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On the Cross-Scale Prospects of the Logarithmically Corrected Gravitational Potential: From Black Hole Singularities to Galactic Rotation

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Abstract

General Relativity (GR) has long been confronted with a fragmentation dilemma regarding black hole singularities and galaxy rotation curves: the former requires undetectable higher-dimensional quantum gravity to circumvent infinite curvature, while the latter similarly relies on undetectable dark matter to provide additional gravitational force. In this paper, we abandon the hypothesis of undetectable entities and reveal that the two challenges may share an intrinsic geometric solution: the universal asymptotic behavior of mainstream dark matter halo models is equivalent to a logarithmically corrected gravitational potential $\Phi(r) \sim -(\ln r + 1)/r$, which originates from the self-response of the curvature divergence at the GR singularity ($R^{r}_{trt} \propto r^{-3}$) via Poisson integration. At the microscopic scale, the sign reversal of $\ln r$ generates a repulsive effect, thereby avoiding the singularity. The constructed logarithmically corrected Schwarzschild metric is rigorously solved via the Lambert W function, revealing a layered internal structure determined by the black hole mass M (with thickness $\propto 1/M$), which realizes the holographic screen of the renormalization group flow under the AdS/CFT correspondence. On this basis, we present parameter-free a priori predictions for the black hole shadows of Sgr A* and M87* that are consistent with Event Horizon Telescope (EHT) observations, and provide rigid falsifiable predictions for unobserved black holes, especially the crucial discriminative prediction for NGC315. On the galactic scale, the logarithmic term can fit the galaxy rotation curves of the Milky Way, Andromeda, and NGC2974 without the additional gravitational force from dark matter, and also successfully passes the test of the gravitational lensing phenomenon of the Bullet Cluster with good agreement with observations. On the other hand, the calculated solar system tidal difference ($\Delta g \sim 10^{-18} \mathrm{m/s^2}$) is far below the current experimental limit, ensuring the validity of the equivalence principle without the need for a shielding mechanism; meanwhile, the Solar System Parameterized Post-Newtonian (PPN) tests are also consistent with GR. This work demonstrates that gravitational phenomena from black holes to galaxies are governed by the spacetime self-response triggered by the GR singularity. It further reveals that macroscopic gravitational systems may be "holographic projections" of quantum topological structures (quantum vortices). This framework thus pulls quantum gravity research from pure mathematical modeling back to the energy scales accessible to contemporary observations, and provides a new direction for thinking about the unification of General Relativity and quantum mechanics.

Keywords: black hole; singularity; general relativity; gravitational potential; quantum gravity; event horizon telescope (EHT); metric; geodesic; AdS/CFT correspondence; quantum vortex; logarithmic correction; einstein field equations; black hole shadow; Sgr A*; M87*; NGC315; high-velocity stars; S62; S4714; dark matter; galaxy rotation curves; milky way galaxy; andromeda galaxy; NGC2974 galaxy; gravitational lensing; bullet cluster; equivalence principle; MOND; modified gravity

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1. Introduction

Modern astrophysics and gravitational theory have long faced two major cross-scale challenges: at the microscopic scale of black holes, singularities predicted by classical general relativity exhibit infinite curvature, violating the finiteness requirement of physical quantities in quantum mechanics, and the "information paradox" triggered by Hawking radiation remains unresolved; at the macroscopic scale of galaxies, the observed rotational velocities of peripheral stars and gas are much higher than the limit sustainable by the gravity of visible matter. The mainstream ΛCDM model relies on the hypothesis of unobserved dark matter halos, and there is tension between small-scale predictions and observations.

Traditional theories explain these two major problems in an isolated manner: black hole physics relies on the Kerr metric (requiring post-hoc fitting of spin and inclination), while galaxy dynamics relies on the dark matter hypothesis, both lacking a unified physical core. More critically, these theories either suffer from inherent incompleteness (such as singularities) or lack direct physical carriers (such as dark matter particles).

By analyzing the dynamics of various dark matter halo models, this paper discovers an overlooked and universal logarithmic asymptotic behavior: a simple logarithmic correction term is generated in the gravitational potential ($\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$), so can it solve both the black hole singularity and galaxy rotation curve problems? Subsequently, we will show a possible physical picture: at the black hole singularity distance ($r < r_* \approx 8.792 \times 10^{-11}\mathrm{m}$), the negative contribution of $\ln r$ makes the quantum gravitational potential repulsive, preventing matter from collapsing into singularities; at the black hole gravitational field and large galactic distances, the positive contribution of $\ln r$ provides additional gravity, offering a possibility to maintain the high-speed revolution of stars and the flattening of rotation curves without dark matter. This mechanism requires no mathematical renormalization or new unobservable entities (such as extra-dimensional strings, dark matter particles, etc.), and can be realized only by the own dynamics of ordinary matter: 1) desingularization; 2) flattening of rotation curves. To explain this mechanism, we provide a possible physical framework: relying on a physically supported carrier—quantum vortices, and a mathematical structure born in string theory to describe non-local entanglement—AdS/CFT correspondence, placing black hole physics and galactic dynamics under the same mechanism, and providing a new scheme for solving cross-scale gravitational problems that is only based on observable general relativity and quantum mechanics but does not rely on current traditional physical models (such as unobservable extra-dimensional strings, dark matter particles, etc.).

2. Universal Logarithmic Asymptotics of Dark-Matter Halo Models

A wide variety of dark-matter halo profiles have been proposed to explain the flat rotation curves of galaxies, including cuspy profiles derived from N-body simulations and phenomenological cored profiles motivated by observations. Despite their apparent diversity, we show that all commonly used halo models converge asymptotically to the same effective gravitational behavior, characterized by a logarithmic potential. This universality strongly suggests that the logarithmic term represents the true physical content of halo modeling, while the detailed density profiles merely encode different regularizations of the same asymptotic structure.

2.1. General Condition for Flat Rotation Curves

For a test particle on a circular orbit, the centripetal acceleration satisfies $\frac{v^2(r)}{r} = g(r) = \frac{GM(r)}{r^2}$, where $M(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$. A flat rotation curve: $v(r) \to v_0 = const$, implies $g(r) \sim \frac{v_0^2}{r}$, $M(r) \sim \frac{v_0^2}{G} r$. Differentiating, $\rho(r) = \frac{1}{4\pi r^2}\frac{dM}{dr} \sim \frac{1}{r^2}$. However, no realistic halo model maintains $\rho \sim r^{-2}$ at arbitrarily large radii, as this would lead to divergent total mass. Consequently, all viable models steepen to $\rho(r) \sim r^{-3} \quad (r \gg r_s)$, which leads to

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$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2} \tag{1}$$

This logarithmic behavior is therefore not model-dependent but a mathematical consequence of mass convergence combined with extended flat rotation curves. In addition, we note that the gravitational acceleration with logarithmic behavior ($g(r) \sim \frac{\ln r}{r^2}$) will have a sign reversal of the logarithmic term "$\ln r$" at extremely short (microscopic) distances, which is likely to form a dynamic mechanism that repels classical gravity ($g(r) \sim \frac{1}{r^2}$) to naturally "desingularize".

2.2. Cuspy Halo Models: NFW and Einasto [1–3]

2.2.1. NFW Profile

The NFW profile: $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$, satisfies $\rho(r) \sim r^{-3} \quad (r \gg r_s)$.

Integrating:

$$M(r) \propto \ln\left(1 + \frac{r}{r_s}\right), \text{ and thus } g(r) = \frac{GM(r)}{r^2} \sim \frac{\ln r}{r^2}.$$

The logarithmic term therefore arises inevitably from the outer density tail, not from any detailed inner structure.

2.2.2. Einasto Profile

The Einasto profile: $\ln \rho(r) = \ln \rho_0 - \left(\frac{r}{r_0}\right)^\alpha, \quad \alpha \ll 1$, admits the expansion:

$$\left(\frac{r}{r_0}\right)^\alpha \simeq 1 + \alpha \ln\frac{r}{r_0} + \mathcal{O}(\alpha^2).$$

Hence: $\rho(r) \approx \rho_0 r^{-\alpha}$, which again steepens toward an effective r^{-3} behavior at large radii, yielding $M(r) \sim \ln r$.

The shape parameter α merely controls how rapidly the logarithmic regime is approached.

2.3. Cored Halo Models: Burkert and Pseudo-Isothermal

Cored profiles replace the inner cusp with a constant-density core but retain the same outer asymptotics.

For example, the Burkert profile: $\rho(r) = \frac{\rho_0}{(1+r/r_0)(1+(r/r_0)^2)}$, satisfies $\rho(r) \sim r^{-3} \quad (r \gg r_0)$, leading again to

$$M(r) \sim \ln r, \quad g(r) \sim \frac{\ln r}{r^2}.$$

Thus, core formation modifies only the inner boundary conditions, leaving the outer logarithmic behavior intact.

2.4. Self-Interacting and Wave Dark Matter [4,5]

Self-interacting dark matter (SIDM) and fuzzy/wave dark matter (FDM) models generate cores through microphysical mechanisms (collisions or quantum pressure). Nevertheless, in all cases the outer halo relaxes to an NFW-like tail, $\rho(r) \to r^{-3}$, ensuring $M(r) \sim \ln r \quad and \quad g(r) \sim \frac{\ln r}{r^2}$.

Hence, these models do not introduce new large-scale gravitational behavior, but merely regulate the inner halo.

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2.5. Universality of the Logarithmic Potential

Since $g(r) = \frac{d\Phi}{dr}$, the asymptotic form $g(r) \sim \frac{\ln r}{r^2}$ corresponds to an effective potential:

$$\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r} \tag{2}$$

We emphasize that this logarithmic potential is not a peculiarity of any specific halo model, but a universal asymptotic structure shared by all viable dark-matter halo parametrizations.

3. Logarithmic Quantum Gravity for Physical Singularity Resolution

Important note: This study first adopts a "bottom-up" effective theory approach, aiming to find the minimal testable gravitational correction that can uniformly describe gravitational phenomena from the black hole to the galaxy scale. Its starting point is the induction of the universal logarithmic asymptotic structure shared by successful dark matter halo models, rather than the deduction from a certain fundamental action. Therefore, the metric constructed in this paper and its microscopic physical interpretation (such as the nested AdS/CFT correspondence) should be regarded as an effective framework that generates testable predictions and inspires future theories. Subsequently, we analyze the metric in a "top-down" manner in Section 4.6, which verifies the validity of the microscopic picture. Furthermore, all core predictions of this theory (such as black hole shadows and high-velocity stars) are directly derived from this logarithmically corrected potential. Its validity is judged by the cross-scale consistency with observations, and does not rely on the deduction from a fundamental action.

3.1. Core Physical Assumptions

This section aims to provide a possible microscopic physical picture for the logarithmic correction jointly derived from the dark matter halo density ($\rho(r) \sim r^{-3}$) induced in Section 2 and the GR singularity behavior ($R^{r}_{trt} \propto r^{-3}$, detailed in Sections 3.2 and 3.3), rather than a rigorous quantum field theory derivation. We adopt a bottom-up effective theory approach, inspired by quantum tornado experiments (where vortices emerge in environments simulating the "near-black hole" regime) [6], and propose the "quantum vortex" as the topological carrier of nonlocal entanglement. We also qualitatively describe the ultraviolet regularization mechanism by means of the nested AdS/CFT correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). The main purpose of this picture is to demonstrate how the logarithmic correction naturally achieves both physical singularity resolution and macroscopic extra gravity simultaneously, rather than serving as a necessary foundation of the theory.

1. **Quantum vortex topological structure**: We describe it as the statistical (geometric) average microscopic topological carrier of fermion fields, boson fields and gauge fields. Under the WKB approximation, its possible operator form is given by an effective composite operator. On the strongly coupled/CFT boundary, it is characterized by the amplitude and phase of its expectation value, which is essentially a nonlocal topological condensate field:

$$\phi_{vortex}(x) = \int d^4 y \sqrt{-g(y)} K(x,y) \mathcal{O}_{vortex}(y) \tag{3}$$

$$K(x,y) = \frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}} \tag{4}$$

$$\mathcal{O}_{vortex}(y) = \langle \langle \bar{\psi}\psi \rangle (y) \phi(y) \mathcal{A}_{\mu\nu}(y) \mathcal{A}^{\mu\nu}(y) \rangle^{1/2} \tag{5}$$

- $\bar{\psi}\psi$: Fermion field, with dimension $[\bar{\psi}\psi] = L^{-3}$
- ϕ: Boson field, with dimension $[\phi] = L^{-1}$

- $\mathcal{A}_{\mu\nu} \equiv (B_{\mu\nu}, W^a_{\mu\nu}, G^a_{\mu\nu})$: Unified field strength tensor, which can be regarded as coupled by electromagnetic, strong, and weak forces (excluding classical gravity) through non-local entanglement, $[A_{\mu\nu}] = [A^{\mu\nu}] = L^{-2}$, so $[\phi_{vortex}] = L^{-4}$, which is the Lagrangian density.
- $g(y)$: metric determinant;
- $K(x,y) = \frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}}$: non-local kernel function (Green's function with vortex phase), with $\Delta \approx 2$ (the coexisting dimension of quantum vortices in four-dimensional spacetime). The non-locality of the statistically averaged vortex phase $e^{iC\theta(x\cdot y)}$ provides a potential mechanism for the path integral $\int d^4y\sqrt{-g(y)}\frac{e^{iC\theta(x\cdot y)}}{|x-y|^4}$ to avoid ultraviolet divergence (mathematically, oscillatory integrals can act as a regularizer in certain cases, similar to the Riemann-Lebesgue lemma, but a rigorous mathematical proof is complex and only a heuristic application for constructing the physical picture is presented here);
- $e^{iC\theta(x,y)}$: Vortex (nonlocal) phase, representing the vortex winding between spacetime points x and y, while O_{vortex} serves as the statistically averaged coarse-grained local order parameter for the excitations of fermions, bosons and gauge fields;
- C: Central charge (topological charge number);
- $\theta(x,y) \sim \arctan\left(\frac{y_2-x_2}{y_1-x_1}\right)$: Topological phase;
- The vortex winding number (quantized winding) is obtained from the central charge C and topological phase $\theta(x,y)$: $W = \oint_C \nabla\theta \cdot dl$, and the conformal dimension relationship under AdS/CFT correspondence ($\Delta \sim \frac{W^2}{4\pi^2 C}$) can directly calculate this winding number W.

It should be noted that quantum vortices (fields) under statistical average do not violate the "Pauli exclusion principle". Firstly, the vortex phase $e^{iC\theta(x,y)}$ in the operator indicates statistical average under non-local entanglement; secondly, the apparent structure of this topological condensate field may be mainly located near black holes with huge spacetime curvature (extending the "quantum tornado" experiment to a more macroscopic cosmic scale). The Heisenberg uncertainty principle: $\Delta p \approx \hbar/\Delta x \to \infty$ (because $\Delta x \to 0$ under huge spacetime curvature), thus the "Pauli exclusion principle" is weakened by the huge spacetime curvature.

The construction of the aforementioned quantum vortices (fields) does not follow the standard quantum field theory treatment, and its mathematical rigor requires further refinement in subsequent work. However, recent experiments such as quantum tornadoes (which exhibit vortex lattice structures in superfluid helium) provide certain support for this physical picture (our construction is precisely inspired by such experiments).

The above expressions for quantum vortices ($\mathcal{O}_{vortex}$) and their scalar fields (ϕ_{vortex}) are heuristic constructions, used to illustrate how nonlocal phase oscillations avoid ultraviolet divergence and generate an effective r^{-3} source term. Strict quantum field theory treatment is left for future work.

2. **Nested AdS/CFT duality**: A hierarchical structure $AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$ [7,8] is adopted to correlate the quantum spacetime inside the black hole with the external classical spacetime through the conformal boundary, realizing the quantitative description of nonlocal entanglement. Thus, there exists a physical mechanism for the path integral $\int d^4y\sqrt{-g(y)}\frac{e^{iC\theta(x,y)}}{|x-y|^4}$ to avoid ultraviolet divergence (combining the nonlocality of the phase $e^{iC\theta(x\cdot y)}$). Using the nested structure $AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$, the AdS_4 bulk spacetime is dual to the CFT_2 boundary: $\int d^4y\sqrt{-g(y)}\frac{e^{iC\theta(x\cdot y)}}{|x-y|^4} \Leftrightarrow \int d^2y\sqrt{-g(y)}\frac{e^{iC\theta(x\cdot y)}}{|x-y|^3}$. It can be seen that the term $|x-y|^{-3}$ appears in the integral ($\int d^2y$), which is similar to the asymptotic density behavior of dark matter halos ($\rho(r) \sim r^{-3}$). Combined with the fact that the logarithmic asymptote of the potential ($\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$) is obtained after integrating the density asymptote (also $\int d^2y$), their "essences" may be similar: the universal logarithmic asymptote of dark matter halos in the AdS_4 bulk spacetime may be the logarithmic behavior after being dual to the CFT_2 boundary, and the sign reversal of the logarithmic term "lnr" at the microscopic

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scale can repel the classical gravitational potential ($-GM/r$) to avoid collapse to a "singularity". In other words, through this "holographic mapping", the ultraviolet divergence problem in the four-dimensional bulk spacetime is transformed into an integral on a two-dimensional boundary, and the output of this integral (after appropriate mapping) exactly gives the r^{-3} source term required in the three-dimensional physical space to generate the logarithmic potential. Thus, the divergence is "avoided".

The nested AdS/CFT correspondence is mainly used here to intuitively understand how the integral form $\int |x-y|^{-4}$ in the four-dimensional bulk spacetime is mapped to the boundary integral $\sim \int |x-y|^{-3}$ through dimensional reduction, and mathematically corresponds to the $\rho(r) \sim r^{-3}$ asymptote of dark matter halos.

3.2. Construction of Key Formulas (with Physical Heuristics; More Rigorous Derivation is Detailed in Appendix A at the End of the Paper)

3.2.1. Modified Poisson Equation

Based on the quantum vortex as the carrier of the microscopic topological structure, we regard its statistical average field (quantum vortex field) ϕ_{vortex} as a dynamic subsystem satisfying the effective field theory under the high-energy background inside the black hole. Considering the nonlocal entanglement characteristics and scale relativity of this system, its dynamics can be described by a modified d'Alembert operator under the CFT boundary approximation: $\Box\phi_{vortex} \approx (k\hbar/c^2)\partial_t^2\phi_{vortex} - \nabla^2\phi_{vortex} = 0$, where k is a dimensionless factor characterizing the strength of nonlocal entanglement. Further analysis shows that in the critical region near the boundary, the time evolution derivative term of the field $\partial_t^2\phi_{vortex}$ may have self-similarity due to non-local entanglement ($\partial_t^2\phi_{vortex} \approx a^2\left(\partial_t\phi_{vortex}\right)^2 \sim a^2(M/t)^2$, similar to the statistical average logic of "Reynolds stress" in turbulence).

Since the additional corrected gravitational potential ($\Phi_{\hbar alo}(r) \sim -\frac{\ln r+1}{r}$) is incompatible with the traditional Newtonian gravitational potential ($-\frac{GM}{r}$), it is necessary to add additional gravity to the original Newtonian gravitational potential ($\Phi(r) = -\frac{GM}{r} + \Phi_{halo}(r)$), which will also change the Schwarzschild metric: $ds^2 = -\left(1-\frac{2GM}{c^2r} + term1_{halo}(r)\right)c^2dt^2 + \left(1 + \frac{2GM}{c^2r} + term2_{halo}(r)\right)dr^2 + r^2 \mathrm{d}\Omega^2$ (the original Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r)$: $\left(1-\frac{2GM}{c^2r}\right)^{-1} = 1 + \frac{2GM}{c^2r} + \left(\frac{2GM}{c^2r}\right)^2 + \cdots \sim 1 + \frac{2GM}{c^2r} + term2_{halo}(r)$). Among them, the time component of the modified metric: $g_{tt} = -\left(1-\frac{2GM}{c^2r} + term1_{halo}(r)\right)c^2$. Since $g_{tt} > 0$ inside the black hole, the spacetime is spacelike: time is radialized like space, so the time evolution of the field can be re-calibrated as spatial behavior.

Therefore, after coordinate re-calibration, the quadratic term contribution ($\propto \frac{M^2}{t^2}$) caused by the self-similarity of the quantum vortex field (statistically averaged field ϕ_{vortex}) inside the black hole is approximately equivalent to an additional gravitational source term inversely proportional to the cube of the distance ($\propto \frac{M^2}{r^3}$). This substitution naturally connects the universal asymptotic density structure $\rho(r) \sim r^{-3}$ obtained for dark matter halos at large radii, because in the quadratic integral operation of solving the Poisson equation $\nabla^2\Phi = 4\pi G\rho$, only this source term ($\propto \frac{M^2}{r^3}$) can uniquely generate a logarithmic potential to reverse the potential direction and prevent collapse; other source terms such as $\propto \frac{M^2}{r^2}$ and $\propto \frac{M^2}{r^4}$ cannot prevent collapse and desingularize. Moreover, we also find that $\rho(r) \sim r^{-3}$ is surprisingly consistent with the boundary behavior $R^{r}_{trt} \propto r^{-3}$ of the Riemann tensor component R^{r}_{trt}, indicating that the divergent behavior of classical general relativity itself near the singularity ($R^{r}_{trt} \propto r^{-3}$) may have inherently contained a source term ($\rho(r) \sim r^{-3}$) that prevents curvature divergence. It also means that spacetime will spontaneously form a response to the divergent behavior near the singularity ($R^{r}_{trt} \propto r^{-3}$), and this response can produce a finite observable result (such as a black hole shadow) in a non-perturbative manner. In the following, we will initially verify the feasibility of this "desingularization" scheme by "a priori predicting" the angular diameter of black hole shadows (consistent with EHT observations) using the logarithmically modified Schwarzschild metric.

Introducing this equivalent source term into the density ρ in the classical Poisson equation ($\nabla^2\Phi = 4\pi G\rho$) gives the CFT_2 boundary modified Poisson equation:

$$\nabla^2\Phi = 4\pi G\left(M\delta^3(r) + \frac{kG_h M^2}{4\pi G r^3}\right) \tag{6}$$

where $M\delta^3(r)$ is the classical gravitational point mass source term ($\delta^3(r)$ is the three-dimensional Dirac delta function), and $\frac{kG_h M^2}{4\pi G r^3}$ is the additional gravitational correction source term (i.e., the source term of the universal asymptotic structure $\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$ derived from dark matter halo models), which we call "quantum gravity". k is the non-local entanglement relative strength factor ($k = \frac{M_{BH,ref}}{M_{BH,topo}}$, where $M_{BH,ref}$ is the reference black hole mass, and $M_{BH,topo}$ is the topological black hole mass (equal to the target black hole mass M in the strong field regime), which provides the quantum gravity background). The Galactic center black hole Sgr A* is usually taken as the reference: $k = \frac{M_{SgrA*}}{M_{BH,topo}}$. If another galactic center black hole is used as the reference, the benchmark G_h needs to be relatively transformed. For example, with M87* as the reference: $G_{h,M87*} = \frac{M_{SgrA*}}{M_{M87*}} G_h$, then $k_{M87*} = \frac{M_{M87*}}{M_{BH,topo}}$, so $k_{M87*}G_{h,M87*} = \frac{M_{M87*}}{M_{BH,topo}} \cdot \frac{M_{SgrA*}}{M_{M87*}} G_h = kG_h$, indicating that the value of kG_h is independent of the chosen reference black hole.

G_h is the quantum gravitational constant ($G_{\hbar} = \hbar c^2 G^3/8 \approx 3.5224 \times 10^{-49}\,\mathrm{kg^{-2}m^3s^{-2}}$). If we substitute the standard dimensionality of the Planck constant into the expression, we obtain $[G_h] = \mathrm{kg \cdot m^2s^{-1} \cdot m^2s^{-2} \cdot kg^{-3}m^9s^{-6} = kg^{-2}m^{13}s^{-9}}$, which breaks the covariance of the quantity. To preserve the dimensional covariance, the "dimensional compactification" from the nested duality must be invoked. Our framework incorporates the nested AdS/CFT correspondence ($AdS_4/CFT_3 \supseteq AdS_3/CFT_2 \supseteq AdS_2/CFT_1$). In this picture, the effective (reduced) Planck constant $\hbar$, which is dual from the AdS_4 bulk spacetime of the microscopic quantum vortex structure to the CFT_2 boundary, undergoes a dimensionality change from $\mathrm{kg \cdot m^2s^{-1}}$ to $\mathrm{kg \cdot m^{-8}s^6}$ due to the compactification (caused by dimensional reduction) of the coupled spacetime dimensions (including the fluctuation dimension and phase dimension of the gauge group). Owing to the intrinsic nature of information transfer in the AdS/CFT correspondence, the numerical value of $\hbar$ remains unchanged when dualized to the CFT_2 boundary, while its dimensionality is modified via dimensional compactification (involving changes in the fluctuation dimension of the gauge group and the phase dimension) by a dimensional compactification factor of $\mathrm{m^{-10}s^7}$. This dimensional conversion is incorporated into the definition of G_h, resulting in its final dimensionality of $\mathrm{kg^{-2}m^3s^{-2}}$ and thus maintaining the overall dimensional covariance. (Experimental evidence supporting this hypothesis: when quantum vortices in superfluid helium are confined in a nanoscale space (to simulate dimensional compactification), their vortex phase oscillation energy satisfies $E \propto \hbar_{eff}\omega$, where $\hbar_{eff} \propto d^{-8}$ (d: confinement scale), consistent with the $\mathrm{m^{-8}}$ dimensionality [9]. Of course, its rigorous mathematical proof is highly complex, and the current stage is still dominated by physical heuristics.).

This work adopts a bottom-up effective theory approach, naturally deriving logarithmic correction from the inherent $R^r_{trt} \propto r^{-3}$ curvature divergence in GR and the summarized universal asymptotic behavior of dark matter halos $\rho(r) \sim r^{-3}$. The microscopic explanation only serves as qualitative support and does not affect the consistency between major predictions and observations.

We acknowledge that the constructions of quantum vortices, nested AdS/CFT correspondence, and modified Poisson equations are more to provide a proposed physical picture for the discovered universal asymptotic behavior of dark matter halos that maximally adheres to Occam's Razor principle (entities should not be multiplied beyond necessity). This explanatory framework is still in its early stages, and rigorous field theory derivation will be the core of future work. However, at the current stage, it does not affect its powerful cross-scale empirical predictive ability and unity. In the history of physics, there are many precedents for constructing effective theories based on profound empirical laws (such as Kepler's laws for Newtonian mechanics, black-body radiation for quantum theory, etc.).

3.2.2. Modified Gravitational Potential with Logarithmic Term

Solving the modified Poisson equation above yields the general solution:

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_hM^2(\ln r + 1)}{r} - \frac{C_1}{r} + C_2$$

With the boundary condition that $\Phi(r) \to 0$ as $r \to \infty$, we have $C_2 = 0$, and the term $-C_1/r$ can be absorbed into the classical Newtonian term $-GM/r$. The expression is thus simplified to

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_hM^2(\ln r + 1)}{r} \tag{7}$$

This equation consists of two terms:

- **Classical gravitational term** $-\frac{GM}{r}$: Dominates conventional gravitational effects, consistent with Newtonian gravity and the weak-field approximation of general relativity.
- **Quantum gravitational logarithmic term** $-\frac{kG_hM^2(\ln r+1)}{r}$ (consistent with $\Phi_{halo}(r) \sim -\frac{\ln r+1}{r}$): Serves as the core cross-scale correction term. Its effect depends on the magnitude of the distance r —exhibiting repulsive behavior at short distances (black hole "singularity" scale) and gravitational enhancement at long distances (galaxy scale). Essentially, it is likely a macroscopic manifestation of nonlocal entanglement of quantum vortices under the hierarchical nested structure ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$).
- Note: In the International System of Units (SI) adopted in this paper, the distance variable r is measured in meters (m). Since the argument of a logarithmic function must be a dimensionless quantity, the expression $\ln r$ should be interpreted as $\ln(r/r_*)$, where r_* is the intrinsic universal scale of $\Phi(r)$ (defined by $\Phi(r_*) = 0$). Writing the logarithmic term as $\ln(r/r_*)$ only introduces an additional term $\frac{kG_hM^2\ln r_*}{r}$, which is equivalent to the renormalization of the GM parameter in the Newtonian gravitational potential (i.e., $C_1 = -kG_hM^2\ln r_*$). That is to say, in astronomical observations where GM is calibrated via dynamical or lensing measurements, this choice does not alter the primary observables (what is actually calibrated observationally is the effective gravitational parameter $GM_{eff} = GM + C_1$). Therefore, the calculation results obtained by normalizing r_* to 1 m and using $\ln(r/1)$ (with the effective GM_{eff} calibrated accordingly) are completely equivalent to those obtained by strictly using $\ln(r/r_*)$ (with the theoretical calibration of $GM + C_1$). In all practical calculations (black hole shadow, high-velocity stars, galaxy rotation curves, gravitational lensing, etc.), directly substituting the numerical value of r in meters and taking its natural logarithm $\ln r$ is equivalent to $\ln(r/1)$. This convention applies throughout the paper and will not be reiterated in subsequent sections.

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- If the quantum gravitational effect under non-local entanglement is not considered ($k = 0$, i.e., ignoring the black hole: $M_{BH,ref} = 0$), the gravitational potential automatically degenerates into the classical gravitational potential: $\Phi(r) = -\frac{GM}{r}$, and the framework also naturally degrades to the classical gravitational framework.

3.3. Cross-Scale Physical Nature of the Logarithmic Term

The unique properties of the logarithmic term $\ln r$ are the key to realizing "short-range repulsion and long-range attraction":

- When $r \to 0$ (black hole core region): $\ln r$ tends to negative infinity, and the quantum gravitational term transforms into a strong repulsive potential. When $r < r_* = e^{-1-\frac{G}{kG_h M}} = e^{-1-\frac{G}{G_h M_{SgrA*}}} \approx 8.792 \times 10^{-11}$m (in the strong field regime, $k = M_{SgrA*}/M_{BH,topo} = M_{SgrA*}/M$), $\Phi(r) > 0$ in the total potential, dynamically preventing matter from collapsing into a singularity.
- When r is sufficiently large (galactic peripheral region): $\ln r$ is a positive finite value, and the quantum gravitational term provides additional gravity logarithmically dependent on distance, compensating for the insufficient gravity of visible matter and maintaining the stable rotational velocity of stars.

This characteristic stems from the monotonicity and boundary behavior of the logarithmic function. No additional adjustment of physical mechanisms is required; a single mathematical form can adapt to the scale transition from the microscopic to the macroscopic, reflecting the simplicity and self-consistency of the theory.

4. Black Hole Scale Application: Physical Avoidance of Singularity, Shadow Prediction and High-Speed Stars

4.1. Physical Avoidance of Singularity

In the core region of a black hole, the quantum repulsive potential dominated by the logarithmic term plays a central role:

- **Suppression of curvature divergence**: The repulsive potential prevents matter from reaching $r = 0$ and avoids the divergence of spacetime curvature, thus realizing the dynamical avoidance of singularities without the need for renormalization.
- **A potential mechanism for resolving the information paradox**: The repulsive potential excites virtual particles from vacuum fluctuations into real particles, which tunnel out of the black hole horizon through the nested AdS/CFT correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). These real particles carry information away from the black hole, and the black hole loses mass synchronously. This mechanism is conducive to making black hole physics satisfy the unitarity of quantum mechanics, namely the principle of information conservation.

4.2. Logarithmically Corrected Schwarzschild Metric and A Priori Prediction of Black Hole Shadows

Based on the modified gravitational potential, the quantum-corrected Schwarzschild metric is derived (substituting $\Phi(\mathrm{r})$ into the relationship between the metric and gravitational potential under the weak-field approximation of general relativity):

$$ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2d\Omega^2$$

$$A(r) \approx 1 + \frac{2\Phi(r)}{c^2} = 1 - \frac{2GM}{c^2 r} - \frac{2kG_h M^2(\ln r + 1)}{c^2 r} \quad (8)$$

Where: $term1_{halo}(r) = -\frac{2kG_h M^2(\ln r+1)}{c^2 r}$

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$$B(r) \approx 1 - \frac{2\Phi(r)}{c^2} = 1 + \frac{2GM}{c^2 r} + \frac{2kG_h M^2(\ln r + 1)}{c^2 r} \tag{9}$$

Where: $term2_{halo}(r) = \frac{2kG_h M^2(\ln r+1)}{c^2 r}$

Analysis of Metric Singularity Resolution

For motion in the equatorial plane with $\theta = \pi/2$ and $\dot{\theta} = 0$, the Lagrangian is given by: $L = -Ac^2\dot{t}^2 + B\dot{r}^2 + r^2\dot{\varphi}^2$. From Killing symmetry, the conserved energy and angular momentum are obtained as: $E = A(r)c^2\dot{t}$ and $L = r^2\dot{\varphi}$ (with respect to the affine parameter λ). The general "first integral" radial equation is derived (where $\sigma = 1$ for timelike geodesics (proper time τ); $\sigma = 0$ for null geodesics (affine parameter λ)):

$$\dot{r}^2 = \frac{E^2 - A(r)\left(\frac{c^2L^2}{r^2} + \sigma c^4\right)}{ABc^2}$$

along with: $\dot{t} = \frac{E}{Ac^2}$ and $\dot{\varphi} = \frac{L}{r^2}$. We define the effective potential as: $V_\sigma(r) = A(r)\left(\sigma c^4 + \frac{c^2L^2}{r^2}\right)$.

Analysis of timelike geodesics ($\sigma = 1$): the effective potential $V_{timelike}(r) = A(r)\left(c^4 + \frac{c^2L^2}{r^2}\right)$ tends to $+\infty$ as $r \to 0$ (since $A(r) \approx 1 - \frac{2GM}{c^2 r} - \frac{2kG_h M^2(\ln r+1)}{c^2 r} \to +\infty$). For any finite energy E and angular momentum L, the radial equation

$$\dot{r}^2 = \left(\frac{dr}{d\tau}\right)^2 = \frac{E^2 - A(r)\left(\frac{c^2L^2}{r^2} + c^4\right)}{ABc^2} = \frac{E^2 - V_{timelike}(r)}{ABc^2}$$

must satisfy $E^2 = V_{timelike}(r_{min})$ at some $r_{min} > 0$—a bounce occurs, meaning the particle cannot physically reach $r = 0$. The analysis for null geodesics ($\sigma = 0$) is essentially the same (the effective potential $V_{null}(r) = A(r)\left(\frac{c^2L^2}{r^2}\right) \to +\infty$ as $r \to 0$), so light cannot physically reach $r = 0$ either.

This metric does not require fitting of black hole spin and inclination; the shadow angular diameter can be predicted solely by the black hole mass M and distance D.

The shadow radius is taken as the geometric mean of the modified event horizon r_h and the modified photon sphere r_{ph}:

$$r_{sh} \approx \sqrt{r_h r_{ph}} \tag{10}$$

and the angular diameter: $\theta_{sh} = 2r_{sh}/D$.

For equatorial null geodesics, let the impact parameter $b \equiv \frac{Lc}{E}$. The closest distance r_0 satisfies $b^2 = \frac{r_0^2}{A(r_0)}$, leading to the logarithmic correction (Gravitational Lensing Deflection Angle):

$$\hat{\alpha}(b) = 2\int_{r_0}^{\infty} \frac{dr}{r}\sqrt{\frac{B(r)}{A(r)}}\left[\left(\frac{r}{r_0}\right)^2 \frac{A(r_0)}{A(r)} - 1\right]^{-1/2} - \pi \tag{11}$$

Analysis shows that in the strong-field regime, the deflection angle diverges as the closest distance approaches the photon sphere $r_0 \to r_{ph}$. The additional logarithmic correction to $A(r)$ causes two key effects: the photon sphere radius shifts outward compared to the standard

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Schwarzschild metric; the divergence point appears earlier, trapping light rays sooner. Thus, any light ray attempting to graze the event horizon will undergo severe deflection and will not actually contribute to the "sharply imaged" light path—multiple diffracted orbits cannot form a stable image. Therefore, the size of the black hole shadow and bright ring is mainly determined by the geometry of the Schwarzschild metric with logarithmic correction, rather than the superposition of numerous deflected light rays. In other words, the truly imaging light paths near the black hole originate from the stable luminous ring at the edge of the shadow (accretion disk or plasma emission), not from complex multiple diffractions. Specifically, the observed annular emission of black holes is almost the real emission distribution from the inner edge of the nearby accretion disk (the region where particles escape the black hole through tunneling via nested AdS/CFT correspondence under nonlocal entanglement due to the repulsive potential $\Phi(r) > 0$ from physical singularity resolution), rather than an illusion formed by "bent and diffracted light". This also means that the critical impact parameter ($b_c = \frac{r_{ph}}{\sqrt{A(r_{ph})}}$) under the Schwarzschild metric with logarithmic correction no longer characterizes the black hole shadow radius.

According to the hierarchical correspondence mechanism we adopted ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$): particles (including but not limited to photons) inside the black hole escape the black hole through quantum tunneling due to the reversal of the total potential direction ($\Phi(r) > 0$), and the imaging interval is between the modified horizon and the modified photon sphere: (r_h, r_{ph}). The quantum term of the logarithmically modified Schwarzschild metric is proportional to $\frac{\ln r+1}{r}$, implying that under non-local entanglement, the logarithmic coordinate $\ln r$ is more natural than the linear coordinate r. Thus, we perform the variable substitution $x = \ln r$, and the tunneling interval (r_h, r_{ph}) is transformed into $(\ln r_h, \ln r_{ph})$ (i.e., (x_h, x_{ph})).

For photons tunneling from r_h to $r\left(\in (r_h, r_{ph})\right)$ for imaging, the tunneling probability density under the WKB approximation is: $P(x) \propto e^{-2S(x)}$ (action: $S(x) \sim \int_{x_h}^{x} \sqrt{2m(V(x)-E)}\frac{dr}{dx}dx$). From the total potential $\Phi(\mathrm{r})$, it is known that the potential barrier originates from the logarithmic term of the quantum gravitational potential: $V(x) \sim V_0 + a(\ln r + 1)/r = V_0 + a\frac{x+1}{e^x}$. In the tunneling imaging interval $r \in (r_h, r_{ph})$, we make a linear approximation: expand $\sqrt{V(x)-E}$ to the first order and approximate it as a linear function in the interval (x_h, x_{ph}): $\sqrt{V(x)-E} \approx \alpha + \beta(x - x_c)$, where x_c is a certain midpoint. Thus, the action S(x) becomes a quadratic function of x: $S(x) \approx S_0 + A(x-x_c) + B(x-x_c)^2$, and therefore: $P(x) \propto e^{-2S(x)} \sim e^{-2B(x-x_c)^2} \cdot f(x)$ (where f(x) is a slowly varying factor). That is to say, in the tunneling imaging interval (r_h, r_{ph}), the tunneling probability follows a Gaussian distribution, so the imaging of photons in the interval (r_h, r_{ph}) becomes a Brownian random equilibrium in the logarithmic interval $(\ln r_h, \ln r_{ph})$ (i.e., (x_h, x_{ph})). Therefore, the tunneling steady state is naturally located at the arithmetic mean of the logarithmic interval $(\ln r_h, \ln r_{ph})$: $\ln r_{sh} \approx (\ln r_h + \ln r_{ph})/2$, and converting back from the logarithmic coordinate to the linear coordinate gives: $r_{sh} \approx \sqrt{r_h r_{ph}}$.

(We emphasize: this shadow radius is derived based on the Schwarzschild metric with logarithmic correction after physical singularity resolution. Although it differs from standard general relativity with singularities, it does not violate its fundamental logic (the theory originates from the spacetime self-response to the curvature divergence behavior near the singularity ($R^{r}_{trt} \propto r^{-3}$)), and as an effective theoretical framework, its ultimate evaluation criterion is observation.)

Similar to the modified gravitational potential, if the quantum gravitational effect under non-local entanglement is not considered ($k = 0$, i.e., ignoring the black hole: $M_{BH,ref} = 0$), the logarithmically corrected Schwarzschild metric strictly degenerates to the Schwarzschild metric, restoring standard general relativity (the Schwarzschild metric $B(r) = 1/A(r)$; Taylor expansion of $1/A(r)$ gives $\left(1-\frac{2GM}{c^2r}\right)^{-1}=1+\frac{2GM}{c^2r}+\left(\frac{2GM}{c^2r}\right)^2+\cdots$, and higher-order terms are omitted, leading to $\left(1-\frac{2GM}{c^2r}\right)^{-1}\sim 1+\frac{2GM}{c^2r}$).

In addition, according to this metric: at "infinite distance" from the black hole: $ds^2\approx -c^2dt^2+dr^2+r^2d\Omega^2$ (four-dimensional flat); near the black hole horizon ($g_{tt}\approx 0$): $ds^2\approx 2dr^2+r^2d\Omega^2$ (three-dimensional flat, because after variable substitution: $dl^2\approx 2\left(d\left(\sqrt{2}\rho\right)^2+\rho^2d\Omega^2\right)$)). According to conformal flatness, the inside and outside of the black hole form an AdS_4/CFT_3 correspondence, that is, the strong-field spacetime properties near the black hole "horizon" are similar to the weak-field spacetime properties at "infinite distance" from the black hole. Therefore, although the logarithmically modified Schwarzschild metric is obtained by substituting the modified gravitational potential into the weak field approximation of general relativity, it is applicable to the entire spacetime of strong and weak fields (the properties of this metric can be further extended to the nested correspondence inside the black hole ($AdS_2/CFT_1\subseteq AdS_3/CFT_2$), which together with the AdS_4/CFT_3 correspondence near the horizon forms a hierarchical correspondence ($AdS_2/CFT_1\subseteq AdS_3/CFT_2\subseteq AdS_4/CFT_3$). Due to the complexity of the strict mathematical proof (needing to further analyze the metric such as $g_{tt}=0$ and $g_{rr}=0$), we verify it by a priori calculating (predicting) the size of the black hole shadow and comparing it with EHT observations.

The Schwarzschild radius remains unchanged: $r_s=2GM/c^2$ $g_{tt}=0\rightarrow$ Horizon equation (where $M=M_{BH,topo}$):

$$c^2r=2GM+2kG_hM^2(\ln r+1) \tag{12}$$

Solving this equation yields the modified event horizon r_h.

For photons, $ds^2=0$, and $\dot{r}=0$ on circular orbits. Satisfying the extremum condition of the effective potential $\frac{d}{dr}\left(\frac{r^2}{A(r)}\right)=0$, the photon sphere equation is obtained (where $M=M_{BH,topo}$):

$$c^2r=3GM+kG_hM^2(3\ln r+2) \tag{13}$$

Solving this equation yields the modified photon sphere r_{ph}.

A Priori Prediction and Verification Results of Observed Black Hole Shadows [10,11]

Black Hole	Mass ($M_\odot$)	k-factor	Theoretical Shadow Angular Diameter (μas)	EHT Measured Value (μas)	Consistency
Sgr A*	4.3×10^6	1	53.3	51.8 ± 2.3	Within observational range

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M87*	6.5×10^9	6.61×10^{-4}	46.2	42 ± 3	1.4σ (reasonable error)

Compared with the traditional Kerr black hole model [12], this theory performs a priori prediction calculations with no free parameters (only the target black hole mass $M_{BH,topo}$ is required; the k-factor $k = M_{SgrA*}/M_{BH,topo}$ is already fixed, and theoretically, the shadow of a black hole of any mass can be predicted). The comparison with the shadows of two black holes observed by EHT verifies the effectiveness of this logarithmically corrected Schwarzschild metric in strong fields and the rationality of the hierarchical duality inside and outside the black hole ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$).

A common problem in fitting black hole shadows with the Kerr model is the non-uniqueness of the fitting combination of spin a and inclination angle i for the same black hole shadow. For example, regarding the observed shadow angular diameter of M87*, both the combination of spin ($a = 0.99$) + inclination angle ($i \approx 17°$) and spin ($a = 0.50$) + inclination angle ($i \approx 65°$) can satisfy the shadow fitting. Similarly, Sgr A* faces the same issue. Although the EHT collaboration later introduced multidimensional observational data (e.g., polarization structure, brightness distribution) to add constraints, this is more of a "patchwork approach" to "lock in" the most plausible solution in practice rather than eliminating degeneracy theoretically. In contrast, the logarithmically corrected Schwarzschild metric calculates black hole shadows without free parameters (only the black hole mass M and the mass ratio k relative to Sgr A* are required), fundamentally eliminating parameter degeneracy.

In summary, since this theory does not require a posteriori fitting of the Kerr black hole spin and inclination, it can uniquely a priori calculate the shadow radius size only by the black hole mass, and predict the observed shadow angular diameter according to the distance.

Based on this, we provide a priori predictions for six EHT candidate black holes ($\lesssim 3.5\sigma$) for reference by the EHT project team to verify this observable prediction (if the mass and distance of the black hole are relatively accurate, and the observed shadow angular diameter is significantly larger than 3.5σ, the theory is falsified).

A Priori Prediction of Shadow Angular Diameters for EHT Candidate Black Holes

Black Hole	**Mass ($M_\odot$)**	**k-factor**	**Distance Range (Mpc)**	**Shadow Radius r_{sh} (m) of Logarithmically Modified Schwarzschild Metric**	**Shadow Angular Diameter Range θ_{sh} (μas)**	**Kerr Fitting Range ($a \in [0, 0.99]$, $i \in [0°, 90°]$) θ_{Kerr} (μas)**
Centaurus A*	5.5×10^7	7.82×10^{-2}	3.4~4.2	4.47×10^{11}	1.4~1.8	1.3~1.7

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Black Hole	Mass ($M_{\odot}$)	k-factor	Distance Range (Mpc)	Shadow Radius r_{sh} (m) of Logarithmically Modified Schwarzschild Metric	Shadow Angular Diameter Range θ_{sh} (μas)	Kerr Fitting Range ($a \in [0, 0.99]$, $i \in [0°, 90°]$) θ_{Kerr} (μas)
NGC315	3.0×10^{9}	1.43×10^{-3}	65~72	2.64×10^{13}	4.9~5.4	3.9~4.8
NGC4261	1.6×10^{9}	2.69×10^{-3}	30~32	1.41×10^{13}	5.9~6.3	4.6~6.1
M84	1.5×10^{9}	2.87×10^{-3}	16~17.5	1.30×10^{13}	9.9~10.9	8.3~10.1
NGC4594	1.0×10^{9}	4.3×10^{-3}	9.0~10.0	8.61×10^{12}	11.5~12.8	9.6~12.0
IC1459	2.0×10^{9}	2.15×10^{-3}	21~30	1.75×10^{13}	7.8~11.1	6.4~9.8

Due to the inherent "parameter degeneracy" of the Kerr model, we selected the spin range $a \in [0,0.99]$ and inclination range $i \in [0°, 90°]$ for a full scan. Using the "area-equivalent diameter" $D_{eq} = 2\sqrt{A/\pi}$ (equivalent to converting the shadow area into the diameter of a circle with the same area), the shadow scale is normalized by GM/c^2, and D_{eq}/M varies slightly with a and i. Referring to references [13–15], the upper and lower bounds can be determined: the circular shadow diameter of a Schwarzschild black hole ($a = 0$) is $D = 6\sqrt{3}M \approx 10.392M$; the diameter can be reduced to approximately $9.6568M$ for extreme spin and axial viewing angles. Thus, the angular diameter interval of the Kerr metric can be estimated using a "geometric coefficient envelope" (which can be combined with the distance range):

$$\theta_{Kerr} \in \left[\frac{9.6568GM}{c^2 D}, \frac{10.392GM}{c^2 D}\right]$$

Finally, the maximum possible interval of shadow sizes for the six candidate black holes under all spin and inclination combinations (with almost no observational constraints) is obtained under the Kerr metric of vacuum geometry.

Subsequently, we compared the a priori prediction range of our theory with the maximum possible fitting interval of the Kerr metric (both our metric and the Kerr metric only consider the vacuum geometric limit). It can be seen that:

- Centaurus A*: The θ_{sh} overlaps the most, making it difficult to distinguish between the maximum fitting interval of the Kerr model and this theory;
- **NGC315 (Recommended Observation Target)**: The θ_{sh} is the easiest to distinguish, because the lower limit of this theory (4.9 μas) is already higher than the maximum fitting upper limit of the Kerr model (4.8 μas). As long as the EHT measures the diameter with a precision of ~2.5%, it will directly distinguish between this theory and the Kerr model; In other words, in contrast to Kerr models, whose shadow diameters can be adjusted over a broad range by spin and inclination, our metric yields a rigid lower bound on the shadow size (4.9 μas) determined solely by the black hole (e.g., NGC315) mass and distance. If future observations cluster near this lower bound ($\gtrsim 4.9\mu as$), the result would favor our geometry without invoking fine-tuned spin–inclination configurations (because when only considering the vacuum geometry of the Kerr metric, no matter how the spin and inclination are adjusted for NGC315, its fitting upper limit of 4.8 μas cannot reach near 4.9 μas). This means NGC315 becomes a crucial experimental source to distinguish our theory from the standard Kerr paradigm, allowing it to be directly and rapidly falsified by future EHT observations.
- NGC4261: The θ_{sh} overlaps more, making distinction relatively difficult;
- M84: If $\theta_{sh} > 10.1$ μas, it favors this theory;
- NGC4594: If $\theta_{sh} > 12.0$ μas, it favors this theory;
- IC1459: If $\theta_{sh} > 9.8$ μas, it favors this theory.

It should be noted that the interval given by our theory is a rigid prediction interval, which is essentially different from the maximum interval value among all possible spin-inclination ($a + i$) combinations given by the Kerr model: in the fitting of the Kerr metric, the observed shadow size will select a specific spin-inclination combination from a broad prior parameter space. In contrast, our metric only determines a narrow shadow range based on M (mass) and D (distance), with no additional degrees of freedom to accommodate observational results.

Width of Angular Diameter Interval

Black Hole	**This Theory (A Priori Prediction) $\Delta\theta$ (μas)**	**Kerr Model (Full Scan of Spin and Inclination) $\Delta\theta_{Kerr}$ (μas)**
Centaurus A*	0.4	0.4
NGC315	0.5	0.9
NGC4261	0.4	1.5
M84	1	1.8
NGC4594	1.3	2.4
IC1459	3.3	3.4

Further analysis shows that for the unobserved black hole shadows with given mass (M) and distance (D), after we perform a full scan of the spin and inclination, the shadow angular diameter interval (non-rigid prediction) given by the Kerr metric is significantly too large (due to "parameter degeneracy" + fluctuations in M and D). In contrast, the logarithmically modified Schwarzschild metric outputs a rigid prediction with an extremely narrow interval (only due to fluctuations in M and D) because there is no "parameter degeneracy", and the theory has strong falsifiability.

Of course, we acknowledge the achievements of the Kerr solution in black hole physics, as it is still very successful in explaining phenomena such as jet precession and gravitational waves. However, its inherent problems such as "singularities" and "parameter degeneracy" have long plagued the physics community. Therefore, while continuously patching up these inherent problems, we may also consider another possible direction: a theory that avoids "singularities" and eliminates "parameter degeneracy" through its own mechanism.

4.3. A Priori Calculation of Perihelion Velocities of High-Speed Stars (Orbiting Black Holes)

From the modified gravitational potential, the gravitational acceleration of the black hole gravitational field (strong field regime) is derived as:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM}{r^2} + \frac{kG_hM^2 \ln r}{r^2} \quad (14)$$

From the modified gravitational potential and the logarithmically corrected Schwarzschild metric, the circular orbital velocity of the black hole gravitational field (including but not limited to accretion disks) is obtained:

$$v_{obs}(r) = \frac{dr}{dt} = \frac{dr}{d\tau}\cdot\frac{d\tau}{dt} = v_{proper}\sqrt{A(r)} = \sqrt{r\frac{d\Phi}{dr}}\cdot\sqrt{A(r)}$$

$$= \sqrt{\frac{GM}{r} + \frac{kG_hM^2 \ln r}{r}}\cdot\sqrt{1 - \frac{2GM}{c^2r} - \frac{2kG_hM^2(\ln r + 1)}{c^2r}} \quad (15)$$

where $\sqrt{A(r)}$ is the time dilation factor of the black hole gravitational field.

Close-range high-speed stars orbiting black holes (such as S4714 and S62 around Sgr A*) are mainly affected by the black hole gravitational field, so their velocities orbiting the black hole can be calculated using Equation (15).

Comparison Between A Priori Calculated Theoretical Velocities of High-Speed Stars and Observations [16,17]

High-Speed Star	Black Hole Mass ($M_\odot$)	Closest Distance to Black Hole r (km)	k	$v(r)$ (km/s)	Observation Value (km/s)	Error
S4714	4.3×10^6	1.89×10^9	1	25943	24000	8.1%
S62	4.3×10^6	2.39×10^9	1	23159	20000	15.8%

It can be seen that the theoretical velocities of S4714 and S62 are within reasonable error ranges (24000 km/s (0.08c) is adopted as the "periastron" velocity for S4714 (cited in multiple studies with little controversy); there are discrepancies in the orbit and "periastron" velocity of S62 under different data processing and source identification schemes, and the conclusion of the GRAVITY Collaboration is inconsistent with the early 9.9-year orbital solution. This paper adopts the commonly used 20000 km/s (0.067c) in the literature as an order-of-magnitude estimate).

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The a priori calculation of the perihelion velocities of high-speed stars uses the same theoretical framework as the a priori prediction of black hole shadows, requiring only the black hole mass M and the distance r between the star and the black hole. Compared with the traditional method, which still needs to adjust the orbital eccentricity e and semi-major axis a to fit the observed velocity after knowing the black hole mass and the distance between the star and the black hole (e.g., orbital velocity based on standard general relativity: $v(r) \approx \sqrt{\frac{GM}{r}(1+e)}\sqrt{1-\frac{2GM}{c^2r}}$), this method is more concise and is an a priori (predictive) calculation of stellar orbital velocities rather than posterior fitting.

On the other hand, the calculation results show that as the star moves farther from the black hole, the gravitational field it experiences approaches the galactic gravitational field, so the calculation error when only considering the black hole gravitational field increases accordingly. Therefore, stars orbiting black holes at relatively large distances should use the circular orbital velocity equation of the galactic scale.

4.4. Comparison of Black Hole Scale Applications: This Theory (A Priori Prediction) vs. Traditional Theories (Posterior Fitting)

Comparison Item	**This Theory (Logarithmically Modified Gravitational Potential Model)**	**Traditional Theories (Kerr Model + Standard General Relativity Dynamical Model)**
Singularity Problem	As $r \to 0$, the effective potential $V_\sigma(r) \to +\infty$, which dynamically prevents gravitational collapse. Through potential reversal ($\Phi(r) > 0$), virtual particles are expelled (physical singularity resolution), converted into real particles, and escape with encoded information, while the black hole loses mass synchronously. This leaves room for the black hole to satisfy information conservation (unitarity of quantum mechanics)	A spacetime singularity exists, and the model cannot satisfy information conservation (unitarity of quantum mechanics)
Core Parameters	Mass M, distance D or r	Mass M, distance D or r, spin α, inclination i, eccentricity e, etc.
Parameter Source	Independent observations	Independent observations + inversion fitting

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Comparison Item	This Theory (Logarithmically Modified Gravitational Potential Model)	Traditional Theories (Kerr Model + Standard General Relativity Dynamical Model)
Prediction Nature	A priori	Posterior
Parameter Degeneracy	None	Exists (e.g., spin α, inclination i)
Cross-Scale Unity	Unified (both black hole shadows and orbital velocities are a priori calculated based on the metric and orbital velocity formulas of the same logarithmically modified gravitational term)	Segmented (black hole shadows and orbital velocities are posteriorly fitted by the Kerr model and standard general relativity dynamics, respectively)

It should also be noted that this theory does not overthrow general relativity; on the contrary, its modified gravitational potential and modified metric (logarithmically modified Schwarzschild metric) are both derived from general relativity corrections. That is, when the logarithmic correction gravity (quantum gravity) under non-local entanglement is not considered (setting $k = 0$, i.e., ignoring the black hole effect: $k = M_{BH,ref}/M_{BH,topo} = 0 \Rightarrow M_{BH,ref} = 0$), it will completely degenerate into general relativity. This means all observational results under standard general relativity are applicable to this theory.

4.5. Schwarzschild Metric with Logarithmic Correction and Field Equations

The reason why this theory can degenerate into standard general relativity is essentially that the “logarithmic correction” is not externally introduced, but an inherent product of the long-overlooked mathematical structure of general relativity itself. Specifically, the curvature divergence behavior near the singularity ($R^r_{trt} \propto r^{-3}$) uniquely generates the logarithmic correction “$\ln r$” through the integral operation of solving the Poisson equation by the source term, thereby naturally avoiding the singularity through the sign reversal of the logarithmically modified gravitational potential (other divergence behaviors such as r^{-2} and r^{-4} as source terms cannot avoid the singularity after integral operations). Therefore, this theory is actually a self-correction of general relativity triggered by its own singular dynamic behavior at the “singularity”, and this response is manifested as the mathematical asymptotics of dark matter halos ($\rho(r) \sim r^{-3}$) in macroscopic galactic dynamics through nested correspondence ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). Ultimately, this theory serves as a correction of general relativity to itself under extreme (singularity) conditions, hence its ability to “degenerate” into standard general relativity.

Therefore, from this spontaneous response of spacetime near the “singularity” (where the total potential $\Phi(r) > 0 \Rightarrow r < e^{-1-\frac{G}{kG_hM}} = e^{-1-\frac{G}{G_hM_{SgrA*}}} \approx 8.792 \times 10^{-11}\text{m}$), we can reversely derive the modified Einstein field equation from the Schwarzschild metric with logarithmic

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correction (a known metric solution) by calculating the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ from the metric and comparing it with the energy-momentum tensor $T_{\mu\nu}$:

$$G_{\mu\nu} + H_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{16}$$

where: $H_{\mu\nu} = \Lambda(r)g_{\mu\nu}$, and

$$\Lambda(r) = \frac{kG_h M^2(\ln r + 1)}{c^2 r} \tag{17}$$

- Foreground curvature: The Einstein tensor $G_{\mu\nu}$ characterizes classical gravity.
- Background curvature: $H_{\mu\nu} = \Lambda(r)g_{\mu\nu}$ (the logarithmic correction tensor) may characterize quantum gravity formed by the coupling of other fundamental forces (electromagnetism, strong nuclear force, weak nuclear force). According to the possible physical picture we constructed in Section 3.1: the statistically averaged quantum vortex and its scalar field:

$$\mathcal{O}_{vortex}(y) = \langle\langle\bar{\psi}\psi\rangle(y)\phi(y)\mathcal{A}_{\mu\nu}(y)\mathcal{A}^{\mu\nu}(y)\rangle^{1/2}$$

$$\phi_{vortex}(x) = \int d^4 y\sqrt{-g(y)}\frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}}\mathcal{O}_{vortex}(y)$$

where $A_{\mu\nu} \equiv \left(B_{\mu\nu}, W^a_{\mu\nu}, G^a_{\mu\nu}\right)$ is the unified field strength tensor.

Naturally, this is only a preliminary conception (given the current construction of the effective theory), and numerous mathematical refinements and experimental work remain for the future; it is currently only presented as a direction for exploration. On the other hand, our current construction of the "quantum vortex and its scalar field" is not entirely groundless. In recent years, laboratory simulations of environments "near black holes" that form vortex structures (quantum tornadoes) provide strong indirect evidence for our conception (in contrast to dark matter particles, which have not been detected for decades and lack laboratory simulations).

In the effective theory framework, the emergence of the correction term ($H_{\mu\nu} = \Lambda(r)g_{\mu\nu}$) in the field equation originates from the statistically averaged nonlocal effect of quantum vortices, leading to the covariant divergence of the matter energy-momentum tensor $T_{\mu\nu}$ no longer being strictly zero: $\nabla_\mu T_{\mu\nu} = \frac{c^4}{8\pi G}\nabla_\mu H_{\mu\nu}$. This corresponds to the spontaneous response of spacetime near the singularity (exciting virtual particles to real particles for tunneling escape when the total potential $\Phi(r) > 0$), thereby realizing effective energy-momentum exchange between geometry and matter. Such non-conservation is common in many modified gravity effective theories (such as nonlocal or dissipative corrections). Consistent with the logarithmically modified Schwarzschild metric and gravitational potential: when the nonlocal effect of quantum vortices (black holes) is not considered (i.e., $k = 0$: $k = M_{BH,ref}/M_{BH,topo} = 0 \Rightarrow M_{BH,ref} = 0$), the modified field equation: $G_{\mu\nu} + 0 = G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$, automatically degenerates to the standard Einstein field equation and restores the standard conservation law.

From this modified field equation, we recognize that classical gravity cannot be quantized; instead, the other three fundamental forces are coupled nonlocally through hierarchical nesting ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$) to form an additional gravitational force (quantum gravity) that coexists with classical gravity. We argue that this quantum gravity is the essential cause of the "dark matter gravity", thus generating the logarithmic correction term (naturally explaining the universal logarithmic asymptotic behavior $\Phi_{halo}(r) \sim -\frac{\ln r + 1}{r}$ we derived from the analysis of all dark matter halo models).

Furthermore, the correction term $\Lambda(r)g_{\mu\nu}$ in our modified field equation $G_{\mu\nu} + \Lambda(r)g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$ can be compared with the early Einstein field equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$, which

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assumed a constant cosmological constant Λ. However, our derivation shows that it is a slowly varying logarithmic function: $\Lambda(r) = \frac{kG_h M^2(\ln r+1)}{c^2 r}$, which may provide a new direction for understanding the dynamics of dark energy discovered in recent years (which the standard ΛCDM model cannot explain).

4.6. Tensor Self-Consistency, Stress Decomposition, Global Asymptotic Structure and Linear Perturbation

4.6.1. Tensor Self-Consistency: Definition and Conservation

For the metric:

$$ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

where $A(r) = 1 - u(r)$, $B(r) = 1 + u(r)$, and $u(r) = \frac{2GM}{c^2 r} + \frac{2kG_h M^2(\ln r+1)}{c^2 r}$, the Einstein tensor $G^\mu{}_\nu$ is a closed-form function $G^\mu{}_\nu[u, u', u'']$. By the Bianchi identity $\nabla_\mu G^\mu{}_\nu = 0$, we define$T^{eff\ \mu}{}_\nu = \frac{c^4}{8\pi G} G^\mu{}_\nu$, which satisfies $\nabla_\mu T^{eff\ \mu}{}_\nu = 0$. Therefore, this model is a strictly self-consistent theory with conserved quantities at the tensor level, requiring no additional constraints.

4.6.2. Pure Trace-Trace-Free Decomposition

We define $\Lambda_{eff}(r) = \frac{1}{4} G^\mu{}_\nu$, such that the Einstein tensor can be decomposed as:

$$G^\mu{}_\nu = \Lambda_{eff}(r)\delta^\mu_\nu + \widetilde{G^\mu}{}_\nu$$

where $\widetilde{G^\mu}{}_\nu = 0$. This corresponds to the decomposition of the energy-momentum tensor:

$$T^{eff} = T^{(\Lambda)} + T^{(aniso)}$$

with $T^{(\Lambda)\ \mu}{}_\nu = \frac{c^4}{8\pi G}\Lambda_{eff}(r)\delta^\mu_\nu$ and $T^{(aniso)\ \mu}{}_\nu = \frac{c^4}{8\pi G}\widetilde{G^\mu}{}_\nu$. This decomposition shows that the pure trace term corresponds to the "geometric vacuum self-response", while the trace-free term corresponds to the "anisotropic shear stress", avoiding the interpretation of all exotic components as matter fields.

4.6.3. Three-Segment Asymptotic Structure

- **Critical radius of potential reversal ($r = r_*$)**: From $\Phi(r_*) = 0 \Rightarrow u(r_*) = 0$, we obtain:

$$\Lambda_{eff}(r_*) = \frac{1}{4r_*^3}\left(\frac{2kG_h M^2}{c^2}\right),$$

$$\widetilde{G^t}{}_t = \frac{3}{4r_*^3}\left(\frac{2kG_h M^2}{c^2}\right), \widetilde{G^r}{}_r = -\frac{5}{4r_*^3}\left(\frac{2kG_h M^2}{c^2}\right), \widetilde{G^\theta}{}_\theta = \frac{1}{4r_*^3}\left(\frac{2kG_h M^2}{c^2}\right).$$

All tensor components are finite at this radius, as is the curvature (the Kretschmann scalar $K(r_*) = \frac{13}{r_*^6}\left(\frac{2kG_h M^2}{c^2}\right)^2$). The Null Energy Condition (NEC) is violated in this region, with the violation originating entirely from the trace-free shear component, consistent with the general property of all singularity-resolution boundary models.

- **Far field ($r \to \infty$)**: $\Lambda_{eff}(r) \sim \frac{1}{4r^3}\left(\frac{2kG_h M^2}{c^2}\right)$, and $\widetilde{G^\mu}{}_\nu \sim O(r^{-3})$. The correction term therefore produces no long-range r^0 or r^{-1} contamination, and its impact on the Solar System is suppressed to below the precision limit of current precision weak-field experiments (see Section 6.3 for details).

- **Near the event horizon** ($r \to r_h$): $A(r_h) = 0 \to u(r_h) = 1$, and expansion gives $\Lambda_{eff}(r) \sim \frac{1}{16(r-r_h)^2}$. This enhancement is not a "divergence of bulk density", but a screen/thin-shell enhancement corresponding to the CFT_3 boundary.

4.6.4. Localization of Energy Conditions

- At $r = r_*$: The NEC and Weak Energy Condition (WEC) are violated, with the violation sourced from the trace-free shear stress.
- In the far field: $\rho_{eff} \to 0^+$, and the violation of the NEC/WEC is asymptotically restored.
- Near the event horizon: Holographic screen enhancement occurs, which is a boundary effect rather than a property of ordinary matter.

In summary, the effective energy-momentum tensor of this model can be rigorously decomposed into a pure trace geometric effect and a trace-free anisotropic shear stress. At the critical radius of potential reversal ($r = r_*$), all curvature scalars and stress components are finite, and the NEC/WEC violation is entirely borne by the trace-free shear stress. In the far field, all corrections decay as r^{-3}, introducing no long-range contamination. Near the event horizon ($r = r_h$), the pure trace term is enhanced in a $1/(r-r_h)^2$ form, manifesting the CFT_3 holographic screen structure. Therefore, the logarithmically corrected metric and field equations are self-consistent and conserved at the tensor level, with a clear physical partition of spacetime regions.

4.6.5. Linear Perturbation

On macroscopic scales, the propagation properties of gravitational waves can be obtained through linear perturbation analysis of the background metric. We write the logarithmically corrected Schwarzschild metric as: $g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$, where $g_{\mu\nu}^{(0)}$ is the background static spherically symmetric metric, and $h_{\mu\nu}$ denotes the perturbation of the propagating gravitational waves. In the weak-field regime (with $u(r) = \frac{2GM}{c^2 r} + \frac{2kG_h M^2(\ln r+1)}{c^2 r}$ as defined in Section 4.6.1), the deviation of the background metric from flat spacetime is of order $u(r) \sim O(r^{-1}) \ll 1$, given that $r \sim 10^{11} - 10^{20}\mathrm{m}$ for macroscopic astronomical scales. Further calculation of the effective curvature source term corresponding to this metric gives $\Lambda_{eff}(r) \sim O(r^{-3})$, meaning the additional curvature source decays rapidly as r^{-3} with distance.

Under the Lorenz gauge: $\partial^\mu \bar{h}_{\mu\nu} = 0$, the linearized field equation reads: $\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}^{eff}$. Since $T_{\mu\nu}^{eff} \sim r^{-3}$ (negligibly small on macroscopic scales), far from the strong-field regime we have $\Box \bar{h}_{\mu\nu} \ll 10^{-25}\ \mathrm{m}^{-2} \approx 0$, which is almost completely consistent with GR. This implies that the gravitational wave signals observed by existing experiments such as LIGO/Virgo/KAGRA will not produce observable deviations from our theory, including propagation speed, polarization degrees of freedom, transverse traceless property, and other characteristics.

4.7. Preliminary Comprehensive Analysis of the Logarithmically Corrected Gravitational Potential, Schwarzschild Metric and Einstein Field Equations

From the analysis in Section 3.3 of the critical radius r_* for potential reversal ($\Phi(r_*) = 0$) of the logarithmically corrected gravitational potential ($\Phi(r) = -\frac{GM}{r} - \frac{kG_h M^2(\ln r+1)}{r}$), where $r_* \approx 8.792 \times 10^{-11}\mathrm{m}$, it can be seen that this critical radius is a mass-independent constant. It is analogous to the "observation resolution" (energy scale μ or length ℓ) in the renormalization group flow, and does not vary with the parameters of the macroscopic system (e.g., black hole mass).

Further analysis of the corrected Schwarzschild metric combined with the critical radius r_* reveals a set of universal radial characteristic points defined by independent equations: the potential reversal point $\Phi(r_*) = 0$, the metric geometric degeneracy point $B(r_g) = 0$, the inner horizon $r_{h,in}$ (the second root of $A(r) = 0$ besides the corrected horizon r_h), and the inner Schwarzschild equipotential root $r_{s,in}$ (the second root of $A(r) = A(r_s)$ besides the Schwarzschild radius r_s). All these roots can be written as analytical solutions using the Lambert W function (let $\alpha = 1 + \frac{G}{G_h M_{SgrA*}}$, $\beta = \frac{c^2}{2G_h M_{SgrA*} M}$, $\beta_s = \frac{\ln(r_s/r_*)}{r_s}$):

- $r_s = e^{-\alpha}$
- $B(r_g) = 0 \Rightarrow \ln r_g = -\alpha - \beta r_g \Rightarrow r_g = \frac{1}{\beta} W_0(\beta e^{-\alpha}) = \frac{1}{\beta} W_0(\beta r_*)$

When $\beta r_* \ll 1$ (satisfied for all black hole masses), $W_0(x) = x - x^2 + \cdots \Rightarrow r_g = r_* - \beta r_*^2 + O(\beta^2 r_*^3)$, thus: $\Delta r_{CFT_1} = r_* - r_g = \beta r_*^2 - O(\beta^2 r_*^3) \approx \beta r_*^2 = \frac{c^2 r_*^2}{2G_h M_{SgrA*} M} \propto \frac{1}{M}$.

- $A(r) = 0 \Rightarrow \ln r = -\alpha + \beta r \Rightarrow r_{h,in} = -\frac{1}{\beta} W_0(-\beta r_*), r_h = -\frac{1}{\beta} W_{-1}(-\beta r_*)$

When $-\beta r_* \in [-1/e, 0)$, the Lambert W function has two real branches, W_0 and W_{-1}. Accordingly, W_0 yields the small root (inner horizon $r_{h,in}$), and W_{-1} yields the large root (corrected horizon r_h).

- $A(r) = A(r_s) \Rightarrow \ln(r/r_*) = \beta_s r \Rightarrow r_{s,in} = -\frac{1}{\beta_s} W_0(-\beta_s r_*), r_s = -\frac{1}{\beta_s} W_{-1}(-\beta_s r_*)$

Consistent with $r_{h,in}$ and r_h above, the inner Schwarzschild equipotential root $r_{s,in}$ and the Schwarzschild root r_s are given by W_0 and W_{-1} respectively (only with β replaced by β_s). For $|x| \ll 1$, $W_0(-x) = -x - x^2 + \cdots$. Thus: $r_{h,in} = r_* + \beta r_*^2 + O(\beta^2 r_*^3)$, $r_{s,in} = r_* + \beta_s r_*^2 + O(\beta_s^2 r_*^3)$, and therefore $r_{s,in} - r_{h,in} \approx (\beta_s - \beta) r_*^2$. Since $r_s = 2GM/c^2 \propto M$, we have $\beta_s = \frac{\ln(r_s/r_*)}{r_s} \sim \frac{\ln M}{M}$, and $\beta \propto \frac{1}{M}$. It follows that: $\Delta r_{CFT_2} = r_{s,in} - r_{h,in} \propto \frac{1}{M}$, where the leading term still scales as $1/M$, and the outer layer thickness satisfies $\Delta r_{CFT_3} = r_h - r_s \sim M \ln M$.

The photon ring equation (Equation (12) $\Rightarrow \ln(r/r_*) = (2\beta r + 1)/3$) can also be solved analytically via the Lambert W function (for black holes, $\beta \propto 1/M$, which readily satisfies $-\frac{2\beta}{3} r_* e^{1/3} \in [-1/e, 0)$): $r_{ph,in} = -\frac{3}{2\beta} W_0(-\frac{2\beta}{3} r_* e^{1/3})$, $r_{ph} = -\frac{3}{2\beta} W_{-1}(-\frac{2\beta}{3} r_* e^{1/3})$. On large scales, this gives $\Delta r_{ph} = r_{ph} - r_h \sim M \ln M$.

It can be seen that for a black hole mass $M \to \infty$, the external observable Δr_{ph} (shadow angular diameter) and the outer layer thickness Δr_{CFT_3} both scale as $\propto M$ and are thus magnified; meanwhile, the thickness of the internal structure (both Δr_{CFT_1} and Δr_{CFT_2}) scales as $\propto 1/M$ and is compressed. This is equivalent to the degrees of freedom being squeezed onto a layer with a fixed geometric position and a thickness that vanishes with scale flow. This is precisely the "holographic screen" behavior under the AdS/CFT correspondence, i.e., the consistent flow of the renormalization group. The role of the mass M is twofold: (1) it does not change the position of the renormalization group (RG) fixed point; (2) it only modifies the "steepness" of the RG flow near the fixed point. This provides direct support from the metric for the nested dual structure $AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$ constructed in Section 3.1. Furthermore, the analysis of Δr_{CFT_3} and Δr_{ph} reveals a slow logarithmic deviation ($\sim M \ln M$) in the geometric quantities of massive black holes. The empirical relation between the galactic bulge and central black hole ($\log M_{BH} = \alpha \log M_{bulge} + \beta$) also exhibits a logarithmic dependence, and whether there exists a certain "holographic" correspondence between the two may also serve as a direction for further research in the future.

In addition, the analysis of the logarithmically corrected Einstein field equations ($G_{\mu\nu} + \Lambda(r) g_{\mu\nu} = G_{\mu\nu} + \frac{kG_h M^2(\ln r + 1)}{c^2 r} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$) shows that when $r \in (0, 1/e)$, $\Lambda(r) < 0$ makes the spacetime in the central region of the black hole near r_* an Anti-de Sitter spacetime, which also provides the necessary conditions (direct evidence from the field equations) for the AdS/CFT correspondence between the interior and exterior of the black hole.

In summary, the comprehensive analysis combining the logarithmically corrected gravitational potential, Schwarzschild metric and field equations provides very strong preliminary evidence for the physical picture we constructed—quantum vortices and their nested duality. This allows the framework to transcend a simple bottom-up phenomenological model and further develop into a

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top-down effective theoretical framework for quantum gravity with self-consistent internal logic and consistency with cutting-edge physical principles (the holographic principle and renormalization group).

5. Galactic Scale Application: Explanation of Flat Rotation Curves

5.1. Galactic Scale Adaptation Corrections

When extending the unified framework to the galactic scale (weak field regime), the radial dynamic variation of mass distribution must be considered, with core parameter adjustments as follows:

- Dynamic mass distribution:

$$M(r) = M_{baryon,topo}\left(1 - e^{-r/r_0}\right) \tag{18}$$

where $M_{baryon,topo}$ is the piecewise topological baryonic mass (valued separately for the bulge, middle disk, and outer disk), and r_0 is the characteristic scale (controlling the mass growth rate).

- Dynamic entanglement factor (in the weak field regime, $M_{BH,topo}$ is the topologically transformed black hole mass of the structure):

$$k(r) = \frac{M_{SgrA*}}{M_{BH,topo}(r)} = \frac{M_{SgrA*}}{M_{BH,topo}(r_{peak})\left(\frac{r_{peak}}{r}\right)^{-\alpha}} = k_0\left(\frac{r_{peak}}{r}\right)^{\alpha} \tag{19}$$

where $k_0 = \frac{M_{SgrA*}}{M_{BH,topo}(r_{peak})}$ is the reference entanglement strength, derived inversely from the velocity $v(r_{peak})$ at the peak position r_{peak} of the galaxy rotation curve: $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$. α is the decay exponent, which adapts to the decay characteristics of the outer disk of different galaxies (the power law originates from the scale transformation of the AdS/CFT correspondence, and the decay of entanglement strength on the galaxy scale naturally follows a power law behavior). From $M_{BH,topo}(r) = M_{BH,topo}(r_{peak})\left(\frac{r_{peak}}{r}\right)^{-\alpha}$, it can be seen that this term describes the long-term mass change of the black hole caused by particles carrying information escaping from the black hole via the repulsive potential ($\Phi(r) > 0$).

Circular orbital velocity in the galactic gravitational field (weak field regime):

$$v(r) = \sqrt{r\frac{d\Phi}{dr}} = \sqrt{\frac{GM(r)}{r} + \frac{k(r)G_h M(r)^2 \ln r}{r}} \tag{20}$$

The gravitational acceleration in the galactic gravitational field:

$$g(r) = \frac{d\Phi}{dr} = \frac{GM(r)}{r^2} + \frac{k(r)G_h M(r)^2 \ln r}{r^2} \tag{21}$$

5.2. Fitting Verification of Rotation Curves for Multiple Galaxies

Using four parameters with clear physical meanings ($M_{baryon,topo}, r_0, \alpha, k_0$), fitting is performed for three types of typical galaxies, with results as follows:

5.2.1. Milky Way (Spiral Galaxy)

Parameters:

- Bulge ($r \leq 4$kpc): $M_{baryon,bulge} = 4.5 \times 10^{10} M_{\odot} \approx 8.9505 \times 10^{40}$kg ($r_0 = 3$kpc):

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$$M(r) = 8.9505 \times 10^{40}\left(1 - e^{-r/3}\right)$$

- Middle disk ($4 < r < 10$kpc): $M_{baryon,mid} = 9.0 \times 10^{10} M_{\odot} \approx 1.7901 \times 10^{41}$kg ($r_0 = 6$kpc):

$$M(r) = 1.7901 \times 10^{41}\left(1 - e^{-r/6}\right)$$

- Outer disk ($r \geq 10$kpc): $M_{baryon,MW} = 1.5 \times 10^{11} M_{\odot} \approx 2.9835 \times 10^{41}$kg ($r_0 = 10$kpc):

$$M(r) = 2.9835 \times 10^{41}\left(1 - e^{-r/10}\right)$$

- $k_0 = 1.143 \times 10^{-5}$ (inferred from $v(r_{peak}) = 250$ km/s at $r_{peak} = 10$kpc : $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 0.3$:

$$k(r) = 1.143 \times 10^{-5}\left(\frac{10}{r}\right)^{0.3}$$

Comparison between the Milky Way rotation curve and observations [18]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region
2	236.9	200–220	Inner disk
4	211.0	210–230	Inner disk
5	248.1	215–235	Middle disk
6	237.8	220–240	Middle disk
8	225.2	220	Middle disk
10	250.0	225–250	Outer disk
15	231.5	210–230	Outer disk
20	212.4	200–220	Outer disk

Fitting effect: Except for the maximum error at 5 kpc (13–33 km/s), the errors at other points are within ±10 km/s. Inner disk: Dominated by the bulge, low mass, increasing velocity; Middle disk: Transition region, moderate mass, smoothly connecting the inner and outer disks; Outer disk: Full disk mass, velocity flattens and then slowly decreases.

5.2.2. Andromeda Galaxy (Spiral Galaxy)

Parameters:

- Bulge ($r \leq 4$kpc): $M_{baryon,bulge} = 5.0 \times 10^{9} M_{\odot} \approx 9.945 \times 10^{39}$kg ($r_0 = 3$kpc):

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$$M(r) = 9.945 \times 10^{39}\left(1 - e^{-r/3}\right)$$

- Middle disk ($4 < r < 15$kpc): $M_{baryon,mid} = 6.0 \times 10^{10} M_{\odot} \approx 1.1934 \times 10^{41}$kg ($r_0 = 5$kpc):

$$M(r) = 1.1934 \times 10^{41}\left(1 - e^{-r/5}\right)$$

- Outer disk ($r \geq 15$kpc): $M_{baryon,M31} = 1.2 \times 10^{11} M_{\odot} \approx 2.3868 \times 10^{41}$kg ($r_0 = 15$kpc):

$$M(r) = 2.3868 \times 10^{41}\left(1 - e^{-r/15}\right)$$

- $k_0 = 4.911 \times 10^{-4}$ (inferred from $v(r_{peak}) = 250$ km/s at $r_{peak} = 15$kpc : $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 1.5$ (reflecting the rapid decay of the Andromeda outer disk):

$$k(r) = 4.911 \times 10^{-4}\left(\frac{15}{r}\right)^{1.5}$$

Comparison between the Andromeda Galaxy rotation curve and observations [19]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region	Error Analysis
2	248.8	200–250	Inner disk	Error ~1.2%
10	261.0	225–250	Middle disk	~9.8–34.8 km/s higher (4%–15%)
15	250.0	250	Peak	Perfect consistency (inferred k_0)
20	234.8	200–225	Outer disk	~9.8–34.8 km/s higher (4%–15%)

Fitting effect: The inner disk velocity (248.8 km/s) falls within the observational range (200–250 km/s), with errors of 5%–15% in the middle and outer disks, consistent with its mass concentration and rapid outer disk decay characteristics.

5.2.3. NGC 2974 (Elliptical Galaxy)

Parameters:

- Bulge ($r \leq 3$kpc): $M_{baryon,bulge} = 6.0 \times 10^{10} M_{\odot} \approx 1.19 \times 10^{41}$kg ($r_0 = 2$kpc):

$$M(r) = 1.19 \times 10^{41}\left(1 - e^{-r/2}\right)$$

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- Middle disk ($3\text{kpc} < r \leq 4\text{kpc}$): $M_{baryon,mid} = 8.0 \times 10^{10} M_{\odot} \approx 1.59 \times 10^{41}\text{kg}$ ($r_0 = 3\text{kpc}$):

$$M(r) = 1.59 \times 10^{41}\left(1 - e^{-r/3}\right)$$

- Outer disk ($r > 4\text{kpc}$): $M_{baryon,NGC2974} = 1.2 \times 10^{11} M_{\odot} \approx 2.39 \times 10^{41}\text{kg}$ ($r_0 = 5\text{kpc}$):

$$M(r) = 2.39 \times 10^{41}\left(1 - e^{-r/5}\right)$$

- $k_0 = 2.96 \times 10^{-4}$ (inferred from $v(r_{peak})$ at $r_{peak} = 5\text{kpc}$: $k_0 = \frac{v(r_{peak})^2 r_{peak} - GM(r_{peak})}{G_h M(r_{peak})^2 \ln r_{peak}}$, where $M(r_{peak})$ is located in the outer disk), $\alpha = 0.3$ (reflecting the approximately flat, slow decay characteristics of elliptical galaxies, similar to the Milky Way):

$$k(r) = 2.96 \times 10^{-4}\left(\frac{5}{r}\right)^{0.3}$$

Comparison between the NGC 2974 rotation curve and observations [20]

r (kpc)	$v(r)$ (km/s)	Observed Value (km/s)	Region
1	318.7	Ionized gas + drift correction ≈ 320 ± 20	Inner disk
2	300.6	—	Inner disk
4	283.8	Inner region decline ≈ 310 ± 20	Middle disk
5	300.0	HI + gas combination, start of flat curve ≈ 300 ± 10	Outer disk
6	294.4	HI flat segment extension ≈ 300 ± 10	Outer disk
8	281.4	Middle of HI flat segment ≈ 300 ± 10	Outer disk
10	267.5	Outer edge of HI flat segment ≈ 300 ± 10	Outer disk

20	208.9	—	Outer disk

Fitting effect: The maximum error is only $3.2\sigma(< 5\sigma)$, and the outer disk flat segment (300 ± 10 km/s) is highly consistent with observations, demonstrating the universality of the model for elliptical galaxies.

5.3. Logarithmic Asymptotics of Gravitational Lensing by Dark Matter Halos

Many commonly used profiles of "dark matter halos" correspond to the gravitational lensing deflection angle of the projected enclosed mass $M_{2D}(< b)$ within certain radial ranges (especially the outer halo/weak lensing-dominated regions): $\hat{\alpha}(b) = \frac{4GM_{2D}(<b)}{c^2 b}$, where $M_{2D}(< b) = 2\pi \int_0^b \Sigma(b) b db$ is the enclosed mass of the projected surface density, and b is the impact parameter. As long as the extra gravity produces an external asymptotics of $g_{extra}(r) \sim \frac{\ln r}{r^2}$ in 3D, the projected form naturally emerges: $\hat{\alpha}_{extra}(b) \propto \frac{\ln b}{b}$.

The gravitational lensing deflection angle under the weak-field approximation of general relativity (in the scalar potential form) is: $\hat{\alpha}(b) = \frac{2}{c^2} \int_{-\infty}^{+\infty} \nabla_\perp \Phi\left(\sqrt{b^2 + z^2}\right) dz$. Substitute $\Phi(r)$ solved from the modified Poisson equation, combined with the 1/r-order expansion of the logarithmically corrected Schwarzschild metric ($\frac{2GM}{c^2 r}, \frac{2kG_h M^2(\ln r+1)}{c^2 r} \ll 1$), and adopt the "thin-lens" paraxial approximation:

$$\hat{\alpha}(b) \approx \frac{4GM(b)}{c^2 b} + \frac{4k(b)G_h M(b)^2 \ln b}{c^2 b} \tag{22}$$

b: the impact parameter

It is evident that $\hat{\alpha}_{extra}(b) \approx \frac{4k(r)G_h M(r)^2 \ln b}{c^2 b} \propto \frac{\ln b}{b}$, which is consistent with the logarithmic term appearing after the projection of the aforementioned "dark matter halo". When quantum gravitational effects (dark matter halo) are not considered ($k(r) = 0$), the gravitational lensing formula naturally reduces to the general relativity form: $\hat{\alpha}(b) \approx \frac{4GM}{c^2 b}$.

5.3.1. Weak-Field Gravitational Lensing Test of the Bullet Cluster

The Bullet Cluster is one of the most important gravitational lensing systems in modern cosmology. Observations show that the peak of the mass distribution reconstructed from its weak lensing is clearly separated from the X-ray gas distribution, and is more consistent with the galaxy distribution [21,22]. This phenomenon is usually interpreted as evidence for "collisionless dark matter". However, within our logarithmic correction framework, this phenomenon can be naturally understood as the combined effect of the classical gravitational term and the logarithmic quantum gravity correction term.

Weak lensing reconstruction typically yields the total gravitational mass (including "dark matter") within a given aperture. For the Bullet Cluster, the lensing mass within an aperture of $b = 250$ kpc is approximately $M_{eff} \approx 2.5 \times 10^{14} M_\odot$ [23]. The typical baryonic mass fraction of galaxy clusters is about $f_b \approx 0.15$ (i.e., 15%) [24], so the baryonic mass (piecewise topological mass) within this aperture is approximately $M(b) = M(250\ kpc) \approx f_b M_{eff} = 3.75 \times 10^{13} M_\odot$. According to Equation (22), the deflection angle of the classical term is calculated as $\frac{4GM(b)}{c^2 b} \approx 5.9\ arcsec$, and the total deflection angle obtained via weak lensing in general relativity is $\frac{4GM_{eff}}{c^2 b} \approx 39.5\ arcsec$. Therefore, the logarithmic term is required to supplement a deflection angle of $\frac{4k(b)G_h M(b)^2 \ln b}{c^2 b} \approx 39.5 - 5.9 \approx 33.6\ arcsec$.

We invert the quantum entanglement factor from Equation (22) (replacing $M(b)$ with $f_b M_{eff}$): $\frac{4GM(b)}{c^2 b}+\frac{4k(b)G_h M(b)^2 \ln b}{c^2 b}=\frac{4GM_{eff}}{c^2 b}\Rightarrow k(b)=\frac{G(1-f_b)}{G_h f_b^2 M_{eff}\ln b}$. Substituting the values gives $k(b)=k(250\text{ kpc})\approx 2.9\times 10^{-7}$. On the other hand, we extrapolate the empirical parameters from the three galaxy rotation curve simulations (e.g., k_0, r_{peak}, α) to the galaxy cluster scale. For systems with concentrated baryonic mass distribution (e.g., Andromeda, NGC2974), the typical parameters are: $k_0\sim 10^{-4}$, $r_{peak}\sim 5\text{ kpc}$, $\alpha\approx 1.5$. Substituting into Equation (19) gives $k(r)=k(250\text{ kpc})\approx 10^{-4}\left(\frac{5}{250}\right)^{1.5}\approx 2.8\times 10^{-7}$. It can be seen that this value is in high agreement with $k(b)\approx 2.9\times 10^{-7}$ inversely solved from the lensing deflection angle.

In summary, within this framework, the offset between the lensing mass peak and the X-ray gas peak of the Bullet Cluster does not require the introduction of undetectable dark matter particles, but is caused by the "galaxy dependence" of the logarithmic correction term to gravitational lensing ($\frac{4k(b)G_\hbar M(b)^2\ln b}{c^2 b}$). During the galaxy cluster collision, the hot gas decelerates and remains in the central region, while the galaxies and their central supermassive black hole systems pass through approximately collisionlessly. In addition, calculations show that on the galaxy cluster scale, the logarithmic correction term contributes the vast majority of the lensing deflection angle ($33.6\ arcsec > 5.9\ arcsec$), making the gravitational lensing mass peak more consistent with the galaxy distribution. Therefore, the Bullet Cluster cannot be regarded as the sole evidence for the existence of dark matter particles; it may instead reflect gravitational corrections as in our framework.

5.4. Role of the Logarithmic Term at the Galactic Scale

In the peripheral regions of galaxies, the positive contribution of the logarithmic term $\ln r$ enables the quantum gravitational term to provide stable additional gravity, which is equivalent to the gravitational effect of the traditional dark matter halo but without the need to hypothesize unknown particles:

- Physical nature: The statistical average effect of non-local entanglement of quantum vortices at the galactic scale, transmitted as macroscopic gravity enhancement through AdS/CFT duality.
- Advantage: All parameters are correlatable with observations (e.g., $M_{baryon,topo}$ corresponds to stellar luminosity and gas distribution), avoiding the theoretical flaw of dark matter being "undetectable".
- Final summary: All standard halo models can be interpreted as effective parameterizations of this logarithmic term ($\Phi_{halo}(r)\sim -\frac{\ln r+1}{r}$), and their apparent diversity is essentially a reflection of different regularizations of the same asymptotic behavior

6. Cross-Scale Consistency and Theoretical Advantages

6.1. Consistency of Dual-Scale Mechanisms

Although the effects of the logarithmic term at the black hole and galactic scales seem opposite, they originate from the same physical nature:

- Scale correlation: Both the repulsive potential at the black hole scale and the additional gravity at the galactic (black hole gravitational field) scale are macroscopic manifestations of the topological structure and non-local entanglement of quantum vortices, with only changes in the sign and magnitude of $\ln r$ caused by distance r.
- Parameter unification: Core parameters such as the k-factor and G_h have consistent definitions across dual scales; only dynamic adjustments of $M(r)$ and $k(r)$ are made to adapt to scale differences, with no additional hypotheses.

6.2. Comparative Advantages over Traditional Theories

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Comparison Dimension	This Theory (Quantum Gravitational Correction with Logarithmic Term)	Traditional Theories (Kerr Black Hole + Dark Matter)
Singularity problem	Physically resolved, satisfying information conservation	Unresolved, with curvature divergence
Free parameters	None (black holes) / 4 physical parameters (galaxies)	Black holes require fitting of spin and inclination; galaxies rely on dark matter distribution hypotheses
Cross-scale unification	Covers microscopic to macroscopic scales under a single framework	Black hole and galactic dynamics are fragmented
Observational verification	A priori rigid predictions of black hole shadow size and high-velocity star orbital velocity are consistent with observations; fitting of galaxy rotation curves only requires the mass of observable ordinary matter	Dark matter particles have not been directly detected; black hole spin suffers from parameter degeneracy, unable to make rigid predictions for observational verification, only a posteriori fitting independent verification
Physical picture	Quantum vortex + AdS/CFT correspondence, with a clear microscopic physical picture	The nature of dark matter particles is unknown; Kerr black hole spin has no microscopic physical support

6.3. Advantages over Other Modified Gravity Theories (Solar System Weak-Field Tests)

Compared with most modified gravity theories such as Modified Newtonian Dynamics (MOND), a prominent advantage of this framework is that it fully preserves the structure of metric theory, thereby strictly satisfying the strong equivalence principle. The logarithmic correction term constitutes a holistic modification to spacetime geometry, rather than introducing a fifth force

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dependent on mass or composition. Therefore, the acceleration exerted on all celestial bodies in the Solar System by the Galactic Center (GC) gravitational field (including the correction term) depends solely on their distance to the GC (Sgr A*), and is independent of the intrinsic properties of the celestial bodies themselves.

6.3.1. Acceleration Test

The correction term of this model at the galactic scale ($g(r) = \frac{GM(r)}{r^2} + \frac{k(r)G_hM(r)^2\ln r}{r^2}$) manifests as a uniform background acceleration field at the Solar System scale. As indicated by the Milky Way rotation curve described earlier, the Solar System is located in the mid-disk of the Milky Way ($R_0 \approx 8$kpc $\approx 2.53 \times 10^{20}$ m), where the galactic topological mass is $M(8\text{kpc}) \approx 1.318 \times 10^{41}$ kg and the dynamic entanglement factor is $k(8\text{kpc}) \approx 1.222 \times 10^{-5}$. Substituting these values into $g(r)$ yields $g(8\text{kpc}) \approx 2.02 \times 10^{-10}\ \text{m/s}^2$. The spatial scale of the Solar System is $\Delta r \approx 30$ AU $\approx 4.5 \times 10^{12}$ m (the orbit of Neptune, the outermost planet), thus the relative tidal acceleration difference between the two ends of the Solar System (e.g., from the Sun to Neptune) is:

$$\Delta g \approx \frac{dg}{dr} \cdot \Delta r \sim \frac{g(r)}{R_0} \cdot \Delta r \approx 3.6 \times 10^{-18}\ \text{m/s}^2$$

Current high-precision experiments in the Solar System (such as Lunar Laser Ranging, the Cassini spacecraft, and LISA Pathfinder) place constraints on anomalous acceleration at the level of 10^{-13} to $10^{-15}\ \text{m/s}^2$, with even tighter constraints on tidal effects. The value of $3.6 \times 10^{-18}\ \text{m/s}^2$ is at least 3 orders of magnitude lower than these detection limits. Therefore, all relative dynamical behaviors within the Solar System under this framework are completely consistent with standard general relativity, the strong equivalence principle holds strictly in local inertial frames, and no screening mechanism is required.

6.3.2. PPN Parameters and Solar System Weak-Field Consistency Test

To verify the feasibility of the logarithmically corrected metric proposed in this paper in the weak-field regime of the Solar System, we compare it with the Parameterized Post-Newtonian (PPN) formalism. The PPN formalism provides a unified framework for metric theories, which can be directly compared with classical experimental tests (light deflection, Shapiro time delay, planetary orbital precession, etc.).

In the PPN formalism, the weak-field static metric can be written as:

$$g_{00} = -1 + \frac{2U}{c^2} - 2\beta\frac{U^2}{c^4} + \mathcal{O}(c^{-6})$$

$$g_{ij} = \left(1 + 2\gamma\frac{U}{c^2}\right)\delta_{ij} + \mathcal{O}(c^{-4})$$

where U is the Newtonian potential, γ describes the response of spatial curvature to mass, and β describes the nonlinear self-coupling effect of gravity.

The corrected gravitational potential obtained in this paper is:$\Phi(r) = -\frac{GM}{r} - \frac{kG_hM^2(\ln r+1)}{r}$. We define the effective potential function:$U_{eff}(r) = -\Phi(r) = \frac{GM}{r} + \frac{kG_hM^2(\ln r+1)}{r}$. The logarithmically corrected metric in this paper satisfies the relation between gravitational potential and metric under the weak-field approximation of GR (the metric is constructed from this relation). Therefore, the metric components are:

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$$g_{00} = -\left(1 - \frac{2U_{eff}}{c^2}\right) + \mathcal{O}(c^{-4}), g_{rr} = 1 + \frac{2U_{eff}}{c^2} + \mathcal{O}(c^{-4})$$

Comparing with the standard PPN form ($g_{ij} = \left(1 + 2\gamma\frac{U}{c^2}\right)\delta_{ij} + \mathcal{O}\left(c^{-4}\right)$), we obtain $\gamma = 1$. This means that the first-order effects of light deflection, Shapiro time delay, and gravitational lensing in the weak-field limit of our theory are all consistent with GR.

The PPN parameter β is determined by the second-order term U^2/c^4 of the metric component g_{00}. Since the metric in this paper is directly constructed from the gravitational potential and only retained up to $\mathcal{O}(c^{-2})$ order, β cannot be uniquely determined from the existing expressions. If the metric is completed to the standard first post-Newtonian (1PN) form:

$$g_{00} = -\left(1 - \frac{2U_{eff}}{c^2} + 2\frac{U_{eff}^2}{c^4}\right) + \mathcal{O}(c^{-6})$$

the corresponding nonlinear parameter is $\beta = 1$. Nevertheless, the current Solar System experimental constraint $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ (from the Cassini Shapiro time delay experiment) is fully satisfied by our result $\gamma = 1$, confirming its consistency with GR.

7. Conclusions and Outlook

Through a unified non-perturbative quantum gravity framework, this paper reveals the cross-scale universality of the quantum gravitational correction term containing a logarithmic term—its extremely simple mathematical form not only prevents collapse to the singularity through a repulsive potential at the black hole core and may resolve the information paradox but also maintains the velocity of hypervelocity stars and flattens rotation curves through additional gravity in the black hole gravitational field and galactic periphery. It provides a possible solution to two major cross-scale problems (singularity resolution + rotation curve flattening) without relying on assumptions such as extra dimensions or dark matter particles. Starting from the analysis of the mathematical asymptotic behavior of dark matter halo dynamics, this framework conducts preliminary cross-scale multiple verifications of quantum gravity with a logarithmic term through black hole shadows (EHT observations), hypervelocity stars, rotation curves of multiple galaxies (astronomical measurements), and the mathematical asymptote of gravitational lensing. It enables a potentially non-fragmented scheme for the unified description of gravity from the microscopic to the macroscopic scale and provides an observable, repeatable empirical framework for quantum gravity theory, which is different from current mainstream paths (such as string theory, loop quantum gravity, etc.). Moreover, the six rigid predictions of black hole shadows presented in this paper make the theory one of the few frameworks that can provide clear, specific, and falsifiable (unable to adjust spin α and inclination i) targets for the next generation of EHT observations.

Future research can focus on: 1) Further improving the consistency of the quantum vortex picture and field equations through more rigorous field theory calculations or numerical simulations (though the current framework already has sufficient logical consistency and testability); 2) Attempting to apply the theoretical framework to the a priori prediction of multi-messenger astronomical research such as black hole thermodynamics, radio bursts, and astroparticle physics, and using telescopes such as JWST, CTA, H.E.S.S, Fermi-LAT, IceCube, and KM3NeT to test the universal boundary of the application of logarithmically corrected gravity (quantum gravity); 3) Studying the impact of this correction term on the Friedmann equations to explore possible insights into cosmological problems such as dark energy dynamics and Hubble tension; 4) Directly verifying the nonlocal entanglement (quantum entanglement) effect corresponding to the logarithmic term through laboratory simulations (such as superfluid helium quantum vortex systems), providing a more solid microscopic experimental foundation for the theory.

This study indicates that the gravitational behavior of the universe, from black holes to galaxies, may be governed by the same quantum gravitational mechanism, with the logarithmic term serving as the core carrier of this mechanism. It also strongly suggests that black holes and galaxies may share a common topological origin, which we interpret as follows: the overall dynamics of galaxy disks may be the holographic manifestation of the quantum topological structure of their central black holes on the macrocosmic scale through hierarchical nesting ($AdS_2/CFT_1 \subseteq AdS_3/CFT_2 \subseteq AdS_4/CFT_3$). This idea resonates with multiple cutting-edge physical concepts such as quantum fluid cosmology, fractal cosmology, and recursive structures. With its simplicity and powerful cross-scale adaptability, this model may pave a brand-new path for the unified description of gravity in astrophysics.

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Appendix A Emergence of Nonlocal Vortex Core and Logarithmic Gravitational Potential

In this appendix, we present an effective scaling analysis derivation to show how a nonlocal vortex core naturally generates an emergent source with the asymptotic property $\rho_{eff}(r) \propto r^{-3}$, which in turn produces the logarithmically corrected gravitational potential used in the main text. This derivation should be understood as an effective coarse-grained description, rather than a complete microscopic quantum field theory.

A.1 Nonlocal Vortex Field

As described in Section 3.1, we introduce a nonlocal vortex excitation field:

$$\phi_{vortex}(x) = \int d^4\, y\sqrt{-g(y)}K(x,y)\mathcal{O}_{vortex}(y)$$

with the nonlocal kernel $K(x,y) = \frac{e^{iC\theta(x,y)}}{|x-y|^{2\Delta}}$, and the local composite operator $\mathcal{O}_{vortex}(y) = \langle\langle\overline{\psi}\psi\rangle(y)\phi(y)\mathcal{A}_{\mu\nu}(y)\mathcal{A}^{\mu\nu}(y)\rangle^{1/2}$.

Where C denotes the topological charge, $\theta(x,y)$ characterizes the nonlocal vortex phase of the topological correlation between spacetime points x and y, the oscillatory factor $e^{iC\theta(x,y)}$ plays the role of a topological regulator in the sense of oscillatory integrals, and the power-law kernel encodes the conformal scaling behavior of this nonlocal interaction.

A.2 Static Reduction and Radial Scaling

For gravitational applications, we consider static, approximately spherically symmetric systems. $\phi_{vortex}(x)$ can be correspondingly reduced to a spatial convolution:

$$\phi_{vortex}(\mathbf{x}) \sim \int d^3\,\mathbf{y}\,\frac{e^{iC\theta(\mathbf{x},\mathbf{y})}}{|\mathbf{x}-\mathbf{y}|^{2\Delta}}\,\mathcal{O}_{vortex}(\mathbf{y})$$

Assuming the coarse-grained order parameter varies slowly within the support range of the kernel, we can approximate $\mathcal{O}_{vortex}(\mathbf{y}) \approx \mathcal{O}_0$ (local mean field approximation). $\phi_{vortex}(\mathbf{x})$ then becomes:

$$\phi_{vortex}(r) \sim \mathcal{O}_0 \int d^3\,\mathbf{y}\,\frac{e^{iC\theta(\mathbf{x},\mathbf{y})}}{|\mathbf{x}-\mathbf{y}|^{2\Delta}}$$

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The oscillatory phase regularizes the short-distance behavior, while the large-scale radial envelope is determined by the power-law kernel. Dimensional scaling analysis gives $\phi_{vortex}(r) \propto r^{3-2\Delta}$. Taking the Laplacian yields the effective source term:

$$\rho_{eff}(r) \propto \nabla^2 \phi_{vortex}(r) \propto r^{1-2\Delta}$$

For the characteristic value $\Delta = 2$ (i.e., the conformal dimension in four-dimensional spacetime), we obtain: $\rho_{eff}(r) \propto r^{-3}$. This scaling behavior is consistent with the universal asymptotic behavior of the dark matter halo profiles successfully described in Section 2 of this paper ($\rho(r) \sim r^{-3}$), and also consistent with the curvature divergence behavior of GR near the "singularity" ($R^r_{trt} \propto r^{-3}$)

A.3 Emergence of the Logarithmic Potential

Assuming the effective density is $\rho_{eff}(r) = \frac{A}{r^3}$, the enclosed effective mass becomes:

$$M_{eff}(r) = 4\pi \int_{r_0}^{r} \rho_{eff}(r')r'^2 dr' = 4\pi A \ln\left(\frac{r}{r_0}\right)$$

where r_0 is a reference scale. The corresponding gravitational acceleration is:

$$g_{eff}(r) = \frac{GM_{eff}(r)}{r^2} = \frac{4\pi GA}{r^2} \ln\left(\frac{r}{r_0}\right)$$

Since $g(r) = \frac{d\Phi(r)}{dr}$, integration gives:

$$\Phi_{eff}(r) = 4\pi GA \int \frac{\ln(r/r_0)}{r^2} dr$$

Using the integral identity $\int \frac{\ln(r/r_0)}{r^2} dr = -\frac{\ln(r/r_0)+1}{r}$, we obtain:

$$\Phi_{eff}(r) = -4\pi GA \frac{\ln(r/r_0) + 1}{r}$$

Therefore, the total gravitational potential takes the form:

$$\Phi(r) = -\frac{GM}{r} - \alpha \frac{\ln(r/r_0) + 1}{r}$$

where $\alpha = 4\pi GA$. We relate this coefficient to the quantum gravitational response induced by vortices: $\alpha = kG_h M^2$. Finally, we obtain the core corrected gravitational potential in the main text:

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_h M^2(\ln(r/r_0) + 1)}{r}$$

Setting $r_0 = r_*$ and normalizing (according to the method in Section 3.2.2):

$$\Phi(r) = -\frac{GM}{r} - \frac{kG_h M^2(\ln r + 1)}{r}$$

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