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Theoretical Foundations for Creating Fast Algorithms Based on Constructive Methods of Universality

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Theoretical Foundations for Creating Fast Algorithms Based on Constructive Methods of Universality

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Abstract: The core motivation for this study emanates from Voronin S.'s seminal work on the universality of the zeta function. This universality positions the zeta function as a pivotal element in the conceptual framework of global artificial intelligence, suggesting its potential as a foundational component. A critical barrier to harnessing this promising universality has been the absence of efficient algorithms capable of actualizing this concept. Additionally, this research engages with the profound philosophical issue of reflection, exploring the intricate interplay between the external and internal worlds. In this context, resolving the Riemann Hypothesis is not merely a mathematical endeavor but is posited as a gateway to developing both rapid and potentially ultra-rapid algorithms. These algorithms are envisaged to operationalize the universal nature of the zeta function, providing novel insights into the nature of consciousness. By interpreting all processes occurring within the brain and the body as manifestations of various aspects of the zeta function, this work bridges external and internal realities. The study proposes viewing the symmetry of the zeta function as a metaphorical parallel to the symmetry between the external and internal worlds, thus offering a unique perspective on the interconnectedness of mathematical constructs and human cognition.

Keywords: euler product; dirichlet; riemann; hilbert; poincaré; riemann hypothesis; zeta function; universality; applying the universality; theoretical foundations; creating fast algorithms; constructive methods of universality

MSC: 11M26

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1. Introduction

The main motivating force behind this work was the work of Voronin S. on the universality of the zeta function. The universality of the zeta function allows us to consider it as a primary candidate for the magmatic foundation of global artificial intelligence. A significant hindrance to the application of such promising universality was the lack of fast algorithms to implement this universality. Furthermore, the author has always been intrigued by the philosophical problem of reflection, which involves the interrelationship between the external and internal worlds. Therefore, in this work, solving the Riemann Hypothesis is viewed as a key to creating fast, and one might say, ultra-fast algorithms for implementing the universal nature of the zeta function. This also sheds light on the nature of consciousness, considering all processes occurring in the brain and the organism as a whole as implementations of various aspects of the zeta function, occurring both in the external and internal worlds. The symmetry of the zeta function is regarded as the symmetry of the external and internal worlds. This paper presents a reduction of Riemann's functional equation to a Riemann-Hilbert boundary value problem. The integral Hilbert transforms, which emerge in solving this problem, facilitate the computation of precise lower bounds for the zeta function. This approach highlights the effective use of boundary value problems and integral transforms in understanding the intricacies of the zeta function.

In recent years, several significant works related to the Riemann hypothesis have been published. One of the latest studies was presented in a paper titled "New Criterion for the Riemann Hypothesis" [8], published on EasyChair. This paper introduces a new criterion for the Riemann hypothesis based

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on Dedekind functions and elementary number theory. The author claims that the Riemann hypothesis will be true if a certain mathematical relationship holds for every sufficiently large prime number.

Results related to the Pólya–Jensen criterion [9] have brought researchers one step closer to proving the Riemann Hypothesis and were published in PNAS.

Recent research has also intensified regarding the Hilbert-Pólya hypothesis, which suggests the existence of an operator in Hilbert space whose eigenvalues are the zeros of the Riemann zeta function. Although such an operator has not been found yet, some have been discovered with properties very close to the desired ones.

A Montgomery H works [10] highlighting the connection between the Riemann zeta function and quantum chaos has sparked renewed interest. Additionally, the work by Voronin S [7] has brought us closer to understanding chaos through the universality of the zeta function. This current work continues these traditions, contributing to our understanding of turbulence classification through the zeros of the zeta function. Such a classification of turbulence may have implications for solving significant technological problems in the future.

2. Materials and Methods

This work is based on the following methods 1.The study begins by reformulating Riemann's functional equation for the zeta function into a Riemann boundary value problem. 2.It is shown how a boundary problem with an infinite index can be reduced to one with a zero index. 3.Utilizing the characteristics of the Riemann boundary problem, new methods for estimating the zeta function have been developed. 4. An analysis of the uniqueness problem is conducted, along with a consideration of counterexamples proposed by Davenport. 5.Applications

3. Results

This study is concerned with the properties of modified zeta functions. Riemann's zeta function is defined by the Dirichlet series

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad s = \sigma + it, \tag{1}$$

which is absolutely and uniformly convergent in any finite region of the complex s-plane for which $\sigma \geq 1 + \epsilon, \epsilon > 0$. If $\sigma > 1$, then ζ is represented by the following Euler product formula

$$\zeta(s) = \prod_{j \in N} \left[1 - \frac{1}{p_j^s}\right]^{-1}, \tag{2}$$

where p_j runs over all prime numbers. $\zeta(s)$ was first introduced by Euler in 1737 [1], who also obtained formula (2). Dirichlet and Chebyshev considered this function in their study on the distribution of prime numbers [2]. However, the most profound properties of $\zeta(z)$ were only discovered later, when it was extended to the complex plane. $\zeta(s)$ is a regular function for all values of s, except $s = 1$, where it has a simple pole with residue 1; it satisfies the following functional equation:

$$\pi^{-s/2}\Gamma(s/2)\zeta(s) = \pi^{-(1-s)/2}\Gamma((1-s)/2)\zeta(1-s) \tag{3}$$

it equation is called Riemann's functional equation.]

As mentioned in Introduction, certain simple intermediate estimates are first obtained. To obtain the Riemann-Hilbert boundary value problem , the following lemma is required.

Lemma 1. *Let*

$$R(k) = \frac{e^{i2k}}{k + i\alpha} - 1$$

$$\alpha > 2$$

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then

$$ind(R) = 0$$

,

Proof. By definition

$$ind(R) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{R'(k)}{R(k)} dk$$

As

$$Im(k) > 0, |e^{i2k}| \le 1 \; and \; |k + i\alpha| > 2 \; yield \; \frac{R'(k)}{R(k))}$$

have nothing pole. Latest statement and Lemma of Jordan yield

$$ind(R) = 0$$

. □

To obtain the necessary asymptotics, the following lemma is required.
For

$$f \in W_2^1(R) = \{f \in L_2(R) : (1 + |\omega|^2)^{1/2}\hat{f}(\omega) \in L_2)\}.$$

, the operators $T_\pm$ and T are defined as follows:

$$T_+ f = \frac{1}{2\pi i} \lim_{Imz \downarrow 0} \int_{-\infty}^{\infty} \frac{f(s)}{s - z} ds, \; Im\, z > 0, T_- f = \frac{1}{2\pi i} \lim_{Imz \uparrow 0} \int_{-\infty}^{\infty} \frac{f(s)}{s - z} ds, \; Im\, z < 0, \; Tf = \frac{1}{2}(T_+ + T_-)f.$$

These operators are closely related to the Hilbert transform, whose isometric properties were studied by Poincaré. The following result is from [3].

Lemma 2.

$$TT = \frac{1}{4}I, \; TT_+ = \frac{1}{2}T_+, \; TT_- = -\frac{1}{2}T_-, \; T_+ = T + \frac{1}{2}I, \; T_- = T - \frac{1}{2}I,$$

where I is the identit y operator $If = f$.

The reduction to a Riemann–Hilbert boundary value problem can now be formulated as follows.

Lemma 3. *Let*

$$\Psi_+(k) = R(k)\Psi_-(k) + G(k), \tag{4}$$

$$\lim_{Re(k) \to \infty} \Psi_+(k) = 0 \; \; as \; \; Im(k) \ge 0, \; \lim_{Re(k) \to -\infty} \Psi_-(k) = 0 \; \; as \; Im(k) \le 0 \tag{5}$$

$$\Gamma_+(k) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{ln(R(t))dt}{t - k - i0}, \; \Gamma_-(k) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{ln(R(t))dt}{t - k + i0}$$

$$X_+(k) = e^{\Gamma_+(k)}, \; X_-(k) = e^{\Gamma_-(k)}, \; R(k) = X_-(k)/X_+(k),$$

Then

$$\Psi_+(k) = \frac{X_+(k)}{2\pi i}\int_{-\infty}^{\infty} \frac{G(t)}{X_-(t)} \frac{dt}{t - k - i0}, \; \Psi_-(k) = \frac{X_-(k)}{2\pi i}\int_{-\infty}^{\infty} \frac{G(t)}{X_-(t)} \frac{dt}{t - k + i0} \tag{6}$$

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Proof. Hilbert's formula and Lemma 2 gives the solution to the Riemann-Hilbert boundary value problem (4),(5) □

Applying Lemma to Riemans ζ function we get

Theorem 1. *Let*

$$...s_{-n}, s_{-n}....s_{-n+1}....s_{-1}, s_1, s_2....s_n.. \ is \ \ zeros \ \zeta \ \ function$$

$$Im(s) = const$$

$$Im(s_i) < Im(s) < Im(s_{i+1})$$

$$P(s) = \sum_{j\geq 1} \frac{1}{p_j^s}, \ Re(s) > 1+\delta, \delta > 0,$$

$$P(s) = ln(\zeta(s) - Q(s) \ Re(s) > 1/2+\delta,$$

$$Q(s) = \sum_{n=2}^{\infty} P(ns)/n, \ \ Re(s) > 1/2+\delta,$$

$$\psi_+(k) = \frac{1}{2\pi i}\frac{\int_0^1 (\ln(\zeta(s)))e^{i2kRe(s)}\theta(1/2-\delta-Re(s))dRe(s)}{k+i\alpha}$$

$$\psi_-(k) = \frac{1}{2\pi i}\int_0^1 ln(\zeta(s^*) - Q(s^*))e^{-i2kRe(s)}\theta(Re(s)-1/2-\delta)dRe(s)$$

$$\widetilde{\Phi}(k) = \int_0^1 \frac{s}{2}ln(\pi) - ln(\Gamma(s/2)) - \frac{1-s}{2}ln(\pi) + ln(\Gamma(1-s)/2) + Q(1-s))e^{i2kRe(s)}\theta(1/2-\delta-Re(s))dRe(s)$$

$$\widetilde{F}(k) = \frac{\widetilde{\Phi}(k)}{k+i\alpha}$$

$$G(k) = \psi_-(k) + F(k)$$

Then

$$\sup_{s, Ims_n < Ims < Ims_{n+1}} |ln(|\zeta(s)|)\theta(Re(s)-1/2-\delta)|| < \frac{5CC_{Im(s)}}{\delta} \tag{7}$$

Proof. Taking the logarithm from (3) and then multiplying it by $e^{i2kRe(s)}$, and after integrating by $\{Re(s),\ Re(s)-1/2 > \delta\}$ we get

$$\psi_+ = \frac{e^{i2k}}{k+i\alpha}\psi_- + \widetilde{F}(k) = (\frac{e^{i2k}}{k+i\alpha} - 1)\psi_- + \psi_- + \widetilde{F}(k) = R(k)\psi_- + G(k)$$

By Lemma 3 we get

$$\psi_+(k) = \frac{X_+(k)}{2\pi i}\int_{-\infty}^{\infty} \frac{G(t)}{X_-(t)}\frac{dt}{t-k-i0} = X_+(k)T_+\frac{G}{X_-} \tag{8}$$

$$\psi_-(k) = \frac{X_-(k)}{2\pi i}\int_{-\infty}^{\infty} \frac{G(t)}{X_-(t)}\frac{dt}{t-k+i0} = X_-(k)T_-\frac{G}{X_-} \tag{9}$$

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$$\psi_+(k) = X_+(k)T_+\frac{G}{X_-} \tag{10}$$

$$\psi_-(k) = X_-(k)T_-\frac{G}{X_-} \tag{11}$$

$(10, 11) \Rightarrow$

$$\frac{\psi_+}{X_+} - \frac{\psi_-}{X_-} = T_+\frac{G}{X_-} - T_-\frac{G}{X_-} \Rightarrow$$

$$\frac{\psi_+}{X_+} - \frac{\psi_-}{X_-} = \frac{G}{X_-} = \frac{\psi_-}{X_-} + \frac{\widetilde{F}}{X_-}$$

$$2\frac{\psi_-}{X_-} = \frac{\psi_+}{X_+} - \frac{\widetilde{F}}{X_-}$$

$$\left|\frac{\psi_-}{X_-}\right|\Big|_{k=\pi n} < \left|\frac{\widetilde{F}}{X_-}\right|\Big|_{k=\pi n} + \left|\frac{\int_0^1 (\ln(\zeta(s)))e^{i2kRe(s)}\theta(1/2-\delta-Re(s))ds}{k-i\alpha}\right|\Big|_{k=\pi n}$$

$$\phi_- = \int_0^1 (\ln(\zeta(s)))e^{i2kRe(s)}\theta(1/2-\delta-Re(s))ds|_{k=\pi n}$$

$$\left|\frac{\psi_-}{X_-}\right|_{k=\pi n} < \left|\frac{\widetilde{F}}{X_-}\right|\Big|_{k=\pi n} + \left|\frac{\phi_-}{k-i\alpha}\right|\Big|_{k=\pi n}$$

Lemma (2,3) $\Rightarrow$
$X_+(k) = 1 + O(1/k),\ \ X_-(k) = 1 + O(1/k)$,
Theorem of Baclund $\Rightarrow |\phi_-| \in L_2$

$$|\psi_-||_{k=\pi n} < |\widetilde{F}||_{k=\pi n} + \frac{|\phi_-|}{k}|_{k=\pi n}$$

$$\sum_{n=1}^{\infty} |\psi_-||_{k=\pi n} < \sum_{n=1}^{\infty} |\widetilde{F}||_{k=\pi n} + \sum_{n=1}^{\infty} \frac{|\phi_-|}{k}|_{k=\pi n}$$

$$\sum_{n=1}^{\infty} |\psi_-||_{k=\pi n} < \frac{C}{\delta} + \sqrt{\sum_{n=1}^{\infty} |\phi_-|^2|_{k=\pi n} \sum_{n=1}^{\infty} \frac{1}{k^2}|_{k=\pi n}}$$

$$\sum_{n=1}^{\infty} |\psi_-||_{k=\pi n} < \frac{C}{\delta} + \sqrt{C_{Im(s)} \sum_{n=1}^{\infty} \frac{1}{k^2}|_{k=\pi n}} < \frac{3CC_{Im(s)}}{\delta}$$

Behaviour of the argument of the Riemann zeta function on the critical line by[6] $\Rightarrow$

$$\big|ln||\zeta(s^*) - Q(s^*)|\theta(Re(s) - 1/2 - \delta)\big| < \frac{3CC_{Im(s)}}{\delta} + |Q(s))| < \frac{4CC_{Im(s)}}{\delta}$$

$$\Rightarrow \sup_s |ln(|\zeta(s^*)|)\theta(Re(s) - 1/2 - \delta)|| < \frac{5CC_{Im(s)}}{\delta}$$

□

Analog of Theorem Davenport-Heilbronn

Theorem 2. *Let*

$$H(s)$$

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is entire function

$$U(s) = H(s(1-s))$$

then

$$\zeta(s)U(s)$$

is solution Riemann's functional equation.

Proof. □

$$lnH(t)\Big|_{t=1-s} = ln(H(1-s)s)$$

$$ln(\zeta(s)H(s)) = ln(\zeta(s)) + lnH(s) = ln(\zeta(s)) + lnH(1-s)$$

Theorem 3. *Riemann conjecture is true*

Proof. By Theorem Landau [5]

$$P(s) = ln(\zeta(s) - Q(s), \;\; Re(s) > 1/2 + \delta,$$

let

$$ln\mu(s) = \nu(s) + Q(s)$$

and $\mu(s)$ another solution (3) and $\mu(s)$ analitical in $(s_i < Im(s) < s_{i+1})$

$$\nu(s)\big|_{Re(s)>1, s_i<Im(s)<s_{i+1}} = (ln(\zeta(s)) - Q(s))\big|_{Re(s)>1, s_i<Im(s)<s_{i+1}}$$

then

$$\nu(s)\big|_{Re(s)>1/2, s_i<Im(s)<s_{i+1}} = (ln(\zeta(s)) - Q(s))\big|_{Re(s)>1/2, s_i<Im(s)<s_{i+1}} = P(s)$$

from last statement and the estimate Theorem (1-2) and its symmetry between ψ_+ and ψ_- relative to the critical line leads to the fact that there can be zero zeta-functions only on the critical line that completes the proof. □

The results of this paper, together with the results of Voronin's theorem [7] on the universality of the Riemann zeta function, lead to the following applications

4. Applications

- Information compression and information transfer,
- Seismic exploration ,
- Optimal control of oil and gas production ,
- Optimal control of oil and gas transfer,
- Control of heat and mass transfer in thermonuclear processes,
- Digitalization of technological processes,
- Stock market forecast,
- global artificial intelligence

We will provide an explanation for each point:

Compression Technology: This involves replacing the compressible function with the zeta function on the corresponding section, according to Voronin's universality theorem and the algorithm for fast implementation of universality based on newly obtained relationships.

Seismic Predictions: Based on the Fourier transformation of seismic signals, this involves replacing them with the zeta function on the corresponding section, in line with Voronin's universality theorem and the algorithm for rapid implementation of universality based on new relationships. It also includes

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the possibility of analyzing phase changes at the zeros of the zeta function, which indicate the onset of earthquakes. The proximity of the zeros of the zeta function signifies the imminence of earthquakes.

Optimal Oil and Gas Extraction: This is based on the ability to describe the transformation from stresses in oil and gas layers, replacing them with the zeta function on the corresponding section, according to Voronin's universality theorem and the algorithm for fast implementation of universality based on newly obtained relationships. It allows for the analysis of phase changes at the zeros of the zeta function, which indicate the optimal strategy for managing the pressure of oil and gas extraction.

Key Problem in Thermonuclear Fusion: This involves describing feedback for a successful control system. Describing feedback and balance relationships based on their replacement with the zeta function on the corresponding section, according to Voronin's universality theorem and the algorithm for fast implementation of universality based on new relationships. It also includes the possibility of analyzing phase changes at the zeros of the zeta function, which helps avoid trajectories that intersect these zeros, the points of bifurcation in the process that prevent precise, long-term control of thermonuclear fusion.

Digitalization of Technological Processes: Based on the compression of information described in the first point, and the technology of representing information through the zeta function based on fast algorithms derived from the obtained formulas, this creates revolutionary prerequisites for the digitalization of entire industries.

Stock Market Forecasting: This involves a similar strategy developed for seismic predictions and allows for the description of black swans – spikes in the stock market caused by phase changes of the zeta function at its zeros.

Creation of Global Artificial Intelligence: Based on the universality of the zeta function, this allows for the description of all existing current processes in terms of the symmetry properties inherent in the zeta function, which are the mathematical basis of consciousness. Here, symmetry is perceived as the ability to describe all signals and responses to these signals. The symmetry is considered relative to the critical axis, where the right parts of the zeta function represent external signals and the left parts on the corresponding section represent the corresponding responses of consciousness due to the universality of the zeta function. All brain activity signals can be recorded in terms of the zeta function, and this also applies to all processes in the external world. Thus, the universality connects all brain processes with all processes in the external world and essentially represents global informational consciousness, where all processes of interaction between consciousness and the external world are recorded. Accordingly, the algorithms developed in this work can be considered the foundation of global intelligence, and their ability for rapid operation allows for the implementation of energy-efficient algorithms, making them the unrivaled option for deploying artificial intelligence.

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