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Article

Insights into IF-Geodetic Convexity in Intuitionistic Fuzzy Graph: Harnessing the IF-Geodetic Wiener Index for Global Human Trading Analysis and IF-Geodetic Cover for Gateway Node Identification

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Abstract: To offer a viewpoint on convexity and connectedness inside intuitionistic fuzzy graphs (IFGs), the paper is devoted to the study of intuitionistic fuzzy geodetic convexity. The paper introduces an algorithm for precise identification and characterization of geodetic pathways in IFGs, supported by a Python program. Various properties of IF-geodetic convex sets such as IF-internal and IF-boundary vertices are obtained. Furthermore, this work introduces and characterizes the concepts of geodetic IF-cover, geodetic IF-basis, and geodetic IF-number. Additionally, the study develops the IF-geodetic Wiener index. The scope of the work explores the application of IF-geodetic cover in wireless mesh networks, focusing on the identification of gateway nodes. A practical implementation of the IF-geodetic Wiener index method in global human trading analysis underscores the real-world implications of the developed concepts.

Keywords: intuitionistic fuzzy graph; IF-geodetic convex; geodetic IF-number; geodetic IF-basis; IF-internal vertices; IF-boundary vertices; IF-geodetic Wiener index

1. Introduction

The concept of fuzzy sets was introduced by Lotfi A. Zadeh [21] in 1965 and addresses ambiguity and imprecision in set theory. Zadeh researched fuzzy relations and how they work. Zadeh's objective was to develop a theory that might regulate the imprecision and vagueness of a certain category in human thought, particularly in the areas of abstraction, information technology, and communication. In his study, Zadeh challenged a basic concept of probability theory. Each element in the universal set is assigned a value of 1 or 0 by the characteristic function in the crisp set. A generalized version of the characteristic function defined in a crisp set can be thought of as fuzzy set theory. Here, the items in the universal set are assigned within a predetermined range, indicating their membership grade. Subsequently, this function is referred to as a membership function, and the set that is produced is known as a fuzzy set. The membership function has a range of values from 0 to 1. Stated otherwise, the membership function's values vary within the unit interval [0, 1].

Rosenfeld [1] is credited with organizing and developing the concept of fuzzy graphs. Yeh and Bang independently established fuzzy graph design in the meantime. Rosenfeld deduced the fuzzy equivalents of multiple concepts in graph theory. There is an extensive range of practical applications for fuzzy graphs. Numerous domains, including cluster analysis, group structure, pattern categorization, networking, management, and others, demonstrate their broad range of applications. Fuzzy graphs are more useful in real-world scenarios due to their extensive range of useful applications.

Numerous other authors like P.S. Nair [5], Sunil Mathew and J.N. Mordeson [7,17], Sunitha and Vijayakumar [18] and R. Rajeshkumar and A. M. Anto [15] carried out many other groundbreaking studies in this area.

Atanassov [2] introduced a number of concepts related to intuitionistic fuzzy graphs (IFGs). The field of intuitionistic fuzzy sets and its applications has grown exponentially worldwide. This idea now encompasses the fields of information technology as well as traditional mathematics. The concept and associated features of intuitionistic fuzzy graphs were introduced by Parvathy and Karunambigai [14]. Additionally, Nagoor Gani covered a few more features of intuitionistic fuzzy graphs in [12]. In [8], Akram et al. offered various concepts and provided a precise definition of IFGs. Anto A.M. and Rajeshkumar R. [15] introduced the idea of IFGs having independent dominances.

This article starts with a brief description of the fundamentals of intuitionistic fuzzy graphs discussed in [10,15,20]. Later, based on a metric established in intuitionistic fuzzy graphs in [19], this study introduces novel ideas and approaches that make a substantial addition to the subject of intuitionistic fuzzy graph theory. We present the concept of intuitionistic fuzzy geodetic convexity, which offers an insightful view of convexity and connectedness in the context of intuitionistic fuzzy graphs (IFGs). A specially designed Python program for precisely finding and characterizing geodetic pathways in IFGs is presented along with a specialized algorithm for geodetic path identification in the study. The investigation also includes IF-internal and IF-boundary vertices in IF-geodetic convex sets. An important development in the field of IFGs is the creation of the Geodetic IF-Basis and Geodetic IF-number, providing tools for a thorough understanding and characterization of geodetic convexity. A novel concept, the Intuitionistic Fuzzy Geodetic Wiener Index (IF_gWT), is introduced, leveraging the framework of IF-geodesics. Furthermore, the application of IF-Geodetic Cover in a wireless mesh network is explored for the identification of gateway nodes, demonstrating its effectiveness in enhancing network connectivity and optimizing node placement for improved efficiency. In a practical application, the IF_gWT method is employed in global human trading analysis.

2. Preliminaries

We use the notion of Intuitionistic Fuzzy Graphs (IFGs) introduced by Karunambigai and Parvathi [14]. In this section, let us have a quick review of the basic concepts in intuitionistic fuzzy graph theory which were established in [8,14,15,19].

An IFG is a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ be the vertex set, $\mu_1 : \mathcal{V} \rightarrow [0, 1]$ and $\gamma_1 : \mathcal{V} \rightarrow [0, 1]$ represent membership degree and nonmembership degree of each element $v_i \in \mathcal{V}$, respectively, and $\mu_1(v_i) + \gamma_1(v_i)$ varies over the range $[0, 1] \forall v_i \in \mathcal{V}$. $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where $\mu_2 : \mathcal{V} \times \mathcal{V} \rightarrow [0, 1]$ and $\gamma_2 : \mathcal{V} \times \mathcal{V} \rightarrow [0, 1]$ are given by $\mu_2(v_i, v_j) \leq \wedge [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \geq \vee [\gamma_1(v_i), \gamma_1(v_j)]$, here $\mu_2(v_i, v_j) + \gamma_2(v_i, v_j)$ varies over the range $[0, 1]$ for all $(v_i, v_j) \in \mathcal{E}$. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be two IFGs, if $\mathcal{V}' \subseteq \mathcal{V}$ and $\mathcal{E}' \subseteq \mathcal{E}$, we call $\mathcal{H}$ as an IF-subgraph of $\mathcal{G}$. There doesn't exist any edge between v_i and v_j if $\mu_2(v_i, v_j) = 0 = \gamma_2(v_i, v_j)$ for some i and j . In an IFG, a path $\mathcal{P}$ is defined as a chain of distinct vertices $v_1, v_2, \dots, v_n$ and which satisfies any one of the three following conditions: (a) $\mu_2(v_i, v_j) > 0$ and $\gamma_2(v_i, v_j) = 0$ for some i and j , (b) $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) > 0$ for some i and j , (c) Both $\mu_2(v_i, v_j)$ and $\gamma_2(v_i, v_j)$ are greater than zero for some i and j ($1 \leq i, j \leq n$). Let $\mathcal{P}$ be a path in an IFG, then we say that $\mathcal{P}$ is a cycle if for $n \geq 3$, $v_1 = v_{n+1}$. If two vertices are linked by a path then we say that they are connected. The μ -strength (S_μ) and γ -strength (S_γ) of a path $\mathcal{P}$ is defined as $\wedge_{i,j} \{\mu_2(v_i, v_j)\}$ and $\vee_{i,j} \{\gamma_2(v_i, v_j)\}$ respectively. The strength of a path $\mathcal{P}$ is the value of an edge with both the values of μ -strength and γ -strength and denoted as $S_{\mathcal{P}}$. $\mu^\infty(v_i, v_j) = \vee \{S_\mu\}$ represents the μ -strength of connectedness and $\gamma^\infty(v_i, v_j) = \wedge \{S_\gamma\}$ represents the γ -strength of connectedness among all possible paths. An IFG is complete if $\mu_2(v_i, v_j) = \wedge [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) = \vee [\gamma_1(v_i), \gamma_1(v_j)] \forall v_i, v_j \in \mathcal{V}$. A connected IFG is an intuitionistic fuzzy tree if it has an intuitionistic fuzzy spanning subgraph $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ which is a tree, where for all arcs not in $\mathcal{H}$, $\mu_2(v_i, v_j) < \mu^\infty(v_i, v_j)$ and $\gamma_2(v_i, v_j) > \gamma^\infty(v_i, v_j)$. In an IFG, the weight of a vertex u is defined by $\mathcal{W}(u) = \frac{1+\mu_1-\gamma_1}{2}$ and also the weight of an edge uv is defined $\mathcal{W}(u, v) = \frac{1+\mu_2(u, v)-\gamma_2(u, v)}{2}$. In an IFG, the distance $d(u, v)$ between two of its

vertices is the length of the shortest path between them, i.e., $d(u, v) = \wedge \{\sum_{i,j} \mathcal{W}(u_i, v_j)\}$. The function d is called an intuitionistic fuzzy graph metric on $\mathcal{V}$.

3. Intuitionistic Fuzzy Geodetic Convexity

The concept of intuitionistic fuzzy geodetic convexity is introduced in this section, along with some of its characteristics. A particular algorithm for geodetic path identification in IFGs is described, with a specially constructed Python program.

Definition 1. Let v_i and v_j be any two vertices in a connected IFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. A $v_i - v_j$ path $\mathcal{P}$ is called an IF $u_i - v_j$ geodesic if $d_g(u_i, v_j) = d(u, v)$ where $d(u, v) = \wedge \{\sum_{i,j} \mathcal{W}(u_i, v_j)\}$.

Definition 2. Given any two vertices v_i and v_j in a connected IFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, the IF-geodetic closed set is defined to be the set which contains all the vertices in all $v_i - v_j$ IF-geodesics together with v_i and v_j and is denoted by $I_g[v_i, v_j]$.

Definition 3. In a connected IFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ the IF-geodetic closure of $\rho \subseteq \mathcal{V}$ is defined by the union of all IF-geodetic closed sets over each pair of vertices $v_i, v_j \in \rho$ and denoted as $I_g[\rho]$.

Definition 4. Let ρ be a subset of $\mathcal{V}$ in a connected IFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and is said to be an IF-geodetic convex if its IF-geodetic closure equals ρ , i.e. $I_g[\rho] = \rho$.

Definition 5. The IF-geodetic eccentricity $IF_g e(v_i)$ of a vertex v_i in an IFG $\mathcal{G}$ is defined by $IF_g e(u_i) = \vee_{v_j \in \mathcal{V}} d_g(u_i, v_j)$.

Definition 6. The IF-geodetic radius ($IF_g r(\mathcal{G})$) of an IFG $\mathcal{G}$ is given by $IF_g r(\mathcal{G}) = \wedge_{v_j \in \mathcal{V}} \{IF_g e(u_i)\}$ and the IF-geodetic diameter ($IF_g diam(\mathcal{G})$) of $\mathcal{G}$ is defined by $IF_g diam(\mathcal{G}) = \vee_{v_j \in \mathcal{V}} \{IF_g e(u_i)\}$.

Definition 7. A vertex v_i is a mid-vertex if $IF_g e(v_i) = IF_g r(\mathcal{G})$. Similarly, a vertex v_j is a geodetic IF-peripheral vertex if $IF_g e(v_j) = IF_g diam(\mathcal{G})$.

Example 1. Consider the following IFG given in Figure 1.

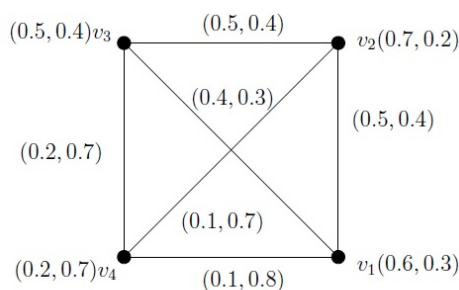


Figure 1.

Here, let $\rho = \{v_1, v_3, v_4\}$ be an IF-geodetic convex set since $I_g[\rho] = \rho$. Similarly, $\{v_1, v_4\}$ and $\{v_3, v_4\}$ are IF-geodetic convex sets. The IF-geodetic eccentricity of a vertex v_1 is $IF_g e(v_1) = \vee \{0.35, 0.4, 0.15\} = 0.4$. Also, $IF_g r(\mathcal{G}) = 0.25$ and $IF_g diam(\mathcal{G}) = 0.45$.

The observations mentioned in the following proposition pass for obvious.

Proposition 1. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an IFG. Then the set of entire vertices, all singleton sets of vertices, and the null set are IF-geodetic convex.

Theorem 1. For an IFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, the intersection between two IF-geodetic convex sets is again IF-geodetic convex.

Proof. Let’s assume $v_i, v_j \in X \cap Y$, we observe that both v_i and v_j are in X and Y . This leads to the desirable conclusion, that all the vertices in each $v_i - v_j$ IF-geodesics are both in X and Y . Therefore, $X \cap Y$ is an IF-geodetic convex. $\square$

Theorem 2. If $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an IF-tree, there exists an IF-spanning subgraph and has a finite collection of nested sets $\mathcal{V}' = \mathcal{V}_n' \supset \mathcal{V}_{n-1}' \supset \mathcal{V}_{n-2}' \supset \dots \supset \mathcal{V}_\infty'$ where each $\mathcal{V}_j'$ is IF-geodetic convex.

Proof. Assume that $\mathcal{G}$ is an IF tree, from the definition, there is an IF spanning subgraph, say $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ and which is a tree. Since $\mathcal{H}$ is a tree, it assures the existence of a single path between any pair of vertices. This leads us to an important observation, that each path connecting any pair of vertices is a unique IF-geodesic. Thus the union of all vertices in every geodetic path is an IF-geodetic convex. Let $\mathcal{H}_1$ be a subgraph of $\mathcal{H}$ that is obtained from the set $\mathcal{V}_{n-1}'$ by removing a terminal vertex from $\mathcal{V}_n'$. As a result, IF-subgraph $\mathcal{H}_1$ is a tree. Similar justifications lead us to the conclusion that $\mathcal{V}_{n-1}'$ is an IF-geodetic convex. We keep going till we obtain a singleton set. Thus, it is undeniably true that the finite collection of nested sets exists in a manner that allows for $\mathcal{V}' = \mathcal{V}_n' \supset \mathcal{V}_{n-1}' \supset \mathcal{V}_{n-2}' \supset \dots \supset \mathcal{V}_1'$. $\square$

4. Algorithm to Compute Geodetic Path of IFGs

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an IFG with n vertices. In the literature of graph theory, various algorithms are available to compute the geodetic path between pairs of nodes. In this part, we created an algorithm for determining the geodetic path of an intuitionistic fuzzy graph and, more specifically, we offered a Python program initially for identifying all geodetic paths of IFGs.

4.1. Python Program for Identifying All Geodetic Paths of IFGs

```
# A program to find Geodetic Paths in an IFG
vertices = str(input("Enter the vertices : "))
# we will take all the vertices in graph
vertices = vertices.split("_")
def path_finder(vertices):
    """Find all the possible source-destinations"""
    ver1 , ver2 = vertices, vertices
    paths = []
    for v in ver1:
        for v_ in ver2:
            if v != v_:
                path_ = str(v)+"_"+str(v_)
                if path_ in paths or str(v_)+"_"+str(v) in paths:
                    pass
                else:
                    paths.append(path_)
    #print(paths)
    return paths
source_dest = path_finder(vertices)
PATH_SUBPATHS = {} # store source-destination : [path1,path2,path3...]
for src_des in source_dest:
    v1,v2 = src_des.split("_")
    print("Please enter the no of possible paths between '{v1}' and '{v2}'"
    ".format(v1 = v1, v2 = v2 ))
    no_path = int(input("=> "))
```

```

sub_path_list = []
for i in range(no_path):
    sub_path = str(input("Enter path {path} : ".format(path = i )))
    sub_path_list.append(sub_path)
PATH_SUBPATHS[src_des] = sub_path_list
print(PATH_SUBPATHS)
CALCULATED_PATHS = {} #{MAIN_PATH1:{PATH:distance},
MAIN_PATH2 : {PATH:distance}}
class Distance:
    """A class for distance calculation"""
    def __init__(self):
        pass
    def get_weight(self,mu2,gama2):
        weight = (1+mu2-gama2)/2
        return weight
    def distance(self,path):
        no_edges = len(path.split("_")) - 1
        weight_sum = 0
        print("Enter the edges of subpath "+ path + " accordingly")
        for i in range(no_edges):
            edge = input("Enter the edge "+ str(i+1) + " : ")
            edge = tuple(float(num) for num in edge.split())
            weight = self.get_weight(mu2 = edge[0], gama2 = edge[1])
            weight_sum += weight
        return weight_sum
for MAIN_PATH in PATH_SUBPATHS.keys():
    paths_mainpath = PATH_SUBPATHS[MAIN_PATH]
    dist = Distance()
    CALCULATED_PATHS[MAIN_PATH] = {}
    for path in paths_mainpath:
        CALCULATED_PATHS[MAIN_PATH][path] = dist.distance(path)
print(CALCULATED_PATHS)
FINAL_GEODETICS_PATHS = {} #{MAIN_PATH:{"GEODESIC_PATH":value}}
for MAIN_PATH in CALCULATED_PATHS.keys():
    distances = list(CALCULATED_PATHS[MAIN_PATH].values())
    keys = list(CALCULATED_PATHS[MAIN_PATH].keys())
    min_distance = sorted(distances)[0]
    min_index = distances.index(min_distance)
    Gdesic_path = keys[min_index]
    FINAL_GEODETICS_PATHS[MAIN_PATH] = {}
    FINAL_GEODETICS_PATHS[MAIN_PATH][Gdesic_path] = min_distance

print(=====GEODESIC PATHS=====)
print(FINAL_GEODETICS_PATHS)

```

4.2. Algorithm

- Step 1. Determine how many vertices there are in the graph.
- Step 2. Identify the number of possible pathways between each pair of nodes.
- Step 3. Make a list of all possible pathways for each source and destination vertices.
- Step 4. Using the Python program to locate geodetic paths from the paths mentioned in Step 3. Enter the edge values of the sub-paths accordingly.

Step 5. A distance class will be constructed using a predetermined distance formula.

Step 6. Construct an $m \times m$ matrix D_g , where each row and column represent the vertices in IFG. If row i corresponds to vertex u_i and column j corresponds to vertex v_j , then the appropriate element in the ij^{th} array is $d_g(u_i, v_j) = \wedge \{ \sum_{i,j} \mathcal{W}(u_i, v_j) \}$.

Step 7. Obtain a list of every geodetic path and its associated distance between two vertices. So, in Step 7, we are able to obtain all of the geodetic pathways.

5. Intuitionistic Fuzzy Geodetic Blocks

In intuitionistic fuzzy graphs, the idea of blocks was established by Muhammad Akram and N. O. Akshehri in 2014. In this section, we present the notion of intuitionistic fuzzy geodetic blocks and some of its characterization.

Definition 8. A connected intuitionistic fuzzy subgraph of $\mathcal{G}$ is called an intuitionistic fuzzy geodetic block (IFGB) if there doesn't exist a path such that $\sum_{i,j} \mathcal{W}(u_i, v_j) \geq d(v_i, v_j) \forall v_i, v_j \in \mathcal{V}'$.

A fact that should be evident from the definition is that in an IFGB there is only one IF-geodetic path between any pair of nodes. For a complete IF-subgraph, the edges themselves act as sole IF-geodetic path.

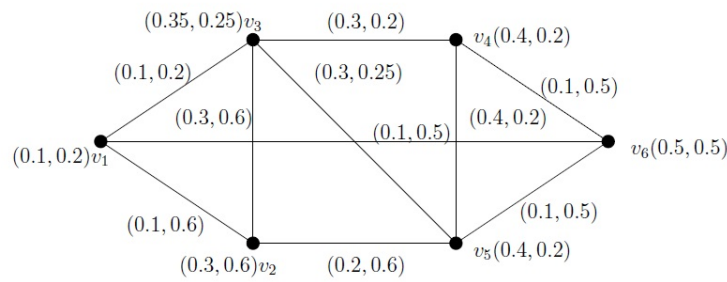


Figure 2.

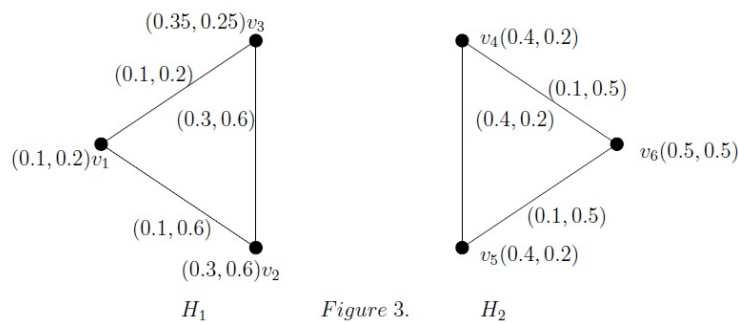


Figure 3.

Example 2. To familiarize ourselves with Definition 8, let's consider an IFG in above Figure 2 and its subgraphs in Figure 3. Here the IF-subgraph $\mathcal{H}_1$ is an IFGB. But on the other hand $\mathcal{H}_2$ is not an IFGB, since there exist more than one IF-geodetic paths in between the vertices v_4 and v_3 .

Theorem 3. If $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be a complete IF-subgraph of a connected IFG $\mathcal{G}$. Then $\mathcal{H}$ is an IFGB if and only if $\{v_i, v_j\}$ is IF-geodetic convex for any $v_i, v_j \in \mathcal{V}'$.

Proof. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represent a connected IFG and $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be a complete IF-subgraph of $\mathcal{G}$. For the forward direction, we begin with the assumption that $\mathcal{H}$ is an IFGB. Since it is an IFGB and complete, every edge that connects two vertices serves as the only IF-geodetic path. This shows that $\{v_i, v_j\}$ is IF-geodetic convex.

To prove the other half of the theorem, suppose that $\{v_i, v_j\}$ is IF-geodetic convex for any $v_i, v_j \in \mathcal{V}'$. To prove there is only one IF-geodetic path between any pair of nodes, we assume for contradiction that there exists another IF-geodesic in $\mathcal{H}$, say $\mathcal{P}$. Suppose that $\mathcal{P}$ has m edges. Infact, $\mathcal{H}$ is complete, thus $\sum_{i,j} \mathcal{W}(v_i, v_j)$ will exceeds $d(v_i, v_j)$, but which is not possible. This contradiction finishes the proof. $\square$

Theorem 4. Let $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be a complete IF-subgraph of a connected IFG $\mathcal{G}$. If $\mathcal{H}$ is an IFGB, then $\mathcal{V}'$ is an IF-geodetic convex.

Proof. Let $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be a complete IF - subgraph of a connected IFG $\mathcal{G}$. Since $\mathcal{H}$ is an IFGB, there doesn't exists a path in $\mathcal{H}$ such that $\sum_{i,j} \mathcal{W}(v_i, v_j) \geq d(v_i, v_j) \forall v_i, v_j \in \mathcal{V}'$. Here, in this case is $\mathcal{H}$ complete, leads to the conclusion that an edge connecting any pair of vertices in $\mathcal{H}$ itself is the only IF -geodesic between them. Thus, the vertices in all IF $v_i - v_j$ geodesics are exactly v_i and v_j . This is obvious for any pair of vertices in $\mathcal{V}'$, hence we are done. $\square$

Theorem 5. If a complete IF-subgraph of a connected IFG is an IFGB. Then every vertex removed IF-subgraph of an IFGB generates an IFGB again.

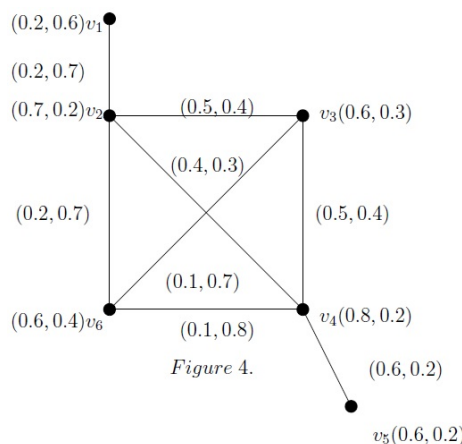
Proof. Assume that $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ is an IFGB and a complete IF-subgraph of a connected IFG $\mathcal{G}$. Let $u_i \in \mathcal{V}'$ and $\mathcal{H}^*$ be the vertex removed IF-subgraph of generated by $\mathcal{V}' \setminus \{u_i\}$. Since $\mathcal{H}$ is complete, leads to the conclusion that an edge linking any two vertices is the single IF-geodesic between them. As a result, removing any vertex has no effect on any IFGB property. This completes the proof. $\square$

6. Geodetic Intuitionistic Fuzzy Boundary and Internal Vertices

In this section, the IF-boundary and IF-internal vertices of IF-geodetic convex sets are expounded. Let's first state the relevant definition, and then look at some examples before their features are get examined.

Definition 9. Given a connected IFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and ρ be a non-empty subset of $\mathcal{V}$, which is an IF-geodetic convex. A node $u_i \in \rho$ is called a Geodetic IF-boundary vertex of ρ if $\rho \setminus \{u_i\}$ is an IF-geodetic convex. Otherwise, u_i is referred to as a Geodetic IF-internal vertex of ρ .

Example 3. Consider an IFG exhibited in Figure 4.



Observe that the direct edge $v_4 - v_6$ is an IF-geodesic and the path $v_4 - v_2 - v_6$ is an IF $v_4 - v_6$ geodesic. At this point, we let $\rho = \{v_2, v_4, v_6\}$ and which is an IF-geodetic convex, since $I_g[\rho] = \rho$. Similarly, $\{v_4, v_6\}$ is an IF-geodetic convex. Now we have $\rho \setminus \{v_2\} = \{v_4, v_6\}$ is an IF-geodetic convex, vertex v_2 is a geodetic IF-boundary vertex of ρ . But vertex v_6 is a geodetic IF-internal vertex.

Example 4. Looking back at our Example 1, $\rho = \{v_1, v_3, v_4\}$ is an IF-geodetic convex. Similarly, $\{v_1, v_4\}$ and $\{v_3, v_4\}$ are IF-geodetic convex. Here, the vertices v_1 and v_3 are geodetic IF-boundary vertices.

The following theorems are devoted to characterizing geodetic IF- boundary and geodetic IF-internal vertices.

Theorem 6. Given any set $\rho \subset \mathcal{V}$ of a connected IFG, and ρ be an IF-geodetic convex. Then a vertex $u_i \in \rho$ is a geodetic IF-boundary vertex of ρ if and only if u_i doesn't cite on any $v_i - v_j$ IF-geodesic for all $v_i, v_j \in \rho \setminus \{u_i\}$.

Proof. Let $\rho \subset \mathcal{V}$ of a connected IFG be an IF-geodetic convex. Assume that u_i is a geodetic IF-boundary vertex of ρ and $v_i, v_j \in \rho \setminus \{u_i\}$. Since u_i is a geodetic IF-boundary vertex of ρ , we have $\rho \setminus \{u_i\}$ is an IF-geodetic convex. By Definition 4, all the vertices in all $v_i - v_j$ IF-geodesics together with v_i and v_j are contained in $\rho \setminus \{u_i\}$. This implies all $v_i - v_j$ IF-geodesics are not dependent on u_i . Hence u_i doesn't cite on any $v_i - v_j$ IF-geodesic for all $v_i, v_j \in \rho \setminus \{u_i\}$.

For the reverse implication, suppose that u_i doesn't cite on any $v_i - v_j$ IF-geodesic for all $v_i, v_j \in \rho \setminus \{u_i\}$. We show that u_i is a geodetic IF-boundary vertex. Definition 9 insists that it is enough to establish that $\rho \setminus \{u_i\}$ is an IF-geodetic convex. By hypothesis, all $v_i - v_j$ IF-geodesics are independent from u_i . Notice that, $(\rho \setminus \{u_i\}) \cup \{u_i\} = \rho$, therefore all the vertices in an IF-geodesic connecting any two vertices of $\rho \setminus \{u_i\}$ are contained in $\rho \setminus \{u_i\}$. Hence we are done. $\square$

Theorem 7. Given any set $\rho \subset \mathcal{V}$ of a connected IFG, and ρ be an IF-geodetic convex. Then a vertex $u_i \in \rho$ is a geodetic IF-internal vertex of ρ if and only if u_i is sited in at least one $v_i - v_j$ IF-geodesic for some $v_i, v_j \in \rho \setminus \{u_i\}$.

Proof. Let u_i be a geodetic IF-internal vertex of ρ and $v_i, v_j \in \rho \setminus \{u_i\}$. Since u_i is a geodetic IF-internal vertex, the set $\rho \setminus \{u_i\}$ fails to be an IF-geodetic convex. By Definition 4, there exists at least one node in any of the $v_i - v_j$ IF-geodesics that is not a member of $\rho \setminus \{u_i\}$. Moreover, we observe that $(\rho \setminus \{u_i\}) \cup \{u_i\} = \rho$. This leads to the conclusion that u_i is a part of at least one $v_i - v_j$ IF-geodesic.

For the converse statement, we may assume that $u_i \in \rho$ lies on a $v_i - v_j$ IF-geodesic for some $v_i, v_j \in \rho \setminus \{u_i\}$. Now it is just a matter of examining that $u_i \in \rho$ is a geodetic IF-internal vertex of ρ . It is sufficient, if we show that $\rho \setminus \{u_i\}$ fails to be an IF-geodetic convex. We begin by observing that the set $(\rho \setminus \{u_i\}) \cup \{u_i\} = \rho$, which is an IF-geodetic convex. By hypothesis, there is a presence of at least one $v_i - v_j$ IF-geodesic which contains u_i . Hence the set $\rho \setminus \{u_i\}$ is not an IF-geodetic convex, as desired. $\square$

Theorem 8. Let $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be a complete IF-subgraph of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. If $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ is an IFGB, then every node of $\mathcal{H}$ is a geodetic IF-boundary vertex of $\mathcal{V}'$.

Proof. Assume that $\mathcal{H}$ is an IFGB. Let u_i be any node in $\mathcal{H}$. In order to show u_i is a geodetic IF-boundary vertex of $\mathcal{V}'$, it suffices to prove $\mathcal{V}' \setminus \{u_i\}$ is IF-geodetic convex. By Theorem 3, $\{v_i, v_j\}$ is an IF-geodetic convex for any $v_i, v_j \in \mathcal{V}'$. Since u_i is arbitrary, it leads to our desired conclusion. $\square$

7. Minimal IF-Geodetic Subgraph

There is no end to the remarkable characteristics of the IF geodesics. The introduction to the geodetic IF-cover, geodetic IF-basis, and geodetic IF-number of an IFG is the primary focus of this section. We introduce the idea of the minimal IF-geodetic subgraph in IFGs and its characterizations are presented.

Definition 10. A subset ρ of $\mathcal{V}$ in a connected IFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is said to be a geodetic IF-cover (IF_{gC}) of $\mathcal{G}$ if $I_g[\rho] = \mathcal{V}$.

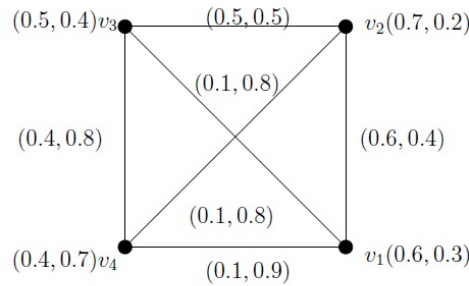


Figure 5.

Definition 11. A Geodetic IF-Basis (IF_{gB}) for $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is any IF_{gC} of $\mathcal{G}$ with the fewest vertices, and the number of vertices in it is termed as the geodetic IF-number and denoted by $IF_{gn}(\mathcal{G})$.

Example 5. To become acquainted with the above definition, let's consider $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ as in Figure 5. Here the path $v_2 - v_4 - v_1 - v_3$ is an IF $v_2 - v_3$ geodesic. Let $\rho = \{v_2, v_3\}$, then $I_g[\rho] = \mathcal{V}$, so $\{v_2, v_3\}$ is an IF_{gC} and observe that $IF_{gn}(\mathcal{G}) = 2$. Also $\{v_2, v_3\}$ serves the role of an IF_{gB} .

Theorem 9. In a complete IFG, the full set of vertex is the only IF_{gC} of $\mathcal{G}$.

Proof. For a complete IFG, each edge between a pair of vertices acts as the sole IF-geodetic path. Hence, the theorem is obvious. $\square$

An immediate corollary to Theorem 9 is as follow:

Corollary 1. For a complete IFG $\mathcal{G}$, $IF_{gn}(\mathcal{G}) = n$.

Theorem 10. If $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is an IF-tree, there exists an IF-subgraph that spans a unique IF_{gB} consisting of its end nodes.

Proof. An observation that should be immediately evident from Theorem 2, there exists an IF spanning subgraph and has a finite collection of nested sets that are IF geodetic convex. By construction of such a finite collection, it guarantees that there exists a unique IF_{gB} for $\mathcal{G}$ consisting of its end vertices. $\square$

Based on the definitions and as well as the findings presented thus far, lower and upper bounds for the $IF_{gn}(\mathcal{G})$ can be determined as follows.

Theorem 11. Consider any non-trivial IFG with n vertices, $2 \leq IF_{gn}(\mathcal{G}) \leq n$.

Proof. In a non-trivial IFG $\mathcal{G}$, IF_{gC} of $\mathcal{G}$ has to consist of at least two vertices in it and thus, $IF_{gn}(\mathcal{G}) \geq 2$. Also, full vertex set of $\mathcal{G}$ is an IF_{gC} of $\mathcal{G}$, clearly $IF_{gn}(\mathcal{G}) \leq n$. This completes the proof. $\square$

Definition 12. An IF-graph $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ is called a minimal IF-geodetic subgraph if there exists an IF-graph $\mathcal{G}$ containing $\mathcal{H}$ such that $IF_{gB}(\mathcal{G}) = \mathcal{V}'$.

Example 6. To get a sense of the above definition, let's once again take a look into the Figure 5 in Example 5 and here, we consider IF-subgraph $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ of $\mathcal{G}$ given in Figure 6.

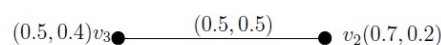


Figure 6.

Clearly $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ a minimal IF-geodetic subgraph of $\mathcal{G}$, since, $\rho = \{v_2, v_3\}$, then $I_g[\rho] = \mathcal{V}$, so $\{v_2, v_3\}$ is an $IF_{gC}(\mathcal{G})$ and at the same time $\mathcal{V}' = \{v_2, v_3\} = IF_{gB}(\mathcal{G})$ with $IF_{gn}(\mathcal{G}) = 2$. Here, $IF_{ge}(v_2) = 0.4$, $IF_{ge}(v_3) = 0.4$ and $IF_{gdiam}(\mathcal{G}) = 0.4$. Therefore, v_2 and v_3 are geodetic IF- peripheral vertices of $\mathcal{G}$.

The next theorem holds for obvious reasons.

Theorem 12. Let $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be a minimal IF-geodetic subgraph of $\mathcal{G}$ with $IF_{gn}(\mathcal{G}) = 2$, then $v_i \in \mathcal{V}'$ are geodetic IF- peripheral vertices of $\mathcal{G}$.

Theorem 13. If $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ with n vertices, $2 \leq IF_{gn}(\mathcal{H}) \leq IF_{gn}(\mathcal{G}) \leq n$.

Proof. Let $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be a non-trivial IF-subgraph of $\mathcal{G}$ with the number of vertices equal to $|\mathcal{V}'|$. Then, using Theorem 11, it is apparent that $2 \leq IF_{gn}(\mathcal{H}) \leq |\mathcal{V}'|$. Since, by hypothesis, $\mathcal{H}$ is a minimal IF-geodetic subgraph of $\mathcal{G}$, $\mathcal{V}' = IF_{gB}(\mathcal{G})$. Thus, $IF_{gn}(\mathcal{G}) = |\mathcal{V}'|$, we conclude that $2 \leq IF_{gn}(\mathcal{H}) \leq IF_{gn}(\mathcal{G})$. Again, by Theorem 7.7, it is clear that $2 \leq IF_{gn}(\mathcal{H}) \leq IF_{gn}(\mathcal{G}) \leq n$. $\square$

The next theorem should be compared with Theorem 9 and Corollary 1.

Theorem 14. The only minimal IF-geodetic subgraph of the complete IFG $\mathcal{G}$ is $\mathcal{G}$ itself.

Proof. To argue for this, we use Theorem 9. For a complete IFG, each edge between pair of vertices acts as sole IF-geodetic path. Hence, $IF_{gC}(\mathcal{G})$ is the entire set of vertex. Therefore, $IF_{gn}(\mathcal{G}) = n$, which completes the proof. $\square$

8. Intuitionistic Fuzzy Geodetic Wiener Index

The Wiener index is a well-known index that is utilized in a variety of sectors such as telecommunication, facility localization, chemical graphs, networking and so on. A connected IFG accurately models a wide range of circumstances. This section introduces the notion of the Intuitionistic Fuzzy Geodetic Wiener Index ($IF_g\mathcal{W}\mathcal{J}$) using the IF-geodesics, and the formal formulation is as follows.

Definition 13. If $\mathcal{P} : v_1, v_2, \dots, v_n$ is a path in an IFG, the Intuitionistic Fuzzy Geodetic Wiener Index is defined as $IF_g\mathcal{W}\mathcal{J}(\mathcal{G}) = \sum_{v_i, v_j \in \mathcal{V}} d_g(v_i, v_j)$ where $d_g(v_i, v_j)$ is the IF-geodetic distance from v_i to v_j .

Example 7. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an IFG (Figure 1). The direct edges $v_1 - v_4$, $v_2 - v_4$ and $v_3 - v_4$ are IF-geodesics. The paths $v_1 - v_4 - v_2$, $v_1 - v_4 - v_3$ and $v_2 - v_4 - v_3$ are IF-geodesics. Here, $IF_g\mathcal{W}\mathcal{J}(\mathcal{G}) = \sum_{v_i, v_j \in \mathcal{V}} d_g(v_i, v_j) = 1.8$.

Removing a node from a graph usually creates a subgraph of the original graph. As a result, the type of graph determines whether $IF_g\mathcal{W}\mathcal{J}(\mathcal{G})$ will increase, decrease, or remain constant. In most cases, the index will decrease. The effect of removing a bridge on $IF_g\mathcal{W}\mathcal{J}(\mathcal{G})$ will be discussed in the upcoming scenario.

Example 8. In Figure 7 the path v_1v_4 and $v_4v_2v_1$ have the strength $(0.3, 0.4)$ and $(0.2, 0.7)$, respectively. Here, the edge that joins v_1 and v_4 serves as both a bridge and a geodesic. In this case $IF_g\mathcal{W}\mathcal{J}(\mathcal{G}) = 2.2$, but $IF_g\mathcal{W}\mathcal{J}(\mathcal{G} - v_1v_4) = 2.25$.

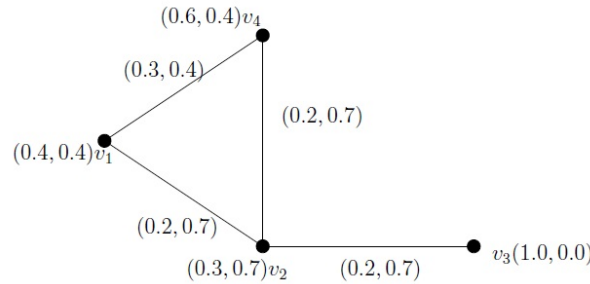


Figure 7.

The above case will be generalized by the following theorem.

Theorem 15. *If a bridge is removed from an IFG, then $IF_g \mathcal{W} \mathcal{J}$ increases only if it is an edge of any geodesic.*

Proof. Assume that there exist a bridge in an IFG, say $v_i v_j$. Now we consider two possibilities as follows. Case 1: Suppose that $v_i v_j$ itself a geodesic for some i and j . Then, $d_g(v_i, v_j) = d(v_i, v_j)$. Thus deletion of $v_i v_j$ will increase the value of $d_g(v_i, v_j)$. Hence from the definition of $IF_g \mathcal{W} \mathcal{J}$, we are done. Case 2: Take $v_i v_j$ as an edge of any geodesic path joins any pair of vertices. Since the deletion of $v_i v_j$ creates new geodesic path. Thus there will be an increase in $IF_g \mathcal{W} \mathcal{J}$. $\square$

Theorem 16. *The $IF_g \mathcal{W} \mathcal{J}$ of two isomorphic IFGs are equal.*

Proof. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ be two isomorphic IFGs. Then there exists a bijective map $\phi : (\mathcal{V}, \mathcal{E}) \rightarrow (\mathcal{V}', \mathcal{E}')$ such that for every $u \in \mathcal{V}$ and $(u, v) \in \mathcal{E}$, $\mathcal{W}(u) = \mathcal{W}(\phi(u))$ and $\mathcal{W}(u, v) = \mathcal{W}(\phi(u), \phi(v))$. Similarly, for every $d_g(u_i, v_j)$ there exists $d_g(\phi(u_i), \phi(v_j))$. As a result, we can categorically state that there is a path in $\mathcal{H}$ that corresponds to the path in $\mathcal{G}$ such that the sum of weights of edges is minimum among all the paths from $\phi(u_i) - \phi(v_j)$. Hence $d_g(u_i, v_j) = d_g(\phi(u_i), \phi(v_j))$. Therefore, $IF_g \mathcal{W} \mathcal{J}(\mathcal{G}) = \sum_{v_i, v_j \in \mathcal{V}} d_g(v_i, v_j) = \sum_{v_i, v_j} d(v_i, v_j) = \sum \wedge \{\sum \mathcal{W}(\phi(v_i), \phi(v_j))\} = \sum_{\phi(v_i), \phi(v_j)} d_g(\phi(v_i), \phi(v_j)) = IF_g \mathcal{W} \mathcal{J}(\mathcal{H})$, as desired. $\square$

9. Applications

9.1. Application of $IF_g \mathcal{C}$ in Wireless Mesh Network

In 2015, mesh networking for small networks made its debut with the promise of resolving Wi-Fi issues through increased coverage, faster networks, and less fuss. In order to prevent inactive and weak spots, it also claimed to eliminate the requirement for base stations to be placed precisely across a house or small workplace. Mesh networks also offer help in figuring out where to locate units. Mesh networks have the ability to scale effortlessly by incorporating additional nodes without requiring substantial alterations to the network infrastructure, making them well suited to extend coverage over expansive areas. In general, wireless mesh networks provide a robust and decentralized networking solution that is applicable in numerous scenarios where traditional wired networks are not viable or feasible.

For instance, in a University campus environment, setting up a wireless mesh network can help increase network coverage and efficiency. Every node in a wireless mesh network communicates with other nodes in the area, creating a scalable and self-healing network. For stability and to prevent wireless congestion, certain nodes could still require a wired Ethernet connection to the main router to accomplish this. In order to plan and configure a wireless mesh network (WMN), follow these general steps. The first step is to survey the area, where we identify areas that need Wi-Fi coverage and make note of any obstacles such as walls and other structural constraints. The second factor, node location, guarantees that the wireless mesh is strategically placed and that the entire region is adequately covered. Lastly, the types of node are crucial in designating specific nodes as "gateway"

nodes that provide a direct Ethernet connection to the main router. The remaining nodes will be "mesh" nodes that establish wireless connections with both the gateway and other nodes. Use Ethernet connections to link gateway nodes to the main router. In a wireless mesh network, not all nodes necessarily need a wired connection. We need to identify gateway nodes, or the nodes that need to be linked to the cable in order to cover the entire area. With the help of geodetic IF-cover, we can now tackle the issue. To make computations easier, we have taken $(\mu_1, \gamma_1) = (0.9, 0.1)$. Take a look at the fuzzified WMN model shown in Figure 8.

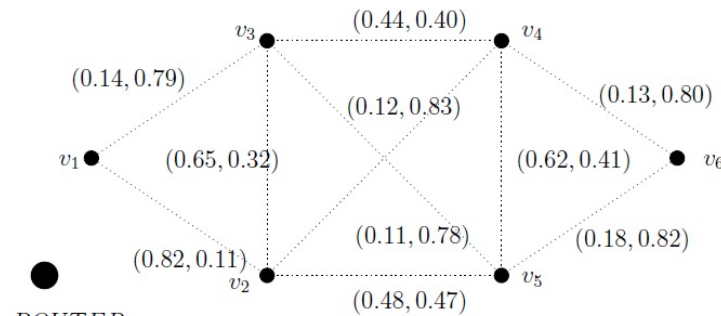


Figure 8.

Here we have, 16 geodetic pathways in total, in particular consider the pathway, $\mathcal{P}_1: v_1 - v_3 - v_5 - v_6 - v_4 - v_2$ is an IF $v_1 - v_2$ geodesic and $d_g(v_1, v_2) = 0.83$, likewise, $\mathcal{P}_2: v_1 - v_3$ is an IF - geodesic. Let $\rho = \{v_1, v_2\}$, then $I_g[\rho] = \mathcal{V}$, so $\{v_1, v_2\}$ is an IF_gC for the WMN mentioned above. Consequently, we may identify v_1 and v_2 as our gateway nodes, enabling them to cover the whole region of the network. Here $\{v_1, v_2\}$ act as IF_gB as well. The following Figure 9 depicts the finalized structure of a wireless mesh network, where gateway nodes are highlighted in red, mesh nodes are identified with green labels, dotted lines in the figure symbolize wireless connections, while solid lines without breaks indicate wired connections with the router.

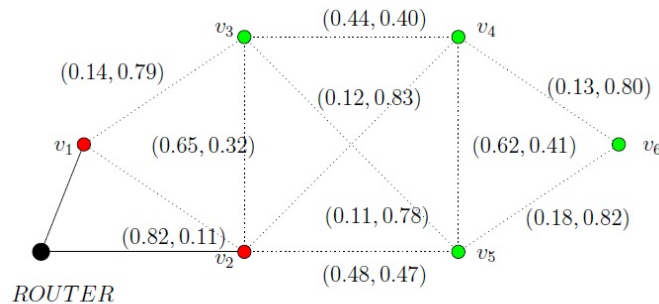


Figure 9 – WMN Configuration.

Note that depending on the hardware and software solutions you select for your wireless mesh network, there may be differences in the precise processes and required equipment. For the finest implementation, it is also a good idea to speak with network or IT specialists. More coverage, better connectivity, and an increase in total network capabilities can be achieved by expanding and improving a WMN to create a more complex and sophisticated network configuration.

9.2. Application of $IF_g\mathcal{W}\mathcal{I}$ in Global Human Trading

Human trading often involves intricate networks of individuals and organizations. Taking into account the ambiguity and uncertainty in the connections, IFG theory can be used to examine and visualize these networks. Law enforcement and policymakers may find this analysis useful in understanding the dynamics and structure of trafficking networks. In this paper, we use the major illegal human trade routes to the United States as mentioned in [3,6].

Several approaches were applied to fuzzy graphs to assess the likelihood of illegal immigration along various routes from the countries of origin to the United States. In this study, we quantify the susceptibility to trafficking with respect to a directed IFG using $IF_g\mathcal{W}\mathcal{J}$. We interpret μ_2 as flow from distinct locations and γ_2 represents government measures to control internal transit fluxes. Consider Figure 11 in [9] as a model of directed IFG. Thus, Table 1 can be constructed as follows.

Table 1.

Country	Vulnerability (μ_1)	Government Measure(γ_1)	(μ_2)	(γ_2)
India	0.46	0.53	0.26	0.73
China	0.36	0.45	0.19	0.68
Russia	0.3	0.42	0.06	0.61
UAE	0.56	0.26	0.17	0.57
Somalia	0.28	0.72	0.16	0.79
Ethiopia	0.42	0.58	0.21	0.79
South Africa	0.49	0.49	0.32	0.65
Nigeria	0.44	0.56	0.31	0.65
Spain	0.71	0.2	0.15	0.46
Cuba	0.21	0.32	0.11	0.61
Colombia	0.53	0.42	0.30	0.66
Brazil	0.66	0.31	0.34	0.55
Ecuador	0.51	0.35	0.29	0.63
Guatemala	0.56	0.42	0.3	0.67
Mexico	0.57	0.43	0.47	0.53
United States	0.82	0.18	0	0

It is important to emphasize that the Wiener indices fall as the effectiveness of a path in preventing trafficking rises. Consider the path from India to U.S., $IF_g\mathcal{W}\mathcal{J} = \sum_{v_i, v_j \in \mathcal{V}} d_g(v_i, v_j) = 0.265 + 0.325 + 0.47 = 1.06$. Using analogous computations along the path from Somalia to our desired destination reveals two distinct pathways. One is through Guatemala with $IF_g\mathcal{W}\mathcal{J}$ is 3.485 and the other one is directly through Mexico from Colombia and its index is 1.745. An $IF_g\mathcal{W}\mathcal{J}$ of 1.37 linked to the route originating from China as the country of origin. Along Nigeria to U.S via Guatemala, $IF_g\mathcal{W}\mathcal{J} = 2.04$. Directly traversing the route via Colombia through Mexico to the United States from country origin Nigeria registered an $IF_g\mathcal{W}\mathcal{J}$ of 1.6. It has been determined that along the path from Ethiopia, there is an $IF_g\mathcal{W}\mathcal{J}$ of 1.84.

Measures of a route’s vulnerability to illegal human trading are established by applying the principles and results. One potential such metric is the $IF_g\mathcal{W}\mathcal{J}$, which is introduced. Three techniques were employed in fuzzy graphs to assess the likelihood of illegal immigration along certain routes from the countries of origin to the United States. The method in this research and the other methods in article [3,6,9] concur that the path leading to the United States with the highest susceptibility originates in Somalia. The source nation with the second-highest likelihood is Nigeria. The newfound method proves to be more accurate, as it takes into account internal transit flow. Upon comparing the average Wiener index in fuzzy graphs with the indexes obtained in this paper, it can be concluded that this method is more precise and accurate.

10. Conclusions

A notion of intuitionistic fuzzy geodetic convexity within the framework of intuitionistic fuzzy graph metric is discussed in this article. This paper culminated in the development of a specialized algorithm for identifying the geodetic path of an IFG. Also we presented a Python program crafted for the precise identification of all geodetic paths. This article is devoted in the development of IF-boundary vertices, IF-internal vertices, geodetic IF-basis and geodetic IF-number within IF-geodetic convex sets. A lower and upper bounds for the geodetic IF-number is also identified in this article. Later, we introduced the idea of the minimal IF-geodetic subgraph in IFGs. We introduced a novel

concept, the intuitionistic fuzzy geodesic Wiener index, leveraging the framework of IF-geodesics. Finally, the application of IF-geodesic cover in a wireless mesh network for identifying gateway nodes. Additionally, the introduction of the intuitionistic fuzzy geodesic Wiener index has found practical application in the domain of global human trading analysis in certain trafficking channels to the U.S. The findings coincide with those reported in earlier studies.

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