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# The Dark Energy Sector of Generalised Proper Time

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**Abstract**

Generalisation from the local 4-dimensional spacetime form for a proper time interval provides an alternative basis for a unified theory contrasting with the more familiar approach of appending extra dimensions of space. In both cases the main goal is to derive a structure of matter that accounts for empirical observations from the properties of the additional components. Previous work has focussed upon the direct and significant connections that this new approach makes with the Standard Model of particle physics while also providing a link with dark matter models. In this paper we elaborate on how a further sector of generalised proper time, incorporating the key component of negative pressure, can account for the apparent dark energy fuelling the accelerating large-scale expansion of the universe. Through comparison with existing dark energy models we elucidate the elementary composition of this sector, its potential relation to an effective cosmological constant term, and the nature of possible interaction with the visible and dark matter sectors.

**Keywords:** Dark energy; dark matter; Standard Model; unification; proper time; extra dimensions

## Contents

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## 1 Introduction

That the rate of expansion of the universe is speeding up was first observed in the late 1990s [1, 2], as subsequently confirmed by further cosmological evidence and analysis (see for example [3], [4] section 28 and references therein). While this phenomenon may seem counter-intuitive, within the general relativistic framework for cosmological models the observations of the large-scale structure of the universe can be collectively accommodated on postulating a significant source of uniform *negative* pressure together with an associated *positive* energy density, generically termed 'dark energy'. The balance of these required pressure and energy density contributions is essentially consistent with the role of a cosmological constant $\Lambda$, that can be introduced as a free parameter in the Einstein field equation. Whether interpreted as a cosmological constant, or a more general source of dark energy, the nature of a microscopic physical origin for this apparently all-pervading invisible entity, that has such a dominant gravitational impact on large-scale cosmology, is one of the major outstanding questions for 21$^{\text{st}}$ century fundamental physics.

The approach to unification based upon generalised proper time can be motivated through comparison with the many models based on extra dimensions of space [5]. Rather than constructing an extended higher-dimensional *spacetime* manifold, the focus is shifted to the local form for proper *time* and its generalised expression. A basis for matter is identified through the additional components and symmetry breaking patterns resulting from the extraction of the local basis for 4-dimensional spacetime from the general form for proper time. This direct and simple approach has been shown to work far better than schemes with extra spatial dimensions in establishing direct connections with the Standard Model of particle physics [6] while also leading to new physics beyond that might be accessible in the laboratory [7].

While based uniquely and simply on local proper time intervals there are several mathematically permitted 'branches' of generalised proper time that augment the 4-dimensional spacetime form. In addition to the branch that leads to Standard Model structures another has been identified that underlies a 'hidden QCD' sector, in parallel with the standard quantum chromodynamics (QCD) of visible particle physics, as the basis for a dark matter candidate [8]. A yet further branch of generalised proper time leads to a potential source for dark energy, and connections with existing dark energy models, as we explore in detail in this paper (building upon the suggestion in [8] section 7), as outlined below.

In section 2 we review the simplifying assumptions typically employed for cosmological models in the framework of general relativity, and in particular summarise several pertinent dark energy models in this setting. The motivation for generalised proper time, as an alternative to extra spatial dimensions, will be reviewed in section 3, where the means of identifying a basis for the elementary structures of matter will also be described. The successes in accounting for structures of the Standard Model will be briefly summarised in section 4, as will the direct connection with dark matter models identified through a parallel branch of generalised proper time.

A further branch of generalised proper time that leads directly to a source of negative pressure, associated with a negative kinetic energy component, and its corresponding capacity to account for the cosmological dark energy sector will then be introduced and considered in section 5. In section 6, guided by comparison with existing dark energy models that incorporate a negative kinetic energy component, we explore the microphysical structure and stability of this sector and associated large-scale phenomenology. There we also assess the possibility of addressing the 'cosmological constant problem' concerning the extremely low value of an effective Λ or dark energy density as empirically required.

A direct mechanism for a close relationship and interaction between the dark energy and dark matter sectors automatically arises in this theory, as may be relevant to the 'cosmic coincidence problem' concerning the similarity in the overall density of these two sectors at the present epoch, as we elaborate in section 7. There we also briefly assess the nature of possible interaction of this unified dark sector with visible matter and any potential relation between this theory and inflationary cosmology in the very early universe. We then summarise the progress made in the final section.

## 2 Cosmological Dark Energy Models

The metric structure of 4-dimensional spacetime can be described by a symmetric metric tensor $g_{\mu\nu}(x)$, on employing a general coordinate system $\{x^\mu\}$ with spacetime indices $\mu, \nu = 0, 1, 2, 3$, with respect to which a local proper time interval $ds$ can be expressed in the quadratic form (in this paper we adopt the convention of summing over repeated upper and lower indices, unless stated otherwise):

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1}$$

Two classes of assumptions are utilised in constructing large-scale Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological models ([9] section 12.2 and references therein). One assumption concerns the symmetries of 4-dimensional spacetime on large scales that allow the introduction of a preferred set of coordinates with a global cosmic time parameter $t$. This 'cosmological principle' asserts that the universe at any particular cosmic time can be taken to a good approximation to be spatially homogeneous and isotropic about any given location. The most general metric compatible with this assumption takes the form with proper time element:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right] \tag{2}$$

with a set of 3-dimensional spatial spherical coordinates $\{r, \theta, \phi\}$. This metric structure is a very special case of equation 1 with metric components $g_{00} = 1$, $g_{11} = \frac{-a^2(t)}{(1-kr^2)}$ and so on. The cosmological principle has then reduced the possible set of ten parameters of $g_{\mu\nu}(x)$ in equation 1 down to just two, namely $a(t)$ and $k$.

The parameter $k$ describes the geometric curvature of the spatial hypersurfaces, which is positive for $k > 0$, negative for $k < 0$ and flat for $k = 0$; corresponding to a three-way classification that is independent of cosmic time. The scale factor $a(t)$ parametrises the metric size of the spatial dimensions as a function of cosmic time $t$. For 'Big Bang' cosmology it is generally taken that $a(t) \to 0$ for $t \to 0$ at the apparent temporal origin of the universe.

The second major assumption of FLRW models is that the gravitational field, described by the metric $g_{\mu\nu}(x)$ of equation 1 as expressed in the form of equation 2, should satisfy the Einstein field equation of general relativity. This equation relates the Einstein tensor $G^{\mu\nu}(x)$, a non-linear function in the spacetime derivatives of $g_{\mu\nu}(x)$, to the energy-momentum tensor $T^{\mu\nu}(x)$, describing the matter content of the universe, directly as:

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = -\kappa T^{\mu\nu} \tag{3}$$

with $\kappa$ a positive normalisation constant (see [9] sections 3.3 and 3.4 for further sign conventions adopted relating to Riemannian geometry, with the term 'Riemannian' here understood as covering also 'pseudo-Riemannian' manifolds and the special case of 'Lorentzian' geometry as employed in general relativity).

On taking the covariant derivative $\nabla_\mu$ and summing over the $\mu$ index the contracted Bianchi identity states that $\nabla_\mu G^{\mu\nu} = 0$ as an intrinsic geometric property of the Einstein tensor. Consistent with the further geometric identity $\nabla_\mu g^{\mu\nu} = 0$ the 'cosmological constant' $\Lambda$ term can be inserted into the Einstein field equation. With this divergence-free property of the left-hand side of equation 3 then necessarily also applying to the right-hand side, the relation $\nabla_\mu T^{\mu\nu} = 0$ can be interpreted as the conservation of energy and momentum in the limit of sufficiently flat local regions of spacetime (as discussed in [9] section 5.2 opening).

The field equation 3 can be derived directly from the Einstein-Hilbert action integral that takes the form ([9] equation 3.79):

$$I = \int \left[\tfrac{-1}{2\kappa}(R - 2\Lambda) + \mathcal{L}\right] \sqrt{|g|}\, d^4x \tag{4}$$

In this integral the 4-dimensional spacetime invariant volume element $\sqrt{|g|}\,d^4x$ incorporates the magnitude of the metric determinant $|g|$. Requiring stationarity $\delta I = 0$ under variations in the metric $\delta g_{\mu\nu}(x)$ the scalar curvature $R$, cosmological constant $\Lambda$ and matter Lagrangian $\mathcal{L}$ in equation 4 map onto the $G^{\mu\nu}$, $\Lambda$ and $T^{\mu\nu}$ terms respectively in equation 3. The cosmological constant $\Lambda$ can be interpreted as a free parameter in general relativity, to be determined by empirical observations or as associated with a further underlying theoretical source to be identified.

While the cosmological constant term in equation 3, with constant and uniform parameter $\Lambda$ and the metric of equation 2, trivially respects the cosmological principle, the most general form of energy-momentum consistent with homogeneity and isotropy can be written as:

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - p g^{\mu\nu} \tag{5}$$

Here $u^\mu = (1, 0, 0, 0)$, in the comoving coordinates $\{t, r, \theta, \phi\}$ of equation 2, represents the 4-velocity of an idealised uniform flow of galaxies in the large-scale universe. The parameter $\rho$ ($\equiv T^0{}_0$) represents the energy density while $p$ ($\equiv -T^1{}_1 = -T^2{}_2 = -T^3{}_3$) is interpreted as pressure, both of which are uniform in space at any given cosmic time (in a more general context, equation 5 describes a 'perfect fluid'). In addition to the energy density the pressure of a fluid or substance also contributes to its gravitational impact in general relativity. In solving the Einstein field equation 3 for the metric of equation 2 and homogeneous energy-momentum of equation 5 the three parameters $a$, $\rho$ and $p$ are functions of cosmic time $t$ as the only independent variable, with the dynamics depending on the three constants $k$, $\kappa$ and $\Lambda$ in these equations.

Adopting the notation $\dot{a} = da/dt$ and $\ddot{a} = d^2a/dt^2$ the Hubble parameter is defined as $H(t) := \frac{\dot{a}(t)}{a(t)}$ and in general, while being uniform in space, varies with

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cosmic time. Solving for the $G^{00}$ and $T^{00}$ components in equations 3 and 5 results in the Friedmann equation (which is independent of $p$, [9] equation 12.9):

$$H^2 + \frac{k}{a^2} = \frac{\kappa}{3}\rho + \frac{1}{3}\Lambda \tag{6}$$

Supernova data and observations of the anisotropy in the cosmic microwave background (CMB) in recent decades indicate that the universe is spatially flat to within a good approximation ([4] section 22.2.2), and hence it is possible to set $k = 0$ in equations 2 and 6.

However, on employing equation 6 for the $\Lambda = 0$ case, empirical observations of the current energy density $\rho_m(t_0)$ of visible and dark matter indicate that they collectively represent only around 31% of that required to be consistent with the present expansion rate of the universe, that is the 'Hubble constant' $H_0 \simeq 70$ km s$^{-1}$ Mpc$^{-1}$ ([4] section 25, see also later in this section). This 31% includes a 5% component from ordinary visible matter and a 26% component from cold dark matter (CDM), the latter required to account for galactic dynamics and gravitational lensing effects ([4] section 27). This is then the first indication of the need for a significant source of 'dark energy', to make up the remaining 69% of the energy density as apparently required by the observation of a spatially flat universe.

Further, observations tracking the evolution of the Hubble parameter, as originally plotted via distant supernovae (as alluded to in the opening of section 1), suggest that the expansion of the universe has actually been *accelerating* for the most recent six billion years or so. In FLRW cosmology the dynamical equations, utilising also the $G^{11}$ and $T^{11}$ components in equations 3 and 5, lead to an expression for the rate of change in the expansion rate of the universe of the form (which does depend on $p$ but is actually independent of $k$, [9] equation 12.18):

$$\frac{\ddot{a}}{a} = -\kappa\left(\frac{\rho + 3p}{6}\right) + \frac{\Lambda}{3} \tag{7}$$

An accelerating expansion, that is with $\ddot{a} > 0$, can then be achieved for the case of either $(\rho + 3p) < 0$ or a cosmological constant $\Lambda > 0$, or a suitable combination of these parameters; and is hence inconsistent with the assumption of a positive energy density matter dominated universe with $p = 0$ and $\Lambda = 0$. By comparison of equations 3 and 5 a cosmological constant $\Lambda$ in itself is equivalent to a 'fluid' with an effective pressure $p_\Lambda = -\Lambda/\kappa$ and energy density $\rho_\Lambda = +\Lambda/\kappa$, that are uniform in space as well as *constant in time* as the universe evolves. In the equation of state:

$$p = w\rho \tag{8}$$

a cosmological constant then corresponds to a parameter $w_\Lambda = p_\Lambda/\rho_\Lambda = -1$.

For $\Lambda > 0$ taking a value sufficient for the total energy density to account for the observed spatial flatness of the universe, with $k = 0$ in equation 6, the associated contribution to an effective negative pressure is consistent with the observed accelerating expansion of the cosmos, via equation 7. These collective observations then describe the 'standard cosmological model' or ΛCDM, also termed the 'concordance model', which overall has provided a successful match to the cosmological data ([4] section 25).

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Giving a broader perspective on ΛCDM, the evolution of the universe for around the first 10,000 years after the Big Bang is believed to have been dominated by electromagnetic radiation with pressure $p_r(t)$, an equation of state parameter $w_r = \frac{1}{3}$ in equation 8, and energy density declining as $\rho_r \propto a^{-4}$ (including an $a^{-1}$ redshift factor). From around 50,000 years the impact of the matter density component with $\rho_m \propto a^{-3}$, and $p_m = 0$ with $w_m = 0$, became comparable and then overtook leading to a long period of evolution essentially with $p = 0$ in equation 5, corresponding to an idealised pressureless fluid. As noted above the CDM component is required to total around five times the amount of visible matter in order to account for the original formation and clustering of visible galaxies as well as the present-day galactic dynamics. Finally, around 8 billion years into the 13.8 billion year history of the universe an all-pervading and uniform 'vacuum energy', as associated with the cosmological constant $\Lambda$ in equations 3 and 4, began to take over as the dominant component with $\rho_\Lambda \propto a^0$ and an equation of state parameter $w_\Lambda = -1$. Such a $\Lambda$ term is sometimes considered a property of *space itself*, given its constant and uniform character.

More generally, the definition of 'dark energy' incorporates any proposed entity driving an accelerating expansion of the universe. With the dark energy density required to be positive for consistency with the observed large-scale spatial flatness of the universe a significant source of an associated *negative* pressure is essentially a defining characteristic for dark energy. Such a negative pressure term is effectively equivalent to an intrinsic uniform 'tension' of space itself. Absorbing any $\Lambda$ contribution into the $\rho$ and $p$ terms, equations 7 and 8 imply that an equation of state parameter $w < -\frac{1}{3}$ is required to generate a cosmic acceleration. While the dynamical impact of dark energy may be somewhat simpler to describe than that of dark matter, the microscopic origin of this effective negative pressure and uniform positive energy density is if anything even more obscure from a theoretical perspective, even if interpreted as an apparent vacuum energy.

The simplest way to construct a *model* for dark energy is to postulate a new scalar field $\phi(x)$ and corresponding Lagrangian function $\mathcal{L}_\phi$ with suitable kinetic $\partial_\mu\phi$ and potential $V$ energy terms of the form:

$$\mathcal{L}_\phi = +\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) \tag{9}$$

The energy-momentum $T_\phi^{\mu\nu}(x)$ associated with this scalar field Lagrangian can be determined directly via equations 3 and 4 in general relativity or by Noether's theorem ([9] equations 3.85, 3.102 and 12.25) and takes the form (with $\rho$ here a further spacetime index):

$$T_\phi^{\mu\nu} = \partial^\mu\phi\,\partial^\nu\phi - \frac{1}{2}\partial_\rho\phi\,\partial^\rho\phi\,g^{\mu\nu} + V(\phi)g^{\mu\nu} \tag{10}$$

This energy-momentum can be equated with that of a perfect fluid in equation 5, with any of the $-T_{\phi}{}^{i}{}_{i}$ ($i = 1, 2$ or $3$, no sum) and the $T_{\phi}{}^{0}{}_{0}$ components used to determine the effective pressure and energy density respectively. Assuming that the scalar field $\phi(x)$ only depends upon time, that is neglecting any contribution from the spatial derivatives $\partial_i\phi$ and with the notation $\dot{\phi} = \partial_0\phi$ for the time derivatives, the corresponding pressure and energy density for the scalar field of equations 9 and 10

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are:

$$\begin{aligned} p_\phi &= \tfrac{1}{2}\dot{\phi}^2 - V(\phi) \\ \rho_\phi &= \tfrac{1}{2}\dot{\phi}^2 + V(\phi) \end{aligned} \tag{11}$$

Hence if the field $\phi(x)$ is also approximately constant in time, that is with the potential energy dominating over the kinetic term, it is possible to achieve an equation of state parameter in equation 8 arbitrarily close to $w_\phi = p_\phi/\rho_\phi = -1$. For a suitable choice of the potential $V(\phi)$ this hence corresponds to an effective cosmological constant $\Lambda > 0$ of arbitrary size.

Indeed, in inflationary models, an extremely large effective $\Lambda$ term is proposed to arise temporarily in the very early universe through a 'false vacuum' state of a specifically constructed potential $V(\phi)$ for an 'inflaton' scalar field $\phi(x)$ utilising equations 9–11, generating an initial rapid increase in the scale factor $a(t)$ in equation 2 via equation 7 ([10, 11], see also [9] section 12.3). Inflation is proposed to account for both the 'horizon problem', that is the near isotropy in the CMB temperature over different regions of the sky that would otherwise have been causally disconnected, and also the 'flatness problem', that is the closeness of our universe to being spatially flat. Without an early inflationary period both of these features would seemingly require a very significant level of fine-tuning. Such an initial inflationary era is then sometimes incorporated as a further feature of the standard ΛCDM cosmological model.

On a very different scale a more modest potential $V(\phi)$ in equations 9–11 can similarly give rise to an arbitrarily small effective $\Lambda > 0$ term (the 'Λ' in 'ΛCDM'), as can be consistent with the present epoch of a more sedately accelerating universe. As an alternative to a cosmological constant, 'quintessence' models propose that the new scalar field $\phi(x)$ represents a 'fifth fundamental force' or fluid for which both the potential *and* kinetic terms of the Lagrangian in equation 9 play a significant role [12, 13, 14]. While again providing the source for negative pressure, with the kinetic terms no longer neglected in equations 11, both $p_\phi(t)$ and $\rho_\phi(t)$ now depend upon cosmic time $t$. In this scenario the equation of state parameter is then typically in the range $-1 < w_\phi < -\frac{1}{3}$, characterising quintessence dark energy models as distinct from the cosmological constant case, and with the ability to achieve a more gentle cosmic acceleration.

The fact that the effective cosmological term associated with a very early period of inflation requires a far larger value for an effective $\Lambda$ than that of the present day ΛCDM cosmic acceleration itself suggests the possibility of an evolving, rather than constant, cosmological $\Lambda$ or dark energy density as a function of cosmic time. In principle then the same field $\phi(x)$ might be linked to both inflation *and* quintessence. While further new physics may be needed to connect with the extremely different scale required for inflation, the parameters in quintessence models can be chosen such that after a very brief and early inflationary period in the Big Bang the ever-decreasing energy density $\rho_\phi(t)$ 'tracks' first the radiation density $\rho_r \propto a^{-4}$ and then the matter density $\rho_m \propto a^{-3}$.

That is, with $\dot{\phi}^2$ and $V(\phi)$ in equations 11 larger in the past, and with $w_\phi$ greater than but not close to $-1$ in the early universe, the quintessence energy density contribution can be only a little subdominant to the radiation and then matter density, until approaching $w_\phi = -1$ and with $\rho_\phi$ overtaking the other contributions in the more recent cosmic history. The motivation for this 'tracking solution' is to account for the apparent 'coincidence problem' in the comparable average values of matter (dark and visible together, as unevenly distributed and heavily clumped in space) and dark energy density (which is uniform in space) at the present epoch. The related 'cosmological constant problem', concerning the seemingly unnaturally low value of the dark energy density as empirically required, might also be addressed in these models.

From the Lagrangian of equation 9 and the resulting nature of equations 11 the value $w_\phi = -1$ seems to be a lower bound for the equation of state parameter. However, still in the context of cosmological FLRW models (hence assuming large scale homogeneity and isotropy), equations 9–11 can be modified for the case of a 'wrong-sign' kinetic energy term for a scalar field, now denoted $\phi_{\text{-}}(x)$, that is with the Lagrangian in equation 9 replaced by:

$$\mathcal{L}_{\phi_{\text{-}}} = -\frac{1}{2}\partial_\mu\phi_{\text{-}}\,\partial^\mu\phi_{\text{-}} - V(\phi_{\text{-}}) \tag{12}$$

resulting, in place of equations 11, in the pressure and energy density contributions:

$$\begin{aligned} p_{\phi_{\text{-}}} &= -\tfrac{1}{2}\dot{\phi}_{\text{-}}^2 - V(\phi_{\text{-}}) \\ \rho_{\phi_{\text{-}}} &= -\tfrac{1}{2}\dot{\phi}_{\text{-}}^2 + V(\phi_{\text{-}}) \end{aligned} \tag{13}$$

This construction is motivated to achieve a negative pressure $p_{\phi_{\text{-}}}$ while retaining an overall positive energy density $\rho_{\phi_{\text{-}}}$ related by an equation of state in equation 8 with parameter $w_{\phi_{\text{-}}} = p_{\phi_{\text{-}}}/\rho_{\phi_{\text{-}}} < -1$. Such a value for $w$ defines the 'phantom' scenario for dark energy and implies a more dramatic cosmic acceleration than the $w_\Lambda = -1$ case for a cosmological constant [15, 16, 17].

The significant issue in this case is the instability of the vacuum associated with the negative kinetic energy term in equation 12. If this sector can interact with other more conventional forms of matter then the vacuum can decay to an arbitrary degree into negative energy 'ghosts' of the phantom field together with positive energy matter states (such as those of the Standard Model if mediated by graviton exchange), while conserving the total energy ([18], see also figure 2(a) in section 6 here). Even within the phantom sector itself the equation of state $w_{\phi_{\text{-}}} < -1$ implies that as the universe expands the overall phantom energy density $\rho_{\phi_{\text{-}}}(t)$ ever increases towards a singular infinite value in finite cosmic time. The main difficulty is then how to achieve stability in these models at least for a sufficient period typically tens of billions of years into the future, before the scale factor $a(t) \to \infty$ and the universe disintegrates in a 'Big Rip' [19].

There is also a range of additional models for dark energy, aimed at addressing the pertinent observations and various issues in cosmology (see for example [20, 21, 22, 23]). As further examples 'quintom' models involve a hybrid of quintessence and phantom models, capable of evolving through the $w = -1$ divide, while in '$k$-essence' models the scalar field Lagrangian incorporates a more general non-linear function of the kinetic energy. While scalar fields are typically employed for simplicity, there are also quintessence models based on the phenomenology of a new Abelian [24] or non-Abelian gauge field [25, 26]. There are also models proposing a link between standard visible QCD [27] or an invisible QCD sector [28] and dark energy. In all cases the non-Abelian gauge groups employed are compact.

Some models have the specific feature of alleviating the Hubble tension. While the Hubble constant as described for equation 6 is now well-constrained, there is a $4\sigma$ or more discrepancy between the slightly lower values obtained from 'early time' measurements and the slightly higher values obtained from 'late time' determinations of $H_0$ that these models seek to address [29, 30]. For example in 'early dark energy' models a short early burst of extra cosmic acceleration is proposed to account for this Hubble tension.

The need for 'dark energy' can in principle even be avoided in 'modified gravity' models, with for example the scalar curvature $R$ in the Einstein-Hilbert action of equation 4 replaced by a suitable function $f(R)$, mimicking the effect of $\Lambda$ and directly generating an accelerating expansion as a large-scale geometric feature of the universe [31]. There are also suggestions that the assumptions of the cosmological principle may not be sufficiently valid and the apparent cosmic acceleration is just an artefact deriving from an incorrect interpretation of the observations [32].

By and large all of the above approaches, including quintessence and phantom energy as reviewed in this section, are essentially 'toy models', in some cases considered a 'crude toy model' [15]. Even the simplest model with a cosmological constant $\Lambda$, as a free parameter in the Einstein field equation, lacks an established underlying physical foundation. However, in attempting to account for the cosmological data these models provide insight into the kinds of principles and properties that might be sought in a theory constructed at a deeper level. In this paper we do not introduce *any* dark energy 'model' as such. Rather we describe a fundamental theory which directly leads to a sector giving rise to an apparent source of negative pressure and other key ingredients desired for dark energy.

We motivate the basis for this fundamental theory in the following section and then review the connections that have been established between the theory and the Standard Model of particle physics as well as a dark matter candidate in section 4. In section 5 we then describe a branch of the new theory that directly generates the proposed source for dark energy and in section 6, making further comparisons with the models briefly reviewed in this section, consider the explicit microphysical form this new candidate for dark energy might take.

## 3 Generalised Proper Time as the Basis for Unification

For any theory of the elementary microscopic structure of matter, such as relating to the Standard Model of particle physics, we are interested in the properties of local symmetries and local particle interactions. This motivates beginning with the local structure of 4-dimensional spacetime as the basis for identifying the properties of matter from the components of an augmented structure. A direct means of augmenting the local spacetime metric structure is provided by the form of a proper time interval $\delta s$ expressed in a local inertial reference frame ([7] section 2, [8] section 3, as reviewed here). Compared with the familiar approach involving extra dimensions of space, the contrast in this starting point and means of augmentation is depicted in figure 1.

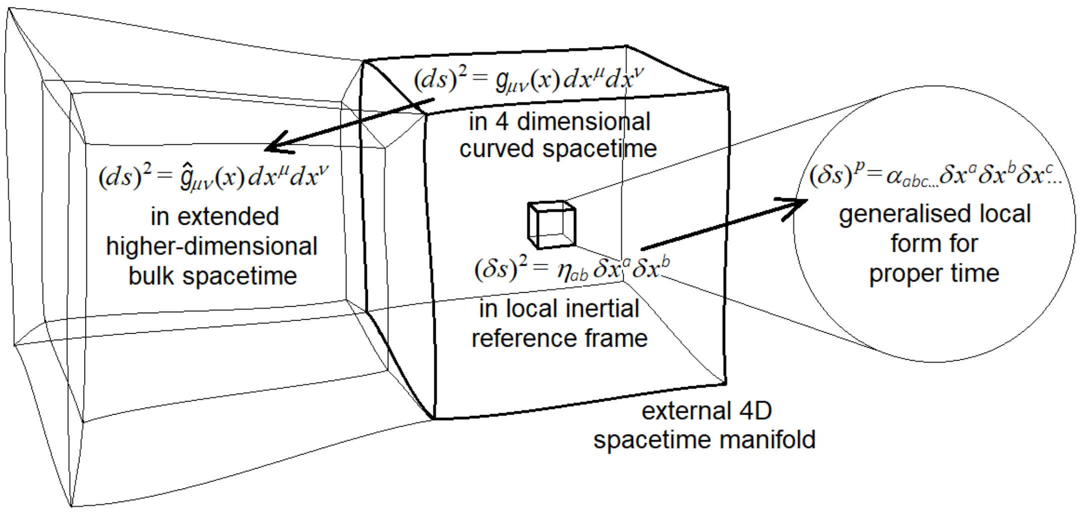


Figure 1: In general relativity the quadratic spacetime form for an infinitesimal proper time interval $ds \in \mathbb{R}$ in 4-dimensional spacetime can be expressed in general global coordinates $\{x^\mu\}$ via the symmetric metric tensor $g_{\mu\nu}(x)$, with $\mu, \nu = 0, 1, 2, 3$ and as described for equation 1, or in terms of a set of local inertial coordinate parameters $\{x^a\}$ and the Minkowski metric $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$, with $a, b = 0, 1, 2, 3$ and in the form of equation 14 for $\delta s \in \mathbb{R}$. These global and local forms lead to contrasting possibilities for generalisation with additional components, as pictured to the left and right respectively, as a potential means of incorporating structures of matter.

Here we use the notation $dx^\mu$ for an infinitesimal interval of a coordinate parameter in an extended spacetime and $\delta x^a$ for an infinitesimal interval of a real parameter more generally, with a one-to-one correspondence between these two notions for the four parameters of a local inertial coordinate frame in 4-dimensional spacetime. A key point for the new theory will be that the additional components $\{\delta x^a; a > 3\}$ are *not* interpreted as coordinate intervals in an extended spacetime structure.

Higher-dimensional spacetime models typically augment the original global metric $g_{\mu\nu}(x)$ of equation 1 by introducing further extended spatial dimensions, with the augmented metric denoted $\hat{g}_{\mu\nu}(x)$ and coordinate indices $\mu, \nu = 0, \ldots, n-1$, in an $(n > 4)$-dimensional extended 'bulk' spacetime (see for example [33, 34]). In the bulk spacetime a proper time interval can be expressed as $ds^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu$ as a direct extension from equation 1. A 4-dimensional spacetime base together with a matter content can then be extracted from the larger bulk spacetime structure, for example by 'compactifying' the extra dimensions.

Here we consider an alternative approach for which it is the local 4-dimensional quadratic form for proper time with local metric $\eta_{ab}$ that is generalised to a $p^{\text{th}}$-order form for an infinitesimal proper time interval $\delta s$ in $(n>4)$ components, that is:

$$\begin{aligned} \text{from} \quad (\delta s)^2 &= \eta_{ab}\delta x^a \delta x^b && \text{with} \quad a, b = 0, 1, 2, 3 && (14) \\ \text{to} \quad (\delta s)^p &= \alpha_{abc\ldots}\delta x^a \delta x^b \delta x^c \ldots && \text{with} \quad a, b, c, \ldots = 0, \ldots, n-1 && (15) \end{aligned}$$

with homogeneous polynomial integer power $p \geq 2$. Each coefficient takes a value $\alpha_{abc\ldots} = -1, 0$ or $+1$ in generalising the metric coefficients $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$, with a sum in equation 15 over each index $a, b, c, \ldots$ for this $n$-component generalised form. While the proper time interval in equation 14 is invariant under Lorentz transformations, with the Lorentz group $\mathrm{SO}^+(1,3)$ acting on the four $\{\delta x^a\}$ components, the interval $\delta s$ in equation 15 in representing generalised proper time is invariant under a larger symmetry group denoted $\hat{G}$ (containing an $\mathrm{SO}^+(1,3)$ subgroup), now acting upon a full set of $n$ $\{\delta x^a\}$ components collectively denoted $\boldsymbol{\delta x}_n \in \mathbb{R}^n$. In particular, since the whole purpose of augmenting from the 4-dimensional spacetime form is to incorporate a structure of *matter*, rather than more *space*, we can drop any restrictive assumption of a quadratic $p = 2$ form for equation 15 as a local generalisation for the form of proper *time*.

As an augmentation from the local 4-dimensional spacetime form of equation 14, the $n$-component form of equation 15 can be written as:

$$\underbrace{(\delta s)^p = \alpha_{abc\ldots}\delta x^a \delta x^b \delta x^c \ldots}_{\substack{a,b,c,\ldots\,=\,0,\ldots,n-1 \\ \text{generalised proper time}}} = \underbrace{(\eta_{ab}\delta x^a \delta x^b)}_{\substack{a,b\,=\,0,1,2,3 \\ \text{spacetime basis}}} \times \underbrace{(\boldsymbol{\delta x}_{n-4})^{p-2} \quad + \quad (\boldsymbol{\delta x}_n)^p}_{\substack{\delta x^c;\, c\geq 4 \qquad \delta x^a;\, a \geq 0 \\ \text{basis for matter}}} \tag{16}$$

where $(\boldsymbol{\delta x}_{n-4})^{p-2}$ and $(\boldsymbol{\delta x}_n)^p$ represent the appropriate polynomial expressions in the corresponding components (with $\boldsymbol{\delta x}_{n-4}$ a subset of $\boldsymbol{\delta x}_n$) as consistent with equation 15. The full symmetry $\hat{G}$ of equation 15 and the left-hand side of equation 16 is broken in *extracting* four preferred components $\boldsymbol{\delta x}_4 \equiv \{\delta x^a; a = 0,1,2,3\}$ as the basis for the local external 4-dimensional spacetime, upon which the Lorentz $\mathrm{SO}^+(1,3) \subset \hat{G}$ symmetry acts, leaving a residual internal symmetry $G \subset \hat{G}$. This internal Lie group $G$ can be defined as the subgroup generated by the subset of the generators of $\hat{G}$ that commute with the external Lorentz group generators in the corresponding Lie algebra. In 4-dimensional spacetime the full symmetry $\hat{G}$ is hence broken down to the local product structure:

$$\text{Lorentz} \times G \subset \hat{G} \tag{17}$$

The internal group $G$ is interpreted as a local gauge symmetry. In identifying an extended 4-dimensional spacetime $M_4$ the surviving symmetries associated with equations 16 and 17, in breaking the full symmetry of the generalised form for proper time, motivate the construction of a 'principal fibre bundle' $P \equiv M_4 \times G$. This is a single manifold $P$ with the geometry of the gauge group $G$ incorporated into the fibres over a 4-dimensional spacetime base $M_4$ ([35] see for example figure 4 therein). This construction is closely analogous to the principal bundle structure as deployed for Kaluza-Klein models with the extra spatial dimensions accommodating a non-Abelian gauge group.

Based on such a construction it is possible to establish an intrinsic geometric relation between the Riemannian geometry and the gauge curvature that can be defined on $P$. This allows the identification of a direct geometric relationship between the curvature of the external 4-dimensional spacetime, as associated with the gravitational field and represented by the Einstein tensor $G^{\mu\nu}(x)$, and the curvature of the internal gauge structure, as described by the gauge field strength $F^\alpha{}_{\mu\nu}(x)$ with $\alpha$ a Lie algebra index. The derivation of this relationship is studied and explained in detail in [35] as outlined here in leading to equation 20 below.

The tangent space at any point $p \in P$ decomposes into a 'vertical subspace', with a basis of vectors assigned indices $\alpha, \beta, \ldots = 1, \ldots, n_G$ tangent to the gauge group

$G$ fibres (with $n_G$ the dimension of the Lie algebra), and a 'horizontal subspace' describing the gauge connection with tangent vectors assigned indices $a, b, \ldots = 0, 1, 2, 3$ as associated with a frame basis on the base space $M_4$ ([35] equation 42 and figure 4). This decomposition is aligned with the orthogonality of the two subspaces as can be defined with respect to a metric $\acute{g}(p)$ constructed on the bundle space ([35] equations 58–60), with vertical subcomponents associated with the Killing metric $K_{\alpha\beta}$ of $G$ and horizontal subcomponents incorporating the original metric $g_{ab}(x)$ as expressed in any tangent space basis on $M_4$.

Coefficients of a linear connection $\acute{\Gamma}(p)$ on $P$ can be defined such that the associated parallel transport along the horizontal subspace directions follows the contours of the internal gauge curvature $F(p)$ on $P$ in the sense of 'Lie dragging', which implies the set of components $\acute{\Gamma}^{\alpha}{}_{ab}(p) = F^{\alpha}{}_{ab}(p)$ for any $p \in P$ ([35] equations 79–83). The full set of linear connection coefficients for $\acute{\Gamma}(p)$ can be defined in a manner that is both compatible with the metric $\acute{g}(p)$ and also gauge covariant on $P$ ([35] equation 88).

The Riemannian geometry defined and constructed on $P$ in this way is then intimately linked with the structure of the internal gauge connection associated with $G$ as well as with the external geometry of $M_4$. The corresponding scalar curvature $\acute{R}(p)$ defined on $P$ is independent of the fibre coordinates and can hence be directly projected onto the base manifold $M_4$ uniquely taking the gauge invariant form ([35] equation 90):

$$\tilde{R}(x) = R_{M_4}(x) + F^{\alpha}{}_{\mu\nu}(x) F_{\alpha}{}^{\mu\nu}(x) \tag{18}$$

Here $R_{M_4}(x)$ is the scalar curvature of $M_4$ itself while $F^{\alpha}{}_{\mu\nu}(x)$, with $\mu, \nu = 0, 1, 2, 3$, are the components of the gauge curvature on the base manifold, now adopting a general coordinate basis on $M_4$. With $R_{M_4}(x)$ defined in the same way as the usual scalar curvature in general relativity on integrating over the 4-dimensional spacetime manifold the quantity:

$$\tilde{I} = \int \tilde{R}(x) \sqrt{|g|}\, d^4x \tag{19}$$

represents an amendment to the Einstein-Hilbert action of equation 4. This can be considered as a *perturbation* from the vacuum case, in the sense of a vacuum with both $\Lambda = 0$ and $\mathcal{L} = 0$ in equation 4, as described for ([35] equation 91). The additional $F^{\alpha}{}_{\mu\nu}(x) F_{\alpha}{}^{\mu\nu}(x)$ term in equations 18 and 19 then effectively plays the role of a matter Lagrangian contribution (namely equation 30 in section 5), but *without* having to postulate this gauge field term independently of the geometric construction.

Stationarity of the action $\delta\tilde{I} = 0$ for equation 19 with respect to variations in the metric $\delta g_{\mu\nu}(x)$ on $M_4$ leads to an expression for the Einstein tensor $G^{\mu\nu}(x)$ of the form ([35] equation 93; with $\chi$ a positive normalisation constant and $\mu, \nu, \rho, \sigma$ 4-dimensional spacetime coordinate indices):

$$G^{\mu\nu} = 2\chi \left( -F^{\alpha\mu}{}_{\rho} F_{\alpha}{}^{\rho\nu} - \frac{1}{4} g^{\mu\nu} F^{\alpha}{}_{\rho\sigma} F_{\alpha}{}^{\rho\sigma} \right) =: -\kappa T^{\mu\nu} \tag{20}$$

Through this direct geometric link between the external spacetime curvature and internal gauge curvature a corresponding effective energy-momentum tensor $T^{\mu\nu}(x)$ for the latter is defined on the right-hand side, as consistent with the Einstein field equation 3. From this equation solutions for the corresponding global 4-dimensional metric $g_{\mu\nu}(x)$ describe a non-trivial gravitational field as originating from the local augmentation

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for a proper time interval $\delta s \in \mathbb{R}$ as described for figure 1 and the resulting residual internal gauge symmetry $G$ of equation 17 and associated gauge field curvature.

The full symmetry $\hat{G}$ of equation 15 is not a symmetry of the physics, rather the symmetry breaking described for equations 16 and 17 is *required* in order to construct 4-dimensional spacetime itself and *any* physics it contains. Hence for example the surviving external and internal group product structure in equation 17 is consistent with the demands of the Coleman-Mandula theorem for any relativistic particle scattering theory that arises within this framework (see for example [6] discussion of equation 83).

The fragmented components of $\boldsymbol{\delta x}_n$ in equation 16, transforming under the surviving broken symmetry of equation 17, form the basis for the local structure of matter and physical particle states themselves, with the local gauge symmetry $G$ associated with the local interactions of these particle states. The corresponding matter fields can be considered to take values in an 'associated' fibre bundle structure, related to the above principal bundle $P \equiv M_4 \times G$, and will also impact the external spacetime geometry generalising further from equation 20 as consistent with general relativity and a further apparent source of energy-momentum. In all cases the gauge and matter fields are identified through the symmetry breaking in equations 16 and 17 and the composition of the extended 4-dimensional spacetime manifold $M_4$ as constructed under the geometric form of $G^{\mu\nu}(x)$, rather than through specific forms of energy-momentum as posited via Lagrangian or $T^{\mu\nu}(x)$ terms.

The local generalisation of proper time, with the basis for matter identified directly alongside the basis for local 4-dimensional spacetime through the composition of equations 16 and 17, then provides a very direct means of deriving local structures of matter. These forms of matter are intimately connected with the geometry of the external 4-dimensional spacetime in a manner consistent with the Einstein field equation 3 as described above. While the conceptual basis of generalised proper time is well-motivated, as described for figure 1 in comparison with extra spatial dimensions, the new approach has also proved notably more successful in providing direct connections with both the Standard Model of particle physics and a dark matter candidate, as we review in the following section.

## 4 The Standard Model and Dark Matter Sectors

A unique sequence of augmentations to the local 4-dimensional spacetime form of equation 14, consistent with the generalised form for proper time in equation 15 with $p > 2$, can be identified on first expressing equation 14 in the matrix form:

$$(\delta s)^2 = \eta_{ab}\delta x^a \delta x^b = \det(\boldsymbol{\delta x}_4) \quad \text{with} \quad \boldsymbol{\delta x}_4 = \begin{pmatrix} \delta x^0 + \delta x^3 & \delta x^1 - \delta x^2 i \\ \delta x^1 + \delta x^2 i & \delta x^0 - \delta x^3 \end{pmatrix} \in \mathrm{h}_2\mathbb{C} \tag{21}$$

The Lorentz symmetry group $\mathrm{SO}^+(1,3)$ of equation 14 is correspondingly replaced by its double cover with elements $S \in \mathrm{SL}(2,\mathbb{C})$ acting upon the $2 \times 2$ complex Hermitian matrices $\boldsymbol{\delta x}_4 \in \mathrm{h}_2\mathbb{C}$ as $\boldsymbol{\delta x}_4 \to S\boldsymbol{\delta x}_4 S^\dagger$ in the standard way.

The form of equation 21 can then be augmented directly to the determinant of $3\times3$ complex Hermitian matrices $\mathrm{h_3\mathbb{C}}$ with a full $\hat{G} = \mathrm{SL}(3,\mathbb{C})$ symmetry, representing a *cubic* form for proper time $(\delta s)^3$ now over nine real components ([7] equation 16). Again consistent with equation 15, further direct mathematical extensions, involving the octonions and exceptional Lie groups, lead to $\hat{G} = \mathrm{E_6}$ and $\hat{G} = \mathrm{E_7}$ symmetries for yet higher-dimensional forms for proper time through to quartic order $(\delta s)^4$, as listed in table 1.

| proper time $\delta s$ | space of $\boldsymbol{\delta x}_n$ | symmetry $\hat{G}$ | comments |
|---|---|---|---|
| $(\delta s)^2 = \det(\boldsymbol{\delta x}_4)$ | $\boldsymbol{\delta x}_4 \in \mathrm{h_2\mathbb{C}}$<br>4-dimensional | $\mathrm{SL}(2,\mathbb{C})$<br>quadratic form | Local form of<br>4D spacetime |
| $(\delta s)^3 = \det(\boldsymbol{\delta x}_9)$ | $\boldsymbol{\delta x}_9 \in \mathrm{h_3\mathbb{C}}$<br>9-dimensional | $\mathrm{SL}(3,\mathbb{C})$<br>cubic form | Matrix algebra<br>extension |
| $(\delta s)^3 = \det(\boldsymbol{\delta x}_{27})$ | $\boldsymbol{\delta x}_{27} \in \mathrm{h_3\mathbb{O}}$<br>27-dimensional | $\mathrm{E_6} \equiv \mathrm{SL}(3,\mathbb{O})$<br>cubic form | Division algebra<br>extension |
| $(\delta s)^4 = q(\boldsymbol{\delta x}_{56})$ | $\boldsymbol{\delta x}_{56} \in F(\mathrm{h_3\mathbb{O}})$<br>56-dimensional | $\mathrm{E_7}$<br>quartic form | Exceptional Lie<br>group extension |
| $(\delta s)^8 = Q(\boldsymbol{\delta x}_{248})$ | $\boldsymbol{\delta x}_{248} \in \mathcal{T}$<br>248-dimensional | $\mathrm{E_8}$<br>octic form | Hypothetical<br>ultimate form |

Table 1: Series of higher-dimensional forms for generalised proper time and their symmetries, building upon equation 14 as expressed in the form of equation 21 and in all cases consistent with equation 15. All of these forms utilise well-known mathematical structures, apart from the final $\mathrm{E_8}$ octic level over a 248-real-component space $\mathcal{T}$ which is anticipated by extrapolation from the other forms [6].

This progression in mathematical structures involving an augmentation from the 4-component space $\mathrm{h_2\mathbb{C}}$ through to the 56-component space of the Freudenthal triple system $F(\mathrm{h_3\mathbb{O}})$ is well-known and has been employed in a range of other theoretical applications (such as described in [36, 37, 38]). For example these mathematical structures and associated symmetries have also been studied in the context of 'generalised spacetimes' [39]. However, here these homogeneous polynomial forms, from quadratic to quartic order as listed in the first column of table 1, are interpreted within the framework of 'generalised proper time' ([6] section 4 and table 3).

For the extensions to cubic and higher-order forms in table 1 an external Lorentz $\mathrm{SL}(2,\mathbb{C}) \subset \hat{G}$ subgroup factor acts upon the four original components extracted as a basis for the local 4-dimensional spacetime as described for equation 16, breaking the full symmetry as discussed for equation 17. The residual internal symmetry $G \subset \hat{G}$ and fragmented components of $\boldsymbol{\delta x}_n$ transforming under the broken symmetry can be interpreted as the basis for a structure of matter as described for equations 16 and 17 towards the end of the previous section. The mathematical analysis of the symmetry breaking patterns has been carried out in detail through to the $\mathrm{E_7}$ level of table 1, with the results summarised in table 2 and the discussion below.

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| $^{56}\diagdown^{\mathrm{E}_7 \supset}$ | Lorentz × | $\mathrm{SU}(3)_c$ × | $\mathrm{U}(1)_Q$ | matter |
|---|---|---|---|---|
| $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ | vector | $\mathbf{1}$ | 0 | $\boldsymbol{\delta x}_4$ 'Higgs' |
| | $\underline{\text{vector}}$ | $\mathbf{1}$ | 0 | '$\nu_L$' |
| $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ | Dirac | $\mathbf{1}$ | 1 | $\begin{pmatrix} e_R \\ e_L \end{pmatrix}$ |
| $\begin{pmatrix} 6 \\ 6 \end{pmatrix}$ | $\underline{\text{scalar}}$ | $\mathbf{3}$ | $\frac{2}{3}$ | $\begin{pmatrix} \text{'}u_R\text{'} \\ \text{'}u_L\text{'} \end{pmatrix}$ |
| $\begin{pmatrix} 12 \\ 12 \end{pmatrix}$ | Dirac | $\mathbf{3}$ | $\frac{1}{3}$ | $\begin{pmatrix} d_R \\ d_L \end{pmatrix}$ |
| $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ | scalar | $\mathbf{1}$ | 0 | } Yukawa coupling |
| 2 | scalar | $\mathbf{1}$ | 0 | |

Table 2: The 56 components of $\boldsymbol{\delta x}_{56} \in F(\mathrm{h}_3\mathbb{O})$ of the $\mathrm{E}_7$ quartic form for proper time in table 1 are partitioned under the broken symmetry in extracting a local Lorentz $\mathrm{SL}(2,\mathbb{C})$ subgroup acting upon the local external spacetime $\boldsymbol{\delta x}_4$ 4-vector subcomponents. The final column lists the matter state interpretation linking these structures with the Standard Model of particle physics ([6] section 4).

The central columns in table 2 result from the direct mathematical analysis of the symmetry breaking. This structure is *found* to be rich in Standard Model properties. Lorentz $\mathrm{SL}(2,\mathbb{C})$ Dirac spinor structures, $\mathrm{SU}(3)_c$ singlets $\mathbf{1}$ and triplets $\mathbf{3}$ and a pattern of $\mathrm{U}(1)_Q$ fractional charges, as listed by their magnitudes $\{1, \frac{2}{3}, \frac{1}{3}\}$ in table 2, motivate the associations with Standard Model colour $\mathrm{SU}(3)_c$ and electromagnetic $\mathrm{U}(1)_Q$ gauge symmetries, as denoted by the subscripts $c$ and $Q$, together with the lepton $(\nu, e)$ and quark $(u, d)$ states as indicated in the final column.

Significant elements of an electroweak $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry structure (with $L$ left-handed and $Y$ hypercharge) in relation to the internal $\mathrm{SU}(3)_c \times \mathrm{U}(1)_Q$ symmetry in table 2 can be identified on temporarily omitting the external Lorentz component ([6] section 4.3, [9] section 8.3). This observation might be anticipated since $\mathrm{E}_6$ and $\mathrm{E}_7$ form part of a well-known sequence of groups of interest in Grand Unified Theories that focus purely on the internal symmetry properties ([4] section 94.2.4). Here these elements of electroweak symmetry breaking point to the external components $\boldsymbol{\delta x}_4$, and the associated scalar $\det(\boldsymbol{\delta x}_4)$, as being closely connected with the Standard Model Higgs, with the contingent nature of this connection indicated by the quote marks on the 'Higgs' assignment in the table. Further, by comparison of terms in the expansion of the $\mathrm{E}_7$ quartic form for proper time under the symmetry breaking with Standard Model Lagrangian mass terms, the scalars at the bottom of table 2 are provisionally associated with Yukawa couplings ([7] subsection 4.2).

A left-right asymmetry arises from the extraction of the Lorentz $\mathrm{SL}(2,\mathbb{C})$ symmetry from the unification group $\hat{G} = \mathrm{E}_7$ and the asymmetric embedding of the external spacetime $\boldsymbol{\delta x}_4$ components in this 56-component form for proper time. The upper entries in the brackets in the left-hand column of table 2 collectively list the 27-component substructure of the symmetry breaking for the intermediate $\hat{G} = \mathrm{E}_6$ level of table 1. These 27 components, corresponding to one $\mathrm{h}_3\mathbb{O}$ subspace of $F(\mathrm{h}_3\mathbb{O})$, are considered to reside in the 'right-handed sector' of the theory. The $F(\mathrm{h}_3\mathbb{O})$ space at the $\mathrm{E}_7$ level contains a second $\mathrm{h}_3\mathbb{O}$ subspace, describing a 'complex conjugate' representation of the $\mathrm{E}_6 \subset \mathrm{E}_7$ subgroup with respect to the first, with the corresponding 27 components residing in the 'left-handed sector' and listed in the lower entries of the brackets in table 2. It is the necessary extraction of the external spacetime components $\boldsymbol{\delta x}_4$ from one, and only one, of these two complementary sectors that breaks the left-right symmetry ([7] subsection 3.1).

Overall, these structures identified in this very direct symmetry breaking of generalised proper time manifest a significant resemblance to structures of the Standard Model. However, the discrepancies in the spinor structure, as underlined in table 2 (and prompting the quote marks on the '$\nu_L$'-neutrino and '$u_{R,L}$'-quark states), the need to incorporate the full Standard Model internal gauge symmetry including a complete electroweak structure *alongside* the external Lorentz symmetry, and the need to describe a full three generations of lepton and quark states, firmly imply that a yet further augmentation from the $\mathrm{E}_7$ level in table 2 should be sought.

The symmetry group $\mathrm{E}_8$ is uniquely the largest exceptional Lie group and has a smallest non-trivial representation of 248 dimensions. These and other relevant mathematical properties, such as the existence of an $\mathrm{E}_8$ octic invariant [40], motivate the proposal of a potential $8^{\mathrm{th}}$-order form $(\delta s)^8 = Q(\boldsymbol{\delta x}_{248})$ for generalised proper time with a $\hat{G} = \mathrm{E}_8$ symmetry, as listed at the bottom of table 1. Based on the 248-dimensional space denoted $\mathcal{T}$ a symmetry breaking pattern for $\mathrm{E}_8$ acting on the fragmented components of $\boldsymbol{\delta x}_{248} \in \mathcal{T}$ in equations 16 and 17 for this further level would in principle be large enough to encompass all the required Standard Model external and internal symmetries and a full three generations of states. It is well known that it is *not* possible to identify such an $\mathrm{E}_8$ symmetry breaking pattern through standard Lie group representation theory, although many significant features of the Standard Model can be identified this way [41, 42]. However, here with the construction heavily involving the octonion algebra the resulting structures can differ from those of the standard analysis of simple Lie groups.

Already for the cubic $\mathrm{E}_6$ and quartic $\mathrm{E}_7$ forms in table 1 the octonion algebra is key to the construction of both the representation space and the symmetry actions, and in turn octonions are also anticipated to be central to the explicit construction for the proposed octic $\mathrm{E}_8$ level. Hence a non-standard representation space $\mathcal{T}$ can be obtained, with the non-associativity of the octonions also implying symmetry compositions that may not in general directly form a group structure (for which composition associativity is a key axiom). The symmetry breaking patterns obtained can then diverge from the expectations deduced from standard Lie group theory, and explicit calculation is required for these octonionic structures as has been the case for the $\mathrm{E}_6$ and $\mathrm{E}_7$ levels in constructing table 2. The existence of such an octonion-based $\mathrm{E}_8$ structure and symmetry breaking pattern that accommodates the full set of Standard Model states constitutes a well-defined mathematical prediction for this approach [6, 43]. This ambition closely relates to ongoing developments in the understanding of the algebraic connections between the octonions, $\mathrm{E}_8$ and the Standard Model (see for example [44, 45] and references therein).

On the physics side, with the 'Higgs' $\boldsymbol{\delta x}_4$ and left-handed neutrino '$\nu_L$' states provisionally associated with complementary components in the two $\mathrm{h}_3\mathbb{O}$ subspaces embedded within $F(\mathrm{h}_3\mathbb{O})$ at the $\mathrm{E}_7$ level as listed in the top rows of table 2, it is suggested that in any augmentation to a three-generation structure with $3\times\nu_L$ states the Higgs will be effectively embedded in a right-handed neutrino $\nu_R$ sector. With spinor components also needing to be identified for the neutrino states under this augmentation, it is proposed that the physical scalar Higgs state, while associated with the external 4-vector $\boldsymbol{\delta x}_4$ components, may be non-elementary and effectively composed from some of the right-handed neutrino spinor degrees of freedom, leaving no more than $2\times\nu_R$ physical states, consistent with a number of models [7]. As for those related models this can lead to predictions that could be testable in the laboratory. For example a composite Higgs, or other form of non-standard or extended Higgs sector, is a prime candidate for new physics that may account for the $7\sigma$ discrepancy between the recent $W^{\pm}$ gauge boson mass measurement and Standard Model electroweak theory expectation [46].

The sequence of forms for generalised proper time in table 1 through to the proposed $\hat{G}=\mathrm{E}_8$ symmetry with a central role for the octonions, hence involving both the largest exceptional Lie group and the largest normed division algebra, describes a unique augmentation beyond the local 4-dimensional spacetime form of equation 14 as consistent with the generalised form for proper time in equation 15 for $p>2$. These structures have been found to make highly non-trivial connections with the Standard Model and new physics beyond, as reviewed above. However, equation 15 itself is of a more general character, permitting the identification of further possible 'branches' for the augmentation of local proper time intervals and implying the possibility of further forms of matter.

For example, equation 14 *can* simply be extended to the $(n>4)$-dimensional quadratic spacetime form, as also consistent with equation 15 for the $p=2$ case:

$$(\delta s)^2 \;=\; |\boldsymbol{\delta x}_n|^2 \;=\; \hat{\eta}_{ab}\delta x^a\delta x^b \quad \text{with} \quad \boldsymbol{\delta x}_n\in\mathbb{R}^{1,n-1} \tag{22}$$

This form applies to the $n$ components $\boldsymbol{\delta x}_n\in\mathbb{R}^{1,n-1}$, with augmented Lorentz metric $\hat{\eta}_{ab}=\mathrm{diag}(+1,-1,\ldots,-1)$, indices $a,b=0,\ldots,n-1$, and a full $\hat{G}=\mathrm{SO}^+(1,n-1)$ higher-dimensional local Lorentz symmetry.

The basis for a corresponding structure of matter can be identified from the residual components over the $\boldsymbol{\delta x}_4=(\delta x^0,\delta x^1,\delta x^2,\delta x^3)\in\mathbb{R}^{1,3}$ subset extracted for the local external spacetime basis as described for equations 16 and 17. A similar analysis as applied for table 2 now results in the symmetry breaking structure of table 3, as can be analysed through to the $n\to\infty$ limit.

The common $\boldsymbol{\delta x}_4$ components of the external spacetime have already been associated with the Higgs in the context of the Standard Model branch as described for table 2. Here the internal local gauge symmetry $\mathrm{SO}(m)_D$, with $m=n-4$, together with the residual components $\boldsymbol{\delta x}_m=(\delta x^4,\ldots,\delta x^{n-1})\in\mathbb{R}^m$ in table 3, in lacking any common gauge symmetry interactions with the Standard Model sector, for which a full internal $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ is sought in augmenting from table 2, has an inevitable interpretation as a dark 'hidden QCD' sector. For this resulting dark matter sector the components $\boldsymbol{\delta x}_m$ can be considered a basis for 'scalar dark quarks', by analogy with the familiar fermionic quark states of standard QCD.

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| $n$ \ $\mathrm{SO}^+(1,n-1) \supset$ | Lorentz | $\times$ $\mathrm{SO}(m)_D$ | matter |
|---|---|---|---|
| 4 | 4-vector | invariant | $\boldsymbol{\delta x}_4$ external |
| $m$ $(= n-4)$ | scalar | $m$-vector | $\boldsymbol{\delta x}_m$ dark |

Table 3: The breaking of the full $\hat{G} = \mathrm{SO}^+(1,n-1)$ symmetry of the quadratic form for proper time of equation 22 on extracting the local external 4-dimensional spacetime and Lorentz symmetry results in a residual internal symmetry and matter structure that has a natural interpretation as a basis for dark matter ([8] sections 5 and 6).

The non-Abelian gauge symmetry $\mathrm{SO}(m)_D$ in table 3 with $m$ arbitrarily large (see for example [47] for the QCD-like properties of this gauge group) could result in the formation of massive 'dark glueballs' or 'scalar dark quark nuggets' in the early universe, by analogy with existing dark matter models [48, 49]. With neither Coulomb repulsion nor Fermi pressure from the constituents for support extremely heavy glueball or nugget states might also be prone to collapse into 'primordial black holes' as a component of this dark matter candidate [50, 51].

Whether as glueballs, nuggets or black holes, these relatively massive states would naturally describe a candidate for *cold* dark matter, hence conforming with the standard ΛCDM cosmological model. Further, in exhibiting at most only the very short-range internal $\mathrm{SO}(m)_D$ gauge interaction in addition to the gravitational force, and being more sparsely distributed than would be the case for lower-mass dark matter candidate states, the self-interactions between these dark glueball-based states would also be relatively insignificant, other than in the earliest stages of the evolution of the universe. Since a high degree of self-interaction should imply a contraction of expanses of dark matter to smaller volumes, this is again consistent with the observational constraints on the nature of dark matter as distributed in giant haloes far larger in extent than visible galaxies. On the other hand there are features of relatively short-scale structure that might be accounted for by some degree of self-interaction, such as associated for example with dark glueball-like states of a hidden confining non-Abelian gauge theory ([52] section VI.D).

A full classical gravitational interaction between the Standard Model branch described for tables 1 and 2 and the dark matter branch of equation 22 and table 3 can be readily identified. This is established through the common external 4-dimensional spacetime geometry with the same local root of equation 14 and as constructed on an extended scale as described for equation 20 and its generalisation to incorporate all matter component contributions as discussed in section 3 ([8] section 6). Given that the common local 4-dimensional spacetime components $\boldsymbol{\delta x}_4$ have been associated with the Higgs as discussed for table 2, there may be the potential for some form of 'Higgs-portal' interaction between the two sectors (as suggested in [8] section 6). This might tentatively provide the most plausible means of detecting this dark matter candidate in a terrestrial experiment.

Any further mathematically possible branches of equation 15 sharing this common 4-dimensional spacetime root will give a further dark gravitational impact on the visible matter in our universe, as we consider in the following sections. In section 7 we shall return to further develop the conception of dark matter as realised in this theory. In particular there we shall reassess the likelihood of any 'portal'-like or other non-gravitational interactions between dark matter and visible matter in the context of the possible interactions between each of these sectors and a dark energy candidate.

## 5 Basis for a Dark Energy Sector

In the previous section we described how rewriting the original 4-dimensional quadratic spacetime form of equation 14 in the $2 \times 2$ matrix form of equation 21 allowed a natural augmentation of the form for proper time to a cubic $3 \times 3$ matrix expression with a $\hat{G} = \mathrm{SL}(3, \mathbb{C})$ symmetry, leading on directly to higher-dimensional forms with exceptional Lie group symmetries as listed in table 1. However, the $2\times2 \to 3\times3$ matrix structure extension could also be further augmented to the $p^{\mathrm{th}}$-order determinant of $p \times p$ complex Hermitian matrices for any integer $p > 3$:

$$(\delta s)^p = \det(\boldsymbol{\delta x}_{p^2}) \quad \text{with} \quad \boldsymbol{\delta x}_{p^2} \in \mathrm{h}_p\mathbb{C} \tag{23}$$

with a full $\hat{G} = \mathrm{SL}(p, \mathbb{C})$ symmetry acting upon the $n = p^2$ real components of $\boldsymbol{\delta x}_{p^2}$, as consistent with the generalised form for proper time in equation 15. The extraction of the external 4-dimensional spacetime components $\boldsymbol{\delta x}_4$, in common with the other branches as described for tables 2 and 3, in this case results in the symmetry breaking pattern listed in table 4, as can be taken to the $p \to \infty$ limit.

| $p^2$ \ $\mathrm{SL}(p,\mathbb{C}) \supset$ | Lorentz | $\times$ | $\mathrm{SL}(k,\mathbb{C})_D$ | $\times$ $\mathrm{U}(1)_D$ | matter |
|---|---|---|---|---|---|
| 4 | 4-vector | | invariant | 0 | $\boldsymbol{\delta x}_4$ external |
| $k^2$ $(k = p-2)$ | scalar | | $\boldsymbol{\delta x}_{k^2} \to S_k \boldsymbol{\delta x}_{k^2} S_k^\dagger$ | 0 | $\boldsymbol{\delta x}_{k^2}$ dark |
| $4k$ | Weyl | | $\boldsymbol{\delta x}_{4k} \to S_k \boldsymbol{\delta x}_{4k}$ | 1 | $\boldsymbol{\delta x}_{4k}$ dark |

Table 4: The breaking of the full $\hat{G} = \mathrm{SL}(p, \mathbb{C})$ symmetry of the branch of generalised proper time in equation 23 over the external $\boldsymbol{\delta x}_4$ components. The resulting basis for matter states includes a set of $k^2$ scalars (with $k = p - 2$), transforming as the matrix $\boldsymbol{\delta x}_{k^2} \in \mathrm{h}_k\mathbb{C}$ under determinant-preserving internal $S_k \in \mathrm{SL}(k, \mathbb{C})_D$ actions and neutral under $\mathrm{U}(1)_D$, together with a set of $k$ 2-component complex Weyl spinors, transforming under the standard representation of $\mathrm{SL}(k, \mathbb{C})_D$ acting on the $k \times 2$ complex matrix $\boldsymbol{\delta x}_{4k}$ and charged under the internal dark $\mathrm{U}(1)_D$. The non-compact nature of the gauge group $\mathrm{SL}(k, \mathbb{C})_D$ supplies the link with dark energy models, as originally suggested in ([8] section 7) and analysed in detail in this paper.

At first sight, in lacking any internal gauge interactions in common with the visible sector described for tables 1 and 2, that is in being independent of the internal $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ associated with the Standard Model, this new branch of table 4 might appear to present a further dark matter sector in addition to that of table 3. However, as a new feature in table 4 there is a non-Abelian internal gauge symmetry $\mathrm{SL}(k, \mathbb{C})_D$ that is *non-compact* for $k = p - 2 \geq 2$.

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In order to assess the significance of this feature we first review the simpler case of an Abelian gauge theory, such as associated with the electromagnetic $\mathrm{U}(1)_Q$ symmetry of table 2 or even the dark $\mathrm{U}(1)_D$ of table 4 (as *would* be related under a restriction to the common $\mathrm{h}_3\mathbb{C}$ substructure of table 1 and equation 23 for the $p = 3$ case). The field strength $F_{\mu\nu}(x)$ is a simple function of the U(1) gauge field $A_\mu(x)$, with kinetic energy Lagrangian $\mathcal{L}_\mathrm{M}$ and corresponding energy-momentum tensor $T^{\mu\nu}_\mathrm{M}(x)$ respectively of the form (the subscript 'M' for Maxwell acknowledges the link with electromagnetism, while $\mu, \nu, \rho, \sigma$ are 4-dimensional spacetime coordinate indices):

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{24}$$

$$\mathcal{L}_\mathrm{M} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{25}$$

$$T^{\mu\nu}_\mathrm{M} = +F^\mu{}_\rho F^{\rho\nu} + \frac{1}{4} g^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \tag{26}$$

This expression for $T^{\mu\nu}_\mathrm{M}(x)$ can be obtained for example by substituting the 'matter' Lagrangian of equation 25 into the Einstein-Hilbert action of equation 4 for general relativity ([9] equation 3.105). That the energy density component $T^{00}_\mathrm{M} = \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2)$, in terms of electric $\boldsymbol{E}$ and magnetic $\boldsymbol{B}$ 3-vector fields in a local inertial reference frame (with field strength components $F_{0i} = E_i$ and $F_{ij} = -\varepsilon_{ijk} B_k$, where $i, j, k$ are spatial component indices, [9] equation 5.23), is always non-negative justifies the minus sign for the kinetic energy Lagrangian term in equation 25.

In particular, in the relativistic theory, in considering the spatial components of the gauge field $A_i(x)$ ($i = 1, 2, 3$) to contain the physical degrees of freedom of the electromagnetic field and with the metric signature for $\eta_{ab} = \mathrm{diag}(+1, -1, -1, -1)$ adopted in equation 14, the minus sign in equation 25 corresponds to positive definite kinetic energy in the $\dot{A}_i = \partial_0 A_i$ terms, that is: $\mathcal{L}_\mathrm{M} = -\frac{1}{2}\partial_0 A_i \partial^0 A^i = -\frac{1}{2}\dot{A}_i \dot{A}^i \geq 0$. Equating equation 26 with equation 5 for any of the $T_{\mathrm{M}}{}^i{}_i$ (no sum over $i = 1, 2$ or 3) and the $T_{\mathrm{M}}{}^0{}_0$ components in the context of FLRW models effectively leads to:

$$\begin{aligned} p_r &\equiv -\tfrac{1}{6}(\dot{A}_i)^2 \geq 0 \\ \rho_r &= -\tfrac{1}{2}(\dot{A}_i)^2 \geq 0 \end{aligned} \tag{27}$$

with $(\dot{A}_i)^2 = \dot{A}_i \dot{A}^i$ (summing over $i = 1, 2, 3$), and hence with both $p_r$ and $\rho_r$ positive. This corresponds to an equation of state with $w_r = p_r/\rho_r = +\frac{1}{3}$ in equation 8 as consistent with the relativistic case of radiation pressure $p_r$ in relation to the corresponding energy density $\rho_r$, as alluded to in the discussion following equation 8.

This analysis for an Abelian gauge theory, leading from equation 25 to equations 27, is very similar to that for a single scalar field leading from the Lagrangian of equation 9, with emphasis on the time derivatives of $\phi(x)$, to equations 11. Before considering the case for a non-Abelian gauge field, we first describe a generalisation for the scalar field case. For an $n_\phi$-component real scalar field $\phi^a(x)$, with $a = 1, \ldots, n_\phi$, the kinetic energy term in equation 9 can be directly generalised to:

$$\mathcal{L}_{\phi^a} = +\frac{1}{2} D_{ab}\, \partial_\mu \phi^a\, \partial^\mu \phi^b \tag{28}$$

Through a choice of an appropriate linear combination and normalisation of the $\phi^a(x)$ field components the matrix $D_{ab}$ can be diagonalised with the diagonal elements taking values $\pm 1$. However, the case with any diagonal element $D_{aa} = -1$ is generally avoided as it implies that the kinetic energy does not have a lower bound. The overall case with matrix $D_{ab} = +\delta_{ab}$ is then usually a required condition to construct a consistent theory. On the other hand, the seemingly pathological case of negative kinetic energy is precisely the situation adopted for models of phantom dark energy involving a single scalar field, via the 'wrong-sign' kinetic term introduced in equation 12.

For non-Abelian gauge theory, in generalising from the above Abelian U(1) case, there are necessarily now a set of gauge field components $A^{\alpha}{}_{\mu}(x)$ with $\alpha = 1, \ldots, n_G$, where $n_G$ is the dimension of the Lie algebra. The corresponding standard generalisation from equations 24–26 for the field strength $F^{\alpha}{}_{\mu\nu}(x)$, Lagrangian $\mathcal{L}_{\rm YM}$ and energy-momentum $T^{\mu\nu}_{\rm YM}(x)$ are respectively (the subscript 'YM' denotes Yang-Mills, while $\alpha, \beta, \gamma$ are indices in the Lie algebra and $c^{\alpha}{}_{\beta\gamma}$ the associated structure constants):

$$F^{\alpha}{}_{\mu\nu} = \partial_\mu A^{\alpha}{}_{\nu} - \partial_\nu A^{\alpha}{}_{\mu} + c^{\alpha}{}_{\beta\gamma} A^{\beta}{}_{\mu} A^{\gamma}{}_{\nu} \tag{29}$$

$$\mathcal{L}_{\rm YM} = -\frac{1}{4} F^{\alpha}{}_{\mu\nu} F_{\alpha}{}^{\mu\nu} = -\frac{1}{4} K_{\alpha\beta} F^{\alpha}{}_{\mu\nu} F^{\beta\mu\nu} \tag{30}$$

$$T^{\mu\nu}_{\rm YM} = +F^{\alpha\mu}{}_{\rho} F_{\alpha}{}^{\rho\nu} + \frac{1}{4} g^{\mu\nu} F^{\alpha}{}_{\rho\sigma} F_{\alpha}{}^{\rho\sigma} \tag{31}$$

In particular equation 30 is considered the appropriate gauge and Lorentz invariant generalisation from the Maxwell Lagrangian of equation 25 for Yang-Mills gauge theory. This scalar Lagrangian is also invariant under general coordinate transformations in general relativity. The above energy-momentum can again be obtained from this Lagrangian via the Einstein-Hilbert action of equation 4 ([35] equation 40).

For a compact gauge group, such as $SU(3)_c$ or $SU(2)_L$ in the Standard Model, employing the usual convention in particle physics with a basis of Hermitian Lie algebra elements, the Killing metric $K_{\alpha\beta}$ in the kinetic energy term of equation 30 is positive definite. A Lie algebra basis for the compact case can then be chosen in which the Killing metric takes the diagonal form $K_{\alpha\beta} = +\delta_{\alpha\beta}$. Hence in the sum over the Lie algebra indices in equation 30 each term in the time derivatives of the non-Abelian gauge field spatial components $\dot{A}^{\alpha}{}_{i}$ makes a non-negative contribution to the kinetic energy, similarly as described for equation 25 for the Abelian case and again justifying the minus sign in equation 30. The role of the positive definite Killing metric $K_{\alpha\beta}$, and the corresponding requirement of a *compact* gauge group, is then closely analogous to the requirement of a positive definite matrix $D_{ab}$ for the scalar field case of equation 28, in terms of constructing a consistent theory.

For a non-compact gauge symmetry, such as $SL(k, \mathbb{C})_D$ in table 4, by definition the 'non-compact' Killing metric of the Lie algebra in equation 30 is *indefinite*. That is, Lie algebra bases can be found with each element $K_{\alpha\beta} = \pm\delta_{\alpha\beta}$ but there is no basis in which as a whole $K_{\alpha\beta} = +\delta_{\alpha\beta}$. This implies an inevitable source of negative kinetic energy without a lower bound and a seeming instability of the vacuum in the corresponding quantum field theory (see for example [53] discussion of equation 1.1). For this reason non-compact gauge symmetry groups are generally avoided and this branch of generalised proper time of equation 23 and table 4 would seem to be problematic or simply non-physical.

However, a source of negative kinetic energy is a central ingredient in models for 'phantom dark energy' [15, 16, 17] as reviewed in section 2 for equations 12 and 13 and recalled above after equation 28. In such models a 'wrong-sign' kinetic term $\mathcal{L}_{\phi_-} = -\frac{1}{2}\partial_\mu\phi_-\partial^\mu\phi_-$, together with a suitable potential term $V(\phi_-)$, is proposed for a postulated scalar field $\phi_-(x)$. In this paper we are then considering whether the negative kinetic energy contribution implicit for a non-compact gauge group might also be related to the dark energy sector.

Here $A_-(x)$ will denote the gauge field component with negative kinetic energy, as associated with all diagonal Killing metric elements with $K_{\alpha\alpha} = -1$, while $A_+(x)$ will denote the remaining positive kinetic energy contribution, as associated with generators of the Lie algebra with corresponding $K_{\alpha\alpha} = +1$ components in equation 30. (More generally, '+' or '−' subscripts on fields in this paper will always denote positive or negative kinetic energy respectively). In the following discussion we make an initial assessment of any possible connection between the $A_-(x)$ field and a dark energy candidate, focussing on new features that these negative kinetic energy components introduce into this gauge sector.

The Lagrangian term of equation 30 that contributes the source of positive and negative kinetic energy for the $A_\pm(x)$ fields of a non-compact gauge group also appears as a factor in the $g^{\mu\nu}(x)$ term for the corresponding energy-momentum tensor of equation 31. On assuming this energy-momentum can be modelled by the form of a general perfect fluid in equation 5, and taking care of a consistent set of sign conventions, the gauge field strength in the $g^{\mu\nu}(x)$ term of equation 31 makes a *contribution* to the pressure $\not{p}_{A_\pm} = -\frac{1}{4}F^\alpha{}_{\rho\sigma}F_\alpha{}^{\rho\sigma}$ (with the 'slash' indicating that this is an incomplete assignment, from only *part* of the energy-momentum). Hence any negative contribution to the kinetic energy Lagrangian term in equation 30, as arises for a non-compact gauge group, also makes a *negative* contribution to the pressure, with $\not{p}_{A_-} \leq 0$, which is the key feature required for dark energy. This is a significant observation in terms of the potential for the structures of table 4 to relate to the dark energy sector.

We can also note that for a non-compact gauge group the 'wrong-sign' kinetic energy part from equation 30 when substituted into the $g^{\mu\nu}(x)$ term in equation 31 has the 'right-sign' to describe a positive cosmological constant in the Einstein field equation 3, alongside other matter term contributions to $T^{\mu\nu}(x)$. That is, if such a term by itself for the $A_-(x)$ field were sufficiently uniform in cosmological space and constant through cosmic time within an FLRW model it could mimic the role of a positive cosmological constant $\Lambda$ in equation 3 and underlie the observed acceleration of the universe with an equation of state parameter $w = -1$.

This suggests a somewhat different mechanism to that in phantom dark energy models. As can be seen from equations 13 for phantom models it is the $V(\phi_-)$ potential term that acts as the main source of dark energy, with the negative kinetic energy of the $\phi_-(x)$ field employed to achieve an equation of state with $w_{\phi_-} < -1$ (similarly as $w_\phi > -1$ is obtained for positive kinetic energy in quintessence models, as seen for equations 11). By contrast for the present theory with a non-compact gauge symmetry it is the negative pressure directly associated with the negative kinetic energy of the $A_-(x)$ contribution itself, involving the $g^{\mu\nu}(x)$ term in equation 31, that is proposed to *drive* a dark energy effect, with the other fields and terms determining the overall equation of state and any relation to the $w = -1$ cosmological constant case.

Hence here one question will be how closely a negative kinetic energy contribution from equation 30 can be a factor in generating the case $w = -1$ in the context of the ΛCDM cosmological model. This conclusion is based on analysis of the $g^{\mu\nu}(x)$ term *alone* in equation 31, for which we have the part contributions:

$$\begin{aligned} -p\!\!\!/_{A_\pm} &= +\rho\!\!\!/_{A_\pm} = +\Lambda_{A_\pm} \equiv +\frac{1}{4}F^{\alpha}_{\ \mu\nu}F_{\alpha}^{\ \mu\nu} \\ \text{with} \quad &+\frac{1}{4}F^{\alpha}_{\ \mu\nu}F_{\alpha}^{\ \mu\nu} \geq 0 \quad \text{for } A_-(x) \text{ fields} \\ \text{and} \quad &+\frac{1}{4}F^{\alpha}_{\ \mu\nu}F_{\alpha}^{\ \mu\nu} \leq 0 \quad \text{for } A_+(x) \text{ fields} \end{aligned} \tag{32}$$

However, this implies an apparent *positive* energy density $\rho\!\!\!/_{A_-} > 0$ associated with the $A_-(x)$ component of this non-compact gauge sector, seemingly in contradiction with the assumption here that the associated contribution to the Lagrangian term in equation 30 is acting as a source of *negative* kinetic energy. Hence while the second term, proportional to $g^{\mu\nu}(x)$, in equation 31 for a non-compact gauge group might provide a very direct source for a positive cosmological constant this can not represent the full picture.

This inconsistency is resolved through the need to take into account *both* quadratic terms in the field strength in equation 31, and not the latter only. The kinetic energy term in equation 30 has a similar form as for the case of electromagnetism in equation 25. In considering the kinetic terms in the gauge fields for the non-Abelian case we can again focus upon the time derivatives of the spatial components $\dot{A}_{\pm\ i}^{\ \alpha}$, as described after equation 31, by analogy with the $\dot{\phi}$ contribution in the scalar field models as described for equations 9–13 and similarly as for the Abelian case for $\dot{A}_i$ in equations 24–27. The direct generalisation from equations 27 implies:

$$\begin{aligned} p_{A_\pm} &\equiv -\tfrac{1}{6}(\dot{A}_{\pm\ i}^{\ \alpha})^2 && \leq 0 \ \text{ for } A_- \text{ fields} \quad (\geq 0 \ \text{ for } A_+ \text{ fields}) \\ \rho_{A_\pm} &= -\tfrac{1}{2}(\dot{A}_{\pm\ i}^{\ \alpha})^2 && \leq 0 \ \text{ for } A_- \text{ fields} \quad (\geq 0 \ \text{ for } A_+ \text{ fields}) \end{aligned} \tag{33}$$

where $(\dot{A}_{\pm\ i}^{\ \alpha})^2 = \dot{A}_{\pm\ i}^{\ \alpha}\dot{A}_{\pm\alpha}^{\ \ i} = K_{\alpha\beta}\dot{A}_{\pm\ i}^{\ \alpha}\dot{A}_{\pm}^{\ \beta i}$ (summing over the relevant Lie algebra $\alpha, \beta$ indices, with the minus sign of the $K_{\alpha\alpha} = -1$ components associated with the $A_-$ states cancelling the minus sign from the contraction over the $i = 1, 2, 3$ spatial components within the metric convention). For the $p_{A_\pm} = -T_{\mathrm{YM}\ 1}^{\ \ 1}$ component from equation 31 the first term provides a contribution of opposite sign, but smaller in magnitude, to the second. However, for the $\rho_{A_\pm} = T_{\mathrm{YM}\ 0}^{\ \ 0}$ component the first term provides a contribution of opposite sign and twice the size of the second; hence flipping the net sign for the energy density in comparison with equation 32.

This then indeed implies an overall *negative* energy density $\rho_{A_-} < 0$, as well as negative pressure $p_{A_-} < 0$, consistent with the negative kinetic energy assumption for the $A_-(x)$ field. For a non-compact gauge group the source of negative kinetic energy from the $A_-(x)$ component could then in *itself* through equation 7 underlie a significant acceleration in the expansion of the universe, although *not* of a form consistent with a cosmological constant term. Indeed, a negative $\rho_{A_-}$ as the dominant component alone would be inconsistent with the observational demand for a positive dark energy density contribution to account for the spatial flatness of large-scale cosmic structure as reviewed in section 2. However, there are further contributions to both the pressure and energy density for this sector, including of course from the $A_+(x)$ field, that will also have an impact.

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While further analysis will be described in the following section, there are clearly both similarities and a range of significant differences with phantom dark energy models based on a postulated new scalar field. Rather than the *negative definite* kinetic energy term for the scalar field $\phi_{\text{-}}(x)$ in equation 12 here for the non-compact gauge field the kinetic energy term of equation 30 is *indefinite*, that is with both negative and positive energy contributions from the $A_{\text{-}}(x)$ and $A_{+}(x)$ fields to take into account. There will also be further matter field contributions to $T^{\mu\nu}(x)$ deriving from this branch of generalised proper time based on the symmetry breaking structures listed in table 4, as will also be significant for the development of the theory in the next section.

Here the theory is rooted in the elementary underlying concept of the local generalisation of proper time in equation 15, with the connections to physics following from a very direct analysis of the mathematical structures arising from the symmetry breaking in extracting the local external 4-dimensional spacetime form in equations 16 and 17. This is the case for the Standard Model branch of table 2 and dark matter branch of table 3 described in the previous section, as also for the proposed dark energy branch of table 4. That is, there is no reliance on a *postulated* field such as $\phi_{\text{-}}(x)$ or an *invented* Lagrangian such as equation 12 as there is in the phantom toy models, rather here the non-compact gauge symmetry $\mathrm{SL}(k,\mathbb{C})_D$ and its relation to the external spacetime geometry arises as an inherent feature of the theory based on equation 23 and table 4. An issue with scalar field models based on equation 9 or 12 is that the potential function $V$ is essentially arbitrary, whereas for the present theory the dynamics is driven by kinetic terms that take a well-defined form.

Further, unlike the contrived situation of a postulated scalar field $\phi_{\text{-}}(x)$ and specific Lagrangian function of equation 12 for the phantom models, here an *intrinsic* geometric relation can be identified between the external spacetime curvature and internal gauge field curvature in the form of equation 20. That is, rather than introducing the Lagrangian kinetic energy term of equation 30, this field strength term for the non-Abelian gauge field arises in the geometry of the principal bundle $P \equiv M_4 \times G$ as a perturbation to the vacuum case of the 4-dimensional spacetime Einstein-Hilbert action of equation 4 as reviewed for equations 18 and 19. This scalar invariant term $F^{\alpha}_{\ \mu\nu}(x)F_{\alpha}^{\ \mu\nu}(x)$ can hence be afforded a purely geometric interpretation, rather than necessarily considered to represent 'kinetic energy'.

The energy-momentum tensor $T^{\mu\nu}(x)$ for the gauge field is *defined* through equation 20, as consistent with the Einstein field equation 3 of general relativity, similarly as for non-Abelian Kaluza-Klein theories as reviewed in section 3. Hence a *direct* geometric relation between the external spacetime curvature and internal gauge curvature can be identified in equation 20 consistent with equation 31 but *without* considering a specific form for energy-momentum to act as an apparent *intermediary*. The consequences of the non-compact gauge symmetry in table 4 will then be intimately tied to the structure of 4-dimensional spacetime itself. This construction is then compatible with the notion of dark energy as associated with a property of space itself, as alluded to shortly after equation 8 in section 2.

To end this section we note some further connections between this new approach and variations of non-Abelian Kaluza-Klein models. Early attempts to construct a non-Abelian Kaluza-Klein theory with a compact gauge group led to a problematically very large cosmological constant. While the construction of the principal fibre bundle $P \equiv M_4 \times G$ utilised for that theory (see for example [54] figure 1) is very similar to

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that for the present theory as described following equation 17, this problem arose from the adoption of the unique Levi-Civita torsion-free linear connection defined on the bundle space $P$.

This results in a further modification to the perturbed scalar curvature $\tilde{R}(x)$ of equations 18 and 19 defined on the base space $M_4$ with a further contribution $R_G$, the scalar curvature of the gauge group manifold itself, appended to the right-hand side ([54] equations 17 and 24). In the Einstein-Hilbert action of equation 4 this additional $R_G$ term itself acts directly as a cosmological constant $\Lambda$ (again with attention required to ensure a consistent set of sign conventions for the geometric and group structures involved). Assuming Planck length dimensions for the group space in the absence of any argument to the contrary, this source for $\Lambda$ in the Einstein field equation 3 would be too large by a factor of around $10^{120}$ ([54] section 9).

Subsequent developments, allowing a linear connection with torsion to be defined on the principal bundle space were constructed, and to some degree motivated, to have the property $R_G = 0$ and hence remove this cosmological term altogether (see for example [55, 56]). These variants of non-Abelian Kaluza-Klein theory in turn have guided the construction and geometric properties of the principal bundle space utilised in the present theory, as described in section 3 in leading to equations 18, 19 and 20 (see [35] table 2 and discussion for details). These equations hence contain no explicit cosmological constant term, whether for a compact or non-compact gauge group.

Also in the 1980s, another means of removing the problematic large cosmological constant in non-Abelian Kaluza-Klein theory proposed employing a non-compact internal space [57]. In this case it is possible to achieve $\Lambda = 0$ or a continuous spectrum of solutions, including extremely small arbitrary non-zero values for $\Lambda$. However, with solutions corresponding to saddle-points rather than minima in the internal scalar potential, which is hence unbounded below, this construction comes at the expense of an apparent instability. A similar result for $\Lambda$ could be achieved on allowing some extra dimensions to be timelike rather than spacelike [58], although with the need to deal with ghost states and the potential further issue of causality violation. Several more recent studies have indicated that comparable systems with negative energy ghost states are not necessarily unstable [59, 60].

These models with a non-compact internal space or timelike extra dimensions are analogous to the case with a non-compact internal gauge group, as associated with indefinite kinetic energy, arising in the present theory as described in this section. Since here the internal space of the Lie group is at no point interpreted as a 'physical space' (see discussion in [35] sections 4 and 5.1) there is no conceptual problem regarding causality (see also [5]). However, by incorporating a source of negative kinetic energy unbounded below, there is very much an issue with stability that will need to be addressed for this new theory.

The above discussion of equations 32 and 33 has, through comparison of features common to models with a cosmological constant or a form of phantom dark energy, provisionally described how the non-compact gauge group in this sector of generalised proper time of equation 23 and table 4 might plausibly underlie the accelerating expansion of the cosmos. In particular, the non-compact gauge group provides a seemingly inevitable source of negative pressure as a crucial element required for dark energy. Here the differences in the construction and features of this theory in comparison with phantom dark energy, and other models reviewed in section 2 and

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above, will be key to addressing the question of stability. This will be elucidated in the following section where the microscopic structure of the dark energy source proposed, and its potential relation to a cosmological constant, will be investigated further.

## 6 Physical Structure of the Dark Energy

In the overall framework for the theory presented here the constraints implied in constructing matter in an *extended* 4-dimensional spacetime via the *local* structure of equations 16 and 17 replace the standard role of Lagrangian terms ([61] section 5.1). Rather than *beginning* with an extended 4-dimensional spacetime manifold $M_4$, such as in the centre of figure 1, here such an arena is *built up* from the local elements of spacetime geometry and matter fields that are identified from the $\boldsymbol{\delta x}_n$ components in equations 15 and 16, under the broken symmetry of equation 17, as a substructure of the generalised form for proper time intervals [5]. The energy-momentum is defined through equation 20, and its generalisation that includes contributions from matter fields as well as gauge fields, as reviewed at the end of section 3 (see also [61] equations 30, 31, 33 and 34 and discussion). There is an ambiguity, or local degeneracy, in the matter and gauge field components underlying the same local spacetime geometry, as constructed within these constraints.

This local degeneracy is proposed to underlie the interactions between the corresponding states of matter and gauge fields and the observed quantum indeterminacy in their behaviour. While classical probabilities are proportional to the 'number of ways' something can happen *in* spacetime, here quantum probabilities are proportional to the 'number of ways' a single 4-dimensional spacetime *itself*, with its associated matter composition, can be constructed. This construction is the key to identifying a framework unifying gravitation with quantum phenomena, with gravity itself remaining 'unquantised' in this proposed approach to a 'quantum gravity' theory ([61], [5] section 4).

The gravitational field and extended 4-dimensional spacetime are hence smooth and continuous, with only the matter and gauge fields 'quantised' through this construction of an extended spacetime geometry. A standard formalism for quantum field theory (QFT) is to be recovered in the limiting approximation of a flat spacetime background. QFT as employed for example in calculations of Standard Model phenomena observed in the laboratory is considered a low-energy effective theory, as a limiting approximation to the fundamental origin of the physics in the construction of matter in 4-dimensional spacetime from a basis in generalised proper time, with the Standard Model particle content obtained as reviewed in the discussion of table 2 in section 4. The full derivation of quantum properties more generally will be needed to determine more completely the physical implications of the theory, incorporating the detailed assessment of the visible matter branch of table 1 as well as the dark matter branch deriving from equation 22.

This full picture may also be needed to fully assess the suitability of the branch of equation 23 and table 4 to account for the dark energy sector. The quantitative nature of dark energy for this branch and any interpretation in terms of a 'negative kinetic energy' component as provisionally suggested from the Lagrangian perspective of equation 30, might then be elucidated. A determination will be required of the large-scale average effective $p_V$ and $\rho_V$ 'vacuum' contribution terms in equation 5 for this sector and their impact in the context of an FLRW cosmological model such as described in section 2. The corresponding effective cosmological equation of state $p_V = w_V \rho_V$, as potentially underlying the cosmic acceleration and its relation to ΛCDM, might then be fully assessed. We explore some of the key features that might determine this state in this section.

With the effective 'vacuum' corresponding to the lowest *net* energy state any determination of $p_V$ and $\rho_V$ should incorporate both the positive $A_+(x)$ and negative $A_-(x)$ field contributions for the non-compact non-Abelian gauge symmetry case described for equations 33. Those equations only involved the time derivatives of the spatial components of the gauge fields. However, in addition to possible contributions from spatial derivatives the situation is complicated in the non-Abelian case by terms of cubic and quartic order in the gauge fields in equations 30 and 31 arising from the final term in equation 29 for the field strength.

These cubic and quartic contributions hence imply a significant degree of *self-interaction* for the non-Abelian gauge field from an apparent 'kinetic energy' term. Now including non-derivative terms in the gauge fields, involving the structure constants of the Lie algebra in equation 29, these additional contributions may in general be of indefinite sign even for the case of a compact non-Abelian gauge symmetry. Nevertheless, as a gauge and Lorentz invariant scalar equation 30 has a consistent form to act as a Lagrangian term and also arises naturally in the study of Riemannian geometry on a principal fibre bundle in leading via equations 18 and 19 to equation 20 as described in section 3 and emphasised towards the end of the previous section. This is an example of an intrinsic constraint on the relation between the external spacetime geometry and the structure of matter arising directly *without* the need to introduce an intermediary energy-momentum tensor or specific Lagrangian term in equation 3 or 4 respectively, as described in the opening of this section.

The problem of stability, also discussed at the end of section 5, arises for phantom dark energy models with negative kinetic energy unbounded below, since *any* form of local interaction between phantom states and the positive energy states of visible matter implies a catastrophic unlimited decay of the vacuum. Various models of phantom dark energy seek to avoid or suppress this pathology of vacuum decay which, as typically presumed, could be mediated by 'graviton' exchange between phantom and Standard Model states via the process depicted in figure 2(a) below.

On the one hand, in the present theory with gravity itself remaining unquantised, as reviewed above, there are *no gravitons* to mediate this form of instability via a decay of the vacuum into negative energy $A_-$ states and the visible states of matter. That is, without the possibility of even graviton exchange this dark energy sector, deriving from table 4, may be *sealed off* from the visible matter sector, associated with table 2, other than through the classical gravitational interaction via the curvature of the common external 4-dimensional spacetime as essentially fully consistent with general relativity.

On the other hand, the present theory seems potentially even more unstable than the phantom models since the indefinite Killing metric for the non-compact gauge symmetry, as discussed for equation 30, implies the existence of *both* positive energy gauge bosons $A_+$, associated with each $K_{\alpha\alpha} = +1$ element, as well as the negative

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energy $A_-$ gauge bosons, associated with the $K_{\alpha\alpha} = -1$ elements, that can *mutually* interact via the self-interactions described by equations 29–31. This implies a *limitless* energy-momentum conserving generation of these $A_+$ and $A_-$ states out of the vacuum, via processes such as depicted in figure 2(b), from within this proposed dark energy sector alone.

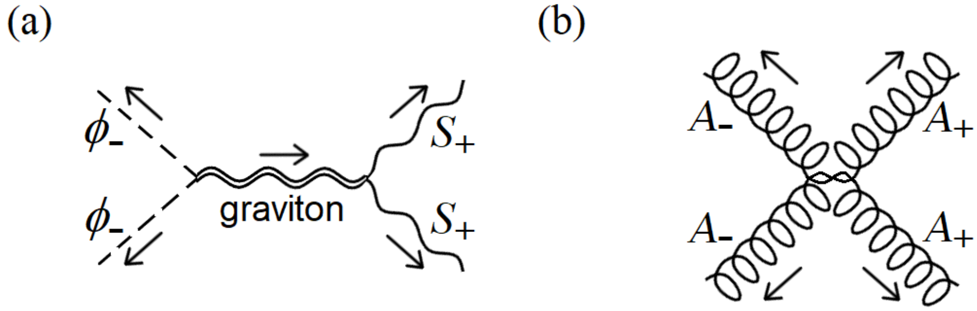


Figure 2: (a) Production of negative energy phantom model states, denoted $\phi_-$, together with positive energy Standard Model states such as photons, denoted $S_+$, out of the vacuum via graviton coupling and conserving total energy-momentum (see for example [18] figure 1). (b) Production of negative energy $A_-$ and positive energy $A_+$ gauge bosons of a non-compact gauge symmetry out of the vacuum, as mediated directly through the self-interactions of the non-Abelian gauge sector. As distinct from more typical Feynman diagrams here all arrows indicate the forward direction of time and the creation of both positive and (literally) negative energy states.

However, the spontaneous creation of these states throughout spacetime, via processes such as that in figure 2(b), would result in a *sea* of $A_+$ and $A_-$ gauge bosons and a degree of *mutual annihilation* would inevitably ensue, removing some of the positive and negative energy contributions. With more and more positive and negative energy taken out of the system through this annihilation as the magnitude of the respective density contributions $\rho_{A_+}$ and $\rho_{A_-}$ increases, and assuming a constant rate of creation, a stable equilibrium might be rapidly attained through the cubic and quartic self-couplings. The nature of the *indefinite* kinetic energy and the *self-interactions* within this dark energy sector, together with the lack of coupling to the visible sector, then leads to a means of addressing the stability issue that is rather different to that for phantom toy models, and as represented in figure 4(a) below.

The construction of equations 33 with an implied equation of state parameter $w_{A_+} = p_{A_+}/\rho_{A_+} = w_{A_-} = p_{A_-}/\rho_{A_-} = +\frac{1}{3}$ clearly does not represent the whole story. Only the time derivatives of the gauge fields were there included and the interactions within and between these $A_+$ and $A_-$ contributions from the cubic and quartic terms of equation 30 were neglected. However, by the construction in equations 29–31 for the general case there is a *symmetry* between the positive energy $K_{\alpha\alpha} = +1$ and negative energy $K_{\alpha\alpha} = -1$ contributions. This symmetry suggests that the corresponding positive and negative pressure contributions, $p_{A_+} = w_{A_+}\rho_{A_+}$ and $p_{A_-} = w_{A_-}\rho_{A_-}$ respectively, would be equivalent and complementary, individually satisfying the same *effective* equation of state for which $w_{A_+} = w_{A_-}$ can be assumed. An equilibrium ground state for figure 4(a) could then be achieved with $p_{A_-} = -p_{A_+}$ and $\rho_{A_-} = -\rho_{A_+}$. That is, by the symmetry of this construction the net impact would be expected to cancel, hence with total pressure and energy density for the vacuum state of this non-compact gauge sector:

$$\begin{aligned} p_V &= p_{A_+} + p_{A_-} = 0 \\ &\equiv w_{A_+}\rho_{A_+} + w_{A_-}\rho_{A_-} = 0 \\ \text{and} \quad \rho_V &= \rho_{A_+} + \rho_{A_-} = 0 \quad \text{for} \quad w_{A_+} = w_{A_-} \end{aligned} \tag{34}$$

Hence this dark sector might after all be expected to be completely benign, lacking even a net impact upon the classical gravitational field through equations 3 and 5, and remain utterly undetectable from within the visible matter sector of the Standard Model. While it would be possible to achieve a state with net $p_V > 0$ and $\rho_V > 0$ (or $p_V < 0$ and $\rho_V < 0$), corresponding to a net excess of $A_+$ (or $A_-$) states and consistent with $w_{A_+} = w_{A_-}$, in both cases this would describe a state with $w_V = p_V/\rho_V > 0$ and hence the large-scale gravitational impact would not be consistent with the cosmological demands for dark energy.

However, as a further feature for the proposed dark energy sector of generalised proper time the structures arising from the symmetry breaking of equation 23 as described for table 4, while not interacting non-gravitationally with the visible sector of table 2, incorporate further sources of matter in addition to the gauge fields associated with the non-Abelian gauge symmetry $\mathrm{SL}(k,\mathbb{C})_D$ that *do* directly couple to the $A_+(x)$ and $A_-(x)$ fields. These include a set of $k^2$ Lorentz scalar matter fields based on the $\boldsymbol{\delta x}_{k^2}$ components in table 4 and now denoted $\boldsymbol{\phi}(x)$ and a set of $k$ Weyl spinor matter fields based on the $\boldsymbol{\delta x}_{4k}$ components and denoted $\boldsymbol{\psi}(x)$, transforming under the internal symmetries that include the Abelian gauge symmetry $\mathrm{U}(1)_D$ and associated gauge field denoted $\mathring{A}(x)$. Since these further matter fields are all presumed to have positive energy they can be collectively denoted $M_+ = \{\boldsymbol{\phi}_+, \boldsymbol{\psi}_+, \mathring{A}_+\}$. This implies these new states of matter can *also* be created out of the vacuum, via interactions with the negative energy $A_-$ states, as depicted in figure 3.

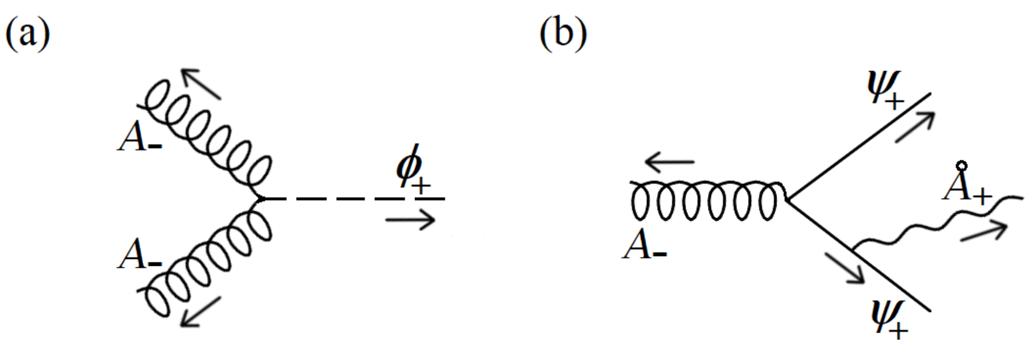


Figure 3: Creation out of the vacuum of negative energy $A_-$ gauge bosons as coupled to positive energy (a) $\boldsymbol{\phi}_+$ and (b) $\boldsymbol{\psi}_+, \mathring{A}_+$ matter states as an additional contribution over figure 2(b). As for figure 2 the arrows indicate forward progression in time and the production of both positive and negative energy states.

The exact nature of the interactions between the $A_-$ and $M_+$ states in figure 3, and of any further interactions in this sector, will depend upon the implications of the constraints as discussed in the opening of this section (see for example [61] discussion of

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equations 58–67 and figure 3 therein). The possibility and rates of the processes such as in figures 3(a) and (b) will depend upon whether the $M_+$ states gain mass and whether they are considered real or virtual in calculations with higher-order interactions. With the reverse annihilation processes again possible between the $A_-$ and $M_+$ states, the question will then concern the net properties of a vacuum state stable equilibrium between the $A_-$ and collective $\{A_+, M_+\}$ states.

The creation events of figures 2(b) and 3, together with the associated annihilation processes between the positive and negative energy states, collectively imply that it is the equilibrium vacuum state not for figure 4(a) but rather for that depicted in figure 4(b) that will generate the empirical phenomena for this dark energy sector. While containing a myriad of real $\{A_-, A_+, M_+\}$ interacting particles this will still be considered a 'vacuum' state in the sense of having the lowest net energy in the context of general relativity.

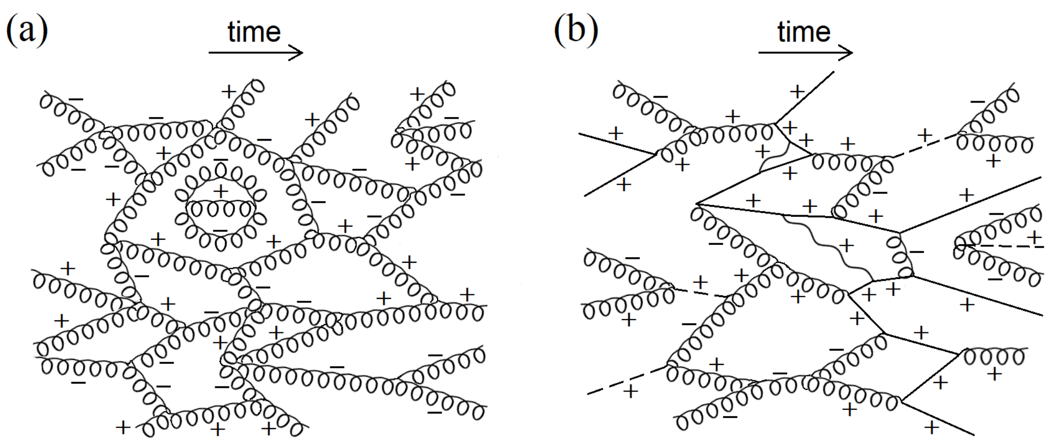


Figure 4: (a) As the dark vacuum is filled by a sea of $A_+$ and $A_-$ gauge bosons (the coiled lines labelled '+' and '−' for the positive and negative energy states), through the processes described for figure 2(b), a stable situation may arise in which the creation of these states is balanced by their mutual annihilation, with net zero pressure and energy density as described for equations 34. (b) On including the further states in this sector $M_+ = \{\boldsymbol{\phi}_+ \text{(dashed lines)}, \boldsymbol{\psi}_+ \text{(straight lines)}, \mathring{A}_+ \text{(wavy lines)}\}$, all of positive energy, a network of interactions, including those of figure 3, might again result in a stable equilibrium vacuum state, but now in principle with an asymmetry in the total energy density and pressure balance for this system as described for equations 35–37. In these figures time flows from left to right for all states.

The $M_+ = \{\boldsymbol{\phi}_+, \boldsymbol{\psi}_+, \mathring{A}_+\}$ matter component with positive energy density will also presumably make a positive contribute to the pressure, similarly as for the $A_+$ states of the non-compact gauge field. The negative contribution to the energy density from the $A_-$ field for the non-compact gauge group $\mathrm{SL}(k, \mathbb{C})_D$ might then be compensated by positive energy contributions from both the $A_+$ and $M_+ = \{\boldsymbol{\phi}_+, \boldsymbol{\psi}_+, \mathring{A}_+\}$ fields in this sector. If a net negative pressure $p_{A_-}$ could be retained from the $A_-$ contribution together with a net positive energy density $\rho_{AM_+}$ from the $\{A_+, M_+\}$ states, the overall impact could be of a dark energy sector consistent with cosmological observations.

This is possible since the effective equation of state parameter $w_{AM_+}$ for the collective positive energy $\{A_+, M_+\}$ states will most likely *differ* from the equation of state parameter $w_{A_+} = w_{A_-}$ for either the $A_+$ or $A_-$ gauge fields described for equations 34. For the highly self-interacting system considered here the situation is clearly very far from that of an 'ideal gas' for which collisions between molecules are purely elastic and the total pressure is a simple linear sum of the partial pressures of the individual gases in the mixture. However, symmetry arguments led to the zero net pressure and energy state described for equations 34, from which we are seeking to introduce a perturbation.

The further contribution from the $M_+ = \{\phi_+, \psi_+, \mathring{A}_+\}$ states provides the source of this perturbation from the symmetry balance of the total pressure and energy density in equations 34. That is, since this new contribution is *only* on the positive energy and pressure side this inevitably introduces an asymmetry in the dynamics and hence for the stable vacuum state of figure 4(b) in general it will *not* be possible to generate both a total pressure $p_V = 0$ and energy density $\rho_V = 0$ as for equations 34, in place of which we *can* now have:

$$\begin{aligned} p_V &= p_{AM_+} + p_{A_-} < 0 \\ &\equiv w_{AM_+}\rho_{AM_+} + w_{A_-}\rho_{A_-} < 0 \\ \underline{\text{and}}\quad \rho_V &= \rho_{AM_+} + \rho_{A_-} > 0 \quad \underline{\text{if}}\quad w_{AM_+} < w_{A_-} \end{aligned} \tag{35}$$

That is, for a suitable effective equation of state parameter $w_{AM_+} < w_{A_-}$ a stable vacuum of $\{A_-, A_+, M_+\}$ states could result with a net excess in *positive* energy density $\rho_V > 0$, coming from the $\{A_+, M_+\}$ states, together with a residual *negative* pressure $p_V < 0$ due to the $A_-$ contribution. Hence the 'dark energy' results from the near, but not complete, cancellation of the components of figure 4(b), in a manner that would not be possible for the $\{A_+, A_-\}$-alone vacuum state of figure 4(a) and equations 34. In this way a net vacuum equation of state can hence be obtained:

$$p_V = w_V \rho_V \quad \text{for} \quad w_V < 0 \tag{36}$$

with negative $p_V$ and positive $\rho_V$, and both in principle arbitrarily small in magnitude. In particular, this can be consistent with the $w_V = -1$ case of an effective cosmological constant $\Lambda$, which results if:

$$\left|\frac{\rho_{AM_+}}{\rho_{A_-}}\right| = \frac{1 + w_{A_-}}{1 + w_{AM_+}} > 1 \qquad (\Rightarrow\ w_V = -1) \tag{37}$$

Similarly as for $w_{A_+}$ and $w_{A_-}$ as described before equations 34, here for equations 35 and 37 the effective equation of state parameters $w_{AM_+}$ and $w_{A_-}$ may not in themselves have well-defined values, or even meaning, since the associated states are components in a mutually interacting system. However, the purpose of these equations is to demonstrate the *asymmetry* between the positive and negative energy components that the $M_+$ states introduce. This perturbs the symmetry of the cancellation in equations 34 such that it is now possible to have net $p_V < 0$ together with net $\rho_V > 0$, and even with $p_V = -\rho_V$, as described for equations 35–37. In fact $w_V < 0$ in equation 36 can be essentially arbitrary and could be compatible not only with the cosmological constant case of $w_V = -1$ but also with the quintessence $w_V > -1$ or phantom $w_V < -1$ scenarios, and even evolve across the divide between them.

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Aspects of the contrast between the dark energy sector of the present theory and the models with positive energy $\phi_{+}(x)$ quintessence and negative energy $\phi_{-}(x)$ phantom scalar fields can be further elaborated on comparing the associated energy-momentum tensors, as modifications to equations 31 and 10 respectively:

$$T^{\mu\nu}_{\mathrm{YM}'} = K_{\alpha\beta}F^{\alpha\mu}{}_{\rho}F^{\beta\rho\nu} + \frac{1}{4}g^{\mu\nu}K_{\alpha\beta}F^{\alpha}{}_{\rho\sigma}F^{\beta\rho\sigma} + f(\boldsymbol{\phi}_{+}, \boldsymbol{\psi}_{+}, \mathring{A}_{+}) \tag{38}$$

$$T^{\mu\nu}_{\phi_\pm} = \pm\partial^\mu\phi_\pm\partial^\nu\phi_\pm \mp \frac{1}{2}\partial_\rho\phi_\pm\partial^\rho\phi_\pm\, g^{\mu\nu} + V(\phi_\pm)g^{\mu\nu} \tag{39}$$

The role of the $M_{+} = \{\boldsymbol{\phi}_{+}, \boldsymbol{\psi}_{+}, \mathring{A}_{+}\}$ fields could be considered analogous to the positive contribution from the potential energy $V(\phi_\pm)$ for the quintessence and phantom toy models as reviewed for equations 9–13, although the overall physics is more complex for the new theory. For this theory the properties of these further matter components identified for this sector of generalised proper time of equation 23 as described for table 4, making the contribution $f(\boldsymbol{\phi}_{+}, \boldsymbol{\psi}_{+}, \mathring{A}_{+})$ to the energy-momentum $T^{\mu\nu}_{\mathrm{YM}'}(x)$ in equation 38 and interacting with the non-Abelian gauge fields $\{A_{+}(x), A_{-}(x)\}$, should also then be fully taken into account.

For quintessence and phantom models the potential term $V(\phi_\pm)$ typically dominates and drives the cosmological acceleration similarly as for a cosmological constant $\Lambda$, with the kinetic terms in equation 39 considered a perturbation to achieve an equation of state with $w_{\phi_+} > -1$ or $w_{\phi_-} < -1$. However, here in equation 38 there are significant contributions of positive kinetic energy, from the $K_{\alpha\alpha} = +1$ components, and negative kinetic energy, $K_{\alpha\alpha} = -1$ components, both together from the non-compact non-Abelian gauge fields $A_{+}(x)$ and $A_{-}(x)$, with the impact of the additional $M_{+}(x)$ fields considered a perturbation with a view of identifying a net solution that may be close to a $\Lambda$ cosmological effect as described for equations 35–37.

The question of stability is a common issue with phantom dark energy models. As discussed by comparison with figure 2(a), for the present theory one significant factor is the potential lack of *any* particle-like interactions between the visible matter and dark energy sectors of generalised proper time. Within this dark energy sector itself the mechanism of stability for the full structure of figure 4(b) comes from the balance between the mutual creation of the $A_{-}$ and $\{A_{+}, M_{+}\}$ states via figures 2(b) and 3 and their mutual annihilation, 'catching' the $A_{-}$ states before the negative energy density contribution can cascade down to an arbitrary unbounded value. Having both positive and negative kinetic energy elements from the matter and non-compact gauge fields, rather than strictly negative as for the phantom scalar models, together with the mutual interactions within this sector, are then major factors in this mechanism to stabilise the negative energy density, and associated negative pressure contribution, in equilibrium with the positive contributions, and prevent a runaway situation of vacuum decay.

As the universe expands a corresponding dilution of the overall density of the states in figure 4(b) would lower the mutual annihilation rate between the negative and positive energy components. However, the creation processes of figures 2(b) and 3, with an ever constant production rate, would immediately replenish any such drop in the density of states, in principle maintaining a uniform net energy density $\rho_V$ and pressure $p_V$ for the vacuum state through the relations of equations 35 and 36. This constant 'fuelling' of the 'plasma-like' vacuum, through creation of real $\{A_{-}, A_{+}, M_{+}\}$

particles, hence implies an essentially 'steady state' as the cosmos expands. Through the creation process no region of spacetime can be devoid of this 'dark vacuum-plasma'. This uniformity is a key feature required in general for dark energy, in particular if taking a form consistent with a cosmological constant term under the condition of equation 37.

While this vacuum-plasma consists of a raging sea of real $\{A_-, A_+, M_+\}$ states undergoing a seething interplay of creation and annihilation the classical gravitational effect, over many orders of magnitude in scale from the atomic to the galactic, will be calm and benign, with almost all the positive and negative contributions cancelling to leave an arbitrarily low net energy density and pressure. On more or less subnuclear scales local fluctuations in real $\{A_-, A_+, M_+\}$ state production and propagation, as depicted in figure 4(b), with corresponding local fluctuations in energy density and pressure, could be associated with mild ripples in the spacetime geometry. As a small-scale minor gravitational effect this would likely be unobservable, unless via tiny perturbations to the properties of Standard Model states propagating through spacetime, for example as might accumulate over large distances. On cosmic scales, however, a small net negative pressure $p_V$ and positive energy density $\rho_V$ could add up to a significant gravitational effect, namely that of the accelerating expansion of the universe as a consequence of this dark energy sector.

The nature of the 'vacuum-plasma' proposed here is very different to that of the unconfined states of a 'quark-gluon plasma' in standard visible QCD, although some comparison may be instructive. It might still seem counter-intuitive that a sufficiently low value of effective energy density can result from any sort of 'plasma-like' system for a non-Abelian gauge theory, even if considered as a vacuum state. Indeed, by comparison a quark-gluon plasma in the visible Standard Model sector has an energy density around $10^{44}$ times larger than would be required for any realistic form of dark energy.

There is also a notorious issue for 'vacuum-like' systems. The 'cosmological constant problem' concerns the origin of the extremely low empirical value of $\Lambda$, in the context of the $\Lambda$CDM model, compared with what might be anticipated from a calculation of the vacuum energy in standard QFT. In such a framework a contribution to the vacuum energy is expected of order $\Lambda \sim m^4$ as associated with each particle state, where $m$ is the mass of the particle. From the Higgs state alone, with mass $m_H \simeq 125\,\text{GeV}$, this is too large to be empirically realistic by a factor of $\sim 10^{54}$. Worse than that, as typically conceived for any 'quantum gravity' theory with a quantisation scheme applied to gravitation the relevant mass scale is expected to be set by the Planck mass, $m_{Pl} \simeq 1.2 \times 10^{19}\,\text{GeV}$. The consequent $\Lambda \sim m_{Pl}^4$ is famously too large by a factor of more than $\sim 10^{120}$ to be consistent with cosmological observations (with analogous dimensional arguments employed towards the end of section 5 in the context of non-Abelian Kaluza-Klein models).

The structure of the QFT vacuum energy would then seemingly require an enormous degree of fine-tuning, to cancel out most contributions and stand any chance of accounting for a cosmological constant. How this might be possible, and the true nature of the vacuum in QFT, is not at all understood. With such a vacuum source for a cosmological constant standing at the interface of classical general relativity as provisionally amalgamated with standard quantum particle interaction theory, the construction of a definitive and consistent unification of gravitation with quantum

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theory is likely to be needed before these questions can be satisfactorily addressed (as noted in [61] subsections 2.3 and 7.1).

However, the nature of the vacuum energy of the standard QFT calculation is very different to that of the dark energy sector considered here. The composition of the former vacuum can be conceived of in terms of the quantum fluctuations and the fleeting production of *virtual* particle states, all of *positive* energy. The vacuum-plasma state of figure 4(b) considered here concerns the production of *real* particle states of both *positive and negative* energy, created out of the vacuum through the processes of figures 2(b) and 3, the contributions of which to the total energy *do* largely cancel, hence with far less need for 'fine-tuning'. Further, the mutual annihilation between these states quite literally involves an *annihilation*, with a reduction in the magnitude of both the positive and negative energy contributions and the total number of physical states. This is unlike the case of matter-antimatter 'annihilation' within the Standard Model, such as between real electron and positron states, which actually involves a *transformation* into other states. Hence the composition, and calculation, of the vacuum energy density $\rho_V$ and pressure $p_V$ for the dark vacuum-plasma proposed here is quite unlike that of the standard QFT vacuum approach.

Not only are the microphysical components very different, but the approach to unifying quantum theory with gravity, and the implications for the microphysics, also differs significantly from frameworks that attempt some form of quantisation of gravity itself, with the Planck scale being less relevant here. As noted in the opening of this section, in this theory gravity is not 'quantised', rather through the construction of the Einstein field equation an essentially classical gravity has 'surveillance' over all matter fields and quantum phenomena (see for example [9] discussion on pages 101, 296 and 345). The energy-momentum tensor is *defined* through equation 20 and its generalisation to incorporate the underlying interplay and degeneracy of field states such as represented in figure 4(b). With all particle and quantum phenomena 'enveloped' within a consistent solution for a continuous extended 4-dimensional spacetime geometry, this means of 'quantum gravity' construction could itself suppress pathological behaviour and be a factor in generating an intrinsically stable vacuum structure, and even help constrain the low value for an effective cosmological constant.

Here with 4-dimensional spacetime being smooth the contracted Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ and corresponding divergence-free property discussed for equation 3 applies for both the classical relation of equation 20 and also its generalisation to include further matter fields and degenerate solutions incorporating the quantum phenomena of all matter states, and applies in particular for any vacuum or plasma state. Any possible network of production and annihilation of $\{A_-, A_+, M_+\}$ states as described for figure 4(b) must be consistent with this overall geometric constraint. A large-scale vacuum-plasma solution for the dark energy sector, as the lowest net energy state profusely active with real $\{A_-, A_+, M_+\}$ particles, might then take a form based on the Einstein-Hilbert action of equation 4 without any explicit matter Lagrangian and with $\Lambda$ considered a free parameter to be determined. That is, a vacuum equation for general relativity as consistent with equation 3 might be realised for this dark energy candidate of the geometric form:

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = 0 \tag{40}$$

Such a uniform cosmological term may be preferred over the more general form of energy-momentum of equation 5, with independent energy density $\rho$ and pressure $p$ contributions, as the vacuum-plasma is macroscopically structureless in the sense of lacking any consistent flowing entity as might be associated with a 4-vector field $u^{\mu}(x)$, similarly as for a standard QFT vacuum. In this framework the construction of an extended 4-dimensional spacetime geometry might then itself frame a solution for a vacuum-plasma state of dark energy in a form such that on large scales a cosmological constant term dominates as described by equation 40.

There are further constraints in the QFT limit for this theory, as also noted in the opening of this section, that take the place of those of a Lagrangian-based field theory or of a model with a posited form of energy-momentum. Whether the overall constraints in constructing a cosmological 4-dimensional spacetime solution may contribute to the stability, as well as constrain the level of consistency with the $w_V = -1$ case of a cosmological constant in equations 37 and 40, might then be addressed. In the meantime the possibility remains of a connection with quintessence or phantom models, and a more general value for $w_V \neq -1$ in equation 36, as consistent more generally with equation 5.

Whether or not the cosmological constant $\Lambda$ case is preferred, the further question remains concerning why such small values in the magnitudes for $p_V$ and $\rho_V$ in equations 35 should be obtained. It could simply be that very small values $p_V \sim 0$ and $\rho_V \sim 0$ are energetically preferred in the context of determining a 'lowest energy' *vacuum* solution in general relativity (see also the following section). However, while there is also an intrinsic geometric relation between the spacetime curvature and any gauge field sector through equation 20, these factors may not be sufficient in themselves to achieve the appropriate minimal values.

In conjunction with the very different approach to unifying gravity and quantum theory in the present framework there are, however, several significant factors that suggest such a low value for the vacuum-plasma energy density might be obtained, as consistent with empirical observations and at the level required for the $\Lambda$CDM cosmological model. A central feature is that this vacuum state can be considered as a *perturbation* from the $p_V = 0$ and $\rho_V = 0$ equilibrium described for equations 34. This concerns the generalisation from the structure of figure 4(a) to that of figure 4(b), with the following properties:

- The mutual annihilation between the $A_- \leftrightarrow \{A_+, M_+\}$ states could be so rapid that each of the effective $\rho_{A_-}, \rho_{AM_+}$ contributions in equations 35 to the net energy density of the vacuum-plasma state of figure 4(b) is itself much lower in magnitude than the energy density of a Standard Model quark-gluon plasma that involves only positive energy states. With a more modest energy scale there is less demand for a 'fine-tuned' cancellation.

- The gauge boson states $A_-$ and $A_+$ derive from the non-Abelian gauge theory of the internal symmetry $\mathrm{SL}(k, \mathbb{C})_D$ in the $k \to \infty$ limit and are hence *highly* self-interacting (with $2(k^2 - 1)$ $A_\pm$ states, $k^2$ scalars $\boldsymbol{\phi}$ and $k$ Weyl spinors $\boldsymbol{\psi}$ in table 4). The production rate for $A_+$ states via the cubic and quartic self-couplings, such as pictured in figure 2(b), may be much higher than the production rate for the $M_+$ states via the processes in figure 3, which will depend upon the kinematic and other factors noted in the paragraph below figure 3. In turn the effective energy and pressure contribution from the $M_+ = \{\phi_+, \psi_+, \mathring{A}_+\}$ states, that introduces the asymmetry, might be a relatively minor fraction of the total and not as significant as sketched in figure 4(b).

- If the effective equation of state $p_{AM_+} = w_{AM_+}\rho_{AM_+}$ for the $\{A_+, M_+\}$ states (even if the $M_+$ contribution is not small) has an effective parameter value $w_{AM_+}$ very similar to $w_{A_+} = w_{A_-}$ in the equation of state $p_{A_\pm} = w_{A_\pm}\rho_{A_\pm}$ for the $A_-$ or $A_+$ components alone, the impact would only be that of a minor perturbation from the net-zero values $p_V = 0$ and $\rho_V = 0$ for the vacuum state of equations 34 to the finite values in equations 35 and 36.

For this branch of generalised proper time in particular a fuller working out of the complete theory, including the framework unifying quantum theory with gravitation, may be required to fully assess its practical suitability to account quantitatively for the associated cosmological observations. Despite these open questions, by comparison with the nature of the vacuum energy calculation in standard QFT as associated with visible matter and phantom dark energy models that employ a source of negative kinetic energy for a posited scalar field, the structures arising in the present theory, including a non-compact internal symmetry group and as developed in the previous and present sections, provide an ideal candidate for the dark energy driving the observed accelerating expansion of the large-scale structure of the universe.

In summary, the physical form of matter deriving from this third branch of generalised proper time of equation 23 and table 4 exhibits the collective qualitative properties of darkness, uniformity, $\Lambda$-like constancy, and the means of generating an extremely low positive energy density together with negative pressure; all of which are features desired of any proposed source for the dark energy sector. In the following section we consider further features of this dark energy candidate, in particular in relation to the dark matter candidate reviewed in section 4, as potentially collectively encompassed within a unified dark sector that may be identified for this theory.

## 7 Interaction between Dark Energy and Dark Matter

There may also be yet further possible branches of generalised proper time to consider. In generalising from the common 4-dimensional spacetime form for proper time in equation 14 ultimately a systematic study will be needed of all possible branches of equation 15 and their collective physical contributions to the dark sector and the consequences in relation to the visible sector. This full picture might then be compared not only with existing models for dark energy and dark matter, but also with models of a unified dark sector (see for example [20, 21, 22, 23]).

While for this theory the 'dark sector' may be sealed off from the Standard Model sector, other than through an essentially classical gravitational interaction, there is an open question concerning how the dark energy and dark matter components might themselves be interrelated. Indeed, the extent to which the physics deriving from the generalised proper time branch of equation 23 and table 4 might augment the dark sector generally, both in terms of dark energy and even with an effective dark matter component, can itself be investigated.

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The non-compact internal $\mathrm{SL}(k,\mathbb{C})_D$ symmetry in table 4 has a total of $2(k^2-1)$ generators, $(k^2-1)$ of which are non-compact while $(k^2-1)$ are compact. This symmetry, with an equal number of non-compact generators associated with Killing metric diagonal elements $K_{\alpha\alpha} = -1$ and negative energy $A_-$ states and compact generators associated with $K_{\alpha\alpha} = +1$ elements and positive energy $A_+$ states, further augments the nature of the symmetry between these states described for figures 2(b) and 4(a) and equations 34. The $(k^2-1)$ non-compact generators provide the source of negative pressure as well as negative kinetic energy as originally considered as a basis for dark energy. The remaining $(k^2-1)$ compact generators are associated with the maximal compact subgroup $\mathrm{SU}(k)_D \subset \mathrm{SL}(k,\mathbb{C})_D$. This compact non-Abelian $\mathrm{SU}(k)_D$ might itself potentially provide a basis for a further 'hidden QCD' contribution to dark matter; as an additional component to that based upon the internal $\mathrm{SO}(m)_D$ in the original dark matter branch of table 3.

This would lead to further dark matter features, with for example the resulting 'dark glueballs' accompanied also, for this confining $\mathrm{SU}(k)_D$ gauge symmetry, by a new variety of 'dark quark nuggets' now containing the component matter fields based upon table 4. Incorporating spinor states $\boldsymbol{\psi}(x)$, associated with the $\boldsymbol{\delta x}_{4k}$ components and charged under the internal $\mathrm{U}(1)_D$, this 'dark QCD' sector would be more akin to standard visible QCD, with 'spinor dark quarks' potentially confined in 'dark hadron' states. The production of massive nugget states for this branch with both a net charge and Fermi pressure would make the formation of primordial black holes less likely in this case, compared with the branch described for table 3.

On the other hand, in addition to the quadratic form of equation 22 and table 3 a related quadratic form with a *mixed* internal metric signature and corresponding non-compact internal symmetry $\mathrm{SO}(m_+, m_-)_D$, with $m_+ + m_- = n - 4$, can be identified (again there is a symmetry between the possible values of $m_+$ and $m_-$ and the associated positive and negative energy gauge boson states). The internal $\mathrm{SO}(m)_D$ gauge group of table 3, as a basis for dark matter, might then be considered a compact subgroup, with $m = m_+$, of the more general non-compact $\mathrm{SO}(m_+, m_-)_D$ internal symmetry, that is with $\mathrm{SO}(m = m_+)_D \subset \mathrm{SO}(m_+, m_-)_D$. With $\mathrm{SO}(m_+, m_-)_D$ incorporating $m_-$ non-compact generators, as associated with $K_{\alpha\alpha} = -1$ Killing metric diagonal elements in equation 30, this provides a further potential source of negative kinetic energy and negative pressure via equations 31–33, as a basis for a form of dark energy similarly as constructed in section 6, for this case still in the quadratic branch of generalised proper time. This would mean that rather than dark matter and dark energy residing in the two independent branches of generalised proper time, of equation 22/table 3 and equation 23/table 4 respectively, *each* of these branches could contain *both* dark sector components.

For the branch of table 4 the stable dark energy 'plasma' of figure 4(b) is considered a 'vacuum' state in that it corresponds to the lowest energy state in the context of general relativity and the field equation 40. Any significant net excess of positive energy and positive pressure states, involving gauge bosons $A_+$ from the compact subgroup $\mathrm{SU}(k)_D \subset \mathrm{SL}(k,\mathbb{C})_D$ in interaction with $M_+ = \{\boldsymbol{\phi}_+, \boldsymbol{\psi}_+, \mathring{A}_+\}$ states, might then form a 'non-vacuum' dark matter component in this sector, over and above the dark energy vacuum-plasma state. In a sea of vacuum-plasma dark energy $\{A_-, A_+, M_+\}$ states such a non-vacuum excess of $\{A_+, M_+\}$ states, forming dark glueball, hadron or nugget states, might provisionally be considered analogous to lumps of ice floating on an otherwise uniform pool of water.

These would be different states made of the same underlying components, with exchange between the underlying particle constituents of these dark matter and dark energy states permitted, albeit with the glueball-based states then seemingly somewhat more unstable than their ice-lump analogues. For example $A_+$ states in a dark matter glueball could annihilate with $A_-$ states in the dark energy sea, incorporating the reverse process of figure 2(b), leaving the net excess of positive energy in $\{A_+, M_+\}$ states in the sea that now in fact contribute to the dark matter; always conserving energy-momentum. Such a dark glueball-based state might then behave more like a small drop of oil placed on a water surface and spreading out to form the thinnest possible oil film layer with a thickness of one molecule, as for example in experiments that estimate the size of the molecules. With individual dark glueball or nugget states spilling out and seemingly rapidly losing any sense of individual identity, gravitating 'islands' of dark matter with a stable overall structure might still form with the requisite empirical properties.

In the very early universe the energy density of the dark sector, exceeding that of the visible sector, would be almost uniformly distributed. As the universe expanded and the matter density diluted small variations in the positive energy excess of states could seed the formation of extended gravitational islands that we recognise today as dark matter haloes. The residual vacuum-plasma state with very low positive energy density, and very low but net negative pressure, filling the voids between and extending throughout space would then form the dark energy component of this sector. As a residual spill-out from the initial dark matter density, this might also be a factor in generating the extremely low dark energy density, as appended to the factors listed towards the end of the previous section and as suggested by the above 'oil film' analogy. That is, the dark energy is the thinnest possible spread of dark sector material deriving from energy-momentum conserving interactions with the dark matter component, shedding further light on solving the 'cosmological constant problem'. At around half the present age of the universe the cosmic acceleration associated with this dark energy residue could then begin to dominate the cosmic evolution, and prevent further formation of structure on the larger scales, as consistent with observations.

As noted in [8] having a dark matter sector based on a hidden QCD might underlie the apparent coincidence in the similarity in cosmological energy density with that of the visible sector as dominated by visible QCD; with the dark and visible matter sectors composing 26% and 5% of the total as reviewed in section 2. Having in turn a significant interplay between the dark matter and dark energy sectors, with the latter composing 69% of the total energy density at the present epoch, might then also address the 'cosmic coincidence problem' concerning the similarity in the dark matter and dark energy density contributions at the current stage of cosmic evolution. That is, this framework may result in a dark energy density, or effective cosmological constant term, that evolves with cosmic time, providing the possibility of a mutual 'tracking' between dark energy and dark matter analogous to that of quintessence models as review in section 2. This more generally then encompasses the three-way coincidence covering all three sectors of the 'cosmological pie chart' with all sectors based on non-Abelian gauge theories.

This picture is analogous to that of 'diffusive' dark sector models similarly involving an interaction between the dark matter and dark energy components [62, 63, 64]. While ΛCDM assumes that dark matter and dark energy are non-interacting, an interactive transfer of energy between these two sectors can be modelled by the dynamics of diffusive effects. Typically in this case the effective cosmological 'constant' $\Lambda$ will then be instead a running parameter '$\Lambda(t)$' with a non-trivial dependence on the cosmic scale factor $a(t)$ as a function of cosmic time $t$, while the evolution in the energy density of dark matter will deviate from the usual $\rho_m \propto a^{-3}$ scaling law discussed after equation 8. In this manner such diffusion cosmology models can in principle address both the coincidence problem and the cosmological constant problem.

A more fluid-like 'cloud' of dark matter for the present theory might also provide a possible link with models of a dark matter 'superfluid' that exists in two interacting distinguished states [65, 66]. A unified dark sector based on a ground state and excited state of such a dark matter superfluid can model both the late-time acceleration of the universe as well as the phenomenology usually associated with dark matter. Moreover, the low temperature superfluid properties within the galactic scale can reproduce the successes of MOdified Newtonian Dynamics (MOND) approaches (within the class of modified gravity models noted near the end of section 2) while retaining the successes of ΛCDM on the cosmological scale. Superfluid dark matter models can be constructed from the properties of dark baryons and a hidden non-Abelian gauge theory sector [67], making further possible connection with the dark sector described for the theory proposed in this paper.

The unified dark sector proposed here may also relate to further approaches in which dark energy and dark matter are unified in a fluid-like model, in some cases with phantom and inflationary components [68, 69, 70]. Models with some form of interaction between dark energy and dark matter provide a degree of flexibility that may accommodate further cosmological observations, such as in addressing or alleviating the Hubble tension (also alluded to in section 2, see for example [29]).

Despite the profuse interactions between dark matter and dark energy, for the present theory in order to achieve a stable dark vacuum state there should be no particle interactions with the visible states of the Standard Model, as discussed for figure 4 in section 6. As reviewed in the opening of that section, in this theory with gravity itself not being quantised there are no 'graviton' states to mediate such an interaction as there are typically presumed to be for phantom models as reviewed for figure 2(a). On the other hand here the components $\boldsymbol{\delta x}_4$ of the local external spacetime, the smooth and continuous extended geometric curvature of which describes classical gravity, are closely linked with the phenomena of the Higgs particle, as listed in and described for table 2 (see also [61] equations 41 and 42). In some sense then the physical Higgs particle is here the quantum particle state most closely associated with gravity.

Given the universal and common nature of the external $\boldsymbol{\delta x}_4$ spacetime components as central to the symmetry breaking, relevant not only to the visible sector of table 2 but also the dark matter sector of table 3, the potential for an interaction between these sectors analogous to a 'Higgs portal' might naturally be considered as suggested at the end of section 4. However, if such a portal interaction were possible between visible and dark matter it seems it should also connect to the dark energy sector, with the same $\boldsymbol{\delta x}_4$ components common to all branches including that of table 4. In fact this very likely must be the case since, as noted above, *both* dark matter and dark energy contributions can be associated with *both* the branch of generalised proper time of table 3, as well as with the branch of table 4.

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If such a Higgs portal were available between the Standard Model and dark energy sectors that seemingly would imply a catastrophic decay of the vacuum from our perspective, via processes analogous to that in figure 2(a) but incorporating a Higgs-type, rather than graviton, interaction. Compared with graviton exchange this could involve a more complicated process, with the negative energy $A_-$ states coupled via $M_+$ states (as pictured in figure 3) and in turn through an intermediate Higgs portal connection to $S_+$ Standard Model states. The apparent empirical stability of the dark energy sector may then suggest that such a Higgs portal interaction is highly unlikely, with the only interaction between the visible and dark sectors indeed of a classical gravitational nature, and without even a Higgs portal from the Standard Model to the dark matter component.

This would also presumably render the detection of dark matter states in the laboratory, as considered at the end of section 4, via any form of non-classical and non-gravitational impact impossible. This may then also rule out any form of very weak interaction such as via a 'kinetic mixing' portal between the Standard Model and the dark sector (as considered in [8] towards the end of section 6, although thought unlikely therein). Such kinetic mixing might otherwise be conceivable via the relation between $\mathrm{U}(1)_Q$ and $\mathrm{U}(1)_D$ alluded to before equation 24, or between other partially related gauge interactions in the respective sectors.

The empirical stability of the dark energy vacuum-plasma could itself also be considered evidence that gravity is indeed *not* quantised, ruling out decay processes via 'graviton' exchange similar to that in figure 2(a). With the approach to unifying quantum theory and gravity here indeed *not* involving any 'quantisation' of gravity itself, this aspect of the theory is then fully mutually consistent with the dark energy candidate proposed within this overall framework. This leaves only the classical gravitational interaction of general relativity to connect the visible and dark sectors, at least at the present epoch of cosmic evolution.

Through the Einstein field equation 3, and for example the properties of the energy-momentum tensor in equation 31 for a non-compact gauge group, general relativity itself seems equally compatible with either a positive or negative energy structure of matter. Indeed a purely negative energy source of matter by itself would be as consistent as the more familiar positive energy forms of matter are in themselves. It is only when these two opposite forms of matter are coupled via particle interactions that the mutual issue of instability through decay of the vacuum is raised. For the present theory such a sector originates from the branch of generalised proper time of table 4, and can attain a benign equilibrium state as described in the previous section provided it is detached from the Standard Model sector other than through a universal classical gravitational influence.

The values for $p_V$ and $\rho_V$ in equations 35 and 36 for the dark energy sector, as well as the relation to the ΛCDM cosmological model and the equation of state value for $w_V$ in equations 36 and 37, and their possible dependence on cosmic time, remain to be determined; now with the added complication of a possible independent dark energy contribution from the non-compact gauge group augmentation for the branch of table 3 in addition to the branch of table 4. However, the stability issue and potential for a tracking solution also raises the question of whether a far larger negative pressure, and effective positive Λ term, could have been achieved in the extremely early universe for this theory. In that case branches of generalised proper time with a non-compact internal symmetry might even be associated with an inflationary epoch, as normally attributed to a 'false vacuum' state in the potential $V(\phi)$ of a proposed 'inflaton' scalar field $\phi(x)$ as discussed after equations 11 in section 2 (with reference to [10, 11], [9] section 12.3).

While a range of inflationary models have been proposed many are typically based on such a scalar field and the Lagrangian of equation 9 and hence with the energy-momentum of equation 10 (that is, equation 39 for the $\phi_+(x)$ case). While avoiding the fine-tuning that would otherwise be required to account for the horizon problem and the flatness problem, inflationary models themselves typically involve a high degree of fine-tuning in the assumptions imposed on the form of the potential $V(\phi)$. In inflationary models the dynamical equations for the $\phi(x)$ scalar field dampen oscillations about the minimum in $V(\phi)$ with couplings of $\phi(x)$ to ordinary matter fields that fuel a 'reheating' as the early universe vacuum energy is converted into that of Standard Model states of matter ([11] section 8, [71]), leading to the radiation-dominated era. Local fluctuations in this process lead to variations in matter density that can ultimately seed the formation of galactic structures.

One aim of the present theory is to supplant dark energy models based on a scalar field $\phi(x)$ such as in equations 9–13 and hence, since those models are closely related to inflationary models, in turn a possible connection between this new theory involving non-compact gauge groups with any putative rapid expansion dynamics in the extremely early universe might be considered. However, given the very different scale required for the early inflationary acceleration compared to the present dark energy impact, a basis for significant further new physics might be needed to make this connection. The present theory would then need to address the nature of the interplay between the dark and visible sectors at the earliest epoch and also account for how an enormous effective $\Lambda$ driving such an extremely brief and early period of inflation might have made a transition to the very low value of the present day.

Here there is no direct equivalent of a posited fine-tuned potential $V(\phi)$ introduced in the inflationary models, but there are other forms of matter with a basis in the symmetry breaking structure of equation 23 in table 4 with the resulting structures and interactions taken to the $k = (p-2) \to \infty$ limit. Through the contribution to the energy-momentum of the term $f(\boldsymbol{\phi}_+, \boldsymbol{\psi}_+, \mathring{A}_+)$ in equation 38, in place of the potential $V(\phi)$ of the scalar models in equation 39, the full dynamics for these fields and the present theory can be inherently somewhat more involved. As noted earlier in this section there will also be a contribution from an internal non-compact $\mathrm{SO}(m_+, m_-)_D$ extension from equation 22 and table 3 and the further scalar matter fields, now based upon dark $\boldsymbol{\delta x}_{m_+,m_-}$ components, from that branch of generalised proper time. These new scalar and other matter fields and related features hence provide a source of non-trivial new physics that could play a role in the very early universe and relate to an inflationary era.

Further, as also recalled above, the scalar Higgs of the Standard Model is here closely related to the magnitude $|\boldsymbol{\delta x}_4|$ of the projected external local 4-vector $\boldsymbol{\delta x}_4$ spacetime components, which can take a very different value in the very early universe ([9] section 13.2, in particular figure 13.3). This can also have a radical impact upon the value and role of the Yukawa couplings, discussed for table 2, and the whole electroweak and mass generation sector generally. If for example a scalar field $\phi(x)$ identified in this theory contributes an effective mass term $\sim m_\phi^2 \phi^2$ to the dynamics

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the parameter $m_\phi$ might take a very different value in the very early universe and this term could act not unlike the scalar potential in some inflationary models.

Given the close connection here between the external $\boldsymbol{\delta x}_4$ components and gravity, this construction might also relate to 'Higgs inflation' models that postulate a non-minimal coupling between the Higgs field and gravity [72, 73]. Many open questions remain then, such as whether something like a 'Higgs portal' may come into play at extremely high energy in the very early universe. For this extreme earliest epoch of the Big Bang the potential for interaction between the dark sector and the Standard Model more generally would need to be reassessed, and with a view to describing a transition to the radiation-dominated era.

The degrees of freedom of the early radiation-dominated era were highly thermalised with very little spatial variation in temperature, at a level of around one part in $10^5$ as observed in the CMB maps (and as associated with the subsequent formation of dark matter haloes and galactic structures). On the other hand the apparently highly uniform gravitational field at that epoch indicate that the gravitational degrees of freedom were *not* thermalised resulting in the very special net *low* entropy initial conditions for the Big Bang ([74] sections 28.7 and 28.8). This seeming paradox is again consistent with the approach of the present theory in which gravity itself is not quantised, but rather imposes an external surveillance over the properties of all other fields ([9] page 378). The role played by the amalgamation of quantum theory and gravitation within this framework will then also be important in elucidating the nature of the evolution of the cosmos from the earliest epoch.

The main concern of this paper has been the nature of a dark energy candidate as arising from a theory based upon generalised proper time. This leads directly to an intrinsic means of profuse interaction with the dark matter sector as described in this section. In turn the dynamics of a transition or evolution in the magnitude of the dark energy impact from the very early universe through to the present cosmic epoch might relate to quintessence or quintom models for which a tracking relationship between the dark energy and dark matter density may account for the apparent coincidence problem while also addressing the cosmological constant problem. However, as we have also described in this section, the broader framework of this theory leads to consideration of further significant cosmological questions, including the nature of the Big Bang and inflationary dynamics, linking with further models and potentially accounting for more than the present day cosmic acceleration alone.

## 8 Conclusions

The origin of the accelerating expansion of the observable universe and the nature of the presumed associated 'dark energy' is one of the most significant outstanding mysteries in physics, as remarked in section 1. On applying the Einstein field equation, under the symmetry assumptions of the cosmological principle, several models posit a form of positive energy density and negative pressure that can simulate this empirical observation, as we reviewed in section 2. However, this still leaves open the question of a more fundamental *explanation* for the microscopic physical origin and nature of this dark energy underlying the associated large-scale phenomena.

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The present theory is constructed upon the well-motivated conceptual basis of generalised proper time as described in section 3. The arguments for the construction of the theory, in generalising from the *local* form for 4-dimensional spacetime, can be contrasted favourably with the approach of extra spatial dimensions as based initially on *global* structures. As we reviewed for figure 1 and equations 14 and 15 this new approach to unification, with this change in perspective from a starting point in an extended higher-dimensional spacetime structure to a generalised local form for proper time, is well suited to connect more directly with the *local* symmetries and interactions of interest in particle physics.

The ambition of extracting the basis for forms of *spacetime and matter* from local forms of proper *time* alone, as described for equations 16 and 17, may sound implausible; however, the aim of deriving essentially all the fundamental structures of physics from the *simplest* possible foundation is precisely the *ideal* of unification. Given that *time* is something that we are intimately familiar with, unlike extra dimensions of *space*, this also constitutes a very conservative, as well as simple, starting point for unification. The recording of events in *time* by the values of a *linear* real time parameter $s \in \mathbb{R}$ is indeed a very familiar and long-standing practice. Since the formulation of special relativity the relation between events in a 4-dimensional *spacetime* has been described by a *quadratic* form for *proper* time intervals $(\delta s)^2$, such as employed in equation 14 and as has also become familiar. Here in extending further to a '*spacetime-matter*' theory, incorporating also the material basis for events, we have proposed the generalisation of local proper time intervals allowing for *more-than-quadratic* order $(\delta s)^p$ in equation 15, as a further direct and natural augmentation to the arithmetic form of time.

Through the mathematically possible 'branches' of generalised proper time highly non-trivial connections have already been established with the Standard Model of particle physics while also linking with models for the dark matter component of cosmology, as we have reviewed in section 4. In this paper we have described how analysis of a further branch has led to the possibility of also accounting for the dark energy sector, as argued in detail in sections 5 and 6. This branch of equation 23 and table 4, with a non-compact local symmetry group, provides an intrinsic mechanism for a direct source of negative pressure, as described for equations 29–33, as a key component required for dark energy. The resulting consequences and issues to be addressed are analogous to those faced by some of the proposed dark energy models reviewed in section 2. In particular, in deriving from an internal non-compact gauge group the source of negative pressure here is associated with an implied contribution of negative kinetic energy, sharing that latter characteristic with phantom dark energy models.

However, there are also significant new features. With 4-dimensional spacetime and matter fields having a common basis in generalised proper time in equation 15 as decomposed in equation 16, and with the properties of the gauge field associated with the internal symmetry $G$ in equation 17 intimately tied to the geometry of the external spacetime as described for equation 20, the matter and gauge fields are intrinsically ingrained in the structure and composition of the extended spacetime. Moreover, here the sign of the kinetic energy is *indefinite* and there are *self-interactions* for the non-compact non-Abelian gauge field. These differences are key to addressing the question of stability for the vacuum state, as a common issue with the phantom models, as discussed in relation to figures 2 and 3.

The mechanism through which stability can be achieved for this theory is described for figure 4(a), and on incorporating the further matter fields for this branch of table 4 for figure 4(b). This leads in turn to the question of the proximity to an equation of state with $w_V = -1$ as described for equations 35–37, and an effective cosmological constant $\Lambda$, rather than necessarily implying $w < -1$ as for the phantom models. This led in turn to the question of the specific value this effective $\Lambda$ might take in the context of $\Lambda$CDM cosmology, and of the possibility of addressing the cosmological constant problem concerning the exceptionally low density required for dark energy, as assessed towards the end of section 6.

Hence, from the elementary basis of the theory in generalised proper time and equation 15 a direct path can be drawn, via equations 20 and 23 and table 4, to a potential source for dark energy with the equation of state of equation 36, providing a firm basis for further investigation. Further development will be required to give a fully quantitative account of this sector and the associated empirical observations of the expansion dynamics, as described by cosmic time $t$ dependent parameters $\rho(t), p(t)$ and $w(t)$, or a constant $\Lambda$, in the context of an FLRW cosmology in equations 2–8.

Here we summarise three key features of this branch of generalised proper time, as matching the required characteristics for dark energy and making a credible case for this approach:

- This source for dark energy arises naturally and directly from the basis of a *fundamental theory*, with an independent motivation, rather than as a tailor-made *posited model*, and is intrinsically *dark* since it is identified in a *different* branch of generalised proper time to that of the Standard Model.

- In being ever created out of the vacuum, as described for figures 2(b) and 3, as balanced by annihilation processes a stable equilibrium of positive and negative energy states can result, as discussed for figure 4, with a net energy density and pressure that is essentially *uniform* in space and may be *constant* in time.

- With the positive and negative energy contributions largely cancelling, albeit with an asymmetry in the nature of these states, an arbitrarily small residual net positive energy density $\rho_V$ and negative pressure $p_V$ for this 'dark vacuum-plasma' may result, as described for equations 35 and further in section 7.

The symmetry breaking for the three branches of generalised proper time, of table 1, equation 22 and equation 23, as described for tables 2, 3 and 4, has provided direct connections with the visible matter of the Standard Model, a dark matter candidate and the dark energy sector respectively; as explained in sections 4–6. That non-Abelian gauge theory plays a dominant role in all three sectors provides a further strand of unification identified in this framework, and as may relate to the apparent coincidence in the relative contributions of *all three* sectors to the observed cosmic dynamics at the present epoch. While most of the energy density of visible Standard Model matter is dominated by the standard QCD sector, here dark matter is identified in a parallel hidden QCD sector. In turn the compact gauge group of this 'dark QCD' can now be seen as a subgroup within a larger non-compact gauge symmetry group of the dark energy sector, as we have described in section 7.

In spite of this more intimate relationship and interaction between the dark matter and dark energy sectors, through *sharing* the same branches of generalised proper time, this combined dark sector is still anticipated to be sealed off from the visible matter of the Standard Model other than through the gravitational interplay of classical general relativity. While this may be the case throughout most of the history of the universe, and in particular during the present epoch, there remains the possibility of some form of new physics, with a higher degree of interaction between the visible and dark sectors, coming into play under the very different conditions of the very early universe. That is, given the extremely high density of visible and dark matter the potential for interaction between all sectors, and involving all branches of generalised proper time, would presumably have been greater under the exceptional initial conditions of the Big Bang era.

As also considered in section 7 new physics associated with the new matter fields in the new sectors, beyond the non-compact gauge groups alone, or as associated with the non-standard Higgs in this theory, might also relate to any very early inflationary phase. Understanding how more stable forms of matter could have emerged as the cosmos expanded and how they connect with what we can observe today, both in the laboratory and in cosmology, is then a key question for this theory. This includes the nature and degree of the excess of 'positive energy' over 'negative energy' for the associated particle states in the dark sector, and the corresponding interrelation of dark matter and dark energy, as well as the excess of 'matter' over 'antimatter' states in the visible Standard Model sector, and the question of how these apparent asymmetries might even be related in mutually originating in the Big Bang.

With all structures of matter, and the form of space itself, at the most fundamental level originating from the general form for proper time of equation 15 this approach to unification could be termed a theory of 'chronogenesis' – with time itself as the progenitor underlying the construction of any universe. This stands in contrast with the many theories positing extra spatial dimensions for which forms of matter derive from an appended structure of space, as we discussed in relation to figure 1. Within the framework of extra spatial dimensions there is a 'multiverse' of a vast number of possible worlds that might be theoretically constructed, as reflected for example in the 'landscape problem' in string theory ([75] section 5).

Here for generalised proper time we might also explore of the extent of the 'chronoverse' of all possible worlds that might be constructed within the framework of the new theory. While the generalised expression for proper time in equation 15 is not unique there is a limited set of possible explicit mathematical forms this expression can take as an augmentation from the local 4-dimensional spacetime form of equation 14, with each branch exhibiting unique mathematical structures. While far more limited and hence more predictive compared with the above 'multiverse landscape', the issue of the degree of uniqueness in the 'chronoforming' of our own world, and any role for a degree of anthropic selection might then also be considered ([5] section 5).

In conclusion, this simple augmentation from the local 4-dimensional spacetime form, as initially reviewed in section 3, can in principle collectively account for all three sectors of the cosmological pie chart describing the composition of our observed universe. Given that these three sectors of the Standard Model, dark matter and dark energy have very different empirical properties it is striking that, while sharing the common gravitational root in 4-dimensional spacetime and local substructure of equation 14, they can each have a basis in 'one simple equation', namely equation 15. In particular in this paper we have shown how the esoteric property of the observed large-scale accelerating expansion of the universe, as attributed to dark energy, can be accommodated within this framework. These developments lend further support to this approach of deriving the elementary structures of matter in 4-dimensional spacetime from a generalised form for a local interval of proper time as the basis for a comprehensive unified theory.

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