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Can a Higher Dimensional Model of the Universe Solve the Problem of Dark Matter?

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Abstract: In this letter, we show that the effects usually attributed to the presence of dark matter in the Universe can equally be seen as the "external" effect on our Universe due to extra dimensions. For this, we consider a braneworld scenario where our universe, which is a 3-brane, is immersed in a 6D Minkowski bulk. We derive the $1/r$ force on the 3-brane at large distances as a response of cosmic fluid to external force due to extra dimensions. This scaling reproduces the effect of dark matter, such as the flat rotation curves of galaxies and a higher deflection angle around some gravitational lenses. It also explains the large-scale structure formation of the universe. This approach thus offers a novel perspective on dark matter, suggesting that higher-dimensional physics can account for these cosmic phenomena.

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1. Introduction

Dark matter/DM(see [1] for a detailed review) remains one of the greatest puzzles in our understanding of the cosmos, along with dark energy. The standard model of cosmology called ΛCDM, which assumes that General Relativity(GR) is correct, is based on some yet undetected cold dark matter particles(CDM) and a small but positive cosmological constant Λ. Dark matter is considered to be weakly interacting massive particles(WIMPs). A lower bound on the mass of dark matter in a model-independent way is provided in [2]. The standard model of cosmology is successful in explaining a wide variety of observations, such as flat rotation curves of galaxies, CMB, and large-scale structure formation [3]. However, as of now, there is no direct detection of DM particles. Even the Standard Model of particle physics does not seem to contain good candidates for DM particles. Particles such as axions(see [4] for a review) and branons [5,6], which are massive brane fluctuations, are also proposed to be candidates of dark matter but have no direct evidence as of now. All this has led to alternative theoretical explorations for dark matter. This is usually done by modifying gravity(see [7] for a nice discussion on modifying gravity and [8,9] for detailed reviews). A simple proposal in this regard was given by Milgrom called MOND [10,11]. It argues that Newton's law should be modified at large distances(or at small accelerations). Another theory was given by Moffat called Scalar-Tensor-Vector theory(STV) [12]. This also modifies gravitational dynamics at large distances. Dark matter and dark energy may also be explained using modified $f(R)$ gravity approaches [13]. A different approach in this direction was proposed by Boyle, Finn and Turok [14]. On the grounds of preserving CPT symmetry of the universe as a whole, the authors argued for the existence of an anti-universe which provides an intriguing argument for dark matter. The existence of an anti-universe was also shown to explain cosmic acceleration without dark energy [15], thus providing a unified approach towards explaining dark matter and dark energy. Yet another approach is the spherical reduction of GR at large distances based on dilaton gravity(see [16] for a review). This approach modifies gravity at large distances and solves the problem of flat rotation curves of galaxies [17,18]. In [19,20], it was argued that logarithmic potential can explain phenomena usually attributed to dark matter. The origin of such a term in gravitational systems was given in [21]. In this work, we provide an alternative approach towards explaining dark matter. This is based on two assumptions:

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- GR is the correct theory of gravity in 4D.
- There are no dark matter particles in the universe.

These assumptions rule out both modified gravity approaches and the ΛCDM model. The basic idea is that phenomena usually attributed to dark matter are actually an "external" effect on our universe due to higher dimensions. We, therefore, propose a braneworld model where our universe, which is a 3-brane, is immersed in 6D Minkowski bulk. We elaborate on this idea in the next section.

2. Setup and Emergence of Dark Matter

In what follows, our aim is to achieve a 6D behaviour on the brane at large distances. This would be crucial in explaining dark matter behaviour on the brane. A naive way would be to consider a 6D Minkowski bulk with the 3-brane immersed in it, similar to the DGP model [22]. The assumption we make is that GR is the correct theory on the brane(our universe), with the modification being an "external" effect coming from the higher dimensional bulk in which it is immersed. This leads to the action

$$S = M_6^4 \int d^6X\sqrt{G}R_{(6)} + M_4^2 \int d^4x\sqrt{|g|}R \tag{1}$$

where M_6 is the 6D Planck mass, M_4 is the usual 4D Planck mass. $G(X) = G(x,y,z)$ denotes a 6D metric with $R_{(6)}$ as the 6D Ricci scalar. For simplicity, we consider a 6D scalar field. Then, the action becomes

$$\begin{aligned} S = M^4 &\int d^4xdydz\partial_A\Phi(x,y,x)\partial^A\Phi(x,y,z) \\ + M_P^2 &\int d^4xdydz\delta(y)\delta(z)\partial_\mu\Phi(x,0,0)\partial^\mu\Phi(x,0,0) \end{aligned} \tag{2}$$

Here, A denotes 6D coordinates. We are interested in the effect of this 6D scalar field on the brane. For this, we consider the retarded classical Green's function as

$$\begin{aligned} &(M^4\partial_A\partial^A + M_P^2\delta(y)\delta(z)\partial_\mu\partial^\mu)G_R(x,y,z;0,0,0) \\ &= \delta^{(4)}(x)\delta(y)\delta(z) \end{aligned} \tag{3}$$

Then, the potential mediated by scalar Φ on the brane is given by

$$V(r) = \int G_R(t,\vec{x},y=z=0;0,0,0,0)dt \tag{4}$$

In momentum space, (3) can be written as

$$\begin{aligned} &(M_6^4(p^2 - \partial_y^2 - \partial_z^2) + M_4^2p^2\delta(y)\delta(z))\tilde{G}_R(p,y,z) \\ &= \delta(y)\delta(z) \end{aligned} \tag{5}$$

where p^2 is the square of Euclidean 4-momentum. The Green's function $\tilde{G}_R(p,y,z)$ under appropriate boundary conditions can be written as

$$\tilde{G}_R(p,y,z) = \frac{D(p,y,z)}{M_6^4 + M_4^2p^2D(p,0,0)} \tag{6}$$

where

$$\tilde{G}_R(p,y,z) = D(p,y,z)B(p) \tag{7}$$

$$(p^2 - \partial_y^2 - \partial_z^2)D(p,y,z) = \delta(y)\delta(z) \tag{8}$$

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Here, $B(p)$ is a function that we need to determine. However, for $D > 5$, $D(p, y, z)$ diverges on the brane [23]. This means $D(p, 0, 0)$ is divergent. Only in the special case of $D = 5$, $D(p, 0, 0)$ is well-defined. It might seem that we have no way ahead. However, the answer lies in the problems with the DGP model itself, namely ghost instabilities [24–27]. To make the theory ghost-free, one considers the following setup: A (3+1)-brane embedded in a (4+1)-brane. Both are embedded in 6D bulk. We then have a $\mathcal{Z}_2$ symmetry across both branes such that the coordinates of extra dimensions given by y and z along the 5th and 6th dimension, respectively, range from 0 to ∞. The codimension-1 brane is situated at $z = 0$ while the codimension-2 brane is at $y = z = 0$. This type of setup is called cascading gravity [28–30] and is ghost-free [31]. The corresponding action is

$$\begin{aligned} S = M_6^4 \int d^6x \sqrt{-g_6} R_6 + M_5^3 \int d^5x \sqrt{-g_5} R_5 \\ + M_4^2 \int d^4x \sqrt{-g_4} R_4 \end{aligned} \tag{9}$$

In this case, we have a transition from $4D \to 5D \to 6D$ gravity, and the potential varies as $1/r \to 1/r^2 \to 1/r^3$. The transitions depend on crossover scales. The 4D to 5D crossover scale is

$$m_5 = \frac{M_5^3}{M_4^2} \tag{10}$$

while 5D to 6D crossover scale is

$$m_6 = \frac{M_6^4}{M_5^3} \tag{11}$$

We can now see that we have a 6D behaviour at a large distance, as we required. Therefore, making the theory ghost-free requires we immerse both the brane in 6D bulk with the codimension-1 brane effectively acting as the regulator of the divergence, making the theory ghost-free. Quite remarkably, this mathematical consistency of the theory naturally leads to a 6D behaviour at large distances needed to explain dark matter. Since we are interested in the direct transition from 4D to 6D, we need to ensure this. This can be ensured if $m_5 > m_6$ at the scale $p \sim \sqrt{m_5 m_6}$. Next, we consider the brane plus cosmic fluid to be our system. Then, the force on the fluid due to extra dimensions is external to the system(brane+cosmic fluid). We also take the cosmic fluid to obey the Navier-Stokes equation, which reads

$$\rho \frac{Du}{Dt} = -\nabla p + \nabla \cdot \tau + \rho f_{ext} \tag{12}$$

where the symbols have the usual meaning. Now, with respect to the fundamental observer, the fluid is at rest and assuming no viscosity, we obtain the simple equation

$$\rho f_{ext} = \nabla p \tag{13}$$

Since $f_{ext} = -\nabla V(r)$, we obtain the fluid pressure as[1]

$$p = \frac{kM}{r^3} \tag{14}$$

where k is a constant. This pressure leads to an effective fluid force in a spherical region of radius r as

$$F = p \times 4\pi r^2 = \frac{4\pi Mk}{r} \tag{15}$$

[1] Since f_{ext} is force per unit fluid volume, we have converted this to total force and, therefore, mass M appears instead of mass density ρ.

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Therefore, for large r, we have two scalings: $1/r^4$ directly due to extra dimensions and $1/r$, which is the response of cosmic fluid to external force(due to extra dimensions). To study the dynamics of a galaxy, we should consider how galactic fluid responds to the external force and, therefore, the $1/r$ scaling. This scaling solves the problem of a flat galactic rotation curve and higher deflection angle around some gravitational lenses generally attributed to the presence of dark matter [19,20]. This force profile actually leads to a flat curve for any r greater than r_0, which is the 4D to 6D crossover scale, thereby supporting the recent finding that the rotation curve of galaxies remains flat infinitely [32] since for a spherical galaxy of radius r, the velocity v is given by $F = v^2/r$ which leads to

$$v = \sqrt{4\pi Mk} \tag{16}$$

This is clearly a constant and holds for all $r >> r_0$. Along with galactic morphology and rotation, $1/r$ force also explains large-scale structure formation of the universe [33] generally attributed to dark matter. The origin of such scaling of F at large distances is discussed in [21].

3. Discussion: Dark Matter or Higher Dimension?

Recently, we showed that if the universe is quantum mechanically entangled with an anti-universe with time flow oppositely related, then this explains cosmic acceleration without dark energy [15]. The paper showed that late-time cosmic acceleration is inevitable if a universe-antiuniverse pair exists. The existence of an anti-universe also solves the problem of dark matter [14]. So, this is a unified approach to explain both dark matter and dark energy. Also, in a braneworld scenario, we have shown how variable brane tension with 4D Newton's constant G on brane promoted to a scalar field explains cosmic acceleration without dark energy [34]. And now, in this work, in a cascaded braneworld setup, we have shown that dark matter is simply an external effect of the fact that the universe is immersed in a higher dimensional space. The cascaded gravity also has a consistent self-accelerating solution [35]. Therefore, this, too, is a unified approach similar to the anti-universe theory. All this leads to a particularly bold claim: one either accepts that we live in a 4D universe. GR is a correct theory, leading to the ΛCDM model of cosmology. Then we also have to accept that we have elusive dark matter particles and peculiar dark energy making up 95% of the universe, or we live in a higher dimensional universe, or a universe-antiuniverse pair with no dark matter and dark energy. The reader should remember that this study is based on the assumption that GR(or Newton's law of gravity in the non-relativistic case) is a correct theory of gravity on the brane, which is our universe. The reason for not considering modified gravity theories is both technical and philosophical: Nature seems to prefer beauty and simplicity in selecting a law. The law must be the same everywhere. It should not depend on distances we are probing as is usually done in modified gravity theories such as MOND. Moreover, they introduce extra fitting parameters and fine-tune them to match observations. Therefore, modified gravity theories do not match Nature's criteria for the selection of law, i.e., simplicity and beauty. Technically, modified gravity theories are not always free of pathology. To qualify as a viable theory, any modified theory of gravity must satisfy all of these:

- should not have instabilities and ghosts
- should have a correct Newtonian and post-Newtonian limit
- should be compatible with the data
- should have a well-defined Cauchy problem

For example, a general way of modifying GR is replacing Ricci scalar R with a more general function of R called $f(R)$-gravity. However, this theory faces some serious technical challenges(see [36]). A particular class of such theories is $1/R$ gravity, which proposes to solve the cosmic acceleration problem without dark energy [37]. However, it was shown that it is not compatible with solar system tests [38]. This was generalized to any $f(R)$ theory of gravity in [39]. The reason is that any modified theory of gravity with a function of scalar curvature R is equivalent to a class of scalar-tensor theories of gravity [40] and is therefore inconsistent with solar system tests. Even more general theories of gravity with higher curvature terms $f(R, R_{ab}R^{ab}, R_{abcd}R^{abcd})$ are more problematic since they contain ghost fields. This re-iterates our assumption that GR is the correct theory of gravity on the brane and that dark matter is to be seen as an artefact of higher dimensions on the brane: an "external" effect. Moreover, to our knowledge, there is no relativistic theory of gravity that can exactly reproduce the simple $1/r$ force or, equivalently, a logarithmic potential at large distances necessary to reproduce a flat velocity profile of galaxies. The idea discussed in the paper is also compatible with a recent finding that galactic curves remain flat infinitely [32] since, in this case, after a certain cut-off scale r_0, the force due to fluid is given by $1/r$ and therefore, velocity profile remains constant throughout after this length scale. Obviously, experimental verification remains a potential limitation of this idea.

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