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Article

Asymptotic Microcausality: Deformations of Local Quantum Field Theory

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Abstract

In this paper I formulate and prove an asymptotic microcausality theorem for a broad class of nonlocal deformations of local QFT. I show that the commutator of two such nonlocal observables with spacelike-separated supports obeys a bound of microcausality. These results make precise the sense in which microcausality can be an emergent property of an underlying nonlocal UV theory and then clarify the operational content of locality in this setting in connection with Sorkin's impossible measurements problem.

Keywords: microcausality; locality

1. Introduction

Local relativistic quantum field theory (QFT) is built on a small number of structural principles such as Poincaré invariance, the spectrum condition, existence of a vacuum, and microcausality [1–3]. The latter states that local observables with spacelike-separated supports commute and ensures the impossibility of superluminal signalling and the compatibility of QFT with special relativity [1,2].

At the same time, there is growing evidence that any ultraviolet (UV) completion of QFT and gravity is intrinsically nonlocal. Examples include infinite-derivative gravity and gauge theory, string field theory, nonlocal regulators based on entire functions of the d'Alembertian [4–19]. In these frameworks the kinetic or interaction terms are deformed by form factors $F(\square/M_*^2)$ that smear fields over a fundamental length M_*^{-1} . Such nonlocal deformations generically spoil strict Wightman microcausality at the level of gauge-invariant composite operators. However, the violations are expected to be confined to distances of order M_*^{-1} and to be irrelevant in the infrared (IR). This suggests that microcausality might be an emergent property of an underlying nonlocal UV-complete theory, recovered as an effective locality at distances $\rho \gg M_*^{-1}$ [11–20].

The goal of this paper is to make this statement precise. I formulate and prove an asymptotic microcausality theorem that quantifies how rapidly commutators of nonlocal observables decay at spacelike separation, and how strict locality is recovered in the local limit $M_* \rightarrow \infty$.

A second motivation is operational. Sorkin observed that naively importing the nonrelativistic projection postulate of ideal projective measurements with Lüders state update into relativistic QFT can lead to superluminal signalling in otherwise microcausal theories, a tension now known as the impossible measurements problem [21–24]. This no-superluminal-signalling notion of relativistic causality should be distinguished from EPR or Bell nonlocality where entangled states can violate Bell inequalities while remaining nonsignalling [25–28]. In an entire-function UV completion, however, observables accessible to localized apparatuses are naturally quasi-local as they possess unavoidable non-compact tails and their spacelike commutators are controlled quantitatively. I show that replacing strict localization by this operational notion turns Sorkin's paradox into a controlled, scale-dependent statement, with any spacelike influence bounded by the same asymptotic microcausality estimate proved here.

Following Bohr and Rosenfeld, the idealization of perfectly point-local quantities is not operationally meaningful in relativistic quantum field theory as only observables averaged over finite

spacetime regions admit unambiguous definition and measurement. In this sense, ideal infinitely localized measurements are precisely where causality and measurability tensions can arise [29,30].

Motivated by a conjecture due to Carteret, I show how Sorkin's impossible measurement tension can be understood as an artifact of idealized perfectly overlocalization and the associated ideal projective-update postulate in conventional local QFT; in the entire-function setting, operationally accessible observables are necessarily quasi-local and any spacelike influence is exponentially suppressed.

2. Local QFT and Microcausality

In this section I briefly summarize the aspects of the Wightman framework for microcausality [1, 2,31]. We first let $\mathcal{H}$ be a Hilbert space carrying a unitary representation $U(a, \Lambda)$ of the Poincaré group, and let $\Omega \in \mathcal{H}$ be a Poincaré-invariant vacuum vector. Local fields are operator-valued tempered distributions $\varphi_a(x)$ acting on a common dense domain $\mathcal{D} \subset \mathcal{H}$, such that the smeared fields:

$$\varphi_a(f) := \int d^4x f(x) \varphi_a(x) \quad (1)$$

are essentially self-adjoint operators for real test functions $f \in \mathcal{S}(\mathbb{R}^4)$ [32].

Definition 2.1 (Microcausality). *A local QFT is said to satisfy microcausality if for any two local observables $\mathcal{O}_1(x), \mathcal{O}_2(y)$ and any two test functions f, g with spacelike-separated supports:*

$$\text{supp } f \sim \text{supp } g \implies [\mathcal{O}_1(f), \mathcal{O}_2(g)] = 0, \quad (2)$$

where $\text{supp } f \sim \text{supp } g$ means that every point in $\text{supp } f$ is spacelike-separated from every point in $\text{supp } g$ with respect to the Minkowski metric.

For free fields, microcausality is equivalent to the statement that the Pauli–Jordan commutator function [33]:

$$\Delta(x - y) = \frac{1}{(2\pi)^3} \int d^4p \delta(p^2 - m^2) \text{sgn}(p^0) e^{-ip \cdot (x - y)} \quad (3)$$

has support only on and inside the light cone. For interacting theories, microcausality is encoded in the analyticity properties of Wightman functions and implies, for example, the Jost–Lehmann–Dyson representation and standard dispersion relations [34,35].

In what follows I will assume the existence of a local QFT satisfying the Wightman axioms, including microcausality, which serves as the undeformed theory from which nonlocal observables will be constructed.

3. Entire-Function Deformations of Local Observables

Let $F(z)$ be an entire function on $\mathbb{C}$ with the following properties; that $F(0) = 1$ and $F(z)$ has no zeros on the real axis for $z \leq 0$, $F(z)$ is real and positive for $z \leq 0$, and F is of finite order and type, such that its Fourier transform is well-defined as a distribution with suitable decay in spacelike directions.

It is helpful to interpret these assumptions on F as UV regularity conditions tailored to the operator-distributional structure of relativistic QFT. In particular, choosing F to be an entire function of finite order/type with $F(0) = 1$ and no zeros for $z \leq 0$ so that no new physical poles appear on the Euclidean/negative real axis ensures that the induced kernel $K_F(x, y)$ defining the regulated observable is a well-defined distribution with strong decay in spacelike directions. The net effect is that sharply supported, light-cone-singular distributional structures are replaced by quasi-local objects with a controlled nonlocality length $\ell_* \sim M_*^{-1}$ where M_* is the nonlocality mass scale introduced below. This is conceptually analogous to earlier Heisenberg-picture regularizations of current correlators, where moment constraints on a spectral function are imposed to remove light-cone singular contributions and restore structural consistency [36,37].

Given a mass scale M_* , we consider the operator-valued map:

$$F(\square/M_*^2) = \sum_{n=0}^{\infty} \frac{F^{(n)}(0)}{n!} \square^n M_*^{-2n}, \quad (4)$$

which is formal but can be rigorously defined on test functions via functional calculus or an integral representation of F .

Definition 3.1 (Nonlocal observable). *Let $\mathcal{O}(x)$ be a local observable in the underlying QFT, such as a gauge-invariant composite such as $F_{\mu\nu}F^{\mu\nu}$ or $\bar{\psi}\psi$ [38,39]. The associated nonlocal observable is defined by:*

$$\mathcal{O}^{(F)}(x) := F(\square/M_*^2) \mathcal{O}(x). \quad (5)$$

Equivalently, for $f \in \mathcal{S}(\mathbb{R}^4)$ we define the smeared operator:

$$\mathcal{O}^{(F)}(f) := \int d^4x f(x) \mathcal{O}^{(F)}(x) \quad (6)$$

$$= \int d^4x \mathcal{O}(x) [F(\square/M_*^2)f](x), \quad (7)$$

where $F(\square/M_*^2)$ acts on the test function f .

Under mild assumptions on F one can represent $F(\square/M_*^2)$ as a superposition of heat kernels [40, 41]. For definiteness, suppose that F admits a Laplace-type representation:

$$F(z) = \int_0^\infty d\tau w(\tau) e^{-\tau z}, \quad (8)$$

with a weight function $w(\tau)$ that decays sufficiently fast as $\tau \rightarrow \infty$, then:

$$F(\square/M_*^2) = \int_0^\infty d\tau w(\tau) e^{-\tau \square/M_*^2}, \quad (9)$$

and the action on a test function can be written in terms of a kernel $K_\tau(x, y)$:

$$[e^{-\tau \square/M_*^2} f](x) = \int d^4y K_\tau(x, y) f(y), \quad (10)$$

$$K_\tau(x, y) = \frac{M_*^4}{(4\pi\tau)^2} \exp\left[-\frac{M_*^2(x-y)^2}{4\tau}\right]. \quad (11)$$

Consequently:

$$[F(\square/M_*^2)f](x) = \int d^4y K_F(x, y) f(y), \quad (12)$$

$$K_F(x, y) := \int_0^\infty d\tau w(\tau) K_\tau(x, y), \quad (13)$$

and

$$\mathcal{O}^{(F)}(f) = \int d^4x d^4y K_F(x, y) f(y) \mathcal{O}(x). \quad (14)$$

The salient point is that $K_F(x, y)$ is a smooth function that decays rapidly, typically exponentially in the invariant distance $|x - y|$ on spacelike directions, with a decay rate set by M_* .

The parameter M_* sets the fundamental length scale of nonlocality:

$$\ell_* \sim M_*^{-1}. \quad (15)$$

For $|x - y| \gg \ell_*$ the kernel $K_F(x, y)$ is exponentially small, so nonlocal observables are quasi-local on scales large compared to ℓ_* . In the limit $M_* \rightarrow \infty$ the kernel collapses to a delta function and $F(\square/M_*^2) \rightarrow \mathbb{1}$ on test functions, recovering the original local observable:

$$\lim_{M_* \rightarrow \infty} \mathcal{O}^{(F)}(f) = \mathcal{O}(f). \quad (16)$$

4. Asymptotic Microcausality

We now state and prove the central result, that an exponential bound on spacelike commutators of nonlocal observables constructed as above. We let $f, g \in \mathcal{S}(\mathbb{R}^4)$ be two test functions with spacelike-separated supports:

$$\text{supp } f \sim \text{supp } g. \quad (17)$$

Define the invariant spacelike distance between the supports:

$$\rho(f, g) := \inf_{x \in \text{supp } f, y \in \text{supp } g} \sqrt{-(x - y)^2}, \quad (18)$$

which is strictly positive when the supports are strictly spacelike-separated.

Theorem 4.1 (Asymptotic microcausality). *Let $\mathcal{O}_1(x), \mathcal{O}_2(x)$ be local observables in a Wightman QFT that satisfies microcausality. Let $F(\square/M_*^2)$ be an entire-function deformation satisfying the assumptions of Sec. 3, and define the nonlocal observables $\mathcal{O}_i^{(F)}(x) = F(\square/M_*^2)\mathcal{O}_i(x)$ and their smeared versions $\mathcal{O}_i^{(F)}(f)$.*

Then for any pair of test functions f, g with spacelike-separated supports and any integer $N \geq 0$, there exist constants $C_N(f, g) > 0$ and $\alpha > 0$, independent of M_ , such that*

$$\left\| [\mathcal{O}_1^{(F)}(f), \mathcal{O}_2^{(F)}(g)] \right\| \leq C_N(f, g) (1 + M_*\rho)^{-N} e^{-\alpha M_*\rho}, \quad (19)$$

$\rho \equiv \rho(f, g)$. In particular:

- (i) For fixed M_* , the commutator is exponentially suppressed at large spacelike separations $\rho \gg M_*^{-1}$.
- (ii) For fixed $\rho > 0$, the commutator vanishes in the local limit $M_* \rightarrow \infty$,

$$\lim_{M_* \rightarrow \infty} [\mathcal{O}_1^{(F)}(f), \mathcal{O}_2^{(F)}(g)] = 0, \quad (20)$$

so strict microcausality is recovered.

Using the kernel representation derived in Sec. 3, we can write:

$$\mathcal{O}_1^{(F)}(f) = \int d^4x d^4u K_F(x, u) f(u) \mathcal{O}_1(x), \quad (21)$$

$$\mathcal{O}_2^{(F)}(g) = \int d^4y d^4v K_F(y, v) g(v) \mathcal{O}_2(y). \quad (22)$$

The commutator is therefore:

$$[\mathcal{O}_1^{(F)}(f), \mathcal{O}_2^{(F)}(g)] = \int d^4x d^4y d^4u d^4v K_F(x, u) K_F(y, v) f(u) g(v) [\mathcal{O}_1(x), \mathcal{O}_2(y)]. \quad (23)$$

By microcausality of the underlying local QFT, $[\mathcal{O}_1(x), \mathcal{O}_2(y)]$ vanishes whenever $(x - y)^2 < 0$. Thus only pairs (x, y) with lightlike or timelike separation contribute. For fixed $u \in \text{supp } f$ and $v \in \text{supp } g$, spacelike separation of the supports implies that $(u - v)^2 < 0$ and the invariant distance $\rho(f, g)$ is strictly positive. The Gaussian form of $K_\tau(x, u)$ and the decay properties of $w(\tau)$ imply an estimate of the form

$$|K_F(x, u)| \lesssim M_*^4 \sum_{n=0}^N \frac{a_n}{(1 + M_*|x - u|)^n} e^{-\alpha M_*|x - u|}, \quad (24)$$

and similarly for $K_F(y, v)$, with constants a_n and α that depend on F but not on M_* . Combining these estimates and integrating first over x, y along surfaces of fixed Minkowski distance from u, v , one finds that nonvanishing contributions require x and y to lie within a layer of thickness $\sim M_*^{-1}$ around each other's light cones. For spacelike-separated u, v , this imposes a minimum length of order $\rho(f, g)$ for any such causal chain, which in turn yields an overall factor $e^{-\alpha M_* \rho}$ up to polynomial corrections. The details are analogous to standard Paley–Wiener type bounds for entire functions [42–45].

Carrying through the estimates and integrating over $u \in \text{supp } f$ and $v \in \text{supp } g$ produces the bound (19), with constants $C_N(f, g)$ that depend on seminorms of the test functions and on the growth of F . The stated limits follow immediately.

The theorem can be phrased more abstractly in terms of operator-valued distributions and their wavefront sets [45,46]. The essential input is the support property of the commutator of local fields and the exponential-type decay of the kernel $K_F(x, y)$ in spacelike directions.

The bound (19) shows that nonlocal observables constructed by entire-function smearing of local fields are asymptotically microcausal at spacelike separation. For fixed nonlocality scale M_* , the commutator is exponentially small when the invariant separation ρ of the supports exceeds a few times M_*^{-1} . On macroscopic distance scales, the theory is indistinguishable from a strictly local QFT as far as causality is concerned. In the local limit $M_* \rightarrow \infty$, the commutator vanishes for any fixed spacelike $\rho > 0$, and one recovers the Wightman microcausality axiom.

Thus the theorem provides a mathematically controlled realization of the idea that microcausality can be an emergent property of an underlying nonlocal theory: violations are confined to a microscopic region of size $\sim M_*^{-1}$ around the light cone and are unobservable at accessible scales.

5. Examples of Entire-Function Form Factors

In this section we briefly discuss explicit choices of entire functions F that satisfy the assumptions of Sec. 3 and are commonly used in the nonlocal QFT and quantum gravity literature [4–10,17,19].

A simple example is the Gaussian form factor:

$$F(z) = e^z. \quad (25)$$

In Euclidean space this leads to propagators suppressed as $\exp(-p_E^2/M_*^2)$ at large momenta. The corresponding kernel is a pure heat kernel, and the bound (19) can be made very explicit; in particular, one finds $\alpha = \mathcal{O}(1)$.

A more general class is given by higher-order exponentials:

$$F(z) = \exp[(z + m^2/M_*^2)^N], \quad N \in \mathbb{N}, \quad (26)$$

which can be tuned to improve UV convergence. For $N \geq 1$ these are entire functions of finite order. The Laplace representation and the associated kernel bounds become slightly more involved, but the qualitative behaviour remains the same: exponential decay in spacelike directions with a rate set by M_* , and hence asymptotic microcausality.

6. Operational Locality and Sorkin Impossible Measurements

A standard sufficient condition for relativistic causality in local QFT is microcausality that states that if two observables are localized in spacelike separated regions, they commute, and no local unitary or completely-positive operation in one region can change expectation values in a spacelike separated region. However, Sorkin emphasized that importing the nonrelativistic textbook measurement postulate that an ideal projective measurement with Lüders state-update into relativistic QFT can lead to superluminal signaling in certain natural measurement scenarios [21–23,47–49]. This tension is now commonly referred to as the impossible measurements problem.

From the present perspective, the key issue is not that relativistic QFT fails causality in its local algebraic structure, but that the measurement idealization being imported is too sharp as an

instantaneous Lüders update for a perfectly localized projector presupposes the operational meaning of arbitrarily overlocalized observables. This is precisely the regime in which distributional light-cone singularities and other idealization artifacts are known to produce apparent conflicts with structural principles. A concrete example is Moffat's Heisenberg-picture analysis of current expectation values, where light-cone singular terms in current correlators lead to gauge-noninvariant intermediate expressions unless one works within a regularized class defined by spectral moment constraints [50,51]. Here, the entire-function deformation provides the analogous regularization at the level of observables as operationally accessible interventions are built from the quasi-local algebra generated by regulated observables $O^{(F)}(f)$, with test function f supported in a finite spacetime region, and any influence outside the intended region is quantitatively bounded by the asymptotic commutator estimate at spacelike separation.

To explain light-cone singularities and regularized current correlators let $x^\mu = (t, \mathbf{x})$ and $y^\mu = (t', \mathbf{y})$ be points in Minkowski spacetime $(\mathbb{R}^{1,3}, \eta_{\mu\nu})$ with metric $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$, and define the invariant separation $(x - y)^2 \equiv \eta_{\mu\nu}(x - y)^\mu(x - y)^\nu$. Let $j^\mu(x)$ denote a local Heisenberg-picture conserved current operator, $\partial_\mu j^\mu(x) = 0$, in QED, $j^\mu(x) = e \bar{\psi}(x) \gamma^\mu \psi(x)$. The basic microcausality statement for local currents is that the commutator:

$$C^{\mu\nu}(x - y) \equiv [j^\mu(x), j^\nu(y)] \quad (27)$$

vanishes at spacelike separation, $(x - y)^2 < 0$. Nevertheless, even in strictly local theories the current commutator and related two-point distributions such as vacuum or matrix-element correlators is a distribution whose singular support lies on the light cone $(x - y)^2 = 0$; in particular one expects light-cone singularities and possible contact terms in the short-distance structure of $C^{\mu\nu}$. This viewpoint is formalized by the Jost–Lehmann–Dyson integral representations for causal commutators and their applications to current algebra and deep-inelastic limits [34,36,54–60].

Long before modern entire-function UV completions, Moffat introduced a spectral-function regularization scheme for vacuum expectation values of Heisenberg current operators, imposing conditions on the associated spectral functions, and applied it to Källén-type vacuum polarization in an external electromagnetic field. In the present work, by contrast, the entire-function deformation modifies the UV behavior at the level of the theory's propagators/observables; the goal is to quantify how these deformations soften the light-cone singular structure into controlled nonlocal tails and yield asymptotic microcausality bounds at spacelike separation.

By Heisenberg singularities I mean the fact that, even in a strictly local and microcausal QFT, commutators and correlators of Heisenberg-picture local currents and composite operators are operator-valued distributions whose singular support lies on the light cone. For a conserved current $j^\mu(x)$ one has the causal commutator distribution $C^{\mu\nu}(x - y) \equiv [j^\mu(x), j^\nu(y)]$, which vanishes for spacelike separation $(x - y)^2 < 0$ by microcausality, but typically contains light-cone singular pieces supported on $(x - y)^2 = 0$, and contact terms at $x = y$ once it is paired with test functions or inserted into matrix elements. In practice these appear as $\delta((x - y)^2)$ -type terms, derivatives thereof, principal-value structures, and local contact terms $\propto \delta^{(4)}(x - y)$, familiar from the Jost–Lehmann–Dyson representation and the current-commutator/light-cone literature. These are not violations of relativistic causality but rather, they encode the short-distance/UV structure of local fields in a Lorentz-covariant way and must be treated with proper distributional care. Historically, if one manipulates such expressions as ordinary functions, one can generate formally inconsistent intermediate terms, including gauge-noninvariant pieces, which is why regulated classes of current correlators, for example those defined via spectral moment constraints as in Moffat's Heisenberg-picture analysis are introduced to control or remove the problematic light-cone contributions. In the present work the entire-function deformation $F(\square/M_*^2)$ plays the analogous role at the level of observables: it softens light-cone singular structures into quasi-local kernels with controlled tails, enabling the quantitative spacelike commutator bound of Theorem 3.

Why light-cone Heisenberg singularities occur is due to the key point that local Heisenberg-picture fields and currents are operator-valued distributions, not ordinary functions of x . Accordingly, the pointlike symbol $\mathcal{O}(x)$ is only meaningful through smearing with a test function $f \in C_c^\infty(\mathbb{R}^{1,3})$:

$$\mathcal{O}(f) \equiv \int d^4x f(x) \mathcal{O}(x), \quad (28)$$

and commutators such as $[\mathcal{O}(x), \mathcal{O}(y)]$ are distributions in the separation $x - y$ in the sense of Schwartz and Hörmander [44,45]. In distribution theory it is perfectly consistent for a distribution to vanish on an open set, here the spacelike region $(x - y)^2 < 0$ required by microcausality while still having nontrivial singular support on the boundary of that set, here the light cone $(x - y)^2 = 0$, microcausality constrains the support of the commutator, not its smoothness on the light cone.

Physically, the causal commutator is tied to the difference of retarded and advanced propagation. Already for free fields one has commutators proportional to the Pauli–Jordan distribution $\Delta(x - y)$ (e.g. in covariant QED quantization $[\hat{A}_\mu(x), \hat{A}_\nu(y)] \propto \eta_{\mu\nu} \Delta(x - y)$), which vanishes for spacelike separation but is supported on, and is singular at the light cone. Differentiating such commutators generates light-cone distributions and their derivatives, and for composite operators such as $j^\mu(x) = e\bar{\psi}(x)\gamma^\mu\psi(x)$ the commutator additionally contains local contact terms supported at $x = y$, can be written schematically as $\partial^\mu \delta^{(4)}(x - y)$, reflecting the fact that products of fields at a point require renormalized distributional definitions. Historically, treating these distributional objects as ordinary functions leads to spurious inconsistencies, including gauge-noninvariant intermediate expressions, whereas working within a properly regularized class of current correlators restores a consistent causal structure. This is the precise sense in which Heisenberg singularities are an inevitable consequence of strict pointlike localization in a relativistic QFT, rather than any genuine violation of Einstein causality.

To state the issue cleanly, let $\mathcal{O}, \mathcal{O}' \subset \mathbb{R}^{1,3}$ be bounded spacetime regions with $\mathcal{O} \sim \mathcal{O}'$, it is strictly spacelike separated, and let B be an observable localized in $\mathcal{O}'$. An ideal measurement of an observable A purportedly localized in $\mathcal{O}$ is modeled by a family of orthogonal projections $\{P_a\}$ with $\sum_a P_a = 1$, producing the selective post-measurement state:

$$\rho \mapsto \rho_a := \frac{P_a \rho P_a}{\text{Tr}(\rho P_a)}, \quad p(a) = \text{Tr}(\rho P_a), \quad (29)$$

or the nonselective or outcome-ignored channel:

$$\rho \mapsto \rho' := \sum_a P_a \rho P_a. \quad (30)$$

If the measurement is genuinely localized in $\mathcal{O}$ in the algebraic sense, then $P_a \in \mathcal{A}(\mathcal{O})$ and microcausality implies $[P_a, B] = 0$ for all $B \in \mathcal{A}(\mathcal{O}')$ [2,61], hence:

$$\text{Tr}(\rho' B) = \text{Tr}(\rho B) \quad (\mathcal{O} \sim \mathcal{O}'), \quad (31)$$

and no signaling is possible by choosing whether or not to measure A . This does not preclude Bell-inequality violations between spacelike separated observables as it only forbids controllable superluminal signalling [25–28]

Sorkin's observation is that for sharp localization idealizations and, historically, attempts at relativistic particle localization, one encounters obstructions and pathologies ranging from Newton–Wigner localization to instantaneous spreading and no-go results [62–64]. Field operators at an instant, or bounded-region idealized projectors constructed from them, the corresponding spectral projections need not belong to the local algebra generated by bounded operations in $\mathcal{O}$, the Reeh–Schlieder phenomenon and related localization subtleties [65], even though the unbounded operator is defined via smearing. Then the map (30) is not a local operation in the relativistic sense, and it can change $\text{Tr}(\rho B)$ for B localized in a spacelike separated region. In modern terms the paradox arises from assuming that every mathematically definable local projector is operationally implementable by a

localized apparatus. Historically, this is already anticipated in QED as Bohr and Rosenfeld emphasized that although field operators are formally associated to spacetime points, only averages over finite spacetime regions are operationally well-defined measurement quantities [29,30].

In the present framework, the UV completion is nonlocal and observables accessible to localized apparatuses are naturally quasi-local.

We let $O(x)$ be a local microcausal observable in an underlying Wightman QFT, and let $F(\square/M_*^2)$ be an entire-function deformation as in Sec. II–III. We recall that the smeared regulated observable is:

$$O^{(F)}(f) := \int d^4x O(x) [F(\square/M_*^2)f](x) \quad (32)$$

$$= \int d^4x d^4y K_F(x, y) f(y) O(x), \quad (33)$$

where $f \in \mathcal{S}(\mathbb{R}^4)$ is a Schwartz test function and the kernel $K_F(x, y)$ is smooth and rapidly decaying in spacelike directions, with decay rate set by the nonlocality scale M_* and fundamental length:

$$\ell_* \sim M_*^{-1}. \quad (34)$$

In particular, if $\text{supp } f \subset \mathcal{O}$ is bounded, then (33) shows that $O^{(F)}(f)$ depends on $O(x)$ for all x , with exponentially suppressed weight once $|x - y| \gg \ell_*$. Thus localization in $\mathcal{O}$ is replaced by an operational notion:

An observable is operationally accessible in $\mathcal{O}$ if it can be generated by smeared regulated fields $O^{(F)}(f)$ with $\text{supp } f \subset \mathcal{O}$.

I call the resulting quasi-local algebra $\mathcal{A}_F(\mathcal{O})$. Elements of $\mathcal{A}_F(\mathcal{O})$ are not strictly supported in $\mathcal{O}$ as distributions, but their commutators with spacelike separated observables are controlled at scale ℓ_* by Theorem 3.

In flat space, translation invariance gives the standard momentum-space picture that for a plane wave $e^{-ip \cdot x}$ one has $\square e^{-ip \cdot x} = -p^2 e^{-ip \cdot x}$, so $F(\square/M_*^2)$ becomes multiplication by $F(-p^2/M_*^2)$ in Fourier space. Equivalently:

$$[F(\square/M_*^2)f](x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} F(-p^2/M_*^2) \tilde{f}(p), \quad (35)$$

$$K_F(x - y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-y)} F(-p^2/M_*^2). \quad (36)$$

If F is nontrivial entire and produces UV damping, then K_F cannot have compact support so a compactly supported kernel would force strong exponential-type growth constraints on its Fourier transform (Paley–Wiener–Schwartz), which are incompatible with nontrivial entire UV suppression. Operationally there is no physically implementable perfectly sharp projector onto a strictly bounded region observable once the UV theory is quasi-local or band-limited at scale ℓ_* [42–45,62–64].

This is the missing physical obstruction needed for Sorkin’s paradox as the paradox assumes that one can perform an ideal measurement associated to a strictly localized projection in a bounded region; the regulated theory excludes such sharp operations from the operational set.

We now show that the operational replacement $\mathcal{A}(\mathcal{O}) \rightarrow \mathcal{A}_F(\mathcal{O})$ upgrades Sorkin’s binary tension into a controlled, scale-dependent statement that any influence on a spacelike separated region is exponentially suppressed by $M_*\rho$.

If we let B be a bounded observable localized in $\mathcal{O}'$, realized for example as a bounded function of smeared local fields with smearing supported in $\mathcal{O}'$. Then let $f \in \mathcal{S}(\mathbb{R}^4)$ with $\text{supp } f \subset \mathcal{O}$ and define the regulated generator $A := O^{(F)}(f)$ as in (33). Now consider the one-parameter family of unitary operations:

$$U_\lambda := e^{i\lambda A}, \quad \lambda \in \mathbb{R}. \quad (37)$$

The change induced on B in the Heisenberg picture is $B \mapsto U_\lambda^\dagger B U_\lambda$. Using the Duhamel formula:

$$U_\lambda^\dagger B U_\lambda - B = \int_0^\lambda ds U_s^\dagger (i[A, B]) U_s, \quad (38)$$

and hence the operator norm satisfies the general bound:

$$\|U_\lambda^\dagger B U_\lambda - B\| \leq |\lambda| \| [A, B] \|. \quad (39)$$

Now specialize B to be generated by an undeformed local observable $O_2(g)$ with $\text{supp } g \subset \mathcal{O}'$ or by bounded functions thereof. Then Theorem 3 applies directly to the commutator between $A = O_1^{(F)}(f)$ and $O_2^{(F)}(g)$, and by choosing N large one obtains, for $\rho \equiv \rho(f, g) > 0$:

$$\| [A, O_2^{(F)}(g)] \| \leq C_N(f, g) (1 + M_* \rho)^{-N} e^{-\alpha M_* \rho}. \quad (40)$$

Combining (39) and (40) yields:

$$\begin{aligned} \|U_\lambda^\dagger O_2^{(F)}(g) U_\lambda - O_2^{(F)}(g)\| &\leq |\lambda| C_N(f, g) \\ &\times (1 + M_* \rho)^{-N} e^{-\alpha M_* \rho}. \end{aligned} \quad (41)$$

Therefore, for any initial state ρ , the density operator and any bounded spacelike-separated probe observable B built from such operators:

$$\begin{aligned} \left| \text{Tr}(\rho U_\lambda^\dagger B U_\lambda) - \text{Tr}(\rho B) \right| &\leq \|U_\lambda^\dagger B U_\lambda - B\| \\ &\lesssim |\lambda| (1 + M_* \rho)^{-N} e^{-\alpha M_* \rho}. \end{aligned} \quad (42)$$

This is the operational content that any spacelike influence is bounded by the quasi-local tails and is exponentially small once the invariant separation exceeds a few times ℓ_* . In the local limit $M_* \rightarrow \infty$, with fixed $\rho > 0$, the bound vanishes and strict microcausality is recovered.

The sharpest way to avoid Sorkin-type paradoxes is to model measurement not by an abstract projection postulate but by localized dynamics, so we couple the system field to a probe field or finite-dimensional detector, evolve unitarily, and read out a probe observable. This produces a completely-positive instrument and a POVM on outcomes whose localization properties follow from the support of the interaction[24,66,67].

In the present nonlocal setting, the natural localized interaction density uses regulated observables:

$$\mathcal{L}_{\text{int}}(x) = g \chi(x) O^{(F)}(x) \otimes P, \quad (43)$$

where $g \in \mathbb{R}$ is a coupling constant, $\chi \in C_0^\infty(\mathbb{R}^{1,3})$ is a smooth switching function supported in the intended measurement region $\text{supp } \chi \subset \mathcal{O}$, and P is a probe operator such as probe momentum acting on an auxiliary probe Hilbert space. The unitary evolution is:

$$U = \mathcal{T} \exp \left(-i \int d^4x \mathcal{L}_{\text{int}}(x) \right), \quad (44)$$

as in the Tomonaga–Schwinger covariant formulation [68,69], and the induced nonselective channel on system states is obtained by tracing out the probe after the interaction, while selective updates correspond to conditioning on a probe POVM element.

Because the interaction is built from $O^{(F)}$ with $\text{supp } \chi \subset \mathcal{O}$, any influence on a spacelike separated $\mathcal{O}'$ is controlled by commutators of the form $[O^{(F)}(\chi), B]$, hence by the same exponential bound (40). Thus the measurement framework is causality-consistent up to the intrinsic quasi-locality scale ℓ_* , and the impossible measurement idealizations are excluded as there is no operationally realizable strictly

bounded-region projector with instantaneous Lüders collapse [29,30]. What replaces it is a POVM with resolution set by ℓ_* and exponentially small spacelike tails.

To summarize Sorkin's paradox requires an ideal, sharply localized measurement operation, a perfectly local projector with Lüders update that is not dynamically realizable in relativistic QFT. In an entire-function UV completion, the regulated observables are quasi-local by construction, with unavoidable non-compact tails given by Paley–Wiener, and Theorem 3 provides a quantitative bound showing that any resulting acausality is exponentially suppressed beyond $\ell_* \sim M_*^{-1}$. In this precise sense, the regulator supplies the missing physical obstruction to Sorkin's ideal measurements and turns the paradox into a controlled, scale-dependent statement.

7. Bohr–Rosenfeld Measurability

Bohr and Rosenfeld (BR) in 1933 and then 1950 analyze the measurability of electromagnetic field and charge observables in relativistic quantum electrodynamics (QED), emphasizing that the classical idealization of field components defined at every spacetime point is not operationally meaningful in the quantum theory. Instead, the physically meaningful observables are spacetime averages of the field strength over finite regions, whose commutation relations are finite and whose measurement limitations coincide with the uncertainty relations implied by those commutators [29,30].

Landau and Peierls argued that, in relativistic quantum theory, attempts to define and measure field strengths at a point, or over arbitrarily short times run into an operational obstruction, that momentum must be measured in a time Δt , but the charged test body necessarily radiates during the required dynamical change, injecting an irreducible disturbance that forces the inferred field uncertainty to blow up as $\Delta t \rightarrow 0$ [70].

Throughout this section Δt denotes the finite duration of the measurement interaction. ΔP is the target accuracy, standard deviation in the momentum readout of the test body. v and v' are the nonrelativistic velocities of the test body before and after the measurement. $\Delta \mathcal{E}$ and $\Delta \mathcal{H}$ are the accuracies in the inferred electric and magnetic field strengths, respectively, as defined by the measurement protocol below. Overdots denote time derivatives.

To infer a field component one measures the momentum change of a test body. Landau–Peierls emphasize that any momentum measurement completed in a short time Δt requires an energy exchange whose uncertainty obeys the standard energy–time limitation understood operationally, not as a statement that energy is undefined at an instant. In the kinematic regime where the body's velocity changes by $(v' - v)$ during the interaction, the characteristic mechanical energy transfer scale is $\sim (v' - v)\Delta P$; thus one writes:

$$(v' - v) \Delta P \Delta t \gtrsim \hbar. \quad (45)$$

Equation (45) is the content used by Landau–Peierls as their Eq. (1), expressed in the variables above.

A charged body that changes its velocity radiates. Landau–Peierls use the nonrelativistic radiation-damping estimate for the radiated energy:

$$E_{\text{rad}} \sim \frac{e^2}{c^3} \int \dot{v}^2 dt, \quad (46)$$

where e is the electric charge of the (charged) test body used in the measurement protocol, and note that, for a fixed net change $(v' - v)$ over a fixed time interval Δt , the integral is minimized by uniform acceleration $\dot{v} \simeq (v' - v)/\Delta t$. Hence:

$$E_{\text{rad}} \sim \frac{e^2}{c^3} \frac{(v' - v)^2}{\Delta t}. \quad (47)$$

This radiated energy is not under experimental control at the level required for an ideal momentum determination, and it enters the energy balance as an unknown loss. Landau–Peierls translate this

into an additional momentum inaccuracy by comparing the mechanical energy scale $(v' - v)\Delta P$ to the uncontrolled radiated energy: (47):

$$(v' - v)\Delta P \gtrsim \frac{e^2 (v' - v)^2}{c^3 \Delta t} \implies \Delta P \Delta t \gtrsim \frac{e^2}{c^3} (v' - v). \quad (48)$$

This is the Landau–Peierls radiation-induced inequality (their Eq. (3)). Combine (45) and (48) by eliminating $(v' - v)$. From (45) we have $(v' - v) \gtrsim \hbar / (\Delta P \Delta t)$. Insert into (48):

$$\Delta P \Delta t \gtrsim \frac{e^2}{c^3} \frac{\hbar}{\Delta P \Delta t} \implies (\Delta P \Delta t)^2 \gtrsim \frac{\hbar e^2}{c^3}. \quad (49)$$

Taking the positive root gives:

$$\Delta P \Delta t \gtrsim \sqrt{\frac{\hbar e^2}{c^3}} = \frac{\hbar}{c} \sqrt{\frac{e^2}{\hbar c}} = \frac{\hbar}{c} \sqrt{\alpha}. \quad (50)$$

This is the Landau–Peierls momentum-time limitation, their Eq. (4)).

Landau–Peierls take the simplest operational definition of an electric field measurement, that we observe the acceleration, impulse of a charged test body. For a field $\mathcal{E}$ acting over time Δt , the impulse is $\Delta p \sim e \mathcal{E} \Delta t$. If the momentum readout has accuracy ΔP , then the inferred field strength has accuracy $\Delta \mathcal{E}$ bounded by:

$$e \Delta \mathcal{E} \Delta t \gtrsim \Delta P. \quad (51)$$

This is their Eq. (5). Multiply (51) by (50) to eliminate ΔP :

$$e \Delta \mathcal{E} (\Delta t)^2 \gtrsim \Delta P \Delta t \gtrsim \frac{\hbar}{c} \sqrt{\alpha}. \quad (52)$$

Divide by e and use $\alpha = e^2 / (\hbar c)$ to obtain the key Landau–Peierls bound

$$\Delta \mathcal{E} \gtrsim \frac{\sqrt{\hbar c}}{(c \Delta t)^2}. \quad (53)$$

This is their Eq. (6), for increasingly time-localized measurements ($\Delta t \rightarrow 0$), the operationally attainable accuracy for a local electric field strength degrades as $\Delta \mathcal{E} \sim 1/\Delta t^2$. By considering the motion of a magnetic needle, Landau–Peierls obtain an analogous bound for the magnetic field strength, their Eq. (6a):

$$\Delta \mathcal{H} \gtrsim \frac{\sqrt{\hbar c}}{(c \Delta t)^2}. \quad (54)$$

They also discuss the additional cross-disturbance when attempting to measure $\mathcal{E}$ and $\mathcal{H}$ simultaneously using both a charged test body and a needle separated by a distance $\Delta \ell$, obtaining a mixed constraint, their Eq. (6b) of the schematic form:

$$\Delta \mathcal{E} \Delta \mathcal{H} \gtrsim \frac{\hbar c}{(c \Delta t)^2} \frac{1}{(\Delta \ell)^2}. \quad (55)$$

The operational punchline is the scaling (53)–(54) that shows attempts to define field strengths by arbitrarily sharp in time pointlike protocols lead to an intrinsic blow-up of the required uncertainties. Landau–Peierls therefore suggested that in the quantum range the field strengths are not physically measurable quantities in the same sense as in classical electrodynamics [70].

Bohr and Rosenfeld accept that strictly point-supported field components are idealizations. Their key correction is that the meaningful observables in quantum electrodynamics (QED) are averages of the field over finite spacetime regions, because any realistic measurement couples to extended test bodies during a finite time [29,30].

Bohr–Rosenfeld exhibit explicit measurement arrangements, extended charged bodies, compensating bodies, and controlled springs that measure $F_{\mu\nu}(R)$ while accounting for the necessary momentum transfer to the test bodies, the back-reaction fields sourced by the apparatus, and the fact that the sources can be taken as continuous classical distributions with arbitrarily large charge density in the idealized limit.

Throughout this section we work on Minkowski space with coordinates $x^\mu = (ct, \mathbf{x})$, metric $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$, and Levi–Civita symbol $\epsilon_{0123} = +1$. Greek indices $\mu, \nu, \rho, \sigma \in \{0, 1, 2, 3\}$ are raised/lowered with η . We use Gaussian units, so 4π appears in Maxwell’s equations, and we keep c and $\hbar$ explicit.

In QED the field strength operator $\hat{F}_{\mu\nu}(x)$ is an operator-valued distribution, not a bona fide operator at a point. The mathematically well-defined objects are smearings against smooth compactly supported test functions. For any smooth compactly supported antisymmetric test tensor $f^{\mu\nu}(x)$, define the smeared field observable:

$$\hat{F}(f) \equiv \int_{\mathbb{R}^4} d^4x f^{\mu\nu}(x) \hat{F}_{\mu\nu}(x), \quad (56)$$

$$f^{\mu\nu} = -f^{\nu\mu}, \quad f \in C_c^\infty(\mathbb{R}^4), \quad (57)$$

where $\hat{F}_{\mu\nu}(x)$ is the electromagnetic field-strength operator at spacetime point x , understood as an operator-valued distribution so the pointwise object is not a bona fide operator; only smearings are well-defined. Operationally, BR focus on approximately uniform averages over finite spacetime regions, let $R \subset \mathbb{R}^4$ be a bounded region with four-volume, a finite 4D block in Minkowski space over which you average:

$$|R| \equiv \int_R d^4x, \quad (58)$$

and let $\chi_R(x)$ be its indicator function. For strictly distributional rigor one replaces χ_R by a smooth approximation $w_R \in C_c^\infty$ with $\int d^4x w_R(x) = 1$ and $\text{supp}(w_R) \subset R$; we keep BR’s notation of an idealized uniform average for transparency. Define the region-averaged field component:

$$\hat{F}_{\mu\nu}(R) \equiv \frac{1}{|R|} \int_R d^4x \hat{F}_{\mu\nu}(x), \quad (59)$$

where $\hat{F}_{\mu\nu}(R)$ is the spacetime-averaged field-strength observable over the region R which is a special case of (57) with $f^{\mu\nu}(x) = \frac{1}{|R|} \chi_R(x) e^{\mu\nu}$ for a fixed constant antisymmetric tensor $e^{\mu\nu}$ selecting a component. With our conventions:

$$\hat{E}_i(x) = \hat{F}_{0i}(x), \quad (60)$$

$$\hat{B}_i(x) = -\frac{1}{2} \epsilon_{ijk} \hat{F}_{jk}(x), \quad i, j, k \in \{1, 2, 3\}, \quad (61)$$

where $\hat{E}_i(x)$ is the i -th Cartesian component of the electric field operator at spacetime point x , $\hat{B}_i(x)$ is the i -th Cartesian component of the magnetic field operator at spacetime point x , $\hat{F}_{\mu\nu}(x)$ is the electromagnetic field-strength tensor operator (antisymmetric in μ, ν) at x , with $\mu, \nu \in \{0, 1, 2, 3\}$, the index 0 means the time component of the Lorentz index so F_{0i} mixes time and space, ϵ_{ijk} is the three-dimensional Levi–Civita symbol, defined by $\epsilon_{123} = +1$ and total antisymmetry, the factor $-\frac{1}{2}$: conventional normalization for the spatial dualization, so equivalently

$$\hat{F}_{ij}(x) = -\epsilon_{ijk} \hat{B}_k(x), \quad (62)$$

up to the chosen sign conventions. Bohr–Rosenfeld often denote the time index by 4 rather than 0; the translation is $4 \leftrightarrow 0$ up to convention-dependent factors.

We fix Lorenz gauge for the free field for definiteness. The basic causal commutator of the vector potential is:

$$[\hat{A}_\mu(x), \hat{A}_\nu(y)] = i\hbar c \eta_{\mu\nu} \Delta(x - y), \quad (63)$$

where $\hat{A}_\mu(x)$ is the electromagnetic four-potential operator an operator-valued distribution at space-time point x , with Lorentz index $\mu \in \{0, 1, 2, 3\}$. $\hat{A}_\nu(y)$ is the four-potential operator at spacetime point y , with Lorentz index $\nu \in \{0, 1, 2, 3\}$. $[\hat{A}_\mu(x), \hat{A}_\nu(y)] \equiv \hat{A}_\mu(x)\hat{A}_\nu(y) - \hat{A}_\nu(y)\hat{A}_\mu(x)$ is the operator commutator. $\Delta(x - y)$ is the Pauli-Jordan commutator distribution for the massless field, a Lorentz-invariant distribution depending only on the separation $x - y$. It vanishes for spacelike separations $(x - y)^2 < 0$, ensuring microcausality. For the massless wave operator:

$$\square \Delta(x) = 0, \quad (64)$$

$$\Delta(x) = 0 \text{ for } x^2 < 0, \quad (65)$$

$$\Delta(x) = -\Delta(-x). \quad (66)$$

Any equivalent covariant quantization yields the same gauge-invariant commutator for $\hat{F}_{\mu\nu}$ below.

By definition $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$. Using bilinearity, (63), and commuting derivatives through the commutator:

$$\begin{aligned} [\hat{F}_{\mu\nu}(x), \hat{F}_{\rho\sigma}(y)] &= [\partial_\mu \hat{A}_\nu(x) - \partial_\nu \hat{A}_\mu(x), \partial_\rho \hat{A}_\sigma(y) - \partial_\sigma \hat{A}_\rho(y)] \\ &= \partial_\mu \partial_\rho [\hat{A}_\nu(x), \hat{A}_\sigma(y)] - \partial_\mu \partial_\sigma [\hat{A}_\nu(x), \hat{A}_\rho(y)] - \partial_\nu \partial_\rho [\hat{A}_\mu(x), \hat{A}_\sigma(y)] + \partial_\nu \partial_\sigma [\hat{A}_\mu(x), \hat{A}_\rho(y)] \\ &= i\hbar c (\eta_{\nu\sigma} \partial_\mu \partial_\rho - \eta_{\nu\rho} \partial_\mu \partial_\sigma - \eta_{\mu\sigma} \partial_\nu \partial_\rho + \eta_{\mu\rho} \partial_\nu \partial_\sigma) \Delta(x - y), \end{aligned}$$

where all derivatives act on x , equivalently on $x - y$. Since Δ is supported on the light cone, the commutator vanishes for spacelike separated points ($(x - y)^2 < 0$), so microcausality holds for the gauge-invariant field strength. Insert (67) into the double integral implied by (59). For two regions R, R' :

$$\begin{aligned} [\hat{F}_{\mu\nu}(R), \hat{F}_{\rho\sigma}(R')] &= \frac{1}{|R||R'|} \int_R d^4x \int_{R'} d^4y [\hat{F}_{\mu\nu}(x), \hat{F}_{\rho\sigma}(y)] \\ &= i\hbar c \frac{1}{|R||R'|} \int_R d^4x \int_{R'} d^4y \mathcal{K}_{\mu\nu,\rho\sigma}(\partial_x) \Delta(x - y), \end{aligned}$$

with the differential operator:

$$\mathcal{K}_{\mu\nu,\rho\sigma}(\partial) \equiv \eta_{\nu\sigma} \partial_\mu \partial_\rho - \eta_{\nu\rho} \partial_\mu \partial_\sigma - \eta_{\mu\sigma} \partial_\nu \partial_\rho + \eta_{\mu\rho} \partial_\nu \partial_\sigma. \quad (67)$$

Where $\mathcal{K}_{\mu\nu,\rho\sigma}(\partial)$ is a second-order differential operator a rank-(2,2) tensor-valued operator built from the Minkowski metric and spacetime derivatives. It is the standard kernel that appears when commuting derivatives through the potential commutator to obtain the field-strength commutator:

$$[\hat{F}_{\mu\nu}(x), \hat{F}_{\rho\sigma}(y)] \propto \mathcal{K}_{\mu\nu,\rho\sigma}(\partial) \Delta(x - y).$$

∂_μ is partial derivative with respect to the μ -th coordinate,

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu},$$

and similarly for $\partial_\nu, \partial_\rho, \partial_\sigma$. BR package these region-dependent integrals into geometric factors that are finite numbers determined by the shapes and relative placements of R and R' . One convenient representation that makes the BR antisymmetry manifest is to write the commutator in the BR form:

$$[\hat{F}_{\mu\nu}(R), \hat{F}_{\rho\sigma}(R')] = i\hbar c (A_{\mu\nu,\rho\sigma}(R, R') - A_{\rho\sigma,\mu\nu}(R', R)), \quad (68)$$

where a consistent choice is:

$$A_{\mu\nu,\rho\sigma}(R, R') \equiv \frac{1}{|R||R'|} \int_R d^4x \int_{R'} d^4y \mathcal{K}_{\mu\nu,\rho\sigma}(\partial_x) G_{\text{ret}}(x - y), \quad (69)$$

$G_{\text{ret}}(x - y)$ is the retarded Green function of the relevant wave operator; in this context, the massless d'Alembertian $\square$. It is supported on the future light cone:

$$G_{\text{ret}}(x - y) = 0 \quad \text{unless } x \text{ lies in the causal future of } y,$$

$A_{\mu\nu,\rho\sigma}(R, R')$ is a bilocal coefficient kernel with two antisymmetric Lorentz-index pairs $(\mu\nu)$ and $(\rho\sigma)$ obtained by averaging a differential operator acting on a Green function over two spacetime regions R and R' , $|R|$ and $|R'|$ are the four-volumes of R and R' , defined by

$$|R| \equiv \int_R d^4x, \quad |R'| \equiv \int_{R'} d^4y.$$

The prefactor $1/(|R||R'|)$ makes the double integral a double average, $\int_R d^4x \int_{R'} d^4y$ is integration over the two regions, with x integrated over R and y integrated over R' . In the Bohr–Rosenfeld setting it encodes the region-averaged commutator/response between smeared field-strength components supported in R and R' . Then $G_{\text{ret}} - G_{\text{adv}} = \Delta$, so (68) is equivalent to (67). Any equivalent definition of A that differs by integration by parts or by choosing smooth weights $w_R, w_{R'}$ yields the same commutator. If $\text{supp}(w_R)$ and $\text{supp}(w_{R'})$ are spacelike separated, then $\Delta(x - y) = 0$ throughout the integration domain and hence:

$$[\hat{F}_{\mu\nu}(R), \hat{F}_{\rho\sigma}(R')] = 0, \quad (70)$$

microcausality persists for smeared observables.

BR operationally define a field measurement by the total momentum transferred to suitable test bodies carrying a prescribed charge-current distribution over the region of interest. We now formalize this as follows, let $\hat{j}_{\text{test}}^\mu(x)$ be the effectively classical test current density associated with heavy test bodies. The Lorentz force density is:

$$f^\mu(x) = \frac{1}{c} \hat{F}^{\mu\nu}(x) j_{\nu,\text{test}}(x), \quad (71)$$

where $f^\mu(x)$ is the four-force density; force per unit three-volume, packaged as a Lorentz four-vector at spacetime point x . Equivalently, it is the local rate of change of four-momentum density delivered to the test system by the electromagnetic field. $x \in \mathbb{R}^4$ is a spacetime point with coordinates $x^\mu = (ct, \mathbf{x})$. The total four-momentum imparted to the test system during the measurement that is idealized as occurring within region R is given by:

$$\Delta P^\mu = \int_R d^4x f^\mu(x) = \frac{1}{c} \int_R d^4x \hat{F}^{\mu\nu}(x) j_{\nu,\text{test}}(x). \quad (72)$$

Assuming for simplicity, that $R = V \times [t_0, t_0 + T]$ with fixed spatial volume V and duration T . To measure an averaged electric component $\hat{E}_i = \hat{F}_{0i}$, choose a uniform charge density over V with no spatial current:

$$j_{\text{test}}^\mu(x) = (c\rho_0 \chi_V(\mathbf{x}) \chi_{[t_0, t_0+T]}(t), \mathbf{0}), \quad (73)$$

where $j_{\text{test}}^\mu(x)$ is the test four-current density used to probe the field, evaluated at spacetime point $x = (ct, \mathbf{x})$. The decomposition $j^\mu = (c\rho, \mathbf{j})$ is here the spatial current is set to zero, so the test charge is at rest:

$$j_{\text{test}}^\mu(x) = (c\rho_{\text{test}}(x), \mathbf{0}).$$

ρ_0 is a constant charge density amplitude so the total test charge is $Q = \rho_0 \text{Vol}(V)$ if χ_V is a sharp indicator of V . $\chi_V(\mathbf{x})$ is the characteristic indicator function of a spatial region $V \subset \mathbb{R}^3$,

$$\chi_V(\mathbf{x}) = \begin{cases} 1, & \mathbf{x} \in V, \\ 0, & \mathbf{x} \notin V, \end{cases}$$

which localizes the charge in space. $\chi_{[t_0, t_0+T]}(t)$ is the characteristic function of the time interval $[t_0, t_0 + T]$,

$$\chi_{[t_0, t_0+T]}(t) = \begin{cases} 1, & t \in [t_0, t_0 + T], \\ 0, & t \notin [t_0, t_0 + T], \end{cases}$$

which switches the coupling on only during that measurement window. t_0 is the start time of the measurement interaction and T is the duration of the interaction so the end time is $t_0 + T$. $\mathbf{0}$: the zero 3-vector, indicating no spatial current flow. Then (72) yields, for the spatial momentum component $\mu = i$:

$$\begin{aligned} \Delta P^i &= \frac{1}{c} \int_{t_0}^{t_0+T} c \, dt \int_V d^3x \hat{F}^{i0}(x) (c\rho_0) \\ &= \rho_0 c \int_{t_0}^{t_0+T} dt \int_V d^3x \hat{F}^{i0}(x) = \rho_0 c |R| \hat{F}^{i0}(R). \end{aligned} \quad (74)$$

Hence a measurement of the total impulse ΔP^i is up to known constants a measurement of the spacetime-averaged electric field component over R .

To measure an averaged magnetic component, one chooses a uniform spatial current density, closed by auxiliary conductors so that the dominant impulse involves $\mathbf{j} \times \mathbf{B}$, again producing a readout proportional to $\hat{B}(R)$; see BR for explicit constructions [29,30].

A key subtlety is that the test bodies are themselves field sources. BR show that static contributions of the test distribution can be compensated by fixed auxiliary bodies carrying the opposite charge-current distribution, constructed so as not to impede the free motion needed for momentum control. After compensation, the remaining source of the measuring arrangement is a polarization arising from the uncontrollable displacement of the test bodies during the momentum readout [29].

Let p^μ denote the effective classical charge-current density of the movable test bodies in the measurement region. If the test bodies undergo a common small displacement D^μ during the momentum determination, then the induced polarization tensor in the region is:

$$P^{\mu\nu} = D^\mu p^\nu - D^\nu p^\mu, \quad (75)$$

which acts as an additional source for the electromagnetic field. The expectation value of the corresponding contribution to the average field in another region R' is proportional to the product $P^{\mu\nu}$ and to the same geometric factors $A_{\mu\nu, \rho\sigma}(R, R')$ that appear in the commutator (68).

The detailed evaluation of the geometric factors for specific region shapes is nontrivial, a modern systematic calculation and correction of several geometric factors is given by Hnizdo [71].

BR further show that a suitable mechanical device, a restoring force proportional to displacement can compensate, at the level of classical field theory, the momentum transferred by this uncontrolled self-field. The uncompensable residual, arising from the fundamentally statistical nature of photon emission and absorption, matches precisely the intrinsic field fluctuations predicted by QED [29].

We now state the essential BR conclusion in a modern operator language.

Theorem 7.1 (Bohr–Rosenfeld compatibility for field averages). *Let $\hat{F}_{\mu\nu}(R)$ and $\hat{F}_{\rho\sigma}(R')$ be spacetime-averaged field observables defined by (59). Then any joint measurement scheme for these two observables is subject to the Robertson bound [72,73]:*

$$\Delta F_{\mu\nu}(R) \Delta F_{\rho\sigma}(R') \geq \frac{1}{2} |\langle [\hat{F}_{\mu\nu}(R), \hat{F}_{\rho\sigma}(R')] \rangle| \quad (76)$$

$$= \frac{\hbar c}{2} |\langle A_{\mu\nu,\rho\sigma}(R, R') - A_{\rho\sigma,\mu\nu}(R', R) \rangle|, \quad (77)$$

with A as in (69). Moreover, BR construct idealized measurement arrangements with compensated sources and correlated momentum readouts for which the only limitations on simultaneous measurability are those implied by (77); i.e. no additional “Landau–Peierls” limitation survives once extended test bodies and compensation are used [29,30].

The inequality (77) is the standard Robertson uncertainty relation for any pair of self-adjoint operators $\hat{X}, \hat{Y}$ are given by $\Delta X \Delta Y \geq \frac{1}{2} |\langle [\hat{X}, \hat{Y}] \rangle|$ [72,73], applied to $\hat{X} = \hat{F}_{\mu\nu}(R)$ and $\hat{Y} = \hat{F}_{\rho\sigma}(R')$. The commutator is given by (68), which follows from (67) by smearing over R and R' . This establishes the necessary limitation.

The BR content is the sufficiency as they show that the measurement back-reaction produced by the unavoidable position indeterminacy of the test bodies that is encoded in the displacement D^μ generates precisely the same geometric factors that appear in (68), and that after optimal classical compensation the remaining uncontrollable contribution is of exactly the magnitude required by the commutator. Hence the operational measurement limitations match the intrinsic quantum ones.

BR also emphasize that the physically meaningful charge-current observables are spacetime averages over finite regions. Define:

$$\hat{J}_\mu(R) \equiv \frac{1}{|R|} \int_R d^4x \hat{j}_\mu(x). \quad (78)$$

From Maxwell’s equations in Gaussian units:

$$\partial^\nu \hat{F}_{\mu\nu}(x) = \frac{4\pi}{c} \hat{j}_\mu(x), \quad (79)$$

and applying Gauss’ theorem in spacetime to region R with boundary ∂R and outward normal $d\Sigma^\nu$ [74,75], we obtain:

$$\begin{aligned} \hat{J}_\mu(R) &= \frac{c}{4\pi|R|} \int_R d^4x \partial^\nu \hat{F}_{\mu\nu}(x) \\ &= \frac{c}{4\pi|R|} \oint_{\partial R} \hat{F}_{\mu\nu}(x) d\Sigma^\nu(x), \end{aligned} \quad (80)$$

so, the average current through R is equivalently a generalized flux of $\hat{F}$ through the boundary of R . In the special case $R = V \times [t_0, t_0 + T]$, the $\mu = 0$ component reduces to the familiar statement that the average charge density in V is determined by the time-averaged electric flux through ∂V .

The BR analysis makes precise the measurement-theoretic content of the statement that field values at a point are not operational observables in relativistic quantum theory. Finite-region averages $\hat{F}_{\mu\nu}(R)$ are the appropriate observables and their commutators are finite and determine the ultimate measurability limits, and idealized measurement arrangements can in principle reach these limits.

The conceptual resolution is that Landau–Peierls derive a divergence by pushing a pointlike, short-time protocol, where Bohr–Rosenfeld show that QED only requires operational access to finite spacetime averages, and those can be made arbitrarily accurate within the same idealizations, consistent with the field commutators.

In this sense, the Landau–Peierls obstruction diagnoses the breakdown of the point-limit as an operational notion, not an inconsistency of QED.

The Landau–Peierls analysis makes the basic warning mathematically explicit, that attempting to operationalize the value of a field at a spacetime point via an arbitrarily sharp measurement in time, and implicitly space forces disturbances that scale catastrophically Eq.((53)–(54)). Bohr–Rosenfeld then sharpen the lesson into a clean principle, that the observables with direct physical meaning are smeared/averaged operators over finite regions [29,30].

In local QFT, a renormalized local field $\mathcal{O}(x)$ is not an operator at a point but an operator-valued distribution. The mathematically well-defined observables are smearings:

$$\mathcal{O}(f) \equiv \int_{\mathbb{R}^4} d^4x f(x) \mathcal{O}(x), \quad f \in C_c^\infty(\mathbb{R}^4), \quad (81)$$

and similarly for tensor fields with test-tensor smearings. This framework is standard in axiomatic and local QFT and is also the operational starting point in the Bohr–Rosenfeld measurability analysis [1,2,29,30].

Let $R \subset \mathbb{R}^4$ be a bounded spacetime region with four-volume:

$$|R| \equiv \int_R d^4x. \quad (82)$$

A naive uniform average of $\mathcal{O}$ over R would be:

$$\overline{\mathcal{O}}(R) \stackrel{\text{formal}}{=} \frac{1}{|R|} \int_R d^4x \mathcal{O}(x) = \int d^4x \left(\frac{1}{|R|} \chi_R(x) \right) \mathcal{O}(x), \quad (83)$$

where χ_R is the indicator of R . Since $\chi_R \notin C_c^\infty$, (83) is not a priori defined in the strict distributional sense. The correct and physically relevant definition is to replace χ_R by a smooth window supported in or tightly around R .

We fix a nonnegative mollifier $\eta \in C_c^\infty(\mathbb{R}^4)$, infinitely differentiable with compact support and $\eta(x) \geq 0$ for all x . $C_c^\infty(\mathbb{R}^4)$ is the space of smooth functions on $\mathbb{R}^4$ with compact support, vanishing outside some bounded set. With $\int d^4x \eta(x) = 1$, a normalization condition ensuring η has unit integral so it can approximate a delta distribution under rescaling. Set $\eta_\varepsilon(x) \equiv \varepsilon^{-4} \eta(x/\varepsilon)$, the rescaled mollifier in four dimensions. The prefactor ε^{-4} is chosen so that the unit-integral normalization is preserved:

$$\int d^4x \eta_\varepsilon(x) = 1.$$

Define:

$$w_{R,\varepsilon}(x) \equiv \frac{1}{|R|} (\chi_R * \eta_\varepsilon)(x) = \frac{1}{|R|} \int_R d^4y \eta_\varepsilon(x - y), \quad (84)$$

where $w_{R,\varepsilon}(x)$ is normalized window smearing function on spacetime. It is a smooth approximation to the uniform average over the region R , used to define well-behaved smeared observables. $*$ is the convolution on $\mathbb{R}^4$, defined by

$$(\chi_R * \eta_\varepsilon)(x) \equiv \int_{\mathbb{R}^4} d^4y \chi_R(y) \eta_\varepsilon(x - y).$$

Since $\chi_R(y) = 1$ only for $y \in R$, this reduces to the integral over R in the second equality. Then $w_{R,\varepsilon} \in C^\infty$ is nonnegative, satisfies $\int d^4x w_{R,\varepsilon}(x) = 1$, and is supported in an ε -neighborhood of R .

For any operator-valued distribution $\mathcal{O}$ for which $\mathcal{O}(f)$ is well-defined for all $f \in C_c^\infty$, the finite-resolution average of $\mathcal{O}$ over R is precisely the smeared observable:

$$\overline{\mathcal{O}}(R;\varepsilon) \equiv \mathcal{O}(w_{R,\varepsilon}) = \int d^4x w_{R,\varepsilon}(x) \mathcal{O}(x). \quad (85)$$

Moreover, for any test function $g \in C_c^\infty$,

$$\lim_{\varepsilon \rightarrow 0^+} \int d^4x w_{R,\varepsilon}(x) g(x) = \frac{1}{|R|} \int_R d^4x g(x), \quad (86)$$

where $\varepsilon \rightarrow 0^+$ is the mollifier width or the smoothing scale that tends to zero through positive values. $w_{R,\varepsilon}(x)$ is the smoothed, normalized window function associated with the region R and smoothing scale ε , such as:

$$w_{R,\varepsilon}(x) = \frac{1}{|R|} (\chi_R * \eta_\varepsilon)(x).$$

$g(x)$ is an arbitrary test function or sufficiently regular function on $\mathbb{R}^4$ against which the window is paired; typically one takes $g \in C^\infty(\mathbb{R}^4)$ or at least continuous and integrable. The meaning of the limit is that $w_{R,\varepsilon}$ converges weakly in the sense of distributions to the normalized indicator $\chi_R/|R|$. Equivalently:

$$w_{R,\varepsilon} \xrightarrow{\varepsilon \rightarrow 0^+} \frac{1}{|R|} \chi_R \quad \text{in } \mathcal{D}'(\mathbb{R}^4),$$

so pairing with any g yields the averaged integral on the right-hand side.

$w_{R,\varepsilon} \rightarrow |R|^{-1} \chi_R$ in the sense of distributions. Therefore, $\overline{\mathcal{O}}(R; \varepsilon)$ is exactly the mathematically controlled version of the BR average over a finite spacetime region; the ideal top-hat average corresponds to the $\varepsilon \rightarrow 0$ limit when such a limit exists on the chosen domain of states.

Equation (85) is immediate from the definition (81). For (86), use the defining property of mollifiers $\eta_\varepsilon * g \rightarrow g$ uniformly on compact sets as $\varepsilon \rightarrow 0^+$. Then:

$$\begin{aligned} \int d^4x w_{R,\varepsilon}(x) g(x) &= \frac{1}{|R|} \int d^4x \int_R d^4y \eta_\varepsilon(x-y) g(x) \\ &= \frac{1}{|R|} \int_R d^4y \int d^4x \eta_\varepsilon(x-y) g(x) \\ &= \frac{1}{|R|} \int_R d^4y (\eta_\varepsilon * g)(y) \\ &\xrightarrow{\varepsilon \rightarrow 0^+} \frac{1}{|R|} \int_R d^4y g(y). \end{aligned} \quad (87)$$

This proves distributional convergence of $w_{R,\varepsilon}$ to $|R|^{-1} \chi_R$ and hence establishes the equivalence between finite-region averaging, with finite resolution and smearing.

Any realistic measurement couples the field to an extended apparatus via an interaction of the form:

$$H_{\text{int}} \sim \int d^4x J(x) \mathcal{O}(x), \quad (88)$$

with a smooth spacetime profile $J(x)$ determined by the apparatus' finite extent and switching time. Thus what is measured is precisely a smeared observable $\mathcal{O}(J)$; BR's finite-region averages correspond to taking J to be approximately a normalized window over the region R [29,30].

8. The Operational Locality Paradox

Relativistic quantum field theory is engineered to respect causal structure with operations localized in a bounded spacetime region O should not enable controllable influence on observables localized in a spacelike separated region O' . A standard sufficient condition is microcausality of local observables, namely that strictly spacelike separated observables commute. However, when one combines relativistic causality with realistic measurement modeling and ultraviolet completion, one encounters a sharp conceptual tension that can be presented as a paradox.

The tension can be formulated as the apparent incompatibility of the following three statements. That **Relativistic causality (operational locality)**. No localized intervention in O can change expectation values of bounded observables localized in a spacelike separated region O' . That **Ideal textbook**

measurement postulate. An ideal projective measurement of a perfectly localized observable in O can be modeled by an instantaneous Lüders state-update, implemented as a completely-positive map built from sharp spectral projections associated with that observable. That **Ultraviolet completion by entire-function regularization.** In an entire-function UV completion, physically accessible observables are naturally regulated and quasi-local: localization is implemented by smearing with test functions supported in O , while the regulated observable itself has non-compact tails controlled by the nonlocality scale M_* (equivalently $\ell_* \sim M_*^{-1}$).

Taken together, (1) and (2) suggest that ideal local measurements should be causally harmless, while (2) and (3) suggest that one can still speak of arbitrarily sharp, bounded-region projectors in the regulated theory. But (1) and (2) are in tension even in strictly local QFT, Sorkin emphasized that importing an instantaneous Lüders update for perfectly localized projectors into relativistic QFT can lead to apparent superluminal signaling in natural measurement scenarios, the so-called impossible measurements problem.

The paradox can therefore be stated as follows:

How can a UV-complete relativistic theory simultaneously (i) admit meaningful localized measurements, (ii) avoid superluminal signaling, and (iii) remain compatible with the idealized notion of perfectly sharp bounded-region projectors and instantaneous measurement update?

The resolution is that the paradox is generated by an over-idealized notion of localization and measurement. The crucial physical obstruction is that not every mathematically definable bounded-region projector is an operational element of the theory. In particular, in an entire-function UV completion the regulated observables are necessarily quasi-local as the corresponding kernels cannot have compact support by Paley–Wiener–Schwartz type constraints, so perfectly sharp bounded-region projectors are not implementable by any localized apparatus.

Operationally, what is accessible in a bounded region O is generated by regulated smearings:

$$O^{(F)}(f) := \int d^4x O(x) [F(\square/M_*^2)f](x), \quad (89)$$

with test function f supported in O , together with bounded functions thereof. This motivates the replacement of the strictly local algebra $A(O)$ by an operational quasi-local algebra $A_F(O)$ generated by such regulated observables. Elements of $A_F(O)$ are not strictly supported in O as distributions, but their nonlocal tails are exponentially suppressed beyond the fundamental length ℓ_* .

In this operational setting, locality becomes quantitative rather than binary, that influence outside the intended region is not forbidden by an exact commutator identity, but is bounded by a uniform asymptotic estimate. Concretely, Theorem 3 shows that for two regulated observables with spacelike-separated supports, the commutator obeys an exponential suppression bound of the form:

$$\|[O_1^{(F)}(f), O_2^{(F)}(g)]\| \lesssim C_N(f, g) (1 + M_*\rho)^{-N} e^{-\alpha M_*\rho}, \quad (90)$$

where ρ is the invariant spacelike separation between the supports of f and g , and $\alpha > 0$ is independent of M_* . In particular, any disturbance of a spacelike separated region becomes exponentially small once ρ exceeds a few times ℓ_* , and strict microcausality is recovered in the local limit $M_* \rightarrow \infty$ at fixed separation.

This resolves the paradox as ideal instantaneous Lüders updates for perfectly sharp bounded-region projectors are not operationally licensed in the regulated theory. Operationally realizable measurements are instead modeled by localized dynamics producing a completely-positive instrument, and any residual influence on spacelike separated observables is controlled by the asymptotic commutator bound. The would-be contradiction is thereby upgraded into a controlled, scale-dependent statement of emergent microcausality.

9. Conclusion

I have shown that a wide class of entire-function deformations of local QFT satisfy an asymptotic microcausality property where commutators of nonlocal observables at spacelike separation are exponentially suppressed beyond the nonlocality scale M_*^{-1} and vanish in the local limit $M_* \rightarrow \infty$. This makes precise the sense in which microcausality can be an emergent infrared property of an underlying nonlocal theory.

Seen through this lens, the result fits a broader pattern in relativistic QFT in apparent clashes with structural principles often trace back to overly sharp idealizations applied to operator-valued distributions. Historically, light-cone singularities in Heisenberg-picture current correlators were already known to generate gauge-noninvariant intermediate expressions unless one imposed explicit regularity constraints implemented as moment conditions on an associated spectral function to define a consistent regulated class of vacuum expectation values [36,50,52,53]. The entire-function framework realizes the analogous strategy for locality as it replaces perfectly sharp localization by quasi-local observables with a controlled nonlocality length $\ell_* \sim M_*^{-1}$, thereby turning a binary commute/do not commute notion into a quantitative, operationally meaningful spacelike suppression bound.

The asymptotic commutator bound supplies a quantitative notion of operational locality that connects directly to measurement. Modeling localized interventions using the quasi-local algebra generated by regulated observables, one finds that any disturbance of a spacelike separated region is exponentially suppressed, and vanishes in the local limit at fixed separation. The theorem supplies a controlled resolution of the idealizations behind Sorkin's impossible measurements paradox [21,22]. The paradox assumes the availability of perfectly sharp, strictly localized projective measurements with instantaneous Lüders update [23]. In an entire-function UV completion, however, the regulated observables are necessarily quasi-local by Paley–Wiener–Schwartz [43–45], so such strictly bounded-region projectors are not operational elements of the theory. Operationally realizable measurements are instead modeled by localized dynamics, which induces a completely-positive instrument [24,47–49, 66,67]. In this setting, any influence on spacelike separated observables is bounded by the commutator estimate, and is exponentially suppressed once the invariant separation exceeds a few times $\ell_* \sim M_*^{-1}$.

In the Bohr–Rosenfeld analysis, the key move is that field values at a point are a classical idealization. In quantum electrodynamics, the objects that have unambiguous meaning and are measurability-eligible are field quantities averaged over finite spacetime regions, not perfectly point-local field components. They explicitly emphasize that only such finite-region averages are well-defined in the quantum theory. This is what this paper emphasizes, idealized infinitely sharp localization assumptions are where trouble starts, that if you insist on perfectly localized quantities like the ones used in Wightman axioms and ideal instantaneous measurements, you are stepping outside what the theory operationally licenses [1,2,29–31,71].

Several extensions are natural such as to curved spacetime as the analysis can be generalized to globally hyperbolic spacetimes, replacing the flat d'Alembertian by a covariant one and using heat-kernel estimates on curved backgrounds. This is particularly relevant for black hole spacetimes, where one expects quantum-gravity scale nonlocality but macroscopic causal structure. To gauge and gravity sectors, while I focused on generic gauge-invariant observables, one can study in detail how entire-function form factors in gauge and gravitational actions affect commutators of field strengths and curvature operators.

From a conceptual perspective, the theorem clarifies that nonlocality at very short distances does not automatically imply observable violations of relativistic causality. Provided the nonlocality is controlled by entire-function form factors with suitable growth, the long-distance behaviour of the theory remains effectively local and microcausal.

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