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Article

Beta Decay Properties of Waiting-Point N=50 and 82 Isotopes

Necla Çakmak and Najm Abdullah Saleh *

Department of Physics, Karabük University, Karabük 78050, Türkiye

* Correspondence: najmabdola2014@gmail.com

Abstract: We have performed the microscopic calculation of β-decay properties for waiting point nuclei with neutron-closed magic shells. Allowed Gamow-Teller (GT), and first-forbidden (FF) transitions have been simulated using a Schematic Model (SM) for waiting-point $\mathrm{N} = 50{,}82$ isotopes in the framework of a proton-neutron quasiparticle random phase approximation (pn-QRPA). The Woods-Saxon (WS) potential basis has been used in our calculations. The pn-QRPA equations of allowed GT and FF transition have been utilized in both the particle-hole (ph) and particle-particle (pp) channels in the SM. We solved the secular equations of the GT and FF transitions for eigenvalues and eigenfunctions of the corresponding Hamiltonians. A spherical shape was assigned to each waiting-point nuclei in all simulations. Significantly, this study marks the first time that β-decay analysis has been applied to certain nuclei, including $^{82}_{50}Ge, ^{83}_{50}As, ^{84}_{50}Se, ^{85}_{50}Br\ and\ ^{87}_{50}Rb$ with ($\mathrm{N} =$ 50 isotones) and $^{132}_{82}Sn, ^{133}_{82}Sb, ^{134}_{82}Te, ^{135}_{82}I$ and $^{137}_{82}Cs$ with ($N = 82$ isotones). Since there had been no prior theoretical research on these nuclei, this work is a unique addition to the field. We have compared our results with the previous calculations and measured data, and our calculations agree with the experimental data and the other theoretical results.

Keywords: allowed and first-forbidden transitions; waiting point nuclei; pn-QRPA; woods-saxon potential; schematic model

1. Introduction

Weak reaction r and s-processes are considered among the important phenomena in the nucleosynthesis of heavy elements, especially after the efforts that have been made and are being made to understand the formation of stars [1– 3]. However, the data available for these properties about the process are few and not known experimentally [4,5]. This understanding has improved with time due to modern references [6– 9]. This matter was required by understanding the nuclear properties of many the nuclei rich in neutrons, which are located at a large distance from the stability line, this is a basic requirement in the r-process, which also requires neutron density more than 10^{20} neutrons $/\mathrm{cm}^3$, in addition to data of entropy, density, and high temperature, which is approximately 10^{90} K, and the other information about stellar matter [2,3,8,10,11]. In addition, nuclear safety in reactors depends on data about the beta decay of its core resulting from the fission process and from the acting radioactive ion-beam facilities (RIB): ISOLDE-CERN, ALTO, RIKEN, TRIUMF, NSCL [1], such as its half-life and the possibility of the emission of delayed neutrons, which enables us to see the nuclear structure about the response of the nuclear spin. Many questions raise interest in the development of the nuclear shell in particular when ratios N/Z are high. New sub-shells may be formed as a result of the inversion of the quasi-particle level and the emergence of new magic numbers like $\mathrm{N} = 34$ in Calcium-54 [1,12]. The experimental data available about the waiting point nucleus is insufficient, and the published information about closed neutron shells $\mathrm{N} = 50$ and 82 nuclei [13–15].

The flow of r-process material slows down by waiting for multiple decays before restarting rapid neutron capture. This process is faster than the beta decay processes, through which a neutron-

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rich nucleus is produced with a neutron separation energy greater than or equal to 3 MeV . Due to the high binding energy, discontinuities appear in the separation energy spectrum, especially at the closed nucleus $\mathrm{N}=50$, 82, and 128 isotones [7,16]. The half-life periods of the waiting point nucleus are measured in terrestrial laboratories as a result of rapid development. However, the situation is different for stars where research is still ongoing to obtain more information because the temperatures are high for the r-process to occur, and the possibility that the parent waiting point nucleus is in an excited state, which effects on their decay rates, this demonstrates the importance of the GT transition in both the ground state and the excited state, due to the possibility of capturing the electron to the nucleus of the waiting point, which contributes to increasing the rates of weak-stellar interaction, which was expected to be an exchange of charges [7,18]. Experimentally, a few waiting cores, depend on the ground state [19,20]. Borzov adopted the energy density functional model as well as the continuous quasi-particle random phase approximation (DF + CQRPA) model is based on the self-consistent theory of finite Fermi systems as an alternative to the Skyrme-Hartree Fock (SHF) method [21]. Nabi et al., studied the GT and the unique first-forbidden transitions U1F for the waiting-point $\mathrm{N}=50$ and 82 nuclei in the stellar environment using the pn-QRPA (N) model. The data collected included several aspects of beta decay, such as its properties, half-life, rates of delayed neutron emission probability, and associated energies, as indicated in reference [22]. Caroline et al., calculated the allowed and first forbidden transition FF in the series, $N=82{,}126$, adopting the approach based on the quasi-particle random phase approximation (QRPA) with quasi-particle vibration coupling. It was shown that the available data agree with the obtained data, especially when taking into account the quasi-particle vibrational coupling [23]. Wang et al., have developed the Projected Shell Model (PSM) to describe the allowed and first-forbidden transitions in nuclear beta decay for a wide range of light and superheavy nuclei, including even-even, odd-odd, and odd-mass nuclei. Here, they concentrate on calculating and systematically evaluating 35 significant first-forbidden transitions that are expected to play a major role in solving the problems with the reactor anti-neutrino spectrum. These computations are performed to test how well the PSM works [24]. The β-decay $\log ft$ values for the region surrounding the doubly magic nucleus (e.g., 208 Pb) using the proton-neutron quasi-particle random phase approximation (pn-QRPA) model were estimated. This region has consistently garnered interest for additional research due to its enhanced stability. The schematic model approach was utilized to resolve the pn-QRPA equations. The Woods-Saxon (WS) potential was utilized as a mean-field basis, and spherical nuclei were assumed. An investigation was conducted on allowed Gamow-Teller (GT) and first-forbidden (FF) transitions in the particle-hole (ph) channel. Furthermore, a deformed Nilsson basis is employed to process the random phase approximation (RPA) equations on the particle-particle (pp) and particle-hole (ph) channels. It was anticipated that allowed β-decay and unique firstforbidden (U1F) rates would exist in stellar matter [25]. The rates of β-decay in isotonic nuclear chains is investigated in a study at four distinct neutron shell closures: $\mathrm{N}=50{,}82{,}126$, and 184. The nucleosynthesis of the r-process is dependent on the closure of these casings. This endeavour aims to examine the correlations among nucleons and assess first-forbidden transitions that transpire outside the bounds of the relativistic QRPA. The many-body approach clarifies the relationship between individual and collective degrees of freedom by employing the relativistic QRPA. The study provides evidence that Gamow-Teller transitions take place within the energy range of decay, thereby confirming the decay rates that have been observed. As the system reaches a state of stability, the likelihood of decay via the FF transition diminishes due to the reduction in the coupling between vibrations and quasi-particles. Nevertheless, Gamow-Teller transitions occur during the decay $\mathrm{Q}-$value [26].

The paper is organized as follows. Section 2 briefly describes the formalism of the pn-QRPA model used in this paper. In section 3, we show the calculated results of allowed GT and FF strength distributions, $\log ft$ values for the neutron-closed magic shells 50 and 82 isotopes. In section 4, we finally give a summary of the paper.

2. Mathematical Formalism

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We used the $pn-\mathrm{QRPA(WS)}$ model for the calculation of allowed GT and FF transitions. The formalism is briefly discussed in the following subsections. The $\log ft$ and halflives values for GT and FF transitions were calculated employing a spherical schematic model (SSM). The Woods-Saxon (WS) potential was used to calculate the singleparticle energies and wave functions. The calculations were performed in the particlehole (ph) and particle-particle (pp) channels in allowed GT transition to calculate the eigenvalues and eigenfunctions of the Hamiltonian. The calculation was performed only in the particle-hole (ph) channel in the FF transitions to compute the eigenvalues and eigenfunctions of the Hamiltonian. The Gamow-Teller ($J^{\pi}=1^{+}$) and first-forbidden ($J^{\pi}=0^{-},1^{-},2^{-}$)excitations in odd-odd nuclei were generated from the correlated ground state of the parent nuclei through the charge-exchange spin-spin and spin-dipole interactions.

2.1. Gamow-Teller and First-Forbidden Transitions

The schematic model (SM) Hamiltonian for GT excitations in the quasi-particle representation is usually accepted in the following form

$$H_{SSM}^{GT}=H_{sqp}+h_{ph}^{GT}+h_{pp}^{GT} \tag{1}$$

where H_{sqp} is the corresponding single quasi-particle Hamiltonian of the system and the h_{ph},h_{pp} are the effective interaction operators in the particle-hole and the particleparticle channels, respectively. The single quasi-particle Hamiltonian of the system was given by

$$H_{sqp}=\sum_{j_{\tau}m_{\tau}}\varepsilon_{j_{\tau}m_{\tau}}\hat{\alpha}_{j_pm_p}^{\dagger}\hat{\alpha}_{j_nm_n},(\tau=n,p) \tag{2}$$

where $\varepsilon_{j_{\tau}m_{\tau}}$ is the single quasi-particle energy of the nucleons with angular momentum $j_{\tau}m_{\tau}$. The $\hat{\alpha}_{j_pm_p}^{\dagger}$ and $\hat{\alpha}_{j_nm_n}$ are one quasi-particle creation and annihilation operators, respectively. In quasi-boson approximation, the spin-isospin effective interaction Hamiltonian in the two channels are written in terms of the quasi-boson creation and annihilation operators as follows

$$h_{ph}^{GT}=\frac{2\chi_{ph}}{g_A^2}\sum_{j_{\tau}j_{\tau'}\mu\mu'}\left[b_{j_pj_n}(\lambda)A_{j_pj_n}^{\dagger}(\lambda\mu)+(-1)^{(\lambda+\mu)}\bar{b}_{j_pj_n}(\lambda)A_{j_pj_n}(\lambda-\mu)\right]$$

$$h_{pp}^{GT}=-\frac{2\chi_{pp}}{g_A^2}\sum_{j_pj_nj_{p'}j_{n'}\mu'}\left[d_{j_pj_n}(\lambda)A_{j_pj_n}^{\dagger}(\lambda\mu)-(-1)^{(\lambda+\mu)}\bar{d}_{j_pj_n}(\lambda)A_{j_pj_n}(\lambda-\mu)\right]$$

where χ_{ph} and χ_{pp} represent the particle-hole (ph) and the particle-particle (pp) effective interaction constants. The values of the interaction constants were fixed from the experimental value of the resonance energies.

The Hamiltonian of the first-forbidden transitions was chosen as

$$H_{SSM}^{FF}=H_{sqp}+h_{ph}^{FF} \tag{5}$$

The charge-exchange spin-spin effective residual interaction was determined using

$$h_{ph}^{FF}=\frac{2\chi_{ph}}{g_A^2}\sum_{j_{\tau}j_{\tau'}\mu\mu'}\left[b_{j_pj_n}(\lambda)A_{j_pj_n}^{\dagger}(\lambda\mu)+(-1)^{(\lambda+\mu)}\bar{b}_{j_pj_n}(\lambda)A_{j_pj_n}(\lambda-\mu)\right]$$

The $A_{j_pj_n}^{\dagger}(\lambda-\mu)$ and $A_{j_pj_n}(\lambda\mu)$ in Eqs. (3), (4), and (6) are the quasi-boson creation and annihilation operators, respectively, defined by

$$A_{j_pj_n}^{\dagger}(\lambda-\mu)=\sqrt{\frac{2\lambda+1}{2j_n+1}}\sum_{m_nm_p}(-1)^{j_p-m_p}\alpha_{j_pm_p}^{\dagger}\alpha_{j_n-m_n} \tag{7}$$

and

$$A_{j_pj_n}(\lambda\mu)=\left[A_{j_pj_n}^{\dagger}(\lambda-\mu)\right]^{\dagger}.$$

The $b_{j_pj_n}(\lambda),\bar{b}_{j_pj_n}(\lambda)$ in Eqs. (3) and (6) stands for the reduced matrix elements of the related multipole operators. These operators for $\Delta J^{\pi}=1^{+}$transitions were given by

$$\bar{b}_{j_pj_n}(\lambda)=\frac{1}{\sqrt{2\lambda+1}}\langle j_p(l_ps_p)\|\sigma_k\tau_k^{\pm}\|j_n(l_ns_n)\rangle v_{j_n}u_{j_p}.$$

For $\Delta J^{\pi}=0^{-},1^{-},2^{-}$transitions were described in a general form by

$$\bar{b}_{j_pj_n}(\lambda)=\frac{1}{\sqrt{2\lambda+1}}\langle j_p(l_ps_p)\|\tau_k^{\pm}r_k[Y_1(\hat{r}_k)\sigma_k]_{\lambda}\|j_n(l_ns_n)\rangle v_{j_n}u_{j_p}$$

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and

$$b_{j_p j_n}(\lambda) = \frac{\bar{b}_{j_p j_n}(\lambda)}{v_{j_n} u_{j_p}} u_{j_n} v_{j_p}$$

where v and u are single-particle and hole amplitudes, respectively. The solution of Hamiltonian Eqs. (1) and (5) is briefly described below. The allowed GT and the charge-exchange vibration modes in odd-odd nuclei are considered phonon excitations and are described by

$$|\Psi_i >= Q_i^{\dagger}|0 >= \Sigma_{j_\tau \mu}\left[\psi^i_{j_p j_n} A^{\dagger}(\lambda\mu)_{j_p j_n} - \varphi^i_{j_p j_n} A_{j_p j_n}(\lambda\mu)\right] \mid 0 > \tag{8}$$

where $Q_i^{\dagger}$ is the pn-QRPA phonon creation operator, $\mid 0 >$ is the phonon vacuum which corresponds to the ground state of an even-even nucleus and fulfils $Q_i \mid 0 >= 0$ for all i. The $\psi^i_{j_p j_n}$ and $\varphi^i_{j_p j_n}$ are forward and backward quasi boson amplitudes. Employing the conventional procedure of the $pn-$QRPA we solved the equation of motion

$$[H, Q_i^{\dagger}]|0 >= \omega_i Q_i^{\dagger}|0 > \tag{9}$$

The ω_i is the i th $0^-, 1^-$, and 2^-excitation energy in odd-odd daughter calculated from the ground state of the parent even-even nucleus. Further details of the formalism can be seen in refs. [27–31].

2.2. Extension in the Model for Odd-A Nuclei

We summarise the necessary formalism for treating odd-A nuclei in our model. The Hamiltonian of the odd-A system is chosen as

$$H_{SSM} = H_{sqp} + h^{CC}_{ph} + h^{CD}_{ph}. \tag{10}$$

The effective residual interactions are determined using

$$h^{CC}_{ph} = \frac{2\chi_{ph}}{g_A^2} \sum_{j_\tau j_{\tau'} \mu\mu'} \left[\bar{b}_{j_p j_n}(\lambda) C^{\dagger}_{j_p j_n}(\lambda\mu) + (-1)^{(\lambda+\mu)} b_{j_p j_n}(\lambda) C_{j_p j_n}(\lambda-\mu)\right]$$

$$\times \left[\bar{b}_{j_{p'} j_{n'}}(\lambda) C_{j_{p'} j_{n'}}(\lambda\mu') + b_{j_{p'} j_{n'}}(\lambda) C^{\dagger}_{j_{p'} j_{n'}}(\lambda-\mu')\right] \tag{11}$$

$$h^{CD}_{ph} = \frac{2\chi_{ph}}{g_A^2} \sum_{j_\tau j_{\tau'} \mu\mu'} \left[\bar{b}_{j_p j_n}(\lambda) C^{\dagger}_{j_p j_n}(\lambda\mu) + (-1)^{(\lambda+\mu)} b_{j_p j_n}(\lambda) C_{j_p j_n}(\lambda-\mu)\right] \tag{11}$$

$$\times \left[d_{j_{p'} j_{n'}}(\lambda) D_{j_{p'} j_{n'}}(\lambda\mu') + \bar{d}_{j_{p'} j_{n'}}(\lambda) D^{\dagger}_{j_{p'} j_{n'}}(\lambda-\mu')\right]. \tag{12}$$

The $C^{\dagger}_{j_p j_n}(\lambda-\mu), D^{\dagger}_{j_p j_n}(\lambda-\mu)$ and $C_{j_p j_n}(\lambda\mu), D_{j_p j_n}(\lambda\mu)$ are the quasi-boson creation and annihilation operators, respectively, and given as

$$C^{\dagger}_{j_p j_n}(\lambda-\mu) = \sqrt{\frac{2\lambda+1}{2j_n+1}} \sum_{m_n m_p} (-1)^{j_p - m_p} \alpha^{\dagger}_{j_n m_n} \alpha^{\dagger}_{j_p - m_p} \tag{13}$$

and

$$C_{j_p j_n}(\lambda\mu) = \left[C^{\dagger}_{j_p j_n}(\lambda-\mu)\right]^{\dagger}$$

and

$$D_{j_p j_n}(\lambda\mu) = \left[D^{\dagger}_{j_p j_n}(\lambda-\mu)\right]^{\dagger}.$$

The $b_{j_p j_n}(\lambda), \bar{b}_{j_p j_n}(\lambda), d_{j_p j_n}(\lambda), \bar{d}_{j_p j_n}(\lambda)$ in Eqs. (6) and (7) stand for the reduced matrix elements of the related multipole operators. The charge-exchange spin-spin and spin-dipole transitions are defined by

$$\begin{aligned} \bar{d}_{j_p j_n}(\lambda) &= \frac{1}{\sqrt{2\lambda+1}} \langle j_p(l_p s_p) \| \sigma_k \tau_k^{\pm} \| j_n(l_n s_n) \rangle v_{j_n} v_{j_p} \\ \bar{d}_{j_p j_n}(\lambda) &= \frac{1}{\sqrt{2\lambda+1}} \langle j_p(l_p s_p) \| \tau_k^{\pm} r_k [Y_1(\hat{r}_k)\sigma_k]_{\lambda} \| j_n(l_n s_n) \rangle v_{j_n} v_{j_p}, \end{aligned} \tag{14}$$

and

$$d_{j_p j_n}(\lambda) = \frac{\bar{d}_{j_p j_n}(\lambda)}{v_{j_n} v_{j_p}} u_{j_n} u_{j_p}$$

The wave function of the odd-A (with odd neutrons) nuclei is given by

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$$\left|\Psi^{j}_{j_k m_k} >= \Omega^{j\dagger}_{j_k m_k}\right| 0 >= \left(N^{j}_{j_k} \alpha^{\dagger}_{j_k m_k} + \sum_{j_\nu m_\nu} R^{ij}_{k\nu} A^{\dagger}_{i} \alpha^{\dagger}_{j_\nu m_\nu} \right) | \, 0 > \quad (15)$$

where $\Omega^{j\dagger}_{j_k m_k}$ and $| \, 0 >$ represent the phonon operator and phonon vacuum, respectively. $N^{j}_{j_k}$ and $R^{ij}_{k\nu}$ are the quasi boson amplitudes corresponding to the states and fulfil the normalization conditions. The wave function was formed by the superposition of oneand three-quasi-particle (one-quasi-particle + one phonon) states. The equation of motion is defined by

$$\left[H_{SSM}, \Omega^{j\dagger}_{j_k m_k}\right] \left|0 >= \omega^{j}_{j_k m_k} \Omega^{j\dagger}_{j_k m_k}\right| 0 > \quad (16)$$

The excitation energies $\omega^{j}_{j_k m_k}$ and the wave functions of the GT and FF excitations were obtained from the $pn-\mathrm{QRPA(WS)}$ equation of motion. The details of the solution for the odd-A nuclei can be seen from ref. [25].

2.3. Nuclear Matrix Elements

One of the characteristic quantities for the GT 1^{+}states occurring in the neighbouring odd-odd nuclei is the GT transition matrix elements. The allowed GT $\beta^{\pm}$transition matrix elements are calculated using the following expressions:

$$M^{i}_{\beta^-}(0^+ \to 1^+_i) = \langle 1^+_i, \mu | G^{-}_{1\mu} | 0^+ \rangle = \langle 0 | \left[Q_i(\mu), G^{-}_{1\mu} \right] | 0 \rangle$$

$$M^{i}_{\beta^+}(0^+ \to 1^+_i) = \langle 1^+_i, \mu | G^{+}_{1\mu} | 0^+ \rangle = \langle 0 | \left[Q_i(\mu), G^{+}_{1\mu} \right] | 0 \rangle$$

The $\beta^{\pm}$reduced matrix elements are given by

$$B^{(\pm)}_{\mathrm{GT}}(\omega_i) = \sum_{\mu} \left| M^{i}_{\beta^{\pm}}(0^+ \to 1^+_i) \right|^2 \quad (19)$$

The $\beta^{\pm}$transition strengths ($S^{\pm}$) must fulfill the Ikeda sum rule (ISR).

$$S^{\pm} = \sum_{i} B^{(\pm)}_{\mathrm{GT}}(\omega_i) \quad (20)$$

The FF transition consists of six Nuclear Matrix Elements (NMEs), and these NMEs include relativistic and non-relativistic terms for the $\Delta J^{\pi} = 0^-, 1^-$transitions. The relativistic NMEs were calculated directly without any assumptions. The contribution from the spin-orbit potential was included in the calculation of the relativistic matrix elements. The unique first forbidden (U1F) transitions, $\Delta J^{\pi} = 2^-$, do not contain any relativistic term. The transitions probabilities $\hat{B}(\lambda^{\pi} = 0^-, 1^-, 2^-)$ were specified as [32]

$$\hat{B}(\lambda^{\pi} = 0^-) = \left| < 0^-_i \left\| M^{\mathrm{rank}\ 0} \right\| 0^+ > \right|^2 \quad (21)$$

where

$$M^{rank0} = M(\rho_A, \lambda = 0) - i \frac{m_e c}{h} \xi \hat{M}(j_A, \kappa = 1, \lambda = 0), \quad (22)$$

with

$$M(\rho_A, \lambda = 0) = \frac{g_A}{(4\pi)^{1/2} c} \Sigma_k \tau^{\pm}_k (\sigma_k \cdot \vartheta_k)$$

$$M(j_A, \kappa = 1, \lambda = 0) = g_A \Sigma_k \tau^{\pm}_k r_k [Y_1(\hat{r}_k) \sigma_k)]_{0\mu}$$

where

$$M^{rank1} = M(j_V, \kappa = 0, \lambda = 1, \mu) \pm i \frac{m_e c}{\sqrt{3}\hbar} \xi M(\rho_V, \lambda = 1, \mu)$$

with

$$M(j_V, \kappa = 0, \lambda = 1, \mu) = \frac{g_V}{c\sqrt{4\pi}} \Sigma_k \tau^{\pm}_k (\vartheta_k)_{1\mu}$$

$$M(\rho_V, \lambda = 1, \mu) = g_V \Sigma_k \tau^{\pm}_k r_k Y_{1\mu}(\hat{r}_k)$$

$$M(j_A, \kappa = 1, \lambda = 1, \mu) = g_A \Sigma_k \tau^{\pm}_k r_k [Y_1(\hat{r}_k) \sigma_k]_{1\mu}$$

Where,

$$M^{\mathrm{rank}\ 2} = M(j_A, \kappa = 1, \lambda = 2, \mu) = g_A \Sigma_k \tau^{\pm}_k r_k [Y_1(\hat{r}_k) \sigma_k]_{2\mu} \quad (26)$$

where $\sigma_k, \vartheta_k, Y_1(\hat{r}_k)$ and $\tau^{\pm}_k$denote the Pauli matrices, velocity, spherical harmonics and isospin creation (annihilation) operators, respectively. The $\rho_V(\rho_A)$ and $j_V(j_A)$ are the vector (axial vector) charge and current densities associated with a single nucleon, respectively. These variables are linear

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and depend only on the space point $\mathbf{r}$. They are independent of the velocity. For further details, we refer to Appendix 3D-Beta Interaction in ref. [32]. In Eqs. (22) and (24), the upper and lower signs refer to $\beta^-$and β^+decays, respectively. The multipole operators considered to calculate the reduced NMEs of the first forbidden transitions were defined using

$$M(j_A,\kappa=1,\lambda,\mu)=g_A\Sigma_k\tau_k^{\pm}r_k[Y_1(\hat{r}_k)\sigma_k]_{\lambda\mu} \quad (27)$$

where $\lambda=0,1,2$ and $\mu=(0,\pm1,\pm2,\ldots,\pm\lambda)$ are the corresponding nuclear spin $(\lambda^\pi=0^-,1^-,2^-)$for the transition and its projection, respectively. The ft values of the allowed GT and non-unique first-forbidden transitions (rank0, rank1) were calculated using

$$ft=\frac{D}{(g_A/g_V)^2 4\pi\hat{B}(\lambda^\pi=1^+,0^-,1^-)} \quad (28)$$

and for U1F transitions using [?]

$$ft=\frac{(2n+1)!!}{(n+1!)^2 n!}\frac{D}{(g_A/g_V)^2 4\pi\hat{B}(\lambda^\pi=2^-)} \quad (29)$$

where D is a constant taken as 6295 s. The effective ratio of axial and vector coupling constant was taken as $(g_A/g_V)=-1.254$[33]. The effective/quenched axial-vector weak couplings may be considered depending on different nuclear models. But, the effective ratio of axial-vector weak couplings $0.7\times(g_A/g_V)^2$ was not used in our calculations.

3. Results and Discussion

The matrix elements of a single quasi-particle were calculated using the WS radial wave functions. The parameters were taken from ref. [12]. The proton and neutron pairing gaps were determined using $\Delta_p=C_p/\sqrt{A}$ and $\Delta_n=C_n/\sqrt{A}$, respectively. The pairing strength parameters (C_p and C_n) were fixed to reproduce the experimental pairing gaps [34]. The study of the allowed GT decay nuclear matrix elements was carried out using the schematic Hamiltonian with the ph and pp channels. The parameter values χ_{ph} and χ_{pp} for the allowed GT transitions were used as 1.0 and 0.25 in schematic model calculations. The χ_{pp} effective interaction constant for allowed GT is taken as $\chi_{pp}=0.58A^{0.7}$MeV. The χ_{ph} effective interaction constants for allowed GT, rank 0 , rank 1 , rank 2 transitions are chosen as $\chi_{ph}=5.2A^{0.7}$MeV, $\chi_{ph}=30A^{-5/3}$MeVfm^{-2}, $\chi_{ph}=90A^{-5/3}$MeVfm $^{-2}$, and $\chi_{ph}=350A^{-5/3}$MeVfm $^{-2}$, respectively. These values give better results and are close to the experimental values. The relativistic β - moment matrix elements in the first-forbidden $\Delta J=0$(rank0) and $\Delta J=1$ (rank 1) transitions were calculated directly, without any assumption. Also, the beta transition probabilities of rank0 and rank 1 transitions were performed for within the ξ-approximation.

The computed β-decay half-lives including allowed GT and first-forbidden contributions for waiting point nuclei having N = 50, 82 were shown in Tables 1 and 2. The measured half-lives were taken from the recent available atomic mass data evaluation of reference [35]. It may be seen from Table 1 and Table 2 that our calculated half-lives values are in very good comparison with the experimental data and the previous theoretical results. Calculated the excitation energy (MeV) and log ft values of N = 50,82 waiting-point nuclei with the pn-QRPA (WS) for allowed GT, rank 0, rank 1 and rank 2 transitions were given in Tables 3 and 4, respectively. The ω_i denotes the excitation energy in the daughter nucleus in Tables. The available measured values for each transition are given in Tables 3 and 4. Here, the last column $I(\%)$ -E(MeV)-logft represents the experimental intensity, energy and log ft values, and were taken from NuDAT. The energy dependence of beta decay probabilities of the firstforbidden transitions are generally between $(14-32)$MeV. The dominant contributions for the rank 0, rank 1 and rank 2 transitions in the waiting point nuclei having N = 50,82 are located at energies of the order $(18-25)$MeV, (14-32) MeV, and (17-25) MeV, respectively. The Ikeda Sum Rule values calculated by single quasi-particle (sqp) and pn-QRPA for waiting point isotopes are shown in Table 5. It may be seen that the Ikeda Sum Rule is fulfilled in both even-even and odd-A cases. The compliance is greater than 97%. The calculated Ikeda Sum Rule values are the model-independent. The calculated β-decay half-lives for waiting-point nuclei with $N=50$ and $N=82$, shown in Tables 1 and 2, demonstrate significant concordance with experimental data and

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earlier theoretical models. The experimental half-lives, derived from the latest atomic mass evaluation [35], provide a reliable standard for assessing the precision of the current calculations.

This research utilizes the proton-neutron quasiparticle random phase approximation (pn-QRPA) in conjunction with the Woods-Saxon potential, drawing on theoretical perspectives from significant references. [36] established the foundational formalism for nuclear structure, whereas [37] offered essential values for axial-vector coupling constants, crucial for the accurate derivation of logft values and transition probabilities.

Moreover, the values for pairing strength and effective interaction constants were derived from [38], guaranteeing alignment with experimentally reported pairing gaps. The findings strongly correspond with previous theoretical investigations based on pn-QRPA, including those in [28], which emphasizes the significance of both allowed Gamow-Teller (GT) and first-forbidden (FF) transitions in influencing β-decay properties.

In Table 1, In the absence of prior theoretical predictions, this study provided important new data by calculating β-decay half-lives for specific nuclei at the N=50 waiting point for the first time. Nuclei like Ge-82, As-83, Se-84, Br-85, and Rb-87 did not have previous computational results in the literature, as indicated in Table 1. The pn-QRPA (WS) model-calculated half-lives are in good agreement with experimental observations from Ref. [30]. The β-decay half-life for Ge-82, for example, was found to be 4.82 seconds, which is quite close to the experimental result of 4.31 seconds.

As-83's calculated half-life (12.83 seconds) and the experimental figure of 13.4 seconds both agree quite well. Interestingly, the measured values of 195.6 and 174 seconds, respectively, exhibit a good association with the half-lives of Se-84 (178.6 seconds) and Br-85 (161.2 seconds). The experimental result of 49.7 Gy for Rb-87 is in agreement with the calculated half-life of 47.3 Gy.

These findings close a major gap in the nuclear data landscape by representing the first theoretical study of the β-decay parameters for these nuclei. The strong agreement with experimental values highlights the pn-QRPA methodology's dependability, especially its ability to forecast β-decay properties for nuclei without the need for previous theoretical research. In addition to confirming the durability of the chosen model, this work lays the groundwork for future research on neutron-rich nuclei in the N=50N = 50N=50 area, which is vital for comprehending r-process nucleosynthesis.

Rb-87's unusual nuclear structure and decay mechanism are the main causes of its extraordinarily long β-decay half-life, which is measured in giga-years (Gy). This isotope's exceptionally slow β-decay rate is caused by a number of reasons. Because of its unique nuclear shell structure and quantum states, Rb-87 has a strongly suppressed weak transition matrix element that controls the β-decay probability. Furthermore, Rb-87's half-life is much increased and the accessible phase space for decay is constrained by its comparatively small Q-value, which represents the energy difference between the parent and daughter nuclei. Strict spin-parity selection restrictions that limit permitted transitions exacerbate this suppression and further slowdown the decay rate.

Furthermore, Rb-87 has a very stable nuclear structure and a relatively low decay energy due to its proximity to the line of nuclear stability, which also adds to its long half-life. Rb-87's half-life is estimated by experimental measurements to be around 49.7 Gy, and the study's computed value of 47.3 Gy is in great agreement with experimental data. The accuracy with which the pn-QRPA (WS) model captures the physics of Rb-87 decay is demonstrated by this consistency.

One feature that sets Rb-87 apart from the other nuclei under study is its incredibly lengthy half-life. In geochronology and cosmology, where Rb-87 is a crucial chronometer in the rubidium-strontium dating technique, which is frequently used to determine the age of rocks and minerals, this special characteristic has important ramifications. These results demonstrate the significance of Rb-87 for both basic and applied research.

In Table 2, This study filled a major gap in our understanding of the decay properties of N=82 waiting-point nuclei, including Sn-132, Sb-133, Te-134, I-135, and Cs-137, by calculating their β-decay half-lives for the first time. This work is an important contribution to nuclear physics and astrophysics because there was no previous theoretical or computational data available for these

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nuclei. Table 2 displays the experimental data from Ref. [30] in addition to the outcomes of the current computations, which are based on the pn-QRPA (WS) model.

The theoretical framework's dependability was demonstrated by the calculated half-life for Sn-132, which was 41.2 seconds and nearly matched the experimental value of 39.7 seconds. Likewise, the experimental value of 140.4 seconds is in good agreement with the estimated half-life of 131.7 seconds for Sb-133. With theoretical half-lives of 2401.24 seconds and 22314.56 seconds, respectively, the heavier isotopes Te-134 and I-135 show greater half-lives than their experimental equivalents, which are 2508.00 and 23688.00 seconds, respectively. The half-life of Cs-137, a well-known isotope with important uses in nuclear technology, was determined to be 29.11 years, which is quite close to the experimental value of 30.08 years.

These results highlight the pn-QRPA (WS) model's prediction ability, since it effectively depicts the β-decay processes of these nuclei. The model's resilience in handling nuclei without any previous theoretical research is demonstrated by the agreement with experimental results. Additionally, these N=82 waiting-point nuclei's computed half-lives offer crucial inputs for r-process nucleosynthesis research, especially when simulating the creation of heavy elements in stellar environments. This groundbreaking study lays the groundwork for further investigation and provides important standards for astrophysical modeling and experimental confirmation.

A combination of nuclear structure, decay energetics, and transition characteristics can account for Cs-137's comparatively lengthy half-life (measured in years) in comparison to other N=82 nuclei, but it's much shorter half-life (measured in giga-years) in comparison to Rb-87.

First, Cs-137 slows its β-decay because to the increased stability that the closed neutron shell at N=82 provides. However, because of higher favorable energetics, it decays more quickly than Rb-87 since it is located closer to the nuclear stability line. In particular, the β-decay Q-value of Rb-87 (around 0.28 MeV) is much smaller than that of Cs-137 (about 1.17 MeV). The half-life is shortened by this greater Q-value because it expands the phase space that is accessible for decay.

Furthermore, compared to the severely prohibited transitions seen in Rb-87, the allowed or semi-forbidden transitions involved in the β-decay of Cs-137 are less repressed. As a result, the decay probability increases and Cs-137's half-life is significantly shortened. Because Cs-137 is closer to the stability line and has lower decay Q-values, which slow the decay process, it has a longer half-life than other N=82 nuclei in this study, such Te-134 and I-135.

Cs-137's intermediate half-life emphasizes how important it is for real-world uses like radiological research and environmental monitoring. It is both scientifically intriguing and practically valuable because of its decay rate, which strikes a compromise between the extremely long half-life of isotopes like Rb-87 and the fast decay of highly unstable isotopes.

The strength distributions of permitted Gamow-Teller (GT) and first-forbidden (FF) transitions are significantly revealed by the calculated logft values and excitation energies (ωi) for the waiting-point nuclei with N=50N and N=82, which are shown in Tables 3 and 4. Reliability in capturing the nuclear structural effects governing β-decay is demonstrated by the successful reproduction of the logft values by the pn-QRPA (WS) model.

According to the calculated logft values for N=50 isotones (Table 3), first-forbidden contributions play a secondary but not insignificant part in the decay process, while GT transitions dominate. Notably, the strength of acceptable transitions is confirmed by the logft values for Se-84 and Br-85 falling within the anticipated range. The presence of collective nuclear excitations is confirmed by the excitation energies (ωi), which show that the dominant decay transitions take place at lower energy states. The calculated values for Rb-87 provide additional evidence that the model can include long-lived isotopes with significantly reduced rates of decay.

The logft values show a more intricate interaction between allowed and forbidden transitions for N=82 isotones (Table 4). Given the comparatively greater logft values and wider excitation energy distributions, the results imply that first-forbidden transitions play a major role in the decay process in heavier nuclei like Te-134 and I-135. This outcome supports earlier theoretical predictions and emphasizes how crucial FF transitions are becoming in heavier, neutron-rich nuclei. The pn-QRPA

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framework's accuracy in forecasting decay properties across various nuclear mass regions is further supported by the calculated logft values for Sn-132 and Sb-133, which closely match experimental data.

Overall, the excitation energies and logft values shown in Tables 3 and 4 demonstrate how well the chosen model reproduces the β-decay properties of neutron-rich nuclei. The findings highlight the need of incorporating both first-forbidden and allowed GT contributions in theoretical computations, especially for waiting-point nuclei that are essential for r-process nucleosynthesis.

The Ikeda Sum Rule (ISR) values for the waiting-point nuclei with N=50 and N=82 are shown in Table 5 and were determined using the single quasi-particle (sqp) and proton-neutron quasi-particle random phase approximation (pn-QRPA) approaches. Equation (20) is the Ikeda Sum Rule, which is a basic criterion for confirming that nuclear models in β-decay investigations are consistent. With compliance reaching 97% in both even-even and odd-A scenarios, Table 5's results show that the computed ISR values are in great accord with theoretical expectations.

The calculated ISR values for nuclei like Ge-82, As-83, Se-84, and Br-85 for the N=50 isotones substantially resemble the predicted theoretical values, demonstrating that the model faithfully captures the GT transition intensity. The consistency of the sqp and pn-QRPA results emphasizes even more how well the chosen theoretical framework captures the collective nuclear excitations that cause β-decay.

Likewise, the ISR values for the N=82 isotones—Sn-132, Sb-133, Te-134, I-135, and Cs-137—remain in line with theoretical expectations. The pn-QRPA model provides a thorough description of β-decay properties in neutron-rich nuclei by effectively accounting for both authorized GT and first-forbidden transitions, as evidenced by the reported high compliance rate.

All things considered, Table 5's findings support the pn-QRPA (WS) model's resilience in forecasting β-decay properties while meeting basic nuclear physics requirements. The approach's validity is demonstrated by the high degree of agreement between calculated and theoretical ISR values, which makes it a trustworthy tool for examining the function of waiting-point nuclei in r-process nucleosynthesis.

This work validates the dependability of the pn-QRPA approach and its parameters by integrating these contributions, providing predicted insights into β-decay processes of astrophysical significance, especially for nuclei essential to r-process nucleosynthesis.

Table 1. Comparison of our computed β-decay half-lives (in units of seconds) for $N = 50$ waiting-point nuclei with previous calculations and experimental half-lives. Half-lives mentioned with an asterisk in the last column were adopted from [20].

Nuclei–A	Refs. [31,32] GT	Ref. [33] GT+FF	Ref. [34] GT	Ref. [34] GT+FF	Ref. [22] GT	Ref. [22] GT+FF	Present Work GT	Present Work GT+FF	Exp. [30]
Fe − 76	0.008	0.008	0.045	0.027	0.060	0.012	0.057	0.022	0.003
Co − 77	0.016	0.016	0.013	0.014	0.025	0.016	0.021	0.017	0.015
Ni − 78	0.127	0.150	0.477	0.224	1.210	0.102	0.284	0.173	0.122
Cu − 79	0.222	0.270	0.430	0.157	0.436	0.235	0.321	0.223	0.241
Zn − 80	0.432	0.530	3.068	1.260	0.851	0.557	0.735	0.544	0.562
Ga − 81	0.577	1.030	1.568	1.227	3.387	1.083	1.816	1.174	1.217
Ge − 82	-	-	-	-	-	-	4.82	4.25	4.31
As − 83	-	-	-	-	-	-	14.52	12.83	13.4
Se − 84	-	-	-	-	-	-	211.4	178.6	195.6
Br − 85	-	-	-	-	-	-	197.3	161.2	174
Rb − 87	-	-	-	-	-	-	51.6 Gy	47.3 Gy	49.7 Gy

Table 2. Comparison of our computed β-decay half-lives (in units of seconds) for $N = 82$ waiting-point nuclei with previous calculations and experimental half-lives. Half-lives mentioned with an asterisk in the last column were adopted from [20].

Nuclei−A	Refs. [31,32] GT	Ref. [33] GT+FF	Ref. [34] GT	Ref. [34] GT+FF	Ref. [22] GT	Ref. [22] GT+FF	Present Work GT	Present Work GT+FF	Exp. [30]
Tc − 125	0.009	0.010	0.009	0.009	0.017	0.008	0.025	0.013	-
Ru − 126	0.020	0.020	0.034	0.030	0.027	0.017	0.038	0.021	-
Rh − 127	0.028	0.028	0.022	0.020	0.032	0.029	0.030	0.027	0.028
Pd − 128	0.046	0.047	0.125	0.074	0.042	0.035	0.073	0.044	0.035
Ag − 129	0.070	0.070	0.047	0.032	0.052	0.049	0.051	0.046	0.0499
Cd − 130	0.162	0.164	1.123	0.502	0.135	0.122	0.154	0.117	0.1268
In − 131	0.260	0.248	0.147	0.139	0.286	0.281	0.276	0.247	0.261
Sn − 132	-	-	-	-	-	-	41.2	38.3	39.7
Sb − 133	-	-	-	-	-	-	154.2	131.7	140.4
Te − 134	-	-	-	-	-	-	2731.53	2401.24	2508.00
I − 135	-	-	-	-	-	-	27123.12	22314.56	23688.00
Cs − 137	-	-	-	-	-	-	31.23 y	29.11 y	30.08 y

Table 3. Calculated $\log ft$ values for $\mathrm{N} = 50$ waiting-point nuclei with the pn-QRPA (WS).

	GT					rank 0					rank 1					rank 2				
Nuclei−A	Ωi (MeV)	log ft	I (%)	−E (MeV)	−log ft	Ωi (MeV)	log ft	I (%)	−E (MeV)	−log ft	Ωi (MeV)	log ft	I (%)	−E (MeV)	−log ft	Ωi (MeV)	log ft	I (%)	−E (MeV)	−log ft
Fe-76	1.037	4.63	-	-	-	1.022	6.28	-	-	-	0.816	8.58	-	-	-	2.045	9.53	-	-	-
Co-77	0.902	4.57	-	-	-	1.487	5.84	-	-	-	0.684	8.42	-	-	-	2.314	9.72	-	-	-
Ni − 78	0.827	4.43	-	-	-	1.224	5.87	-	-	-	0.683	7.85	-	-	-	1.674	9.75	-	-	-
Cu − 79	0.287	4.77	-	-	-	1.146	6.02	-	-	-	0.793	8.71	-	-	-	1.204	9.61	-	-	-
Zn-80	0.832	4.58	34	4.61	1.449	0.517	5.73	-	-	-	0.562	8.31	-	-	-	0.865	9.87	-	-	-
Ga-81	1.074	4.76	-	-	-	1.083	6.22	-	-	-	1.217	8.47	-	-	-	2.113	9.84	-	-	-
Ge-82	0.892	4.57	-	-	-	1.032	6.24	-	-	-	0.836	8.43	-	-	-	1.345	9.73	-	-	-
As-83	0.583	4.52	-	-	-	0.846	5.63	-	-	-	1.142	8.52	-	-	-	0.937	9.34	-	-	-
Se-84	1.208	4.37	100	4.04	0.408	1.265	6.04	-	-	-	0.583	7.68	-	-	-	1.562	9.22	< 0.1 >	8.5	0.0
Br − 85	0.755	4.53	-	-	-	0.423	6.52	-	-	-	1.053	7.23	-	-	-	0.875	9.74	-	-	-
Rb − 87	0.748	4.81	-	-	-	1.132	6.13	-	-	-	0.614	8.07	-	-	-	1.026	9.85	-	-	-

Table 4. Calculated $\log ft$ values for $\mathrm{N} = 82$ waiting-point nuclei with the pn-QRPA (WS).

	GT					rank 0					rank 1					rank 2				
Nuclei−A	Ωi (MeV)	log ft	I (%)	−E (MeV)	−log ft	Ωi (MeV)	log ft	I (%)	−E (MeV)	−log ft	Ωi (MeV)	log ft	I (%)	−E (MeV)	−log ft	Ωi (MeV)	log ft	I (%)	−E (MeV)	−log ft
Tc-125	2.356	4.87	-	-	-	1.207	5.83	-	-	-	2.173	7.56	-	-	-	1.058	8.87	-	-	-
Ru-126	1.608	4.65	-	-	-	2.023	6.02	-	-	-	2.713	7.82	-	-	-	2.682	9.73	-	-	-
Rh-127	0.984	4.73	-	-	-	2.124	5.76	-	-	-	1.756	8.45	-	-	-	2.583	9.21	-	-	-
Pd − 128	0.793	4.82	-	-	-	1.157	5.94	-	-	-	2.302	7.85	-	-	-	1.706	9.13	-	-	-
Ag-129	0.858	4.58	-	-	-	1.014	5.76	-	-	-	1.925	8.21	-	-	-	1.314	9.26	-	-	-
Cd-130	2.577	4.78	-	-	-	2.321	5.93	-	-	-	1.804	7.63	-	-	-	1.424	9.26	-	-	-

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In-131	0.538	4.32	90	4.4	2.434	1.274	6.04	-	-	-	1.835	7.12	< 20	> 5.6	0.0	2.341	8.94	-	-	-
Sn-132	0.821	4.12	99	4.02	1.325	2.137	5.81	-	-	-	1.563	8.52	-	-	-	1.253	9.23	-	-	-
Sb-133	1.472	4.68	-	-	-	0.846	6.04	-	-	-	1.673	7.68	-	-	-	2.517	9.42	-	-	-
Te-134	1.275	4.57	42	4.65	0.846	0.748	5.92	-	-	-	1.576	7.23	-	-	-	2.031	9.56	-	-	-
I-135	1.637	4.87	23.6	7.04	1.260	2.136	5.86	0.142	6.01	2.475	2.614	8.07	-	-	-	0.634	9.84	1.9	9.97	526
Cs-137	1.032	4.68	-	-	-	1.172	6.72	-	-	-	0.542	7.86	-	-	-	0.931	9.57	94	9.625	0.661

Table 5. The Ikeda Sum Rule calculated by single quasi-particle (sqp) and pn-QRPA for waiting point isotopes.

Waiting-Point Nuclei			Ikeda Sum Rule (ISR) $= 3(\mathrm{N}-\mathrm{Z})$		
Nuclei	***Z***	**A**	**single quasi-particle**	***pn*-QRPA**	**Theoretical**
Fe	26	76	71.9	72.0	72
Co	27	77	69.1	69.5	69
Ni	28	78	65.2	65.7	66
Cu	29	79	62.4	62.8	63
Zn	30	80	59.3	59.6	60
Ga	31	81	56.1	56.7	57
Ge	32	82	53.3	53.8	54
As	33	83	50.2	50.7	51
Se	34	84	47.1	47.7	48
Br	35	85	44.3	44.6	45
Rb	37	87	38.2	38.8	39
Tc	43	125	116.2	116.5	117
Ru	44	126	113.4	113.8	114
Rh	45	127	110.3	110.9	111
Pd	46	128	107.4	107.6	108
Ag	47	129	105.1	105.3	106
Cd	48	130	101.2	101.6	102
In	49	131	98.3	98.7	99
Sn	50	132	95.3	95.8	96
Sb	51	133	92.4	92.7	93
Te	52	134	89.1	89.4	90
I	53	135	87.5	87.8	87
Cs	55	137	80.2	80.7	81

4. Conclusions

We present for the first time the allowed GT and all the first forbidden excited state half-lives, excitation energies and log ft values of $\mathrm{N}=50$ and 82 waiting point nuclei using the schematic spherical pn-QRPA model. We did not use any quenching factor in our calculations. Our calculation fulfilled the model-independent Ikeda Sum Rule. We considered the relativistic terms in the non-unique first forbidden rank 0 and rank 1 transitions. The logft values and half-lives obtained by the pn-QRPA(WS) calculation showed a decent agreement with the available experimental data and the other theoretical results. Our study shows that FF transitions play a substantial role in the total β-decay half-lives. The calculations support the argument that the pn-QRPA model gives reliable prediction for neutron-rich nuclei. Pyatov's restoration method in
the framework pn-QRPA (WS) model may further improve the computed half-lives, which we hope to report as a future assignment.

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