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Article

Phase Transitions in Disordered LC Systems: A Classical Analog of Quantum Transitions

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Abstract

This paper investigates percolation transitions in a disordered L-C system composed of inductors and capacitors (non-dissipative reactive elements). These transitions occur between different percolating states, resulting in distinct, constant values of the effective conductivity. We employ an exact approach based on the rotational symmetry of two-dimensional DC equations. A new type of phase transition is identified for these non-dissipative systems by analogy to a topological transition. The characteristics of these transitions, which are analogs of topological invariants, are calculated. We propose that these transitions may be considered a classical analog to quantum transitions, such as the quantum Hall effect.

Keywords: phase transition; L-C system; conductivity; inductors; capacitors

1. Introduction. Topological Transitions and Dual Symmetry of Two-Dimensional Systems

Topological phase transitions have been discovered in various materials and have been applied to explain diverse physical phenomena [1–3]. Recently, the theory of topological phase transitions has been used to explain the integer quantum Hall effect [4,5].

The aim of this paper is to study phase transitions in a new class of materials—non-dissipative systems such as disordered LC networks—with potential applications to metamaterials [6,7]. For this purpose, we study a two-dimensional two-phase system, consisting of randomly placed inductors (L) and capacitors (C). At first glance, percolation in such a disordered system might seem straightforward; one would expect a cluster to form from all non-dissipative phases because the principle of minimal Joule heat does not apply here:

$$\min \{Q\} = \min \left\{ \int (\vec{j}, \vec{e}) dx dy dz \right\} \tag{1}$$

where Q is the dissipation energy, and j and E are the local electric current and electric field, respectively. Thus, in the studied non-dissipative case, $Q=0$. However, the actual situation is more complex and interesting: the percolation cluster is formed from phases of only one type due to the shielding of the electric field. The transition from one percolation cluster to another manifests as a transition analogous to a topological phase transition.

The paper is structured as follows. In the second part, the model of a disordered LC system is introduced. The third part presents the results of our investigation, including new characteristics of the phase transition that serve as analogs of topological invariants for this problem. The fourth part provides concluding remarks and a discussion of the obtained results.

2. Model of a Disordered System Consisting of Randomly Connected Inductors and Capacitors

The key feature of the two-dimensional DC equations and Ohm's law, which describe a two-dimensional conducting medium, is their internal symmetry:

$$\operatorname{div} \vec{j} = 0, \quad \operatorname{curl} \vec{e} = 0, \quad \vec{j} = \sigma \vec{e} \quad (2)$$

Here, σ is the conductivity of the medium. These equations exhibit an internal symmetry—invariance under linear rotational transformations:

$$\vec{j} = b[\vec{n}, \vec{e}'], \quad \vec{e} = d[\vec{n}, \vec{j}'] \quad (3)$$

where $\vec{n}$ is a unit vector normal to the plane, b and d are constant coefficients. This symmetry was first described in [8,9] and [10].

To understand the nature of this rotational symmetry in a two-dimensional system, let us consider a chessboard system. This system possesses three permutation symmetries:

Interchange of white and black cells ($1 \leftrightarrow 2$)—see Figure 1.

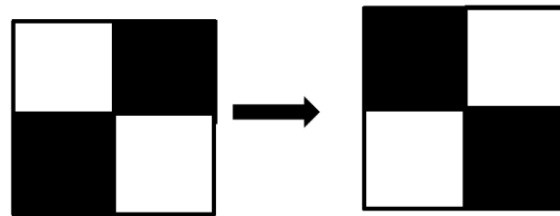


Figure 1. Chessboard system illustrating the interchange of white and black cells.

2) the change of the direction of normal vector $\vec{n}$ only as $\vec{n} \rightarrow -\vec{n}$



Figure 2. a. top view of chessboard. Figure 2b. bottom view of chessboard.

3) an interchange of white and black color cells of chessboard ($1 \leftrightarrow 2$) in its positions and change of the direction of normal vector $\vec{n}$ as ($1 \leftrightarrow 2$) and $\vec{n} \rightarrow -\vec{n}$



Figure 3. a. top view of chessboard. Figure 3b. bottom view of chessboard + interchange of colour cells.

A disordered system with randomly placed cells possesses the same permutation symmetries (see Appendix A). According to Dykhne's method (described in detail in Appendix A), the effective conductivity of a two-phase random medium consisting of non-dissipative capacitive and inductive components is described by the equation:

$$\left(Y^e(\pm\delta) - \frac{(Y_1 + Y_2)}{2} \right)^2 = \left(\frac{(Y_1 - Y_2)}{2} \right)^2 \quad (4)$$

where Y_i is the conductivity of the i -th component; $Y^e(1)$ is the effective conductivity of the percolation cluster formed mainly by resistors of type 1 (Y_1); and $Y^e(2)$ is the effective conductivity of the percolation cluster formed mainly by resistors of type 2 (Y_2). Further details can be found in Appendix A.

However, this solution (4) is not valid at the percolation threshold $p=p_c$, because in this case one must use the rotational transformations (A2) with other coefficients b and d . Consequently, the following expression for the effective conductivity is obtained from equation (A6):

$$Y^e(0) = \sqrt{Y_1 Y_2} = \sqrt{L/C} \quad (5)$$

The key feature of this solution is that the effective conductivity has a real value. This result is unexpected and interesting: although the medium consists of non-dissipative elements (inductors and capacitors), dissipation nevertheless appears. An analogous result was first obtained in [8] (see also [10]). The physical explanation is that the energy dissipation is connected with the excitation of a set of LC circuits. The results (4) and (5) are analogous to those for the quantum Hall effect, where the off-diagonal component σ_{xy} exhibits a plateau and the diagonal component has a non-zero value σ_{xx} at the point of the phase transition. The approach of topological phase transitions was applied to explain these results [11–13].

Below, we introduce a model of a disordered system consisting of randomly connected inductors (L) and capacitors (C) and study its effective conductivity. To analyze this, we apply the exact Dykhne's approach, which is based on the rotational symmetry of a two-dimensional medium (this method is described in detail in Appendix A). According to this approach, calculating the effective conductivity of such a disordered system requires finding the symmetric transformations that map the system to itself, or to its dual system.

3. The Topological Phase Transition

To examine this transition in more detail, let us consider formulas (A5) and (A6) for the case of the third permutation symmetry. In the general case, we obtain the semicircle relation:

$$\left(X^e \right)^2 + \left(Y^e(\pm\delta) + \frac{a}{d} \right)^2 = \frac{b}{d} - \left(\frac{a}{d} \right)^2 \quad (6)$$

This is known as a semicircle relation because we consider only the case with a positive real part of the conductivity, $\text{Re}(Y^e) > 0$ —see Figure 4.

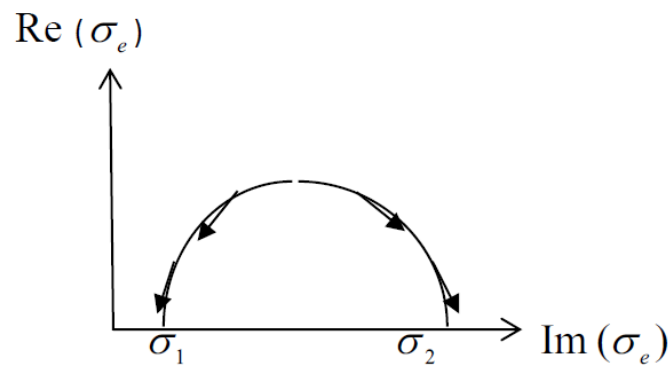


Figure 4. Semicircle relation for conductivity and the flow of solutions toward the stable points Y_1 and Y_2 .

3.1. The Construction of the Percolation Cluster in LC Systems with Non-Dissipative Components

According to formula (6), on the line $(Y_e)=0$, there exist two stable points, Y_1 and Y_2 , corresponding to the solutions in (4). As the value of (Y) decreases, the trajectory toward these solutions follows the semicircle. From the left part of this line, the solution of the equation will flow to the stable point Y_1 , while from the right part, it will flow to Y_2 , as shown in Figure 4.

One solution, $Y_e = Y_1$, corresponds to the creation of a percolation cluster consisting solely of capacitors, while the other solution, $Y_e = Y_2$, corresponds to a percolation cluster formed only of inductors. It is unexpected that the percolation cluster is formed from only one type of phase. As described above, all phases are non-dissipative, and it might seem possible to form a cluster from both phases since there is no restriction from physical laws such as the minimization of Joule heat dissipation (1). To analyze equation (4) and understand why only one of the two solutions is realized, let us consider the distribution of electric fields in the studied disordered two-phase LC system. For this purpose, we calculate the following quantity:

$$A = j - Y_1 e$$

After averaging over the two phases, we obtain:

$$\langle A \rangle = \langle j \rangle - Y_1 \langle e \rangle = Y_e \langle e \rangle - Y_1 \langle e \rangle = (Y_e - Y_1) \langle e \rangle \quad (8)$$

Averaging over the second phase then yields:

$$\langle A \rangle = \langle j \rangle_2 - Y_1 \langle e \rangle_2 = (Y_2 - Y_1) \langle e \rangle_2 \quad (9)$$

In the case where the percolation cluster is formed from phase 1, the effective conductivity is $Y_e = Y_1$ and $\langle A \rangle = 0$. Consequently, from (9) it follows that:

$$\langle e \rangle_2 = 0 \quad (10)$$

This result means that the percolation cluster forms exclusively from the first non-dissipative phase, and no electric current flows through the second phase. In the other case, when the percolation cluster forms solely from phase 2, we calculate a different quantity:

$$B = j - Y_2 e \quad (11)$$

After averaging over the two phases, we obtain:

$$\langle B \rangle = \langle j \rangle - Y_2 \langle e \rangle = Y_e \langle e \rangle - Y_2 \langle e \rangle = (Y_e - Y_2) \langle e \rangle \quad (12)$$

When the percolation cluster is formed from phase 2, the effective conductivity is $Y_e = Y_2$ and $\langle B \rangle = 0$. Subsequent averaging over the first phase gives:

$$\langle B \rangle = \langle j \rangle_1 - Y_2 \langle e \rangle_1 = (Y_1 - Y_2) \langle e \rangle_1 = 0 \quad (13)$$

Thus, the electric field in the first phase is equal to zero:

$$\langle e \rangle_1 = 0 \quad (14)$$

Equations (10) and (14) explain the percolation of electric current in disordered LC systems and the existence of two distinct values of effective conductivity, each determined by the conductivity of only one phase. In other words, although both phases are non-dissipative, the percolation cluster is formed exclusively by one phase, while the other phase does not participate. This non-trivial behavior has a physical reason: the current does not flow through the second phase due to the shielding of the electric field within it. Consequently, one can speak of the existence of two distinct percolation clusters composed of different phases. The existence of these two distinct and non-mixing phases, such as inductors (L) and capacitors (C), is connected to their different physical properties—namely, their different phase shifts relative to the applied electric field $E(t)$. Typically, in classical transport phenomena, phase shift does not influence the conductivity, but in the case studied here, this is not true. The reason for this unusual behavior is the transition between solutions, which takes the form of analog of the topological phase transition.

3.2. The Phase Transition and its Characteristics as Analogs of Topological Invariants

According to formula (4), the effective conductivity has a step-like dependence on the projection onto the plane at a fixed frequency—see Figure 5. Let us consider the transition from one solution to another in (4), which occurs at the percolation threshold $p=p_c$. A graphical representation of these transitions is presented in Figure 5.

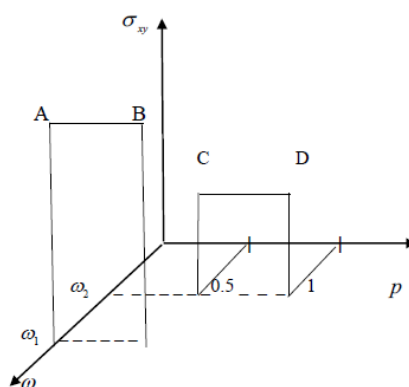


Figure 5. Conductivity as a function of frequency and phase concentration. At a fixed frequency it exhibits a step-like form according to formula (4).

Consider the transition from the plane of inductances L (ABCO) with impedance $Z_L = i\omega L$ to the plane of capacitances C (OEDF) with impedance $Z_C = 1/(i\omega C)$ through the point O, which has a real value. This transition is only possible along the line COE at a fixed frequency $\omega = \omega_0$ from one phase to the other—see Figure 6. This represents a transition between phases analogous to a quantum transition, such as the quantum Hall effect [11–14]. To calculate the topological invariant characterizing this transition, the integration must be performed along a path of constant amplitude where only the phase changes. Thus, a new characteristic of this transition is the phase change, measured in radians, when passing through the point of real conductivity. This is described by the following formula [15–17]:

$$\Delta\varphi = \varphi(Y_2) - \varphi(Y_1) = \frac{\int \ln(f(z)) dz}{2\pi i} \quad (15)$$

where the function $f(z)$ is the effective conductivity. Consequently, the characteristic of the transition, which serves as an analog of a topological invariant in this topological transition between non-dissipative phases, is:

$$n = \varphi(Y_2) - \varphi(Y_1) = \frac{\ln(\exp(i\pi/2)) - \ln(\exp(-i\pi/2))}{2\pi i} = \frac{1}{2} \quad (16)$$

The obtained result can also be visualized by considering a graphical representation of non-dissipative current flow on a cylindrical surface, similar to the current flow diagram in the quantum Hall effect regime [11–14].

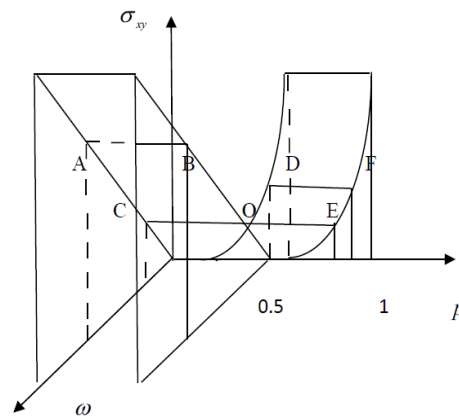


Figure 6. Transition from the plane ABCO, corresponding to one solution, to the other solution in the plane OEFD is only possible along the line COE, which corresponds to the resonant frequency.

Suppose that initially, currents flow in the inductive phase I_L , which leads the applied voltage by $\pi/2$. Then, at the phase transition point, a transition occurs through a dissipative state into a new non-dissipative capacitive phase I_C , where the current lags in phase by $\pi/2$. The current through the dissipative state is in phase with the applied voltage—see Figure 7. However, changing the current direction to the opposite—from inductive alternating current to capacitive alternating current—is not direct. To achieve this, it is necessary to create a local oscillating circuit in which energy is transferred from the inductances to the capacitances. The excitation of these local circuits results in the dissipation.

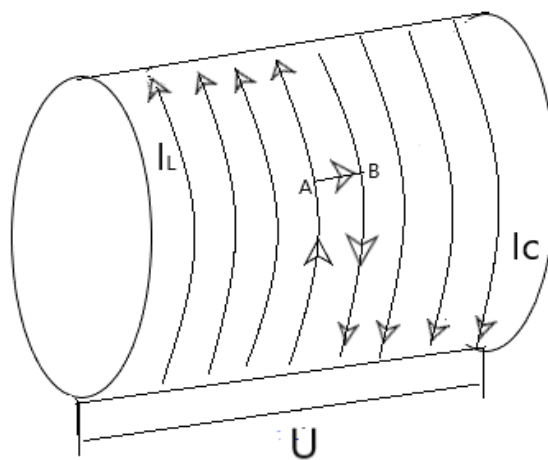


Figure 7. Topological transition from the inductive phase with current I_L , which leads the applied voltage in phase by $\pi/2$, to the capacitive phase with current I_C , which lags the applied voltage in phase by $\pi/2$. This occurs through a dissipative state (segment AB), where the current is in phase with the applied voltage U . This representation was first introduced by Thouless [4].

4. Discussion

In this work, we have studied current flow in a two-dimensional system of randomly connected circuit elements. It has been shown that the effective conductivity is constant (independent of phase concentration) and is equal to either the capacitive reactance or the inductive reactance, as shown by the step-like dependence in Figure 5. Figure 6 illustrates the corresponding transition path between these states. The step-like dependency of the effective conductivity at a fixed frequency and arbitrary phase concentrations appears strange and unexpected. However, as we have shown, this classical quantization in disordered LC systems is connected to the shielding of the electric field. Although all phases are non-dissipative and there is no governing principle like the minimization of Joule heat to form a percolation cluster, the capacitive and inductive phases have different phase shifts relative to the electric field. This difference prevents the formation of a percolation cluster from different non-dissipative components. In other words, our results demonstrate that a different principle governs percolation in non-dissipative systems.

An important feature of the studied phase transition is the appearance of a finite effective resistance R at the percolation threshold $p=p_c$ in a system consisting of randomly placed and connected non-dissipative elements (inductors and capacitors). The emergence of this effective dissipative state can be understood as follows. Let the initial phase through which the current flows consist of inductors; in this case, all the energy is concentrated in the magnetic field of the inductors, $WL=LI^2/2$. For the current to flow through the other phase—in this case, the capacitive phase—this energy must be transferred to the capacitances, $WC=CU^2/2$. This transfer requires the excitation of local oscillations in circuits created by the random connection of inductances and capacitances into local LC resonators. However, exciting these local oscillating circuits necessitates the finite amount of energy. Once excited, the energy transfers from the inductors to the capacitors, and a transition occurs from a percolation cluster consisting of inductances to a new percolation cluster consisting of capacitances. Thus, the mechanism for the transition between percolation clusters, and the associated effective finite Joule heat dissipation, becomes clear.

It would be interesting to generalize these results to multi-phase systems and study topological transitions in such cases.

In our opinion, the obtained results can be applied to the study of metamaterials (materials with a negative refractive index) and effects in nonlinear metamaterials, such as superconducting Josephson-based transmission lines and JTWP arrays [18–21]. As metamaterials are often constructed from inductances and capacitances, our results may help explain some of their properties and could indicate directions for further investigation. For instance, it would be valuable to search for unique features in the properties of metamaterials near the percolation threshold, including nonlinear behavior. Our results suggest step-like behavior and a stable response to alternating electromagnetic fields near this threshold. These results do not apply directly to metamaterials, as they are typically three-dimensional. Nevertheless, physical mechanisms such as the creation and excitation of local oscillatory circuits may be conserved. Consequently, integrated Joule heat dissipation is expected near the percolation threshold. Far from this transition region, local oscillatory circuits may be excited but are not connected into a global percolation cluster; after volume averaging, their contribution to dissipation should be negligible. Therefore, it is interesting to study the properties of metamaterials when their components are near the percolation threshold in concentration and to search for classical analogs of quantum transitions in this class of materials.

A further feature of global connectivity in three dimensions is the potential existence of two independent percolation clusters simultaneously. This circumstance could lead to a new type of transition between these independent clusters. This is not obvious and warrants further research. In other words, the transition from one non-dissipative state of the system, described by the imaginary part of the conductivity, to another non-dissipative state, described by a different value, is possible only through the excitation of the circuit—i.e., through a dissipative state at the transition point. We believe the results obtained here can be used in studying various properties of metamaterials, including their nonlinear properties.

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Conflicts of Interest: The author declares no conflicts of interest.

Appendix A. The Exact Method for Studying Disordered Conducting Media: The Dykhne Approach

In the general case of an anisotropic medium, for example, in a magnetic field, Ohm's law has a tensor form:

$$\vec{j} = \hat{\sigma} \vec{e} \quad (\text{A1})$$

Here, $\hat{\sigma}$ is a two-dimensional conductivity tensor with components $\sigma_{xx}, \sigma_{xy}, \sigma_{yx}, \sigma_{yy}$. To describe the dual symmetry of two-dimensional anisotropic media, we use generalized linear rotational transformations:

$$\vec{j} = a\vec{j}' + b[\vec{n}, \vec{e}'], \quad \vec{e} = c\vec{e}' + d[\vec{n}, \vec{j}'] \quad (\text{A2})$$

In the transformed system, the primed Ohm's law also has a tensor form. It is more convenient to reformulate the problem using complex variables, where the imaginary unit i corresponds to a local rotation by an angle $\pi/2$. This means we use the following notations:

$$j = j_x + ij_y, \quad e = e_x + ie_y \quad (\text{A3})$$

and the complex conductivity [9,10]:

$$Z = X + iY \quad (\text{A4})$$

After applying transformations (A2), the relations between the components of the initial and primed systems in the general case are:

$$X' = \frac{X(ac + bd)}{(Xd)^2 + (Yd + a)^2} \quad (\text{A5})$$

$$Y' = \frac{X^2cd + (cY - b)(dY + a)}{(Xd)^2 + (Y^2d + a)^2} \quad (\text{A6})$$

In the case of capacitors and inductors:

$$X = 0, \quad Y_C = -1/\omega C \quad \text{or} \quad Y_L = \omega L$$

Let us apply this method to study a many-component disordered system consisting of randomly connected inductors (L) and capacitors (C).

There are three possible symmetry transformations, corresponding to permutations of the different component types and the corresponding duality relations for the effective conductivity [17].

1. First permutation symmetry: An interchange of reactance components with odd and even numeration by positions:

$$Y_1'' = Y_2, \quad Y_2'' = Y_1 \quad (1 \leftrightarrow 2) \quad (\text{A7})$$

2. Second symmetry: An interchange of reactances with odd and even numeration in their positions and a change of the direction of the normal vector $\vec{n}$:

$$Y_1'' = Y_2, \quad Y_2'' = Y_1, \quad (1 \leftrightarrow 2) \quad \text{and} \quad \vec{n} \rightarrow -\vec{n} \quad (\text{A8})$$

3. Third symmetry: Only a change of the direction of the normal vector $\vec{n}$:

$$Y_1'' = Y_1, \quad Y_2'' = Y_2, \quad (1 \leftrightarrow 1, 2 \leftrightarrow 2) \quad \text{and} \quad \vec{n} \rightarrow -\vec{n} \quad (\text{A9})$$

Here, the coefficients a_3, b_3, d_3 are determined by the conditions in (A2).

$$a_3 = -c_3 = 1, \quad b_3 = \frac{2Y_1Y_2}{Y_1 + Y_2}, \quad d_3 = \frac{2}{Y_1 + Y_2} \quad (\text{A10})$$

In this case, the primed system is macroscopically equivalent to the original. As a result, the effective conductivity is determined by the equation:

$$d_3 Y_e(\delta)^2 + b_3 - 2a_3 Y_e(\delta) = 0 \quad (\text{A11})$$

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