Brief Report

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Brief Report

The Secret to Fixing Incorrect Canonical Quantizations

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Abstract: Covariant scalar field quantization, nicknamed $(\varphi^r)_n$, where r denotes the power of the interaction term and $n = s+1$ where s is the spatial dimension and 1 adds time. Models such that $r < 2n/(n-2)$ can be treated by canonical quantization, while models such that $r > 2n/(n-2)$ are trivial, or, if treated as a unit, emerge as 'free theories'. Models such as $r = 2n/(n-2)$, e.g., $r = n = 4$, again using canonical quantization also become 'free theories', which must be considered quantum failures. However, there exists a different approach called affine quantization that promotes a different set of classical variables to become the basic quantum operators and it offers different results. It is well-known that the canonical quantization of φ^4_4 fails. Here is how to fix it along with many other problems.

Keywords: canonical quantization; affine quantization; field theory

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I. Introduction

There have been many models with the same 'illness' as that of φ^4_4 [1–7]. The secret to a valid canonical quantization (CQ) is remarkably simple. All you need is the addition of a single, fixed potential, which is not seen in the classical Hamiltonian, but it puts things in proper position elsewhere. This single, additional, potential is just $2\hbar^2/\varphi(x)^2$. Give it a try-out on the model φ^4_4 and see for yourself! You can also give it a new try with φ^8_4 and be surprised! **That additional 'potential', put just after $\hat{\pi}(x)^2$, is all you need to use.**

II. Amazing Results by Just Removing $\varphi(x) = 0$

The special $\hbar$-term has arisen from the fact that $\varphi(x) = 0$ has been removed, which then means that the momentum is no longer self-adjoint. The next step leads to introducing $\hat{\kappa}(x) = [\hat{\pi}(x)^\dagger \varphi(x) + \varphi(x)\hat{\pi}(x)]/2$, and with scaling can lead to become

$$\pi(x)^2 = \kappa(x)^2/\varphi(x)^2 \to \hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) \Rightarrow \hat{\pi}(x)^2 + b\hbar^2/\varphi(x)^2, \tag{2.1}$$

and the factor $b = 2$ has been chosen to fit our particular problem. **Observe that choosing $\varphi(x) \neq 0$ has permitted introducing the 'polynomial'-like term $2\hbar^2/\varphi(x)^2$.**

A. Understanding How Scaling Works

Initially $\hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) = \hat{\pi}^2 + \hbar^2\delta(0)^{2s}/\varphi(x)^2$, where $\delta(0)$ is Dirac's special function, where $\delta(x) = 0$ for all $0 < |x|$, while $\int \delta(x)\, dx = 1$ which leads to $\delta(0) = \infty$. Now our ∞ is reduced to $b\hbar^2 W^2 < \infty$, and W will be sent to ∞ later on.

This now becomes $\hat{\pi}(x)^2 + b\hbar^2 W^2/\varphi(x)^2$. Next $(\hat{\pi}(x)^2 \;\&\; \varphi(x)^2) \to W(\hat{\pi}(x)^2 \;\&\; \varphi(x)^2)$. This leads to $W\hat{\pi}(x)^2 + b\hbar^2 W^2/W\varphi(x)^2$, and now a full multiplication by W^{-1} leads to the final result which is $\hat{\pi}(x)^2 + b\hbar^2/\varphi(x)^2$. Now W can be sent to infinity.

III. Selected Topics of Affine Quantization

A major feature of CQ is that $-\infty < q \;\&\; \varphi(x) < \infty$ either in quantum mechanics where q is position or in scalar field theory where $\varphi(x)$ is the field. It is that fact which affine quantization (AQ) overcomes by introducing a vast variety of parts of incomplete space, such as these retained space, $q > 0,\ |q| > 0,\ q^2 < b^2,\ q^2 > b^2,$ etc. For quantum field theory, the most important change is that $\varphi(x) \neq 0$ **and now that equation has been fully removed.**

Observe, that CQ requires $0 \leq |\varphi(x)| < \infty$, while AQ seeks to find missing equations which shows that **a specific field value, namely, $\varphi(x) = 0$ is removed [8], or explained differently, now $0 < |\varphi(x)| < \infty$.**

A. An Introduction to AQ

Only AQ can correctly solve all examples that have missing space regions, and it can do it correctly only with remaining space examples.

There is something else that CQ can surely fail on, namely the example of the "Particle in a Box", which is an example with missing space, and has been traditionally 'solved' using CQ. However, that very model can, and has, been correctly solved now by using AQ [7].

If you wish to read up on AQ, here are two examples where AQ has been well explained; see [7,9].

IV. Conclusions

There have been many models with the same 'disease' as that of φ^4_4 [1–7]. The secret to a valid canonical quantization is remarkably simple. All you need is the addition of a single, fixed potential, which is not seen in the texts, but it puts things in proper position elsewhere. This single, additional, potential is just $2\hbar^2/\varphi(x)^2$, alongside $\hat{\pi}(x)^2$. It is noteworthy that this potential forces $0 < |\varphi(x)| < \infty$ which leads to $0 < |\varphi(x)|^r < \infty$, and guarantees that almost all other potentials remain finite.

Give it a try on the model φ^4_4 and see for yourself! You can also give it a new try with φ^8_4 and be surprised! **That additional potential is all you will need.**

The secret to this magic has come from affine [10] quantization [9].

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