M_y – harmful torques acting for the respective axes of sensitivity of the gyroscope.

4. Numerical simulation

The system of equations (1) was solved numerically in the Maple environment under zero initial conditions:

$$\Theta_x = \dot{\Theta}_x = 0, \quad \Theta_y = \dot{\Theta}_y = 0. \tag{2}$$

Figures 5 present graphs of the solution of the problem (1) - (2) in the case of an asymmetrical rotor ($A \neq B$). Figure 5a corresponds to the channel for measuring angular velocity (linear accelerations $\dot{V}_x$ and $\dot{V}_y$ in equations (1)are zero), and Figure 5b corresponds to the channel for measuring linear accelerations (components of angular velocity $\dot{\Phi}_x$ and $\dot{\Phi}_y$ in equations (1) are zero).

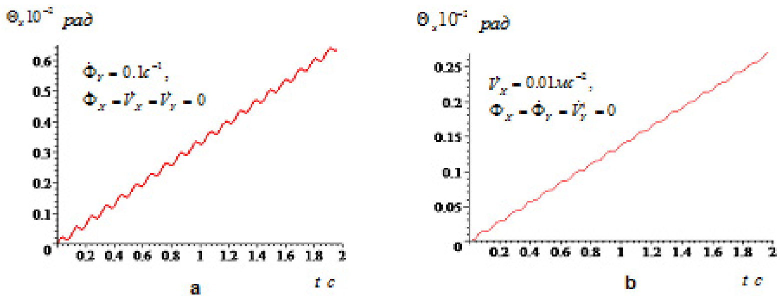

Figure 5. The solution of the problem in the case of an asymmetrical rotor.

Note that information on the linear acceleration measurement channel can be obtained only if the multipliers ($M + \Delta M$) and ($M - \Delta M$) in equations (1) are not equal to zero, i.e. when the parameters ΔZ_1 and ΔZ_2 are not equal to zero (the presence of non-intersection of the torsion axes of the elastic suspension).

The graphs show that the use of an asymmetric rotor leads to its precession, accompanied by nutation both in the channel for measuring angular velocity and in the channel for measuring linear accelerations. Moreover, the amplitude of nutation oscillations at this time interval is quite large (nutation oscillations are observed), that is due to the high-quality properties of the sensitive element.

Figure 6 show similar solutions to the problem (1) - (2) in the case of a symmetric rotor having the same equatorial moments of inertia ($A = B$). A distinctive feature of these graphs is the absence of nutation in the angular velocity measurement channel (Figure 6a), which is due to the dynamics of the input information of the gyroscope, that reacts not only to the angular velocity, but also to the angular acceleration. In the channel for measuring linear accelerations, nutation is observed along with precession (Figure 6b).

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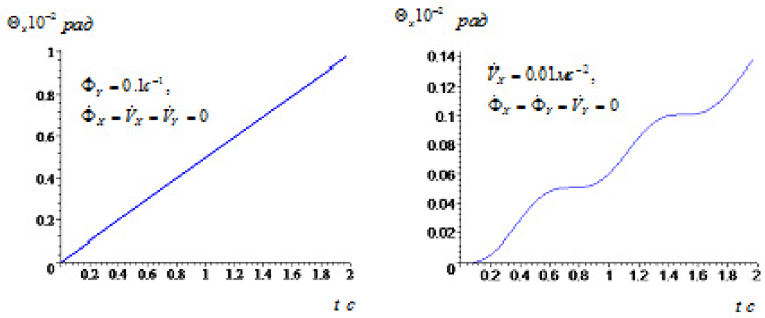

Figure 6. The solution of the problem in the case of an symmetric rotor.

Thus, the precession motion reflects the reaction of the gyroscope sensing element to the angular and translational motion of the base, and nutation vibrations occur only when it is translational.

Note that nutation vibrations become informational, since their amplitude and phase contain information about the magnitude and direction of the measured linear accelerations of a moving object.

5. Method for separating information

In order to use the results obtained in the previous section for metrological purposes, it is necessary to develop an algorithm for separating information by angular and linear parameters. First of all, we will demodulate the information signal taken from the angle sensor, i.e. multiply its output signal by two periodic output signals of the reference generator with the rotor velocity Ω, shifted in phase with respect to each other by $\pi/2$. In this way, we will make the transition from the rotating coordinate system to the coordinate system associated with the base of the device.

Let's analyze the output signals $U_x(t)$,$U_y(t)$ of demodulators (multiplication devices) when the base of the device rotates with angular velocity ($\dot{\Phi}_x\delta(t)$,$\dot{\Phi}_y\delta(t)$) and linear acceleration ($\dot{V}_x\delta(t)$,$\dot{V}_y\delta(t)$), where $\delta(t)$ is the Dirac function.

The corresponding formulas obtained using the high-level modeling environment Maple have the form:

$$U_x(t) = -k_t e^{-dt}\left((1 - \frac{\sin\omega_n t}{T_g\omega_n})\dot{\Phi}_x - \frac{(1-\cos\omega_n t)\dot{\Phi}_y}{T_g\omega_n}\right) + \frac{k_t' e^{-dt}((1-\cos\omega_n t)\dot{V}_x - \dot{V}_y \sin\omega_n t)}{\omega_n}, \quad (3)$$

$$U_y(t) = -k_t e^{-dt}\left((1 - \frac{\sin\omega_n t}{T_g\omega_n})\dot{\Phi}_y + \frac{(1-\cos\omega_n t)\dot{\Phi}_x}{T_g\omega_n}\right) + \frac{k_t' e^{-dt}((1-\cos\omega_n t)\dot{V}_y + \dot{V}_x \sin\omega_n t)}{\omega_n}, \quad (4)$$

where k_t – the transmission coefficient of the information retrieval and conversion system, $k_t' = k_t M/J$, $d = M/2J$ – the damping factor, $T_g = 1/d$ – the time constant of the gyroscope, $\omega_n = \Omega b/2$ – the frequency of nutation oscillations in the coordinate system of the device body, $b = (4J-2C)(2J-C)/(J^2 - \Delta J^2)$.

From the expressions (3) and (4) it follows that the output signals of the demodulator, which determine the response of the device to the angular velocity and linear accelerations in the coordinate system associated with the device body, consist of slowly changing

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components (not containing periodic functions) and components of nutation vibrations. However, the slowly changing values of signals on cross channels are $T_g\omega_n$ several times smaller than those on the main channels, so they will be omitted from consideration in the future. Note that the amplitude of nutation vibrations arising from the influence of angular velocities of rotation of the base is more than an order of magnitude less than the amplitude of nutation vibrations caused by the action of linear accelerations, since the time constant T_g of the device under consideration is greater than or equal to 200 c. Therefore, the influence of nutation vibrations caused by the movement of the device's rotor under the influence of angular velocities $\dot{\Phi}_x$,$\dot{\Phi}_y$ can also be ignored in the future. Then get:

$$U_x(t) = -k_t e^{-dt}\dot{\Phi}_x + \frac{k_t' e^{-dt}(\dot{V}_x - \dot{V}_x \cos\omega_n t - \dot{V}_y \sin\omega_n t)}{\omega_n},$$

$$U_y(t) = -k_t e^{-dt}\dot{\Phi}_y + \frac{k_t' e^{-dt}(\dot{V}_y - \dot{V}_y \cos\omega_n t + \dot{V}_x \sin\omega_n t)}{\omega_n}.$$

Due to the fact that the damping coefficient d of MMG-N is very small, nutation fluctuations in the device are observed during the time determined by the value T_g, and therefore can be used to separate information. For example, to obtain information about the angular velocity $\dot{\Phi}_x$ from the $U_x(t)$ signal, it is necessary to select periodic signals with a nutation frequency ω_n using a selective amplifier, and then demodulate them with a reference signal with a frequency ω_n and perform algebraic addition with slowly changing signals $\dot{\Phi}_x$ and $\dot{V}_x$.

6. Conclusions

The article deals with the actual problem of inertial navigation related to the creation of a multifunctional device that performs the functions of a meter for both angular velocities and linear accelerations.

It has shown that such a device can be built on the base of a hybrid-type modulation micromechanical gyroscope, which solves the problem of "zero offset" inherent in micromechanical gyroscopes, and also increases the metrological accuracy of the device.

It has shown that the measurement of linear acceleration can be implemented due to a small symmetrical separation of the elastic suspension axes relative to the center of mass of the sensor element.

A mathematical model of the device as a high-quality three-dimensional oscillatory system is constructed.

Based on numerical analysis, it has shown that the reaction of the system to the movement of an object has, along with precession, the observed nutation. This allowed us to develop an algorithm for separating information about the angular and linear motion of the base and implement a two-component angular velocity meter and a two-component linear acceleration meter in one device.

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