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Article

Asymptotic Locality: Rectifying Newton–Wigner and Foldy–Wouthuysen Localization

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Abstract

Relativistic quantum theory supports two competing notions of particle locality. Newton–Wigner localization provides a sharp position operator on the one-particle Hilbert space but is not Lorentz covariant and leads to instantaneous spreading for positive-energy states. Foldy–Wouthuysen localization, widely used in precision experiments, yields a robust quasi-classical particle description but is representation-dependent and does not coincide with strict field-theoretic locality. We show that this tension is not a paradox in UV-complete nonlocal quantum field theories whose ultraviolet physics is encoded by an entire-function regulator. In such theories the physically accessible observables form a quasi-local algebra generated by regulated smearings with an intrinsic nonlocality length. Sharp Newton–Wigner projectors are not operationally realistic, and Newton–Wigner and Foldy–Wouthuysen localization become equivalent non-singular notions of locality in the same measurement algebra.

Keywords: locality; microcausality

1. The Localization Problem in Relativistic Quantum Theory

In nonrelativistic quantum mechanics, locality is encoded by a self-adjoint position operator $\mathbf{x}$ whose spectral projectors P_R represent the proposition “the particle is in region $R \subset \mathbb{R}^3$.” In relativistic quantum theory, this particle notion of locality collides with two structural facts.

First, relativistic quantum field theory (QFT) enforces field locality through microcausality:

$$[\mathcal{O}(x), \mathcal{O}(y)] = 0 \quad \text{for} \quad (x - y)^2 < 0, \tag{1}$$

for local observables $\mathcal{O}$. Second, relativistic particles are defined as irreducible unitary representations of the Poincaré group Wigner classification, and there is no unique, fully covariant position observable compatible with positivity of energy and causal propagation. This tension manifests as a split in practice. Theoretical discussions of particle localization often adopt Newton–Wigner (NW) localization, which provides a sharp position operator on the one-particle Hilbert space [1]. Precision experiments and low-energy effective modeling typically adopt Foldy–Wouthuysen (FW) localization, which yields an operationally robust particle picture for the Dirac theory [2]. The question is then *which locality is fundamental, and why do these notions appear incompatible?*

The tension can be stated as a sharp paradox:

A relativistic theory with a stable particle sector admits a positive-energy one-particle Hilbert space in which sharp spatial localization projectors can be defined (Newton–Wigner). Yet the same theory, when formulated as a relativistic quantum field theory, constrains physical observables to arise from local or quasi-local densities obeying causal locality; within such an observable net, sharply bounded projectors are not admissible as fundamental measurements. This obstruction is already foreshadowed in strictly local QFT by the Reeh–Schlieder theorem, which implies that local algebras acting on the vacuum generate a dense set of states, ruling out an interpretation of bounded-region localization as a sharp physical projector [3]. If sharp projectors are

declared physical, causal locality is compromised; if causal locality is declared fundamental, sharp projectors are excluded.

This paper resolves the paradox by identifying the ultraviolet-complete notion of locality with the quasi-local observable net generated by regulated densities. In that framework, exact bounded-region projectors are not elements of the physical observable algebra, and localization is intrinsically finite-resolution, so that Newton–Wigner and Foldy–Wouthuysen localization coincide at accessible scales.

We show that, in UV-complete nonlocal QFTs regulated by entire functions, this apparent conflict dissolves. The theory enforces a stronger operational principle, that physically implementable observables are necessarily quasi-local, with an intrinsic resolution scale ℓ_* , and microcausality emerges asymptotically at distances $\rho \gg \ell_*$, in such a way that this paper shows how microcausality is recovered in the same limit [4].

2. Newton–Wigner Localization and Its Pathologies

Consider a single massive particle of mass $m > 0$ and spin s in the positive-energy Wigner representation. The one-particle Hilbert space can be represented in momentum space by wavefunctions $\psi(\mathbf{p})$ supported on the mass shell $p^0 = E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ with inner product:

$$\langle \psi, \phi \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}} \psi^\dagger(\mathbf{p}) \phi(\mathbf{p}). \quad (2)$$

Newton and Wigner constructed a position operator $\mathbf{X}_{\text{NW}}$ with commuting components and localized eigenstates $|\mathbf{x}\rangle$ at fixed time, uniquely characterized for massive particles by translational covariance, rotational covariance, and orthogonality of localized states. Operationally, NW localization aims to recover sharp projectors P_R analogous to the nonrelativistic case. Conceptually, the cost is that NW localization is tied to an equal-time hypersurface and is not Lorentz covariant in the sense required to define frame-independent “localized events.” A second obstruction is instantaneous spreading. Under mild assumptions, positivity of energy and existence of an initially compactly supported localization, time evolution produces immediate nonzero tails outside any bounded region at arbitrarily small $t > 0$. This is formalized in Hegerfeldt-type results, which show that sharp localization combined with positive energy forces immediate delocalization [5]. The physical lesson is not superluminal signaling by itself, but rather that sharp particle-localization is not compatible with relativistic spectral constraints in any representation that treats localization as a strict projector observable.

3. Foldy–Wouthuysen Localization as an Operational Particle Picture

Foldy and Wouthuysen introduced a unitary transformation that block-diagonalizes the free Dirac Hamiltonian and yields a clean separation between particle and antiparticle sectors. For a Dirac field in the single-particle limit, the Hamiltonian is:

$$H_D = \boldsymbol{\alpha} \cdot \mathbf{p} + \beta m, \quad (3)$$

where $\boldsymbol{\alpha}$ and β are Dirac matrices, $\mathbf{p} = -i\nabla$, and m is the mass.

The FW transformation U_{FW} produces

$$H_{\text{FW}} = U_{\text{FW}} H_D U_{\text{FW}}^{-1} = \beta \sqrt{\mathbf{p}^2 + m^2}, \quad (4)$$

so that positive and negative energy subspaces decouple for the free theory. In this representation the “mean position” operator behaves as the multiplication operator on wavepackets with suppressed zitterbewegung effects, and one obtains systematic $1/m$ expansions for coupling to external fields.

The FW picture is therefore well matched to experiment: real detectors reconstruct coarse-grained positions and spins through interactions that are effectively insensitive to sharply localized projectors.

However, FW locality is still not a fundamental locality principle in the QFT sense. It is a representation-dependent notion tied to a particle-antiparticle split and to low-energy regimes in which pair creation is negligible.

4. Entire-Function Regulated Observables and Quasi-Local Algebras

We now introduce the framework that resolves the NW–FW tension. Let $O(x)$ be a local field observable in a relativistic QFT. We define its regulated (nonlocal) version by an entire function F of the covariant d'Alembertian:

$$O^{(F)}(x) = F(\square/M_*^2) O(x), \quad (5)$$

where $\square = \partial_\mu \partial^\mu$ in flat spacetime (or $\square_g = g^{\mu\nu} \nabla_\mu \nabla_\nu$ in curved spacetime), M_* is the nonlocality scale, and $F(z)$ is entire with UV damping, for example $F(z) = e^{-z}$ or more general entire regulators of finite exponential type. Given a test function $f \in C_c^\infty(\mathbb{R}^{1,3})$, we define smeared regulated observables:

$$O^{(F)}(f) = \int d^4x f(x) O^{(F)}(x). \quad (6)$$

Because $F(\square/M_*^2)$ is nonlocal for nontrivial entire F , $O^{(F)}(f)$ can be expressed as a convolution with a kernel K_F :

$$O^{(F)}(f) = \int d^4x d^4y K_F(x-y) f(y) O(x), \quad (7)$$

where K_F is smooth and exhibits exponentially small spacelike tails set by the length:

$$\ell_* \equiv M_*^{-1}. \quad (8)$$

This motivates the fundamental operational definition of locality in the regulated theory. For any open region $O \subset \mathbb{R}^{1,3}$, define the quasi-local algebra $A_F(O)$ to be the von Neumann algebra generated by all operators of the form $O^{(F)}(f)$ with $\text{supp}(f) \subset O$.

A crucial structural point is that $A_F(O)$ cannot coincide with a strictly local algebra supported only in O . If F is a nontrivial entire UV suppressor, Paley–Wiener type arguments imply that the corresponding kernel K_F cannot have compact support [6]. Therefore a strictly bounded-region projector observable is not physically implementable in the regulated theory: all operationally accessible “local” measurements are quasi-local, with exponentially decaying leakage outside the target region.

5. Asymptotic Microcausality from Entire-Function UV Completion

The key dynamical statement is that, despite intrinsic nonlocality at scale ℓ_* , the theory becomes effectively causal at macroscopic spacelike separations. Let $f, g \in C_c^\infty(\mathbb{R}^{1,3})$ be test functions supported in spacelike separated regions, and define a spacelike distance parameter $\rho > 0$ that lower bounds the separation between $\text{supp}(f)$ and $\text{supp}(g)$.

The asymptotic microcausality theorem states that for broad classes of entire regulators F (including regulators of finite exponential type), the regulated commutator satisfies an estimate of the form:

$$\left\| [O_1^{(F)}(f), O_2^{(F)}(g)] \right\| \leq C_N (1 + M_* \rho)^{-N} e^{-\alpha M_* \rho}, \quad (9)$$

for any integer $N \geq 0$, with constants $C_N > 0$ and $\alpha > 0$ determined by the regulator class and the boundedness properties of the observables. The bound holds uniformly for spacelike separations $\rho \gg \ell_*$.

In momentum space, $F(\square/M_*^2)$ acts as multiplication by $F(-p^2/M_*^2)$. Entire analyticity implies that the regulated two-point functions and commutators extend to analytic functions of complexified momentum variables, with growth controlled by the exponential type of F .

Let K_F denote the position-space kernel associated to $F(\square/M_*^2)$. Entire UV suppression forces K_F to be smooth with rapid decay, but not compactly supported. For spacelike displacement x , one obtains bounds of the schematic form:

$$|K_F(x)| \leq A (1 + M_*|x|)^{-N} e^{-\alpha M_*|x|}, \quad (10)$$

for all N , consistent with the fact that strict localization is unavailable while leakage is exponentially small beyond ℓ_* .

Write the regulated commutator as a double smearing of an underlying distributional commutator kernel $\Delta(x - y)$:

$$[O_1^{(F)}(f), O_2^{(F)}(g)] = \int d^4x d^4y f(x) g(y) \Delta_F(x - y), \quad (11)$$

where Δ_F contains the regulator multipliers and inherits analyticity and decay from F . Spectral positivity prevents exponential growth in physical time directions and allows contour deformation or stationary-phase bounds that expose the exponential spacelike suppression. Combining these ingredients yields Eq. (9).

Taking $M_* \rightarrow \infty$ at fixed $\rho > 0$ yields:

$$\|[O_1^{(F)}(f), O_2^{(F)}(g)]\| \rightarrow 0, \quad (12)$$

so strict microcausality is recovered as an emergent property at macroscopic scales.

6. NW and FW Become One Operational Locality

Now I will make precise the sense in which Newton–Wigner and Foldy–Wouthuysen localization come together in an entire-function regulated, UV-complete QFT. The statement is not that the sharp NW projectors become physical, but rather that the only operationally definable notion of locality is quasi-local, and within that quasi-local measurement algebra any NW–FW differences are unresolvable beyond the intrinsic length $\ell_* \sim M_*^{-1}$.

Let $O \subset \mathbb{R}^{1,3}$ be an open spacetime region. The physically accessible observables in the regulated theory are generated by regulated smearings:

$$A_F(O) \equiv \text{vN}\left(\left\{O^{(F)}(f) : f \in C_c^\infty(\mathbb{R}^{1,3}), \text{supp}(f) \subset O\right\}\right), \quad (13)$$

where $O^{(F)}(f) = \int d^4x f(x) F(\square/M_*^2)O(x)$ and $\text{vN}(\cdot)$ denotes the von Neumann algebra generated by the indicated set.

A localized detector action supported in O is modeled by some bounded operator $D \in A_F(O)$, and the corresponding measurement effect, a POVM element is the positive operator:

$$E \equiv D^\dagger D \in A_F(O), \quad 0 \leq E \leq \mathbf{1}. \quad (14)$$

This captures the operational meaning of measuring in O , that any physically implementable measurement in the regulated theory is an element of $A_F(O)$.

Now we formalize the obstruction to sharp particle-localization. Assume F is a nontrivial entire UV suppressor, so that $F(\square/M_*^2)$ is a genuine nonlocal smoothing operator. Then there does not exist a nontrivial projection $P \neq 0, \mathbf{1}$ in $A_F(O)$ whose action is strictly confined to a bounded spatial region at a fixed time slice, such as a sharp particle-in- R projector.

Every generator $O^{(F)}(f)$ can be written as a convolution against a smooth kernel K_F :

$$O^{(F)}(f) = \int d^4x d^4y K_F(x - y) f(y) O(x), \quad (15)$$

with K_F determined by $F(-p^2/M_*^2)$ in momentum space. If F is a nontrivial entire UV suppressor, Paley–Wiener type bounds imply that K_F cannot have compact support. Consequently, even when $\text{supp}(f) \subset O$, the operator $O^{(F)}(f)$ necessarily contains exponentially small but nonzero tails outside O . Since $A_F(O)$ is generated by such quasi-local elements, it cannot contain a nontrivial projection that is strictly confined to a bounded region on an equal-time slice, because that would require exact compact support at the level of the underlying kernels.

This shows that sharp NW projectors P_R^{NW} are not fundamental observables in the UV-complete theory: they do not belong to $A_F(O)$ for bounded O .

Due to the fact that sharp projectors are operationally excluded, the meaningful localization observables are necessarily coarse-grained. Let $\eta \in C_c^\infty(\mathbb{R}^3)$ be a nonnegative mollifier with $\int d^3x \eta(x) = 1$, and define its ℓ_* -rescaling by:

$$\eta_{\ell_*}(\mathbf{x}) \equiv \ell_*^{-3} \eta(\mathbf{x}/\ell_*). \quad (16)$$

Given a bounded spatial region $R \subset \mathbb{R}^3$ with indicator χ_R , define the coarse-grained window:

$$\chi_{R,\ell_*} \equiv \chi_R * \eta_{\ell_*}. \quad (17)$$

Any operational localization measurement to the region R at time t must be expressed in terms of quasi-local densities such as currents, energy density, number density proxies smeared with χ_{R,ℓ_*} , producing an effect $E_{R,\ell_*} \in A_F(O_{R,t})$ where $O_{R,t}$ is a spacetime neighborhood of the measurement.

The crucial point is that χ_{R,ℓ_*} is fixed by the UV completion scale: it is not an arbitrary experimental imperfection but a structural nonlocality length.

Theorem 1 (Operational NW–FW Equivalence for $\rho \gg \ell_*$).

Let E_{R,ℓ_*}^{NW} and E_{R,ℓ_*}^{FW} be two localization effects intended to represent the particle is detected in R at resolution ℓ_* , constructed from the same regulated detector couplings in $A_F(O_{R,t})$, but expressed in NW or FW one-particle coordinates. Then:

$$\left| \text{Tr}(\rho E_{R,\ell_*}^{\text{NW}}) - \text{Tr}(\rho E_{R,\ell_*}^{\text{FW}}) \right| \leq \|\rho\|_1 \|E_{R,\ell_*}^{\text{NW}} - E_{R,\ell_*}^{\text{FW}}\|, \quad (18)$$

and the operator difference $E_{R,\ell_*}^{\text{NW}} - E_{R,\ell_*}^{\text{FW}}$ is quasi-local and supported only within an $\mathcal{O}(\ell_*)$ neighborhood of the boundary of R . In particular, for any other effect or observable $B \in A_F(O')$ with $\text{dist}(O_{R,t}, O') \geq \rho$, one has the asymptotic causality statement:

$$\| [E_{R,\ell_*}^{\text{NW}} - E_{R,\ell_*}^{\text{FW}}, B] \| \leq C_N (1 + M_* \rho)^{-N} e^{-\alpha M_* \rho}, \quad (19)$$

for $\rho \gg \ell_*$, with the same constants and exponent as in Eq. (9). Hence, for spacelike separated operations, NW and FW localization yield identical operational predictions up to exponentially small corrections.

The bound (18) is the standard trace inequality. It remains to show that the difference is quasi-local and that its operational consequences are exponentially suppressed at separation. First, both E_{R,ℓ_*}^{NW} and E_{R,ℓ_*}^{FW} are realized by detector couplings $D \in A_F(O_{R,t})$ via (14); thus each effect lies in $A_F(O_{R,t})$. Their difference is therefore also an element of $A_F(O_{R,t})$. Second, the distinction between NW and FW is a choice of one-particle coordinatization after projecting onto a stable particle sector. In the free massive case, this relationship is unitary on the positive-energy subspace, and the two position prescriptions agree after the FW diagonalization (equivalently, NW is the unique sharp localization compatible with the Wigner representation, while FW yields the corresponding mean-position in the block-diagonal representation). In the present UV-complete theory, sharp localization is unavailable; both prescriptions are replaced by coarse-grained windows χ_{R,ℓ_*} and regulated kernels K_F that enforce the same minimal resolution ℓ_* . Therefore E_{R,ℓ_*}^{NW} and E_{R,ℓ_*}^{FW} can differ only by terms sensitive to sub-

ℓ_* structure, i.e. by quasi-local contributions concentrated near the boundary of R where different coordinatizations modify the assignment of weight inside versus outside the region.

Finally, since $E_{R,\ell_*}^{\text{NW}} - E_{R,\ell_*}^{\text{FW}} \in A_F(O_{R,t})$, the asymptotic microcausality theorem applies to it as to any other quasi-local element of $A_F(O_{R,t})$. Taking the commutator with $B \in A_F(O')$ at spacelike separation ρ yields the exponential bound (19). This shows that any attempt to distinguish NW from FW by spacelike separated operations is exponentially suppressed for $\rho \gg \ell_*$.

7. Conclusion

Proposition 1 shows that the UV-complete regulated theory forbids the sharp localization projectors that NW seeks to elevate to fundamental observables. Theorem 1 shows that once localization is defined operationally through $A_F(O)$ and the intrinsic coarse-graining scale ℓ_* , the NW–FW distinction reduces to a sub- ℓ_* coordinatization ambiguity with no experimentally accessible content beyond exponentially small corrections governed by Eq. (9). Therefore, locality becomes a single quantitative statement that operational locality is compatible with exponentially suppressed spacelike commutators within $A_F(O)$.

The framework predicts a universal, model-independent structure for relativistic localization with a fundamental localization length $\ell_* = M_*^{-1}$ below which sharp locality is not operationally definable, asymptotic restoration of causality with suppression $\sim e^{-\alpha\rho/\ell_*}$ for $\rho \gg \ell_*$, the apparent NW–FW ambiguity is an artifact of demanding sharp projectors, and disappears once localization is formulated in the quasi-local measurement algebra $A_F(O)$.

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