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Not peer-reviewed version

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Posted Date: 23 January 2026

doi: 10.20944/preprints202601.0883.v2

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Article

Emergent Einstein–Friedmann Dynamics from Universal Wavefunction Geometry

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Abstract

We develop a geometric framework in which an effective spacetime description and an Einstein–Friedmann-type structure emerge from the geometry of a universal wavefunction, without postulating gravitational field equations or introducing matter fields as independent degrees of freedom. Starting from a conserved current associated with the wavefunction, we define a flux hypersurface embedded in a higher-dimensional ambient space and show that, under minimal assumptions of homogeneity and isotropy, its induced Lorentzian geometry is necessarily of Friedmann–Robertson–Walker type. The intrinsic curvature of the induced metric is fixed by the embedding geometry itself. A maximally symmetric hyperboloid corresponds to exact de Sitter spacetime, while more general, physically admissible and normalisable wavefunction envelopes give rise to a time-dependent curvature scale. In this case, the effective cosmological term is approximately constant only in a narrow intermediate regime, where the expansion is transiently quasi–de Sitter, and evolves away from this limit at both early and late times. By identifying a conserved, potential-like geometric invariant inherited from the universal wavefunction, we obtain an effective Einstein–Friedmann structure on the hypersurface without invoking gravitational dynamics. This invariant fixes the scaling of the dominant effective density $\rho_{\text{eff}} \propto a^{-2}$ and determines the effective gravitational coupling. For closed spatial slicing, this matter-like contribution cancels identically against the spatial curvature term in the Friedmann equation, leaving a purely geometric constraint relating the Hubble rate to a residual, time-dependent vacuum-like sector. We show that the apparent tension between a de Sitter-like Friedmann constraint and a nonvanishing \dot{H} is resolved once the effective continuity equation is taken into account: the expansion rate is fixed algebraically at each instant, while its time evolution is governed by the slow variation of the effective cosmological term. As a result, the cosmological evolution exhibits three distinct regimes: a strongly non–de Sitter early-time phase, a transient quasi–de Sitter regime, and an asymptotically coasting late-time expansion with $w \rightarrow -1/3$ emerging dynamically as an attractor. These results position general relativity as an effective geometric description arising from a deeper, wavefunction-based structure, in which spacetime curvature, expansion, and cosmological dynamics are emergent properties of the underlying quantum geometry.

Keywords: emergent gravity; cosmology; friedmann–robertson–walker spacetime; de sitter space; universal wavefunction; hypersurface geometry; spacetime emergence; einstein–friedmann equations; cosmological constant

1. Introduction

Contemporary cosmology is commonly formulated by postulating spacetime dynamics through Einstein's field equations and supplementing them with phenomenological matter components [1,2]. While this framework is highly successful observationally, it leaves open the question of whether spacetime geometry and gravitational dynamics are fundamental or instead emerge from more primitive structures. In particular, the origin of the cosmological expansion law, the physical status of vacuum

energy, the hierarchy between early- and late-time expansion regimes, and the role of spatial curvature are typically addressed at the level of effective field equations rather than derived from underlying geometric or conservation principles. This motivates the exploration of frameworks in which spacetime geometry and its effective dynamics arise from conserved geometric structures, without assuming gravitational field equations or matter content at the outset.

An optional perspective is to regard spacetime itself as an emergent structure, arising from more fundamental geometric or wavefunction-based entities [3,4]. In such a viewpoint, gravitational dynamics need not be postulated *a priori*, but may instead appear as effective relations governing the geometry of certain distinguished hypersurfaces. This approach shifts the focus from dynamical field equations to conserved quantities and geometric identities associated with an underlying universal wavefunction.

In a variety of approaches to quantum foundations and cosmology, the wavefunction is treated not merely as a computational tool, but as a carrier of global geometric and conservation properties [5,6]. In this context, it is natural to consider whether a suitably defined universal wavefunction may admit conserved currents whose geometric structure can be analysed independently of any specific dynamical interpretation. Rather than assigning direct physical reality to the wavefunction itself, we focus on the geometric properties of the flux hypersurfaces associated with such conserved currents, and examine the spacetime structure induced on these hypersurfaces. This allows spacetime geometry to be studied as an emergent, effective concept, rooted in conservation laws and embedding geometry rather than in postulated gravitational dynamics.

In this work, we adopt this perspective by considering the flux hypersurface associated with a conserved current of a universal wavefunction [6]. Treating this hypersurface as an embedded Lorentzian manifold, we show that its intrinsic geometry is determined entirely by the embedding. Under minimal assumptions of homogeneity and isotropy, the induced metric necessarily takes the Friedmann–Robertson–Walker form, with the hypersurface radius playing the role of an effective scale factor. No spacetime dynamics or gravitational field equations are assumed at this stage.

We demonstrate that the intrinsic curvature of the induced geometry, including its Einstein tensor and scalar curvature, follows directly from the embedding geometry. In particular, maximally symmetric constant-curvature embeddings correspond to de Sitter spacetime, while more general embeddings give rise to time-dependent curvature scales. Importantly, the resulting Einstein–Friedmann relations arise as geometric identities rather than as postulated dynamical equations.

A central result of the present framework is the identification of a conserved, potential-like invariant inherited from the universal wavefunction. This invariant fixes the scaling of the dominant effective density and determines the effective gravitational coupling as a geometric conversion factor. Gravity thus emerges not as an independent interaction, but as a consequence of the underlying conservation law and hypersurface geometry.

In this context, recent analyses of large-scale structure, supernova, and baryon acoustic oscillation data have reported mild but persistent indications that the effective dark energy component may deviate from a strictly constant cosmological term, motivating renewed interest in frameworks allowing for time-dependent curvature contributions [7–10]. In the construction developed here, such behaviour is not introduced phenomenologically, but arises as a direct consequence of the geometric evolution of the embedded flux hypersurface, leading to an effective cosmological term that naturally evolves in time. In this sense, a time-dependent effective cosmological term offers a natural way to reinterpret the vacuum energy problem geometrically, by decoupling the observed large-scale curvature from microscopic energy scales.

The resulting cosmological evolution exhibits distinct geometric regimes. At early times, the effective curvature behaves approximately as a constant, leading to de Sitter-like expansion. At intermediate stages, the interplay between curvature and the conserved density governs the evolution. For closed slicing, which is intrinsic to the hypersurface construction, the curvature and density contributions cancel identically in the Friedmann equation, leaving a residual vacuum-like sector that

drives a linear expansion at late times. These regimes emerge directly from the wavefunction geometry and do not rely on imposed equations of state.

Within this framework, the emergence of an effective Einstein–Friedmann description, the scaling of the dominant density component, and the time dependence of the effective cosmological term are all fixed by a single conserved geometric invariant inherited from the universal wavefunction.

The structure of this paper is as follows. In Section 2, we introduce the geometric framework of the embedded flux hypersurface associated with a conserved wavefunction current and derive the induced metric. Section 3 analyses the intrinsic curvature of the induced geometry and establishes the general Friedmann–Robertson–Walker form, including the constant-curvature (de Sitter) limit. In Section 4, we formulate an effective Einstein–Friedmann structure on the hypersurface and derive the associated conservation laws as geometric identities. Section 5 connects these geometric results to a conserved, potential-like wavefunction invariant, clarifying the origin of the effective gravitational coupling and the characteristic scaling of the effective density. In Section 6, we derive the effective acceleration, continuity, and Friedmann equations governing the intrinsic dynamics of the hypersurface. Section 7 provides a detailed analysis of the resulting geometric expansion across different asymptotic regimes, including early-time behaviour, curvature–density interplay, and the emergence of a linear expansion law at late times. Finally, in the Discussion, we summarise the main findings, outline the domain of validity of the effective spacetime description, and discuss implications for the emergence of general relativity.

2. Embedded Hypersurface and Induced Geometry

In this section, we introduce the geometric description of the evolving hypersurface associated with the conserved wavefunction current and derive the intrinsic geometry induced on it by the embedding. The discussion is formulated in a general and model-independent manner, making explicit only the assumptions required for homogeneity and isotropy.

2.1. Euclidean and Lorentzian Hyperboloid Embeddings: A Conceptual Distinction

Before introducing the explicit embedding geometry, it is useful to clarify the geometric status of the hypersurfaces considered in this work and their relation to familiar constant-curvature spacetimes. In particular, it is important to distinguish between generic hyperboloids of one sheet and the special class of hyperboloids that correspond to de Sitter spacetime.

In the geometric construction underlying the universal wavefunction framework, the flow of a conserved current generates a family of hypersurfaces that are hyperboloid-like in shape. These hypersurfaces are analogous to the stream surfaces formed by bundles of Poynting vectors of coherent waves with finite energy in Gaussian optics, where energy flow defines a set of axially symmetric hyperboloids of one sheet. Such hyperboloids are naturally embedded in a space with positive-definite (Euclidean) metric and, in general, do not possess constant intrinsic curvature. Their curvature varies along the surface and reflects the underlying transport dynamics rather than maximal symmetry.

By contrast, de Sitter spacetime corresponds to a very special geometric construction: a hyperboloid embedded in a flat ambient space with Lorentzian signature and fixed Minkowski radius. In this case, the embedding constraint defines a maximally symmetric manifold with constant intrinsic curvature. The constancy of curvature is therefore not a generic property of hyperboloids, but a consequence of both the Lorentzian signature of the embedding space and the invariance of the embedding radius.

The hypersurfaces considered in the present framework interpolate naturally between these cases. When the effective embedding radius is approximately constant, the induced geometry is de Sitter-like. More generally, when the embedding radius evolves in time—as dictated by the wavefunction envelope—the hypersurface remains hyperboloid-like but exhibits time-dependent curvature. This distinction plays a central role in the emergence of an effective, time-dependent cosmological term discussed in subsequent sections.

The hypersurfaces considered in the present framework interpolate naturally between these cases. When the effective embedding radius is approximately constant, the induced geometry approaches the maximally symmetric de Sitter limit. However, such maximally symmetric configurations are intrinsically non-normalisable, reflecting the well-known fact that fully symmetric solutions of wave equations in unbounded spaces generally carry infinite total weight or energy. By contrast, when the hypersurface geometry is generated by a wavefunction envelope, the resulting hyperboloid-like surfaces necessarily break maximal symmetry. This symmetry reduction ensures finite normalisation of the associated wavefunction current and leads to physically admissible configurations with finite total flux. In this sense, the emergence of time-dependent curvature in the present framework is not a deficiency, but a direct consequence of enforcing normalisation and physical finiteness, with de Sitter geometry arising only as an idealised limiting case.

2.2. Ambient Space and Hypersurface Embedding

We consider a flat $(N + 2)$ -dimensional ambient space with coordinates X^A ($A = 0, 1, \dots, N + 1$) endowed with the metric

$$ds_{\text{amb}}^2 = dX_0^2 - \sum_{i=1}^{N+1} dX_i^2, \quad (1)$$

where one coordinate is time-like and the remaining $N + 1$ coordinates are space-like. This ambient space serves purely as a geometric arena for the embedding and is not assumed to have direct physical significance.

The evolving hypersurface \mathcal{H} is realised as a codimension-one Lorentzian submanifold embedded in this ambient space. Motivated by the symmetry properties of the critical hypersurface discussed in [6], we restrict attention to embeddings that preserve homogeneity and isotropy on spatial sections. A closed-slicing parameterisation of such an embedding is

$$X_0 = F(\tau), \quad (2)$$

$$X_i = a(\tau) n_i(\Omega), \quad i = 1, \dots, N + 1, \quad (3)$$

where τ is a monotonically increasing parameter along the hypersurface, $a(\tau)$ is a positive function interpreted as the intrinsic radius of spatial sections, and $n_i(\Omega)$ are coordinates on the unit N -sphere satisfying

$$\sum_{i=1}^{N+1} n_i^2 = 1. \quad (4)$$

The angular variables Ω collectively denote the coordinates on S^N .

This embedding describes a family of $(N + 1)$ -dimensional hypersurfaces whose spatial cross-sections are N -spheres of radius $a(\tau)$. For generic functions $F(\tau)$ and $a(\tau)$, the resulting hypersurface is hyperboloid-like in the ambient space, reducing to the standard de Sitter hyperboloid when the curvature scale is constant [11].

2.3. Induced metric

The intrinsic geometry on \mathcal{H} is obtained by pulling back the ambient metric (1) using the embedding (2)–(3). The tangent vectors to the hypersurface are given by

$$\partial_\tau X^A = (\dot{F}, \dot{a} n_i), \quad (5)$$

$$\partial_\alpha X^A = (0, a \partial_\alpha n_i), \quad (6)$$

where a dot denotes differentiation with respect to τ and α labels angular coordinates on S^N .

Using the identities

$$\sum_{i=1}^{N+1} n_i \partial_\alpha n_i = 0, \quad \sum_{i=1}^{N+1} \partial_\alpha n_i \partial_\beta n_i = \gamma_{\alpha\beta}, \quad (7)$$

where $\gamma_{\alpha\beta}$ is the metric on the unit N -sphere.

The induced line element on \mathcal{H} becomes

$$ds^2 = c^2 \left(\dot{F}^2 - \dot{a}^2 \right) d\tau^2 - a^2(\tau) d\Omega_N^2, \quad (8)$$

with $d\Omega_N^2 = \gamma_{\alpha\beta} d\Omega^\alpha d\Omega^\beta$ denotes the metric on the unit N -sphere, and the lapse factor

$$\mathcal{L}(\tau) \equiv \sqrt{\dot{F}^2(\tau) - \dot{a}^2(\tau)}. \quad (9)$$

The constant c denotes the speed of light in vacuum and is introduced to ensure that the induced Lorentzian metric has the correct physical dimensions in SI.

To express the metric in standard cosmological form, we introduce a proper time coordinate t defined by

$$dt = \mathcal{L}(\tau) d\tau, \quad (10)$$

which is well defined, provided $\dot{F}^2 > \dot{a}^2$.

In terms of t , the induced metric takes the standard Friedmann–Robertson–Walker form [2]:

$$ds^2 = c^2 dt^2 - a^2(t) d\Omega_N^2, \quad (11)$$

regardless of the detailed expansion law, as long as the embedding is isotropic in the spatial directions.

Thus, without assuming any gravitational dynamics, the induced geometry on the embedded hypersurface is locally FRW, with the hypersurface radius $a(t)$ playing the role of the cosmological scale factor.

It is important to emphasise that the parameter τ introduced in the embedding parametrisation (2)–(3) does not represent physical time. Its role is purely geometric, ordering the evolution of the embedding functions $F(\tau)$ and $a(\tau)$. Physical time emerges only after the induced Lorentzian metric on the hypersurface is identified. In particular, the proper time t measured by comoving observers is related to τ through the lapse factor $dt = \mathcal{L}(\tau) d\tau$. Once this identification is made, the induced metric takes Friedmann–Robertson–Walker form and t acquires the interpretation of effective cosmological time. Similar is related to the function $a(\tau)$ in (3) that represents a geometric radius of the hypersurface rather than an ordinary cosmological scale factor. At that stage of the construction, no intrinsic spacetime metric or physical notion of cosmic time has yet been introduced. It is also worth noting that the embedding function $F(\tau)$ does not appear explicitly in the intrinsic spacetime description once the induced metric is written in Friedmann–Robertson–Walker form. This is a direct consequence of the definition of proper time on the hypersurface. The function $F(\tau)$ enters the induced metric only through the combination $\dot{F}^2 - \dot{a}^2$, which defines the lapse factor relating the geometric parameter τ to the emergent proper time t . After the identification (10), the intrinsic geometry is fully characterised by the scale factor $a(t)$, while $F(\tau)$ plays no independent role in the effective spacetime dynamics.

Once the induced Lorentzian metric on the hypersurface is identified and τ is related to the emergent proper time t , the function $a(t)$ may be reinterpreted as an effective cosmological scale factor. If desired, a dimensionless, normalised scale factor defined as a dimensionless ratio of physical distances at different times $\tilde{a}(t) = a(t)/a(t_0)$ can then be introduced without altering the underlying dynamics, and therefore normalisation conditions such as $\tilde{a}(t_0) = 1$ or $\tilde{a}(0) = 0$ can be imposed. In this sense, the absence of standard normalisation conditions at the geometric level is not a deficiency, but a reflection of the fact that cosmological interpretation arises only after the spacetime structure has emerged.

2.4. Hyperboloid Geometry and Curvature Scale

The embedding (2)–(3) implies that the hypersurface satisfies the relation

$$X_0^2 - \sum_{i=1}^{N+1} X_i^2 = F^2(\tau) - a^2(\tau). \quad (12)$$

If the right-hand side is constant, the hypersurface is an exact hyperboloid of constant curvature, corresponding to de Sitter spacetime in closed slicing [2,11]. More generally, the quantity $F^2(\tau) - a^2(\tau)$ defines an effective curvature scale that may evolve with time, leading to a family of hyperboloid-like geometries with time-dependent curvature.

This observation provides an embedding-induced origin for the hyperboloid-like spacetimes within the present framework. Constant intrinsic curvature corresponds to a special case of a maximally symmetric embedding, realised when the embedding radius is strictly invariant. More generally, when the hypersurface evolution is governed by a wavefunction envelope, maximal symmetry is necessarily broken in order to ensure finite normalisation of the associated current. In this case, the resulting hypersurface remains hyperboloid-like but exhibits time-dependent curvature, as realised by the critical hypersurface analysed in the preceding study.

In the following sections, we analyse how the intrinsic curvature of the induced metric (11) is determined by this embedding geometry and how an effective Einstein–Friedmann structure emerges on the hypersurface.

3. Intrinsic and Extrinsic Curvature of the Hypersurface

In this section, we analyse the curvature properties of the embedded hypersurface introduced in Section 2. We show that the intrinsic curvature of the induced Friedmann–Robertson–Walker (FRW) geometry is completely determined by the embedding and, in particular, that de Sitter spacetime arises as the constant-curvature limit of a hyperboloid embedding. This analysis provides the geometric foundation for the emergence of an effective cosmological term.

3.1. Intrinsic Curvature of the Induced FRW Geometry

The induced metric on the hypersurface \mathcal{H} (11) has the standard FRW form with closed spatial sections. Defining the Hubble parameter as

$$H(t) \equiv \frac{\dot{a}}{a}, \quad (13)$$

the nonvanishing components of the Einstein tensor for an $(N + 1)$ -dimensional FRW spacetime are

$$G_{tt} = \frac{N(N-1)}{2c^2} \left(H^2 + \frac{c^2}{a^2} \right), \quad (14)$$

$$G_{ij} = -\frac{1}{c^2} \left[(N-1) \frac{\ddot{a}}{a} + \frac{(N-1)(N-2)}{2} \left(H^2 + \frac{c^2}{a^2} \right) \right] g_{ij}, \quad (15)$$

where g_{ij} denotes the spatial components of the spacetime metric. These expressions are embedding-induced identities following from the form of the induced metric and do not assume any dynamical field equations.

The scalar curvature associated with (11) is

$$R = \frac{N}{c^2} \left[2 \frac{\ddot{a}}{a} + (N-1) \left(H^2 + \frac{c^2}{a^2} \right) \right]. \quad (16)$$

In the special case where the scalar curvature is constant, $R = \text{const}$, the induced spacetime is maximally symmetric and corresponds to de Sitter geometry in closed slicing. The appearance of de

Sitter geometry is therefore not imposed but reflects the maximal symmetry of constant-curvature embeddings.

3.2. Extrinsic Curvature and Embedding Geometry

To relate the intrinsic curvature to the embedding, we introduce the unit normal vector N^A to the hypersurface in the ambient space. For the embedding defined in Section 2, a convenient choice of normal vector is

$$N^A = \left(\frac{\dot{a}}{\mathcal{L}'}, \frac{\dot{F}}{\mathcal{L}} n_i \right), \quad (17)$$

which satisfies

$$\eta_{AB} N^A N^B = -1, \quad \eta_{AB} N^A \partial_\mu X^B = 0. \quad (18)$$

The extrinsic curvature tensor of the hypersurface is defined as

$$K_{\mu\nu} = -\eta_{AB} N^A \partial_\mu \partial_\nu X^B. \quad (19)$$

For the embedding (2)–(3), the nonvanishing components of $K_{\mu\nu}$ are isotropic and take the form

$$K_{tt} = -\frac{1}{c^2} \frac{\dot{a}\ddot{F} + \dot{F}\ddot{a}}{\mathcal{L}}, \quad (20)$$

$$K_{ij} = -\frac{\dot{F}}{a\mathcal{L}} g_{ij}. \quad (21)$$

The isotropy of K_{ij} reflects the assumed symmetry of the embedding and ensures that the induced intrinsic geometry remains homogeneous and isotropic.

3.3. Gauss Relation and the Origin of De Sitter Geometry

For a codimension-one hypersurface embedded in a flat ambient space, the Gauss relation expresses the intrinsic Riemann tensor entirely in terms of the extrinsic curvature:

$$R_{\mu\nu\rho\sigma} = K_{\mu\rho} K_{\nu\sigma} - K_{\mu\sigma} K_{\nu\rho}. \quad (22)$$

Contracting indices yield the scalar curvature

$$R = K^2 - K_{\mu\nu} K^{\mu\nu}, \quad (23)$$

where $K = g^{\mu\nu} K_{\mu\nu}$ is the trace of the extrinsic curvature.

When the embedding satisfies

$$F^2(\tau) - a^2(\tau) = L^2 = \text{const}, \quad (24)$$

the hypersurface is an exact hyperboloid of constant curvature radius L . In this case, the extrinsic curvature components are constant in time, and the intrinsic curvature reduces to

$$R = \frac{N(N+1)}{L^2}, \quad (25)$$

corresponding to de Sitter spacetime [2,11].

More generally, when the quantity $F^2(\tau) - a^2(\tau)$ varies slowly with time, the hypersurface geometry is de Sitter-like but with a time-dependent curvature scale. The intrinsic curvature (16) is then determined by the evolution of the embedding functions $F(\tau)$ and $a(\tau)$, providing a geometric origin for an effective, time-dependent vacuum-like curvature term.

In the next section, we show how this intrinsic curvature may be interpreted in Einstein–Friedmann form and how an effective cosmological term naturally emerges from the geometry of the embedded hypersurface.

4. Emergence of an Effective Einstein–Friedmann Structure

Having established that the induced geometry on the embedded hypersurface \mathcal{H} is of Friedmann–Robertson–Walker type and that its intrinsic curvature is determined entirely by the embedding, we now show how an effective Einstein–Friedmann structure arises naturally. Importantly, this structure is not postulated as a dynamical law but follows from geometric identities once the induced curvature is reinterpreted in a form familiar from relativistic cosmology.

4.1. Geometric Definition of an Effective Cosmological Term

The intrinsic Einstein tensor associated with the induced metric (11) is given by Eqs. (14) and (15). Since these expressions are purely geometric, they may be decomposed without reference to any matter content or gravitational field equations.

A convenient and invariant way to identify a vacuum-like contribution is to extract the part of the Einstein tensor proportional to the metric. We therefore define an effective cosmological term $\Lambda_{\text{eff}}(t)$ by

$$\Lambda_{\text{eff}}(t) \equiv -\frac{1}{N+1} G^{\mu}{}_{\mu}, \quad (26)$$

where $G^{\mu}{}_{\mu}$ denotes the trace of the Einstein tensor constructed from the induced metric.

Using Eq. (16), this definition yields

$$\Lambda_{\text{eff}}(t) = \frac{N}{(N+1)c^2} \left[2\frac{\ddot{a}}{a} + (N-1) \left(H^2 + \frac{c^2}{a^2} \right) \right]. \quad (27)$$

For a constant-curvature embedding, Λ_{eff} is constant, and the induced geometry corresponds to de Sitter spacetime. More generally, $\Lambda_{\text{eff}}(t)$ inherits its time dependence from the evolution of the embedding geometry.

4.2. Residual Curvature and Effective Stress-Energy Tensor

Having identified the vacuum-like contribution, we define the residual Einstein tensor by

$$\tilde{G}_{\mu\nu} \equiv G_{\mu\nu} + \Lambda_{\text{eff}}(t) g_{\mu\nu}, \quad (28)$$

which is traceless by construction,

$$\tilde{G}^{\mu}{}_{\mu} = 0. \quad (29)$$

Introducing an effective gravitational coupling G , we define an effective stress-energy tensor on the hypersurface as

$$T_{\mu\nu}^{\text{eff}} \equiv \frac{c^2}{8\pi G} \tilde{G}_{\mu\nu}. \quad (30)$$

With this definition, the intrinsic curvature identities on \mathcal{H} may be written in the Einstein-like form

$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}^{\text{eff}} - \Lambda_{\text{eff}}(t) g_{\mu\nu}. \quad (31)$$

Importantly, (31) is not postulated as a dynamical field equation but follows identically from the induced geometry of the hypersurface; no variational principle or gravitational action is assumed. This equation serves as a convenient parametrization of the intrinsic geometry in a form directly comparable with standard cosmology.

4.3. Continuity Equation from Geometric Identities

The Einstein tensor satisfies the contracted Bianchi identity,

$$\nabla^\mu G_{\mu\nu} = 0, \quad (32)$$

which holds identically for any metric. Applying Eq. (32) to the decomposition (31) yields

$$\nabla^\mu T_{\mu\nu}^{\text{eff}} = \frac{c^2}{8\pi G} \partial_\nu \Lambda_{\text{eff}}(t). \quad (33)$$

Thus, when Λ_{eff} is constant, the effective stress-energy tensor is locally conserved. When $\Lambda_{\text{eff}}(t)$ varies in time, the residual sector exchanges energy with the vacuum-like geometric component in a manner fixed entirely by the geometry [12,13].

Specializing to the FRW form of the induced metric and assuming isotropy, the effective stress-energy tensor may be written as

$$T^{\text{eff}\mu}{}_\nu = \text{diag} \left(c^2 \rho_{\text{eff}}, -p_{\text{eff}}, -p_{\text{eff}}, \dots \right). \quad (34)$$

The temporal component of Eq. (33) then yields the continuity equation

$$\dot{\rho}_{\text{eff}} + NH \left(\rho_{\text{eff}} + \frac{p_{\text{eff}}}{c^2} \right) = \frac{c^2}{8\pi G} \dot{\Lambda}_{\text{eff}}(t). \quad (35)$$

which reduces to the standard conservation law in the case of constant Λ_{eff} .

In the present formulation, the Einstein tensor is decomposed into a traceless part and a trace part according to Eqs. (28) and (29). The effective stress-energy tensor $T_{\mu\nu}^{\text{eff}}$ is defined from $\tilde{G}_{\mu\nu}$ and therefore describes only the traceless sector of the geometric decomposition. Consequently, the quantities ρ_{eff} and p_{eff} should not be identified with the total physical density and pressure. Thus, Equation (35) represents an exchange relation between the traceless effective sector and the trace component encoded in $\Lambda_{\text{eff}}(t)$. It does not signal a violation of local energy-momentum conservation, but rather reflects the fact that the time-dependent trace term has been separated from $T_{\mu\nu}^{\text{eff}}$ by construction.

Before proceeding, we emphasise that local covariant conservation of the Einstein tensor, $\nabla_\mu G^{\mu\nu} = 0$, holds identically as a consequence of the Bianchi identities. Accordingly, any effective stress-energy tensor introduced through a geometric decomposition inherits local conservation in the appropriate form. The continuity relation derived below therefore reflects a redistribution between different geometric sectors and does not represent a violation of local energy-momentum conservation.

4.4. Effective Friedmann Equations

Finally, the time-time component of Eq. (31) yields an effective Friedmann relation,

$$\frac{N(N-1)}{2c^2} \left(H^2 + \frac{c^2}{a^2} \right) = 8\pi G \rho_{\text{eff}} + \Lambda_{\text{eff}}(t). \quad (36)$$

From the spatial components of the Einstein tensor, one may derive the corresponding acceleration equation for the hypersurface scale factor. Using the (ij) components of the Einstein equations for an $(N+1)$ -dimensional Friedmann-Robertson-Walker geometry and eliminating the geometric combination $H^2 + a^{-2}$ with the aid of Eq. (36), one obtains

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{(N-1)} \left[\frac{(N-2)}{N} \rho_{\text{eff}} + \frac{p_{\text{eff}}}{c^2} \right] + \frac{2}{N(N-1)} \Lambda_{\text{eff}}(t). \quad (37)$$

This relation is identity-level (geometric) in origin and follows directly from the Einstein tensor identities. The effective energy density, pressure, and cosmological term enter only through the decomposition introduced in the preceding subsection.

The trace term $\Lambda_{\text{eff}}(t)$ can be equivalently represented as an effective vacuum component with

$$\rho_{\Lambda}(t) = \frac{\Lambda_{\text{eff}}(t)c^2}{8\pi G}, \quad p_{\Lambda}(t)/c^2 = -\rho_{\Lambda}(t). \quad (38)$$

Accordingly, we define the total effective density and pressure as

$$\rho_{\text{tot}} \equiv \rho_{\text{eff}} + \rho_{\Lambda}, \quad p_{\text{tot}} \equiv p_{\text{eff}} + p_{\Lambda}.$$

For the physically relevant case $N = 3$, Eqs. (35), (36) and (37) reduce to

$$\dot{\rho}_{\text{eff}} + 3H\left(\rho_{\text{eff}} + \frac{p_{\text{eff}}}{c^2}\right) = -\dot{\rho}_{\Lambda} = -\frac{c^2}{8\pi G} \dot{\Lambda}_{\text{eff}}(t). \quad (39)$$

$$\frac{3}{c^2} \left(H^2 + \frac{c^2}{a^2} \right) = 8\pi G \rho_{\text{eff}} + \Lambda_{\text{eff}}(t). \quad (40)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho_{\text{eff}} + \frac{3p_{\text{eff}}}{c^2} \right) + \frac{c^2}{3} \Lambda_{\text{eff}}(t). \quad (41)$$

Equations (40) and (41) should therefore be understood as a Friedmann-type system. In this formulation, the traceless effective variables $(\rho_{\text{eff}}, p_{\text{eff}})$ evolve in the presence of a time-dependent trace curvature term $\Lambda_{\text{eff}}(t)$.

When rewritten in terms of the total quantities $(\rho_{\text{tot}}, p_{\text{tot}})$, the equations reduce to the standard Friedmann equations. Indeed, using Eq. (35) together with $p_{\Lambda} = -c^2\rho_{\Lambda}$, one finds

$$\dot{\rho}_{\text{tot}} + 3H\left(\rho_{\text{tot}} + \frac{p_{\text{tot}}}{c^2}\right) = 0,$$

which is the standard FRW continuity equation for the total effective stress–energy tensor. In the same way, with the definitions Eqs. (38), the dynamical equations may be cast into the conventional FRW form [1]: These expressions are algebraically equivalent to Eqs. (40) and (41). They involve no additional assumptions. Thus, the non-standard form of Eqs. (35), (40) and (41) arises solely from the chosen decomposition and does not indicate any breakdown of local conservation laws.

Together Eqs. (35), Eqs. (40) and (41) provide a complete and effective cosmological description of the intrinsic hypersurface geometry in the present framework. They demonstrate that, once the induced geometry is interpreted in Einstein–Friedmann form, local conservation laws and effective cosmological dynamics follow as geometric identities.

In the next section, we connect these general results to the conserved wavefunction invariant derived in [6] and show how the effective gravitational coupling and late-time expansion law are fixed by the underlying wavefunction geometry.

5. Wavefunction Invariant And Determination Of The Effective Gravitational Coupling

In this section, we connect the effective Einstein–Friedmann structure derived in Section 4 to the conserved geometric invariant obtained from the universal wavefunction in [6]. We show that this invariant uniquely fixes the scaling of the effective energy density on the hypersurface, determines the value of the effective gravitational coupling, and leads naturally to asymptotically linear expansion.

5.1. Surface-type conservation law and density scaling

As demonstrated in [6], the evolution of the critical hypersurface is governed by a conserved quantity associated with the scalar envelope of the universal wavefunction. The corresponding conservation law is expressed as an integral over the two-dimensional boundary of the homogeneous three-ball,

$$\oint_S \rho_{\text{eff}} dS = \rho_{\text{eff}} S = \text{const}, \quad (42)$$

where dS represents a 2-sphere area element over the t -slice intersection and ρ_{eff} is the homogeneous density of the 3-ball restricted by the sphere at fixed time. This may equivalently be written in volumetric form as

$$\frac{\rho_{\text{eff}}V}{a} = \text{const}, \quad (43)$$

where the cosmological scale factor $a(t)$ denotes the hypersurface radius and $V = (4/3)\pi a^3$ is the enclosed volume.

Defining the effective mass–energy content of the hypersurface as $m = \rho_{\text{eff}}V$, Eq. (43) implies

$$\frac{m}{a} = \text{const}, \quad (44)$$

that the effective mass scales linearly with the hypersurface radius $m \propto a$. Consequently, the effective density scales as

$$\rho_{\text{eff}}(a) \propto a^{-2}. \quad (45)$$

This scaling is fundamentally different from that of standard matter or radiation components. It reflects the surface-type nature of the conserved wavefunction invariant.

Combining the geometric scaling $\rho_{\text{eff}} \propto a^{-2}$ with Eq. (35) yields

$$H\left(\rho_{\text{eff}} + \frac{p_{\text{eff}}}{c^2}\right) = -\frac{c^2}{8\pi G}\dot{\Lambda}_{\text{eff}}(t).$$

Assuming $H \neq 0$, the effective pressure can be written in the explicit form

$$p_{\text{eff}} = -\frac{1}{3}\rho_{\text{eff}} - \frac{1}{24\pi G}\frac{\dot{\Lambda}_{\text{eff}}(t)}{H}.$$

Thus the curvature-like equation of state $p_{\text{eff}} = -\rho_{\text{eff}}/3$ emerges as a controlled approximation in regimes where the time variation of $\Lambda_{\text{eff}}(t)$ is slow compared to the expansion rate, i.e. $|\dot{H}| \ll H^2$. In this quasi-adiabatic regime the exchange between the traceless effective sector and the geometric trace term is subleading, and the effective sector approaches zero active gravitational mass without requiring $\dot{H} = 0$ exactly.

5.2. Potential-Like Invariant and Emergence of the Gravitational Constant

The conservation law (43) may be rewritten in the form

$$\frac{2\rho_{\text{eff}}V}{ac^2} = \frac{\phi}{c^2}, \quad (46)$$

where the integrand has the structure of a potential-like quantity ϕ . Using the relations between the characteristic length and mass scales derived in [6], this quantity may be identified with the gravitational coupling constant G ,

$$\frac{\phi}{c^2} = \frac{1}{G}. \quad (47)$$

Equivalently, the gravitational constant may be expressed as

$$G = \frac{c^2 a}{2V\rho_{\text{eff}}}. \quad (48)$$

In terms of the effective mass $m = \rho V$, Eq. (48) yields the dimensionless invariant

$$\frac{Gm}{ac^2} = \frac{1}{2}. \quad (49)$$

Thus, the effective gravitational coupling appearing in the Einstein–Friedmann form of the induced geometry is not introduced phenomenologically, but is fixed by the conserved wavefunction invariant.

In this sense, Newton's constant emerges as a geometric conversion factor between the invariant mass–radius ratio and a dimensionless potential-like quantity.

6. Effective Acceleration, Continuity, and Friedmann Dynamics

6.1. Effective Continuity Equation

Because ρ_{eff} and p_{eff} are defined from the traceless sector, they are not separately conserved when $\Lambda_{\text{eff}}(t)$ is time dependent. Instead one has

$$\dot{\rho}_{\text{eff}} + 3H\left(\rho_{\text{eff}} + \frac{p_{\text{eff}}}{c^2}\right) = -\frac{c^2}{8\pi G} \dot{\Lambda}_{\text{eff}}(t). \quad (50)$$

From the conserved geometric invariant associated with the critical hypersurface, the effective density scales as $\rho_{\text{eff}} \propto a^{-2}$, so that

$$\dot{\rho}_{\text{eff}} = -2H\rho_{\text{eff}}. \quad (51)$$

Substituting Eq. (51) into Eq. (50) yields the exact relation

$$H\left(\rho_{\text{eff}} + \frac{3p_{\text{eff}}}{c^2}\right) = -\frac{c^2}{8\pi G} \dot{\Lambda}_{\text{eff}}(t). \quad (52)$$

6.2. Effective Acceleration Equation

The effective acceleration of the scale factor is governed by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{\text{eff}} + \frac{3p_{\text{eff}}}{c^2}\right) + \frac{c^2}{3} \Lambda_{\text{eff}}(t), \quad (53)$$

which is an exact consequence of the geometric trace–traceless decomposition and contains no approximation. Using Eq. (52) to eliminate the bracket in the acceleration equation (53), we obtain

$$\frac{\ddot{a}}{a} = \frac{c^2}{3} \Lambda_{\text{eff}}(t) + \frac{1}{6} \frac{\dot{\Lambda}_{\text{eff}}(t)}{H}. \quad (54)$$

Since

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 \quad (55)$$

identically, this equation shows explicitly

$$\boxed{\dot{H} + H^2 = \frac{c^2}{3} \Lambda_{\text{eff}}(t) + \frac{1}{6} \frac{\dot{\Lambda}_{\text{eff}}(t)}{H}} \quad (56)$$

In the next subsection we show that the equation (54) is consisted from two formally independent parts.

6.3. Effective Friedmann Dynamics and Curvature Cancellation

In the derivation of the Einstein–Friedmann form in Section 4, the spatial geometry of the hypersurface was taken to be closed, corresponding to the choice of a unit N -sphere and thus to a curvature index $k = +1$. For the purpose of analysing the effective dynamics in a more general and transparent way, it is convenient to reintroduce the curvature parameter k explicitly in the Friedmann equation, treating it as a label of the constant-curvature spatial slicing rather than as an independent dynamical degree of freedom.

With this convention, the effective Friedmann equation (40) may be written in the standard form

$$\frac{3}{c^2} \left(H^2 + \frac{k c^2}{a^2} \right) = 8\pi G \rho_{\text{eff}} + \Lambda_{\text{eff}}(t). \quad (57)$$

Using Eq. (49), the matter-like contribution becomes

$$8\pi G \rho_{\text{eff}} = \frac{3}{a^2}, \quad (58)$$

so that

$$H^2 = \frac{(1-k)c^2}{a^2} + \frac{c^2}{3} \Lambda_{\text{eff}}(t). \quad (59)$$

This result reveals a remarkable structural property in the effective dynamics of the critical hypersurface associated with the universal wavefunction. For the density scaling $\rho_{\text{eff}} \propto a^{-2}$ inherited from the conserved geometric invariant, the matter-like contribution to the Friedmann equation has precisely the same a^{-2} dependence as the spatial curvature term. As a consequence, for the closed slicing ($k = +1$) inherent to the hypersurface description of the universal wavefunction, the matter-like contribution cancels the curvature term identically, leaving the constraint

$$\boxed{H^2 = \frac{c^2}{3} \Lambda_{\text{eff}}(t)}. \quad (60)$$

This cancellation follows directly from the geometric origin of ρ_{eff} and its fixed normalisation. In the present framework, the effective energy density is intrinsically tied to the curvature of the hypersurface rather than representing an independent dynamical matter component. Thus, Eq. (60) should be understood as a constraint following from the geometric normalisation of ρ_{eff} and the choice $k = +1$, rather than as an independent evolution equation for $H(t)$.

Differentiating Eq. (60) gives

$$\boxed{\dot{H} = \frac{c^2}{6} \frac{\dot{\Lambda}_{\text{eff}}(t)}{H}}. \quad (61)$$

which shows that the time derivative \dot{H} is totally controlled by the time variation of $\Lambda_{\text{eff}}(t)$. Moreover, it is fully consistent with the acceleration law Eq. (54). Indeed, applying Eq. (60) to Eq. (56) leads to the exactly similar to Eq. (61) result.

Together, Eqs. (60) and (61) demonstrate that the effective Friedmann description decomposes naturally into two complementary statements. Equation (60) acts as a geometric constraint that fixes the instantaneous value of H^2 in terms of the residual curvature term $\Lambda_{\text{eff}}(t)$, reflecting the cancellation between spatial curvature and the geometrically normalised matter-like contribution on the critical hypersurface. By contrast, Eq. (61) governs the actual time evolution of the expansion rate and shows that departures from exact de Sitter behaviour are entirely encoded in the time dependence of $\Lambda_{\text{eff}}(t)$.

In this way, the cosmological dynamics is not driven by an independent matter sector but by the evolution of the hypersurface geometry itself: the expansion rate is constrained algebraically by geometry at each instant, while its temporal variation is controlled by the slow evolution of the effective cosmological term. This separation resolves the apparent tension between a de Sitter-like Friedmann constraint and a nonvanishing \dot{H} , and makes explicit that the accelerated expansion in the present framework is a geometric effect rather than the consequence of a conserved vacuum energy.

7. Asymptotic Regimes of the Geometric Expansion

For the explicit evolution law obtained in [6],

$$a(t) = a_0 \sqrt{1 + \frac{t^2}{t_0^2}}, \quad (62)$$

the corresponding Hubble relations are

$$H(t) \equiv \frac{\dot{a}}{a} = \frac{t}{t^2 + t_0^2}, \quad \dot{H}(t) = \frac{t_0^2 - t^2}{(t^2 + t_0^2)^2}. \quad (63)$$

A convenient dimensionless measure of the relative importance of the two terms entering $\ddot{a}/a = \dot{H} + H^2$ is

$$\frac{\dot{H}}{H^2} = -1 + \frac{t_0^2}{t^2}. \quad (64)$$

7.1. Late-Time Coasting

At late times $t \gg t_0$, for the explicit solution (62), the expansion becomes asymptotically linear, $a(t) \propto t$, implying $\ddot{a} \rightarrow 0$ and hence $\rho_{\text{tot}} + 3p_{\text{tot}}/c^2 \rightarrow 0$:

$$H(t) \simeq \frac{1}{t}, \quad \dot{H}(t) \simeq -\frac{1}{t^2} = -H^2, \quad \frac{\ddot{a}}{a} = \dot{H} + H^2 \simeq 0, \quad (65)$$

In this asymptotically linear (coasting) expansion $a(t) \simeq (a_0/t_0)t$, the effective cosmological term decays as

$$\Lambda_{\text{eff}}(t) \propto (ct)^{-2}, \quad (66)$$

This implies that the late-time expansion is governed entirely by the residual vacuum-like sector encoded in $\Lambda_{\text{eff}}(t)$.

In standard FRW language this late-time behaviour is equivalent to an effective equation-of-state parameter approaching asymptotically the curvature-like value.

$$w \rightarrow -\frac{1}{3}, \quad (67)$$

since $a(t) \propto t^n$ implies $w = 2/(3n) - 1$ and here $n \rightarrow 1$ asymptotically.

It is important to emphasise that the curvature-like equation of state $w = -1/3$ emerges dynamically as a late-time attractor of the geometric evolution. This behaviour reflects the geometric origin of the effective matter sector and shows that the coasting regime arises naturally as an attractor of the hypersurface dynamics, rather than from the assumption of a conserved fluid with a fixed equation of state.

7.2. Quasi-De Sitter Regime

An intermediate early-time regime of particular interest occurs around $t \simeq t_0$, where the solution (62) comes closest to de Sitter-like behaviour. From Eq. (63), one finds

$$H(t_0) = \frac{1}{2t_0}, \quad \dot{H}(t_0) = 0,$$

so that at this instant the kinematic identity $\ddot{a}/a = \dot{H} + H^2$ reduces to $\ddot{a}/a = H^2$, formally identical to the de Sitter relation. In a neighbourhood of $t = t_0$, writing $t = t_0 + \delta$ with $|\delta| \ll t_0$, one obtains

$$\dot{H}(t) \simeq -\frac{\delta}{2t_0^3}, \quad \frac{\dot{H}}{H^2} \simeq 2\frac{\delta}{t_0},$$

showing that deviations from de Sitter behaviour grow linearly with the distance from t_0 . Consequently, the expansion is quasi-de Sitter only within a narrow temporal window $|t - t_0| \ll t_0$, outside of which the time dependence of H becomes significant. This transient quasi-de Sitter regime separates the strongly non-de Sitter early-time evolution from the asymptotically coasting late-time phase and does not correspond to an extended inflationary epoch.

7.3. Early-Time Behaviour

In the early-time regime $|t| \ll t_0$, we have $H \simeq t/t_0^2$ and $\dot{H} \simeq 1/t_0^2$, so that $|\dot{H}|/H^2 \simeq t_0^2/t^2 \gg 1$, and therefore \ddot{a}/a is controlled predominantly by \dot{H} rather than by H^2 . The early-time behaviour of

the solution (62) is markedly different from both the late-time coasting regime and exact de Sitter expansion. For $|t| \ll t_0$ the scale factor admits the expansion

$$a(t) \simeq a_0 \left(1 + \frac{t^2}{2t_0^2} \right),$$

so that

$$H(t) \simeq \frac{t}{t_0^2}, \quad \dot{H}(t) \simeq \frac{1}{t_0^2}.$$

In this regime the Hubble rate grows linearly from zero, while its time derivative remains approximately constant. As a result,

$$\frac{\dot{H}}{H^2} \simeq \frac{t_0^2}{t^2} \gg 1, \quad (|t| \ll t_0),$$

indicating that the dynamics is far from de Sitter, for which $\dot{H} = 0$. Nevertheless, the acceleration satisfies

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 \simeq \frac{1}{t_0^2},$$

so that the expansion is initially driven by an approximately constant acceleration. The evolution therefore starts from a time-symmetric, non-singular minimum of the scale factor, with vanishing H but finite \ddot{a}/a , rather than from an inflationary de Sitter phase. This distinguishes the present geometric construction from standard inflationary scenarios and highlights the intrinsically non-de Sitter character of the early-time regime.

These results demonstrate that the effective Einstein–Friedmann dynamics on the hypersurface are fully determined by the geometry of the universal wavefunction. The emergence of linear expansion at late times, the value of the effective gravitational coupling, and the decay of the vacuum-like term all follow from a single conserved surface-type invariant, without invoking additional matter components, fine-tuned equations of state, or a fundamental cosmological constant. In the following Discussion, we examine the domain of validity of this effective description, clarify the circumstances under which standard Einstein–Friedmann dynamics are recovered, and identify the regimes in which departures from classical cosmology are expected.

8. Discussion

The results presented in this work support the view that key features of cosmological dynamics may be understood as emergent consequences of geometry and conservation, rather than as manifestations of fundamental spacetime dynamics. In particular, the appearance of an effective, time-dependent cosmological term arises naturally from the evolution of the embedded flux hypersurface and does not require the introduction of additional matter fields or modified gravitational equations. This perspective is especially timely in light of recent observational indications that the dark energy component may deviate from a strictly constant cosmological term, and suggests that long-standing conceptual issues—most notably the vacuum energy problem—may admit a geometric reinterpretation. Within the present framework, the observed large-scale curvature is decoupled from microscopic vacuum energy scales and instead reflects the global structure and conserved properties of the underlying wavefunction geometry.

Relation to de Sitter and Λ CDM Cosmology

In standard relativistic cosmology, accelerated expansion is usually attributed to a fundamental cosmological constant or to a dark-energy component with negative pressure. In the present framework, no such ingredients are postulated. Instead, the large-scale cosmological behaviour considered here emerges geometrically as a limiting case of the embedding geometry of the flux hypersurface associated with the free-evolving universal wavefunction.

As shown in Sections 4 and 5, exact de Sitter spacetime corresponds to a maximally symmetric hyperboloid with constant curvature scale, for which the effective cosmological term Λ_{eff} is strictly constant. More generally, physically admissible and normalisable wavefunction envelopes lead to hypersurfaces with a time-dependent curvature scale. In this case, $\Lambda_{\text{eff}}(t)$ varies slowly only in a narrow intermediate regime, where the expansion is transiently quasi-de Sitter, but evolves away from this limit at both early and late times.

A key result of the present work is that the effective Friedmann equation on the hypersurface decomposes naturally into a geometric constraint and a dynamical evolution law. For closed spatial slicing ($k = +1$), the conserved surface-type geometric invariant inherited from the wavefunction fixes the effective density to scale as $\rho_{\text{eff}} \propto a^{-2}$. This matter-like contribution cancels identically against the spatial curvature term in the Friedmann equation, leaving a purely geometric constraint relating the Hubble rate to $\Lambda_{\text{eff}}(t)$. The time dependence of the expansion, however, is governed by the effective continuity equation, which explicitly accounts for the exchange between the traceless sector and the evolving vacuum-like curvature term.

This structure resolves the apparent tension between a de Sitter-like Friedmann constraint and a nonvanishing \dot{H} . While H^2 is fixed algebraically at each instant by $\Lambda_{\text{eff}}(t)$, the evolution of H is controlled by the slow variation of this effective cosmological term. As a result, de Sitter behaviour appears only transiently, rather than as a stable or eternal phase.

At late times, the decay of $\Lambda_{\text{eff}}(t)$ together with the geometric cancellation between curvature and the conserved a^{-2} sector drives the expansion towards an asymptotically linear regime, $a(t) \propto t$. In standard cosmological language, this corresponds to an effective equation-of-state parameter approaching $w \rightarrow -1/3$. Importantly, this value is not assumed a priori, but emerges dynamically as an attractor of the geometric evolution. The resulting late-time behaviour is kinematically similar to that discussed in $R = ct$ cosmological models [14], but arises here from a fundamentally different physical origin: a geometric cancellation dictated by the structure of the universal wavefunction rather than from a postulated global equation of state.

From this perspective, both de Sitter and Λ CDM cosmologies appear as effective, phenomenological descriptions valid over limited temporal regimes where the curvature scale of the hypersurface varies slowly. The present framework clarifies why such descriptions can be remarkably successful locally, while also identifying the conditions under which systematic deviations are expected, particularly in the early-time and asymptotic late-time limits.

9. Conclusions

In this work, we have shown how an effective spacetime description and an Einstein–Friedmann structure emerge naturally from the geometry of a universal wavefunction, without postulating gravitational field equations or introducing matter fields as independent dynamical entities. By treating the flux hypersurface associated with a conserved wavefunction current as an embedded Lorentzian manifold, we demonstrated that its induced geometry is necessarily of Friedmann–Robertson–Walker type under minimal assumptions of homogeneity and isotropy.

A central outcome of this analysis is the identification of de Sitter spacetime as a special, limiting case of the embedding geometry, realised only when the curvature scale of the hypersurface remains constant. For more general and physically relevant wavefunction envelopes, including the critical hypersurface analysed in [6], the curvature scale evolves in time. This leads to a transient quasi-de Sitter regime rather than a sustained inflationary phase, followed by a gradual departure from de Sitter symmetry.

The dominant contribution to the effective Friedmann dynamics arises from a conserved, surface-type geometric invariant inherited from the universal wavefunction. This invariant fixes the scaling of the effective density as $\rho_{\text{eff}} \propto a^{-2}$ and determines the effective gravitational coupling. For closed spatial slicing, this contribution cancels identically against the spatial curvature term, leaving a geometric constraint that relates the Hubble rate directly to a residual, time-dependent vacuum-like curvature

term. The cosmological evolution is then governed by the variation of this term, as encoded in the effective continuity equation.

As a result, the expansion exhibits a clear sequence of regimes: a strongly non-de Sitter early-time phase, a narrow intermediate interval of quasi-de Sitter behaviour, and an asymptotically coasting late-time expansion with $w \rightarrow -1/3$ emerging dynamically as an attractor. These features arise without invoking dark energy, inflaton fields, or fine-tuned equations of state, and follow directly from the geometric structure of the universal wavefunction.

Within this framework, general relativity appears logically an effective, local description of the intrinsic geometry of the hypersurface. The Einstein–Friedmann equations emerge as geometric identities that acquire physical meaning once reinterpreted in the language of relativistic cosmology, with local consistency ensured by the contracted Bianchi identity even in the presence of a time-dependent vacuum-like curvature term. Taken together, these results suggest that standard cosmological dynamics may be understood as an emergent, hydrodynamic limit of a deeper wavefunction-based geometry, offering a new perspective on the origin of spacetime, curvature, and cosmic expansion.

We emphasise that the present framework is not intended to provide a detailed description of structure formation, perturbation growth, or astrophysical processes, but rather to establish the geometric origin and large-scale evolution of the effective spacetime background emerging from the underlying wavefunction structure.

Funding: This research received no external funding.

Data Availability Statement: All results presented in this work are derived analytically. No new datasets were generated or analyzed.

Conflicts of Interest: The author declare no conflicts of interest.

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