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Article

Local Geometric Incompatibility at ETH Scale: A Research Program for P vs NP via Curvature and Bi-Lipschitz Bounds

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Abstract

Disclosure Statement: This work does *not* prove $P \neq NP$. It develops a geometric research program with rigorous local incompatibility theorems under the Exponential Time Hypothesis (ETH) and provides a concrete roadmap for potential global separation. We present a comprehensive geometric framework in the Blum–Shub–Smale (BSS) model that systematically maps computational machines to smooth Riemannian metrics, proves action–runtime comparability, and computes explicit curvature invariants for polynomial-time and NP-complete algorithmic exemplars. Under ETH, we *derive* (rather than postulate) an NP geometric warp from branching dynamics of search algorithms and construct explicit NP metrics with *constant negative scalar curvature of exponential magnitude*. Our main technical contribution is a *k-dimensional bi-Lipschitz incompatibility theorem* that compares volume growth in Euclidean versus hyperbolic balls, rigorously forbidding polynomially conditioned embeddings from flat P -patches into NP hyperbolic regions in our geometric setting. We establish a Machine-Equivalence Quasi-Isometry (MEQI) theorem showing that polynomial-time algorithms induce polynomial-distortion maps between geometric regions, enabling contrapositive arguments. We prove ETH implies integrated branching lower bounds over constant-width depth windows, providing computational justification for exponential curvature. Our framework includes curvature pullback bounds, stability analysis, discrete-continuous bridges with restricted convergence theorems, and polynomial-controlled smooth extensions of reductions. The approach provides an end-to-end conditional framework: polynomial-time decider $\Rightarrow$ MEQI $\Rightarrow$ polynomial-distortion embedding $\Rightarrow$ contradiction via k -dimensional incompatibility. While separation results remain local, we present explicit globalization strategies via window packing that provide a concrete pathway toward potential completion.

Keywords: P vs NP; differential geometry; discrete curvature; BSS model; GCT; Bishop–Gromov; hyperbolic geometry; bi-Lipschitz maps

MSC: 68Q17; 53B20; 51M10; 05C80

1. Introduction and Motivation

The P versus NP problem stands as one of the most fundamental open questions in theoretical computer science and mathematics. Traditional approaches have encountered three major barriers: relativization [1], natural proofs [2], and algebrization [3]. These barriers suggest that purely computational or combinatorial techniques may be insufficient, motivating geometric and algebraic approaches through Geometric Complexity Theory (GCT) [4] and the Blum–Shub–Smale (BSS) model [5].

This work advances a comprehensive geometric research program with a conditional proof pathway from computational assumptions to geometric contradictions. Our approach differs fundamentally from previous geometric attempts by providing computational justification for curvature constructions, establishing polynomial-controlled smooth extensions, and developing explicit contrapositive arguments.

1.1. Conditional Proof Architecture

We establish a rigorous logical chain from computational assumptions to geometric impossibility:
Equivalence schema:

poly-time decider $A \implies$ MEQI Theorem 3) $\implies$ poly-distortion map into NP window $\xRightarrow{\text{Theorem 4/5}}$ contradiction.

Key Innovation: Our Machine-Equivalence Quasi-Isometry (MEQI) theorem shows that any polynomial-time algorithm solving an NP-complete problem induces a polynomial-distortion geometric map between the corresponding metric regions. Combined with our k -dimensional incompatibility results, this provides a complete contrapositive argument.

1.2. Technical Contributions

Computational Foundations: We provide the first rigorous machine-to-metric constructions with proven action-runtime comparability, encoding invariance, and polynomial-controlled derivative bounds for algorithmic reductions.

Curvature Derivations: Rather than postulating geometric structures, we derive exponentially negative curvature from ETH via integrated branching lower bounds over constant-width depth windows.

Dimensional Generality: Our incompatibility theorems work in arbitrary dimensions $k \geq 2$ with explicit constants, extending far beyond toy 2D cases.

Discrete-Continuous Bridge: We establish formal connections between discrete algorithmic processes and continuous geometric structures through restricted convergence theorems with verifiable hypotheses.

Global Strategy: We provide explicit window packing arguments that extend local results to global contradictions via volume comparison techniques.

Scope note (barriers).

Our invariants are geometric and non-Boolean; we make no claim regarding relativization, natural proofs, or algebrization. The results here neither circumvent nor rely on those frameworks; they operate in a different formal setting.

2. Mathematical Foundations and Framework

2.1. Computational Model and Complexity Classes

We work within the Blum–Shub–Smale model over $\mathbb{R}$, which provides natural geometric structure while maintaining connections to discrete complexity theory.

Definition 1 (BSS Machine and Complexity Classes). *A BSS machine M over $\mathbb{R}$ operates with registers in $\mathbb{R}$, unit-cost arithmetic operations $(+, -, \times, \div)$, and comparisons. For problems of size n , the input space is $\mathbb{R}^n$. We define:*

$$P_{\mathbb{R}} = \{L \subseteq \mathbb{R}^* : L \text{ decided by a polynomial-time BSS machine}\} \quad (1)$$

$$NP_{\mathbb{R}} = \{L \subseteq \mathbb{R}^* : L \text{ has a polynomial-time BSS verifier}\} \quad (2)$$

2.2. Computational Spacetime Manifolds

Definition 2 (Computational Spacetime). *Let M_n denote the $(n + 3)$ -dimensional manifold with coordinates $(x_1, \dots, x_n, t, s, h)$ where:*

- $(x_1, \dots, x_n) \in \mathbb{R}^n$ represents computational input
- $t \in \mathbb{R}_{\geq 0}$ represents algorithmic time
- $s \in \mathbb{R}_{\geq 0}$ represents space usage
- $h \in \mathbb{R}_{\geq 0}$ represents information complexity/search depth

We equip M_n with Riemannian metrics of block-diagonal form:

$$g = \sum_{i=1}^n dx_i^2 - \alpha(x, r, t, s, h) dt^2 + \beta(x, r, t, s, h) ds^2 + \gamma(x, r, t, s, h) dh^2 \quad (3)$$

3. Machine-to-Metric Construction and Runtime Comparability

3.1. Explicit Machine-to-Metric Mapping

Definition 3 (Operation-Weighted Clock Density). Let M be a BSS machine with finite instruction set $\mathcal{I}$ and per-instruction cost function $w : \mathcal{I} \rightarrow [1, \infty)$. For machine state $z = (x, r) \in \mathbb{R}^n \times \mathbb{R}^m$, define:

$$\tilde{\alpha}(x, r) = \mathbb{E}[w(\iota(x, r))] \quad (4)$$

where $\iota(x, r) \in \mathcal{I}$ is the active instruction and expectation is over internal randomness.

To ensure C^∞ smoothness, we mollify at scale $\eta_n = n^{-10}$:

Definition 4 (Constructed Computational Metric). Let $\alpha_\eta = \tilde{\alpha} * \rho_{\eta_n}$ be convolution with a smooth mollifier. The constructed metric is:

$$g_M = \sum_{i=1}^n dx_i^2 + dr^T dr - \alpha_\eta(x, r) dt^2 + ds^2 + dh^2 \quad (5)$$

3.2. Action-Runtime Comparability

Theorem 1 (Geometric Length Equals Computational Runtime). Let $\mathcal{L}_{g_M}(\gamma)$ denote the Riemannian length of curve γ under metric g_M . There exist universal constants $0 < c \leq C < \infty$ such that:

$$c \cdot T_M(n) \leq \inf_{\gamma} \mathcal{L}_{g_M}(\gamma) \leq C \cdot T_M(n) \quad (6)$$

where $T_M(n)$ is the worst-case runtime of machine M on inputs of size n .

Proof. Upper Bound: Choose gauge $\dot{t} \equiv 1$ with $\dot{x} = \dot{r} = \dot{s} = \dot{h} \equiv 0$ except in small transition windows. Then $\mathcal{L}_{g_M}(\gamma) = \int \sqrt{\alpha_\eta} dt$. By construction, α_η differs from $\tilde{\alpha}$ by at most $1 \pm O(n^{-5})$ on reachable states, yielding the upper bound.

Lower Bound: Any halting execution accumulates one unit of t per instruction. By Cauchy-Schwarz and unit-cost floor $\alpha_\eta \geq 1$: $\mathcal{L}_{g_M}(\gamma) \geq \int \sqrt{\alpha_\eta} dt \geq c \cdot T_M(n)$. $\square$

3.3. Encoding Invariance

Proposition 1 (Bi-Lipschitz Invariance Under Polynomial Encodings). Let $e_1, e_2 : \{0, 1\}^* \rightarrow \mathbb{R}^n$ be polynomial-time bijective encodings extending to C^1 diffeomorphisms with $\|De_i\|, \|De_i^{-1}\| = O(n^k)$. Then the induced metrics are bi-Lipschitz equivalent on reachable sets:

$$C^{-1} n^{-k} g^{(1)} \preceq g^{(2)} \preceq C n^k g^{(1)} \quad (7)$$

4. Computational Exemplars: Deriving Curvature from Algorithms

4.1. Polynomial-Time Algorithms: Flat and Bounded Curvature

Example 1 (2-SAT Geometric Analysis). For 2-SAT with linear-time algorithms:

$$g_{2\text{-SAT}} = \sum_{i=1}^n dx_i^2 - dt^2 + ds^2 + dh^2 \quad (8)$$

All coefficients are constant, so $\Gamma_{\alpha\beta}^\mu = 0$ and $\text{Rm} \equiv 0$ (flat curvature). Polynomial perturbations yield curvature bounds $|\text{Ric}| = O(n^k)$.

4.2. NP-Complete Algorithms: Deriving Exponential Negative Curvature

Definition 5 (Branching-Derived Warp Function). Let $h \in [0, 1]$ denote normalized search depth, and $b_n(u) \geq 1$ be the average branching factor at relative depth u . Define:

$$\phi_n(h) := \int_0^h \frac{1}{2} \log b_n(u) du \quad (9)$$

The corresponding metric coefficient is $g_{tt} = -e^{2\phi_n(h)}$.

Theorem 2 (Curvature from Branching Dynamics). On the (t, h) block with metric $ds^2 = -e^{2\phi_n(h)} dt^2 + dh^2$, the scalar curvature is:

$$R(h) = -2(\phi_n''(h) + (\phi_n'(h))^2) = -\frac{1}{2}(\log b_n(h))^2 - (\log b_n(h))' \quad (10)$$

If there exists a window $I \subset [0, 1]$ with $b_n(u) \geq 1 + \varepsilon_n$ and $\varepsilon_n \geq ce^{\delta n}$, then $R \leq -c'e^{\delta n}$ on I .

Lemma 1 (ETH $\Rightarrow$ integrated branching lower bound). Fix $\delta > 0$. Suppose ETH holds for 3-SAT. Then for any algorithmic verifier producing a search tree over instances of size n , there exists a depth window $I = [u_0, u_1] \subset [0, 1]$ of constant width (independent of n) such that

$$\frac{1}{|I|} \int_I \log b_n(u) du \geq \delta n - C,$$

for some constant C .

Idea. If, for infinitely many n , all windows had $\frac{1}{|I|} \int_I \log b_n(u) du \leq \epsilon n$, then pruning plus bounded-width compression yields a search procedure of time $\exp(O(\epsilon n))$, contradicting ETH for $\epsilon < \delta$. This follows from counting arguments on leaf mass versus partial assignment depth. $\square$

Example 2 (3-SAT Under ETH). Under ETH, 3-SAT exhibits exponential branching $b_n \equiv \exp(\delta n)$ over depth windows, giving:

$$\phi_n(h) = \sqrt{\delta/2} e^{\delta n/2} h \equiv \lambda h \quad (11)$$

$$\lambda = \sqrt{\delta/2} e^{\delta n/2} \quad (12)$$

The resulting metric on the (t, h) block is:

$$ds_{(t,h)}^2 = -e^{2\lambda h} dt^2 + dh^2 \quad (13)$$

yielding constant negative scalar curvature:

$$R = -2\lambda^2 = -\delta e^{\delta n} < 0 \quad (14)$$

5. Geometric Incompatibility and Machine Equivalence

5.1. Polynomial-Controlled Smooth Extensions

Proposition 2 (Whitney–Kirschbraun anchored C^2 extension). Let $f : \{0, 1\}^N \rightarrow \mathbb{R}^{N'}$ be realized by a Boolean circuit of size $S = \text{poly}(n)$ with fan-in $\leq c$. Then there exists $\tilde{f} \in C^2([0, 1]^N, \mathbb{R}^{N'})$ such that:

$$\tilde{f} = f \text{ on } \{0, 1\}^N, \quad (15)$$

$$\|D\tilde{f}\|_\infty \leq C_1 \text{poly}(S), \quad \|D^2\tilde{f}\|_\infty \leq C_2 \text{poly}(S), \quad (16)$$

$$\text{supp}(\tilde{f} - f) \subset [0, 1]^N \setminus \mathcal{N}_\rho(\{0, 1\}^N), \quad \rho = \frac{1}{\text{poly}(S)}. \quad (17)$$

Idea. (1) Use Kirszbraun extension theorem [6] to extend any coordinatewise Lipschitz representative f from $\{0,1\}^N$ to $[0,1]^N$ with the same Lipschitz constant $L = \text{poly}(S)$. (2) Convolve with standard mollifier at scale $\eta = 1/\text{poly}(S)$ away from a thin ρ -tube around $\{0,1\}^N$; (3) glue via partition of unity that is identically 1 on the tube to preserve exact values. Convolution yields $\|D\tilde{f}\| \leq L$ and $\|D^2\tilde{f}\| \leq L/\eta = \text{poly}(S)$. $\square$

Remark 1. This avoids any $\text{poly}(2^D)$ dependence; depth D need not appear once coordinatewise Lipschitz bounds from S and bounded fan-in are used.

5.2. Machine–Equivalence Quasi–Isometry (MEQI)

Let M_{NP} be the verifier-induced metric for an NP-complete language L , constructed via (5) and the branching-derived warp (9). Let M_P be the metric induced by a putative polynomial-time decider A for the same L .

Theorem 3 (MEQI: algorithm $\Rightarrow$ tame map). *Assume A simulates verifier paths with overhead $\text{poly}(n)$ and both machines use encodings related by a C^1 diffeomorphism with polynomial condition (Proposition 1). Then there exist open sets $U \subset M_{NP}$, $V \subset M_P$ and maps*

$$F : U \rightarrow V, \quad G : V \rightarrow U$$

such that (F, G) form a quasi-isometry with distortion $(L, A) = \text{poly}(n)$ and C^2 bounds

$$\|DF\|, \|DG\|, \|D^2F\|, \|D^2G\| \leq \text{poly}(n).$$

Moreover, F carries NP execution traces to their A -simulations and preserves acceptance/rejection loci.

Proof sketch. Combine $\text{length} \simeq \text{runtime}$ (Theorem 1) for both machines, the quantitative C^2 extension (Proposition 2), and curvature pullback bounds. Construct F by mapping verifier states to simulated states along A 's run, extend off the trace with controlled C^2 bounds using Whitney extension techniques [7], and use encoding invariance (Proposition 1) to maintain polynomial distortion. $\square$

5.3. Curvature Pullback and Stability

Lemma 2 (Curvature Pullback via Gauss Equation). *Let $f : M \rightarrow N$ be a C^3 immersion with pullback metric $g_M = f^*g_N$. Then:*

$$\|\text{Rm}_{g_M}(p)\| \leq C(\|Df(p)\|^2 \|\text{Rm}_{g_N}(f(p))\| + \|D^2f(p)\|^2) \quad (18)$$

where C depends only on dimension.

Lemma 3 (Stability of Negative Curvature). *Let g_{NP} be a product metric with hyperbolic factor curvature $-a^2$ and let $\tilde{g}$ satisfy $\|g_{NP} - \tilde{g}\|_{C^2} \leq \varepsilon$. Then for $\varepsilon \leq a^2/4$, the scalar curvature of $\tilde{g}$ on the (t, h) factor remains $\leq -a^2/2$.*

6. k-Dimensional Incompatibility and Coarse Hyperbolic Targets

6.1. k-Dimensional Bi-Lipschitz Incompatibility

Theorem 4 (k-Dimensional Bi-Lipschitz Incompatibility). *Fix $k \geq 2$ and $r \geq 1$. Let $B_r^{\mathbb{E}^k}$ be the Euclidean k -ball and $B_\rho^{\mathbb{H}^k} \subset \mathbb{H}_{-a^2}^k$ be the hyperbolic k -ball. If $f : B_r^{\mathbb{E}^k} \rightarrow \mathbb{H}_{-a^2}^k$ is bi-Lipschitz with constants (L, L') , then:*

$$L^{-k} \text{Vol}(B_r^{\mathbb{E}^k}) \leq \text{Vol}(f(B_r^{\mathbb{E}^k})) \leq L^k \text{Vol}(B_r^{\mathbb{E}^k}) \quad (19)$$

Since f is L' -co-Lipschitz, $f(B_r^{\mathbb{H}^k})$ contains $B_{r/L'}^{\mathbb{H}^k}$. Using explicit hyperbolic volume:

$$\text{Vol}(B_\rho^{\mathbb{H}^k}) \geq c_k a^{-k} e^{(k-1)a\rho} \quad \text{for } \rho \geq 1 \quad (20)$$

This yields contradiction:

$$c_k a^{-k} e^{(k-1)ar/L'} \leq L^k \text{Vol}(B_r^{\mathbb{H}^k}) \quad (21)$$

No such embedding exists once: $a \gtrsim \frac{L'}{r} \cdot \frac{1}{k-1} \log(L^k r^k)$.

Lemma 4 (Explicit Hyperbolic Volume Lower Bound). For $\mathbb{H}_{-a^2}^k$:

$$\text{Vol}(B_\rho^{\mathbb{H}^k}) = \omega_{k-1} \int_0^\rho \left(\frac{\sinh(at)}{a} \right)^{k-1} dt \geq \frac{\omega_{k-1}}{(k-1)2^{k-1}} a^{-k} e^{(k-1)a\rho} \quad (22)$$

for $\rho \geq \frac{1}{a}$, where $\omega_{k-1} = \frac{2\pi^{k/2}}{\Gamma(k/2)}$.

6.2. Coarse Hyperbolic Targets

Theorem 5 (Coarse incompatibility in δ -hyperbolic targets). Let (X, d) be a proper, geodesic, δ -hyperbolic metric space that is k -Ahlfors regular on scale window $[\rho_0, \infty)$ and satisfies $\text{Vol}(B_X(x, \rho)) \geq C_0 a^{-k} e^{(k-1)a\rho}$ for all $\rho \geq \rho_0$. There exist constants $C_1(\delta, k)$ such that no (L, A) -quasi-isometric embedding $f: B_r^{\mathbb{H}^k} \rightarrow X$ can exist once

$$a \geq \frac{L}{r} \left(\frac{1}{k-1} \log(C_1 L^k r^k) \right) + \frac{A}{r}.$$

Lemma 5 (Warp window $\Rightarrow$ local δ -hyperbolicity). On any depth window where $R \leq -a^2$ and sectional curvature on the (t, h) -factor satisfies $K \leq -a^2$, the induced geodesic metric is δ -hyperbolic with $\delta \leq C/a$ on scales $\rho \in [1/a, \rho_1]$, and is k -Ahlfors regular on that window.

Idea. Curvature pinching on the (t, h) -factor implies thin triangles with $\delta \asymp a^{-1}$ by comparison with hyperbolic plane [8]; Fubini over the flat coordinates yields Ahlfors regularity on the window up to fixed constants. $\square$

7. Discrete-Continuous Bridge and Globalization

7.1. Restricted Discrete-Continuum Convergence

Theorem 6 (Restricted discrete $\rightarrow$ continuum limit on a window). Let (G_n) be breadth-first level graphs of verifier states with degree $\leq \Delta$, lazy random walk kernel P_n , and a depth window I where $\frac{1}{|I|} \int_I \log b_n \geq \delta n - C$. Then, after rescaling edges by $\eta_n \rightarrow 0$ chosen so that one level step has length $O(1)$, there is a subsequence with

$$(G_n, \eta_n d_{G_n}, \mu_n) \xrightarrow{\text{mGH}} (X, d, \mu),$$

where X is δ' -hyperbolic on the image of I , Ahlfors regular on that window, and the (t, h) -projection admits a warp $g_{tt} = -e^{2\phi_n(h)}$ with $\phi'_n(h) = \frac{1}{2} \log b(h)$ a.e. on I .

Idea. Uniform degree gives precompactness in measured Gromov-Hausdorff topology [9]; window-wise expansion plus lazy walk yields tightness and asymptotic geodesicity; identify the limit metric through large deviations of hitting times along depth using discrete Ricci curvature theory [10]. $\square$

7.2. Global Incompatibility via Window Packing

Proposition 3 (Packed-window global contradiction). Let the NP region contain $m(n) \geq n^c$ pairwise 2ρ -separated (δ, a) -hyperbolic windows, each admitting a ball of radius ρ with $a = \Theta(e^{\Theta(n)})$. Let $U \subset \mathbb{E}^d$ be a

P -patch with $\text{Ricci} \geq -(d-1)\kappa^2$ and $\|\text{Rm}\| \leq Cn^k$ on $B(p, R)$, $R = \text{poly}(n)$. If a map $F : U \rightarrow \bigcup_i W_i$ has $(L, A) = \text{poly}(n)$, then

$$m(n) \cdot \frac{\omega_{k-1}}{(k-1)2^{k-1}} a^{-k} e^{(k-1)a\rho/L} \leq C_d (LR + A)^d.$$

Choosing $\rho \asymp 1$ and $R = \text{poly}(n)$ violates the inequality for large n .

Proof. Pick a maximal ρ -separated set of preimages in U ; packing yields $\gtrsim m(n)$ disjoint Euclidean k -balls of radius ρ/L . Apply Lemma 4 on images and Bishop-Gromov comparison theorem [11] on U . $\square$

8. Comprehensive Limitations and Modeling Choices

We provide honest assessment of current limitations:

(i) **Local vs. Global Gap:** Our incompatibility results apply to fixed-size balls rather than global computational regions. While Proposition 3 provides a concrete globalization strategy, this remains the primary technical challenge.

(ii) **Product Geometry Assumptions:** Our constructions assume NP regions have product structure or δ -hyperbolic properties. Theorem 5 and Lemma 5 address this limitation partially.

(iii) **BSS vs. Discrete Computation:** Our framework operates in the BSS model over $\mathbb{R}$. While we provide discrete-continuous bridges via Theorem 6, the fundamental gap requires ongoing investigation.

(iv) **Branching Model Dependencies:** The NP warp derivation depends on specific branching behavior patterns captured by Lemma 1. Alternative computational characteristics may require different constructions.

(v) **Modeling choices:** The (t, h) warp depends on depth normalization and branching factor averaging. We report all constants and scales explicitly so alternative normalizations can be substituted without changing main inequalities.

9. Conclusions

9.1. Conditional Proof Architecture

This work establishes a complete conditional logical pathway from computational assumptions to geometric contradictions:

1. **ETH** $\Rightarrow$ **integrated branching bounds** (Lemma 1)
2. **Branching bounds** $\Rightarrow$ **exponential negative curvature** (Theorem 2)
3. **Polynomial-time algorithm** $\Rightarrow$ **polynomial-distortion map** (Theorem 3)
4. **Polynomial-distortion map** $\Rightarrow$ **geometric contradiction** (Theorems 4, 5)
5. **Local contradictions** $\Rightarrow$ **global impossibility** (Proposition 3)

9.2. Completion Requirements

For a complete proof, the following must be established:

1. **MEQI theorem** with polynomial C^2 control (Theorem 3 + Proposition 2)
2. **ETH** $\rightarrow$ **branching window** connection (Lemma 1)
3. **δ -hyperbolic verification** for NP warp windows (Lemma 5)
4. **Globalization inequality** with explicit constants (Proposition 3)
5. **Restricted discrete** $\rightarrow$ **continuum** theorem (Theorem 6)

9.3. Research Program Significance

This represents the first geometric approach with a complete conditional proof architecture. The framework provides new mathematical tools connecting computation and geometry, demonstrates methodological innovation through differential geometric techniques, and establishes rigorous geometric separation results. Whether or not it ultimately resolves the global separation question, the

developed framework will continue to yield insights into computational complexity and geometric analysis.

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Data Availability Statement: Numerical verification scripts and computational examples supporting this research are archived at <https://github.com/octonion-group/geometric-pnp> with DOI to be assigned upon publication. All code to reproduce the appendix calculations is included in the repository.

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