

Article

Not peer-reviewed version

Toward a Spectral Principle: Extending the TEQ Framework\(\ \)

[David Sigtermans](#) *

Posted Date: 24 June 2025

doi: 10.20944/preprints202506.1832.v1

Keywords: entropy geometry; entropy-stabilized spectra; analytic continuation; renormalization; zeta regularization; entropy-weighted action; resolution scale; quantum field theory; entropy curvature; spectral comparison; distinguishability; Casimir effect; TEQ framework; entropy flow; quantization



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a Creative Commons CC BY 4.0 license, which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Disclaimer/Publisher's Note: The statements, opinions, and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.

Article

Toward a Spectral Principle: Extending the TEQ Framework

David Sigtermans

Independent Researcher, The Netherlands; david.sigtermans@protonmail.com

Abstract

We reformulate the Total Entropic Quantity (TEQ) framework using two axioms, extending the second to include spectral comparison via analytic continuation. This extension formalizes the treatment of renormalization, vacuum energy suppression, and spectral anomalies as structural consequences of entropy geometry. Using the extended Minimal Principle, we derive the exact Casimir energy, explain the chiral anomaly, and reinterpret zeta regularization as a physically grounded method for comparing entropy-curved spectra. Appendices confirm that core quantum corrections—including the Lamb shift and the running of the coupling constant α —remain derivable from the original two axioms. Crucially, these results are obtained *without recourse to ad hoc regularization, arbitrary subtractions, or postulated operator structure*; instead, regularization and anomaly arise as necessary features of entropy geometry and analytic continuation. These results reinforce TEQ's explanatory economy: a single resolution-based variational principle governs not only quantum dynamics but also spectral comparisons and anomalies. This work preserves axiomatic minimality while extending the empirical and structural reach of the TEQ framework.

Keywords: entropy geometry; entropy-stabilized spectra; analytic continuation; renormalization; zeta regularization; entropy-weighted action; resolution scale; quantum field theory; entropy curvature; spectral comparison; distinguishability; Casimir effect; TEQ framework; entropy flow; quantization

Meta-Abstract

This section provides a concise structural guide to the logic, assumptions, and derivational flow underpinning this work.

- Axioms and Principles:** The framework is based on two explicit axioms: (i) **Entropy Geometry** (Axiom 0), and (ii) the **Minimal Principle of Stable Distinction** (Axiom 1), extended to include spectral comparison via analytic continuation. These are introduced in Section 1 and motivated in Sections 2 and 3.
- Derivation Pathway:** Section 2 shows that the unextended Minimal Principle fails to recover the correct Casimir energy. Section 3 presents the analytic extension of Axiom 1, and Section 4 recovers the exact Casimir result from zeta-regularized entropy-stabilized spectra.
- Technical Justification:** Derivations use entropy-weighted action extremization and analytic comparison of spectra. The key expression appears in Eq. (9), and the Casimir result in Eq. (16). No traditional renormalization techniques are assumed.
- Assumptions and Limitations:** The analytic continuation procedure is treated as a structural extension of Axiom 1, required when comparing entropy-stabilized spectra across configuration spaces (see Sections 3 and 7). This extension is justified internally rather than introduced as a separate axiom. An intuitive conceptual guide to analytic continuation, including its role in resolving epistemological–ontological tension, is provided in Appendix A.
- Section References:**
 - Failure of two-axiom TEQ (unextended):** Section 2

- **Spectral extension of Axiom 1:** Section 3
 - **Casimir derivation:** Section 4
 - **Vacuum energy via zeta sums:** Section 5
 - **Chiral anomaly:** Section 6
 - **Philosophical implications:** Section 7
6. **Supporting Material:** Appendices B and C show that Axioms 0 and 1, without the spectral extension, suffice to derive the Lamb shift and the running of the electromagnetic coupling α , respectively.
 7. **Comparative Clarity:** Section 7 contrasts TEQ's entropy-based logic of structural selection with standard renormalization methods, reinterpreting regularization as a geometric filter on resolution-stable comparisons. The connection to analytic continuation as a bridge between local discernibility and global structure is made explicit in Appendix A.

Precise references to key derivations and operator definitions are found in Sections 4, 5, 6, and Appendices B–C. This meta-abstract serves as a map for the logical structure and derivational flow of the paper.

A meta-abstract has been included to clarify which results are derived versus extended or interpreted, preempting misunderstandings by readers who skim the paper. No other changes or additions were made.

Prelude Toward a Spectral Principle for TEQ

What determines which physical patterns persist, and which dissolve? Classical physics assumes that its laws are fixed, written into the fabric of reality. The TEQ (Total Entropic Quantity) framework proposes a deeper principle: nature selects for survival those patterns that remain distinguishable under finite entropy flow.

This means that physical law is not imposed from above but *emerges* from the geometry of resolution—a curved space of distinctions shaped by information constraints. TEQ frames physics not as a catalog of ingredients, but as a theory of what can be resolved—and thus what can endure—in a universe with limited capacity for distinction.

At the heart of TEQ are two axioms. Axiom 0 is so-called to emphasize that entropy geometry precedes the selection principle; numbering follows the convention that metric structure is logically prior to dynamics.

1. **Axiom 0: Entropy Geometry.** The configuration space of physical systems is equipped with a local entropy metric $G_{ij}(\phi)$. This metric quantifies how hard it is to distinguish between nearby configurations, and induces a curvature that governs which patterns are stable under entropy flow.
2. **Axiom 1: Minimal Principle.** Physical evolution selects those paths $\phi(t)$ that extremize an entropy-weighted effective action:

$$S_{\text{eff}}[\phi] = \int dt (L(\phi, \dot{\phi}) - i\hbar\beta g(\phi, \dot{\phi})), \quad (1)$$

where L is the classical Lagrangian and $g(\phi, \dot{\phi})$ captures the local entropy flux along the path. The factor β sets the resolution scale. The term $i\hbar\beta g$ acts as an entropic filter: paths that would dissolve under coarse-graining are suppressed.

This deceptively compact structure leads to a wide array of consequences. In prior work, TEQ has been shown to recover the Schrödinger equation, Born rule, and even the structure of Hilbert space without postulation—these emerge from entropy-curved variation, not assumptions [1,9].

Perhaps more remarkably, TEQ also accounts for gravitational and cosmological structures. It explains the suppression of vacuum energy [6], the entropy peak in cosmic evolution [3], and the

entropy-stabilized emergence of time and locality. In this view, *spacetime itself* emerges as a resolution-stable regime within the entropy geometry. Where the entropy dimension $D_S \geq 4$, causal structure and ordering can persist. Below this threshold, distinctions blur, and spacetime dissolves into entropic noise.

Crucially, we show in the appendices that these two axioms alone suffice to derive the Lamb shift, the running of the electromagnetic coupling, black hole entropy, and the chiral anomaly—demonstrating both quantum and gravitational consistency within the same structural framework (that is, a framework in which these results emerge directly from the internal logic and geometry of the theory, rather than from added postulates or external prescriptions).

The Logic of Resolution Geometry

TEQ reframes physics as the study of resolution-stable structure. A particle is not a fixed object, but a local attractor in the entropy flow. A quantum measurement is not a metaphysical collapse, but a redistribution of entropy, sharpening an entropy gradient and narrowing the ensemble of contributing paths. Wave behavior arises when multiple entropy-stabilized paths contribute equally to resolution; collapse occurs when that symmetry breaks due to entropy flow.

This shift opens a path to unify quantum, gravitational, and thermodynamic behavior under a single selection principle: only those structures that remain stable under entropy flow—and resolvable within finite-resolution geometry—can manifest as physical phenomena.

Terminological note: Throughout this paper, the term “resolution principle” is used informally to refer to the general structural logic of TEQ: that physical phenomena emerge only from what remains distinguishable under finite entropy flow. It is not a formal axiom. Rather, it summarizes the effect of the two core axioms: the entropy geometry (Axiom 0) defines local distinguishability, and the Minimal Principle (Axiom 1) selects those trajectories that remain stable under entropy-weighted variation. Together, they define a geometry of resolution in which physical law is no longer imposed but selected.

Empirical Reach and Structural Power

Despite its conceptual shift, TEQ yields concrete predictions across domains:

- **Galactic rotation curves** and the Baryonic Tully–Fisher relation follow from entropy curvature at low resolution scales [3].
- **Dark energy suppression** emerges from entropy peaks in cosmic history, without invoking fine-tuning [6].
- **Quantum decoherence, interference suppression, and the quantum eraser** are explained as entropy-weighted path transitions [4,5].
- **Vacuum energy bounds** arise from zeta-regularized entropy spectra [6].
- **Hilbert space structure** emerges from the entropy-stabilized modes of the entropy curvature operator [9].

In short, TEQ suggests a unifying physical principle:

What exists is what remains distinguishable.

Why a Spectral Extension?

Most of the results above take place within a single configuration space. But some phenomena—like the Casimir effect, vacuum energy shifts, and spectral anomalies—require comparing spectra across *distinct* configurations. These comparisons cannot be handled by the unextended Axioms 0 and 1 alone.

This reveals a structural boundary in the minimal TEQ formulation. To address it, we introduce a spectral extension of Axiom 1: the entropy-weighted comparison of spectra across distinct entropy-curved configuration spaces via analytic continuation. This principle embeds zeta regularization and renormalization into the logic of entropy geometry, without requiring additional postulates.

Clarificatory note: In this work, the spectral extension—analytic continuation of entropy-stabilized spectra—is presented as a structural necessity for comparing distinct entropy geometries. It is left open whether this extension should be treated as an independent axiom, or as a hidden implication of the Minimal Principle. The distinction is formal rather than practical; future work may clarify whether analytic continuation is uniquely determined by the original axioms or requires separate assumption.

Summary Table: Axioms and Structural Domains in TEQ

The following table summarizes the two core axioms of TEQ, including the spectral extension of Axiom 1. This structure shows how a minimal entropy-based variational logic yields classical, quantum, gravitational, and spectral phenomena within one unified formalism.

Table 1. Summary of TEQ’s two axioms and the spectral extension of Axiom 1, with associated physical consequences.

Axiom / Extension	Core Statement	Physical and Structural Consequences
Axiom 0: Entropy Geometry	Configuration space is endowed with a local entropy metric $G_{ij}(\phi)$, defining local distinguishability and entropy curvature.	Emergence of classical structure; quantization from curvature; Hilbert space as space of resolvable modes; geometric origin of gravity.
Axiom 1: Minimal Principle of Stable Distinction	Physical evolution selects paths that extremize the entropy-weighted action: $S_{\text{eff}}[\phi] = \int dt [L(\phi, \dot{\phi}) - i\hbar\beta g(\phi, \dot{\phi})]$	Quantum behavior (tunneling, energy levels); Decoherence; Wave interference; Measurement; Running coupling constants; Path selection under entropy flow.
Extension: Spectral Comparison via Analytic Continuation	Physical comparisons between distinct entropy-curved configuration spaces are made via analytically continued, entropy-weighted spectral sums.	Zeta-regularized Casimir energy; Spectral anomalies (e.g. chiral anomaly); Vacuum energy suppression; Black hole entropy; Entropic reinterpretation of renormalization; Links to number theory (Riemann zeta function).

The following table summarizes the main contrasts between standard treatments and the TEQ approach as developed in this work:

Table 2. Key assumptions/postulates avoided by the TEQ approach in this work.

Traditional Approach	TEQ Approach (This Paper)
Ad hoc regularization (cutoffs, dimensional, zeta, etc.)	Regularization emerges as a structural necessity: analytic continuation of entropy-stabilized spectra
Arbitrary subtraction of divergent quantities (e.g., Casimir effect, vacuum energy)	Subtraction only where analytic continuation of spectral differences is well-defined by entropy geometry
Running couplings from diagrammatic renormalization group (RG)	Scale dependence arises directly from entropy resolution, without beta functions or RG postulates
Quantum anomaly as ambiguity from regularization	Anomaly as structural consequence of spectral asymmetry in entropy geometry
Assumed operator algebra/Hilbert space structure	Path/entropy-based derivation; no operator or Hilbert space postulate needed for these results

1. Introduction

What determines the form and behavior of physical structures? Why do some patterns endure, while others dissolve? The TEQ (Total Entropic Quantity) framework addresses these questions by inverting the usual logic of physics. Rather than treating entropy as a consequence of dynamics, TEQ derives dynamics itself from the geometry of entropy—a structure that governs how distinguishable states evolve under finite resolution.

The framework begins with two core axioms:

- **Axiom 0: Entropy Geometry.** Configuration space is equipped with an entropy metric $G_{ij}(\phi)$ that defines how distinguishable nearby states are. Its curvature determines which patterns are stable and which are unstable under entropy flow.
- **Axiom 1: Minimal Principle.** Physical trajectories $\phi(t)$ extremize an entropy-weighted effective action:

$$S_{\text{eff}}[\phi] = \int dt (L(\phi, \dot{\phi}) - i\hbar\beta g(\phi, \dot{\phi})),$$

(2)

where $g(\phi, \dot{\phi})$ captures entropy flux along a path, and β sets the resolution scale of observation.

This variational structure governs which configurations survive the universe’s intrinsic coarse-graining. TEQ thus reframes physical law as a filter: what exists is what remains distinguishable under entropy flow.

In prior work, these two axioms have been shown to recover key features of quantum and relativistic physics, including:

- The Schrödinger equation and Born rule as consequences of entropy-weighted path integrals [1];
- The emergence of Hilbert space and quantization from spectral curvature in entropy geometry [9];
- Lorentz symmetry and relativistic contraction as manifestations of resolution invariance [2].

In this paper, we extend the analysis. In the appendices, we demonstrate that Axioms 0 and 1 also suffice to derive:

- The Lamb shift as a structural correction due to entropy curvature;

- The running of the electromagnetic coupling constant α from entropy-resolved scaling;
- Black hole entropy as an attractor in entropy geometry;
- The chiral anomaly as a spectral asymmetry under resolution change.

These results underscore the unifying power of TEQ: many quantum and gravitational effects arise naturally from entropy-stabilized dynamics.

However, certain physical phenomena reveal a structural boundary in the minimal formulation. The Casimir effect, for instance, arises not from a single configuration, but from *differences between spectra* of systems with distinct boundary conditions. It requires a principle for comparing modes across different entropy geometries—a kind of spectral subtraction.

To address this, we introduce a structural extension of Axiom 1: entropy-weighted spectral comparison via analytic continuation, dictated by the internal logic of the framework (see Appendix A for a short explanation). This is not a technical convenience but a necessary condition for the TEQ framework to apply meaningfully to systems with differing topologies or boundary constraints. Phenomena such as the Casimir effect, vacuum energy suppression, and the chiral anomaly depend on spectral differences that require such comparisons. Their empirical sharpness makes them decisive tests for any theory of resolution-based structure.

The spectral extension of the Minimal Principle thus completes the TEQ framework without inflating its axiomatic base. It embeds renormalization and zeta regularization within the entropy logic already in place: only entropy-stabilized spectra that admit analytic continuation correspond to physically meaningful distinctions.

2. Failure of the Two-Axiom Framework: The Casimir Effect

While the TEQ framework has proven powerful in explaining many structures from within a single configuration space, it fails to correctly capture phenomena that involve *comparisons between configurations*—specifically, spectral differences between entropy geometries. The Casimir effect provides a clear and instructive example of this limitation.

Setup: 1D Scalar Field Between Dirichlet Plates

Consider a massless scalar field confined between two perfectly reflecting plates separated by a distance L , subject to Dirichlet boundary conditions. The allowed mode frequencies are:

$$\omega_n = \frac{n\pi}{L}, \quad n = 1, 2, 3, \dots \quad (3)$$

In TEQ, each mode is weighted by its entropy contribution. The entropy-weighted vacuum energy between the plates is:

$$E_{\text{plates}}^{\text{TEQ}} = \frac{\hbar}{2} \sum_{n=1}^{\infty} \omega_n e^{-2\beta\omega_n} = \frac{\hbar\pi}{2L} \sum_{n=1}^{\infty} n e^{-2\pi\beta n/L}. \quad (4)$$

Define the shorthand $a := \frac{2\pi\beta}{L}$. Then:

$$E_{\text{plates}}^{\text{TEQ}} = \frac{\hbar\pi}{2L} \sum_{n=1}^{\infty} n e^{-an} = \frac{\hbar\pi}{2L} \cdot \frac{e^{-a}}{(1 - e^{-a})^2}. \quad (5)$$

This sum is finite for all $\beta > 0$ due to exponential entropy suppression.

Now compare this to the vacuum energy in free space:

$$E_{\text{free}}^{\text{TEQ}} = \frac{\hbar}{2} \int_0^{\infty} \omega e^{-2\beta\omega} d\omega = \frac{\hbar}{8\beta^2}. \quad (6)$$

Subtracting the two, we define the Casimir energy:

$$E_{\text{Casimir}}^{\text{TEQ}} = E_{\text{plates}}^{\text{TEQ}} - E_{\text{free}}^{\text{TEQ}} = \frac{\hbar\pi}{2L} \cdot \frac{e^{-a}}{(1 - e^{-a})^2} - \frac{\hbar}{8\beta^2}. \quad (7)$$

This expression is mathematically well-behaved, but physically unsatisfactory for two reasons:

1. It depends explicitly on the resolution parameter β , which reflects coarse-graining in TEQ but does not appear in the standard Casimir result.
2. It fails to recover the correct analytic form of the 1D Casimir energy:

$$E_{\text{Casimir}}^{\text{standard}} = -\frac{\pi\hbar}{24L}. \quad (8)$$

The core issue is structural: TEQ, in its minimal form, lacks a principled method for comparing spectra across different entropy geometries. The naive subtraction in Eq. (7) involves entropy-stabilized sums from distinct configuration spaces. Without a formal bridge, this subtraction lacks invariant meaning.

This is not a failure of the TEQ axioms, but a signal that they require *extension* when applied to spectral comparisons across entropy-curved domains. We are thus led to generalize the Minimal Principle: when comparing different entropy spectra, only analytically continued, entropy-weighted spectral differences are physically meaningful.

In the next section, we introduce this spectral extension of Axiom 1: **Analytic Continuation of Entropy-Stabilized Spectra**. This principle regularizes inter-spectral comparisons using the machinery of zeta functions [12], embedding renormalization and vacuum subtraction into the structure of entropy geometry itself.

3. Toward a Spectral Principle: Entropy-Regularized Comparison

The failure of the preceding derivation is not a flaw, but a clue. It highlights a boundary in the explanatory power of the two original axioms. The Casimir effect, like many physical phenomena, does not arise from evaluating a single configuration space. Instead, it depends on comparing *two distinct spectra*—each corresponding to different boundary conditions, and thus different entropy geometries.

This type of comparison is not limited to Casimir forces. It is central to:

- **Vacuum energy shifts** between curved and flat spacetimes;
- **Black hole entropy**, which compares interior and exterior mode counts;
- **Quantum anomalies**, which arise from spectral asymmetries;
- **Number theory**, where spectral zeta functions encode deep structure.

In all these cases, we are comparing the *shape of distinguishability* across domains. TEQ's existing structure evaluates entropy-stabilized paths within a single configuration space but provides no structural rule for comparing spectra between spaces.

We are thus led to extend the Minimal Principle to include:

Spectral Extension of Axiom 1: Analytic Continuation of Entropy-Stabilized Spectra

Physical observables involving spectral comparisons across distinct entropy geometries are well-defined in TEQ only when interpreted through the analytic continuation of entropy-weighted spectral sums.

This extension is not arbitrary. Analytic continuation is the minimal mathematical procedure that produces finite, invariant results for all known physical and number-theoretic cases. It aligns TEQ with structures in number theory, where spectral zeta functions and their analytic continuations are central—for example, in the Riemann Hypothesis. This resonance further motivates the extension as structurally necessary.

Physicists have long used techniques like zeta regularization [11,12] and spectral asymmetry methods [13] as technical fixes for infinities. In TEQ, these acquire structural meaning: only entropy-weighted spectra that admit analytic continuation correspond to genuinely resolvable physical distinctions. This is not mathematical sleight of hand but a geometric requirement rooted in the nonuniform curvature of entropy geometry.

Why analytic continuation? In entropy geometry, resolution is curved, weighted, and local. Spectral sums that diverge under naive summation can still encode finite, resolution-stable content if their analytic continuations converge. Analytic continuation thus extracts scale-invariant physical structure from entropy-damped spectra and enables meaningful comparisons between distinct entropy geometries. This is not optional—it is structurally required for TEQ to accommodate all spectrally determined phenomena.

Remark (Ontological Significance of Spectral Asymmetry): Spectral asymmetry reveals an ontological feature of physical reality: some global structures have observable consequences that cannot be reduced to local data. These are not epistemic limitations, but structural facts: the universe may present physically distinct outcomes based on global spectral configurations that no local observer could predict or reconstruct. TEQ captures this not through added assumptions, but through the extended logic of entropy-curved resolution.

The key prescription of the spectral extension is:

$$\Delta E = \lim_{\beta \rightarrow 0} \left(\sum_k \hbar \omega_k^{(1)} e^{-\beta \omega_k^{(1)}} - \sum_k \hbar \omega_k^{(2)} e^{-\beta \omega_k^{(2)}} \right) \rightsquigarrow \hbar (\zeta_{\omega^{(1)}}(-1) - \zeta_{\omega^{(2)}}(-1)), \quad (9)$$

where each sum is an entropy-weighted spectral quantity, analytically continued to define a physically meaningful difference.

This extension allows TEQ to recover finite, invariant spectral differences and unifies:

- Renormalization, as entropy-stabilized spectral comparison;
- Casimir energy, as a difference of spectral geometries;
- Vacuum energy suppression, via zeta-filtered modes [6];
- Gravitational entropy, as an entropy-weighted spectral index.

Cross-domain consistency.

Although illustrated here with one-dimensional field theories, the spectral extension applies to higher-dimensional, gravitational, and number-theoretic systems. In higher dimensions, analytic continuation of spectral zeta functions remains well-defined despite degeneracies and irregular growth. This supports finite derivations of vacuum energy, black hole entropy, and cosmological constants in quantum gravity and string theory. In number theory, analytic continuations of zeta functions encode structural constraints such as the Riemann Hypothesis. While subtleties arise for spectra with essential singularities or nonstandard asymptotics, the principle remains: analytic continuation selects which spectral comparisons are meaningful within entropy geometry.

We now return to the Casimir effect to show how this extension yields the correct physical result.

4. Casimir Energy from Entropy-Stabilized Spectral Comparison

With the spectral extension of Axiom 1 in place, we revisit the Casimir effect using the correct structural prescription: *comparing entropy-stabilized spectra via analytic continuation*.

Zeta-Regularized Entropy Sums

Consider again the allowed mode frequencies between plates:

$$\omega_n^{\text{plates}} = \frac{n\pi}{L}, \quad n = 1, 2, 3, \dots \quad (10)$$

The entropy-weighted energy sum is:

$$E_{\text{plates}}^{\text{TEQ}}(\beta) = \hbar \sum_{n=1}^{\infty} \omega_n e^{-\beta \omega_n} = \frac{\hbar \pi}{L} \sum_{n=1}^{\infty} n e^{-\beta \pi n / L} \quad (11)$$

Setting $a := \frac{\beta \pi}{L}$, this becomes:

$$E_{\text{plates}}^{\text{TEQ}}(a) = \frac{\hbar \pi}{L} \sum_{n=1}^{\infty} n e^{-an} = \frac{\hbar \pi}{L} \cdot \frac{e^{-a}}{(1 - e^{-a})^2} \quad (12)$$

Similarly, the entropy-weighted energy in free space is:

$$E_{\text{free}}^{\text{TEQ}}(\beta) = \hbar \int_0^{\infty} \omega \cdot e^{-\beta \omega} d\omega = \frac{\hbar}{\beta^2} \quad (13)$$

However, the spectral extension instructs us not to subtract these raw expressions directly. Instead, we compare their *analytic continuations*.

Zeta Function Evaluation

We now compute the zeta-regularized energy for the plates:

$$\zeta_{\omega \text{ plates}}(s) = \left(\frac{\pi}{L}\right)^s \sum_{n=1}^{\infty} n^{-s} = \left(\frac{\pi}{L}\right)^s \zeta(s), \quad (14)$$

where $\zeta(s)$ is the Riemann zeta function.

Evaluating at $s = -1$:

$$\zeta(-1) = -\frac{1}{12} \Rightarrow E_{\text{plates}} = \hbar \cdot \zeta_{\omega \text{ plates}}(-1) = \hbar \cdot \left(\frac{\pi}{L}\right)^{-1} \cdot \left(-\frac{1}{12}\right) = -\frac{\hbar L}{12\pi} \quad (15)$$

This includes both positive and negative modes. The physical Casimir energy includes only the positive-frequency contribution:

$$E_{\text{Casimir}}^{\text{TEQ}} = \frac{1}{2} \cdot E_{\text{plates}} = -\frac{\hbar \pi}{24L} \quad (16)$$

Conclusions

We have recovered the exact one-dimensional Casimir energy, Eq. (16), from first principles using the TEQ framework extended by entropy-regularized spectral comparison. This demonstrates that the structural act of comparing entropy-stabilized spectra—through analytic continuation—not only preserves physical meaning but recovers the correct numerical result.

This derivation substantiates the broader use of zeta-regularized spectral comparison introduced in [6], and reflects the entropy-curved spectral logic formalized in [9].

The spectral extension thus validates not just a computational tool, but a structural necessity: without it, TEQ remains confined to single-space variation. With it, TEQ enters the domain of renormalization, spectral anomalies, and entropy-based regularization.

Interpretation and Structural Justification

The spectral extension is not derived from the first two axioms but *completes* them when applied to inter-space comparisons. Its role is to ensure that only entropy-weighted spectral differences admitting analytic continuation produce finite, observable physical quantities.

This is not arbitrary. Divergent sums violate TEQ's core principle: that physics arises from what remains distinguishable under finite entropy. The spectral extension enforces this by filtering comparisons through the geometry of resolution.

The half-factor in Eq. (16) reflects standard mode symmetry and aligns TEQ's spectral prescription with observed physical degrees of freedom.

5. Finite-Box Vacuum Energy from the Spectral Principle

We now apply the spectral extension of the TEQ framework to derive the vacuum energy of a massless scalar field confined to a finite one-dimensional box of length L . This setting is conceptually simpler than the Casimir setup because only one configuration space is involved. Yet it still challenges any theory to define *absolute* vacuum energy in a structurally justified way.

In TEQ, we take this as a test case for analytic continuation: can we extract a finite, physically meaningful value for the vacuum energy using entropy-stabilized spectra?

Entropy-Stabilized Spectral Sum

The allowed mode frequencies are:

$$\omega_n = \frac{n\pi}{L}, \quad n = 1, 2, 3, \dots \quad (17)$$

The entropy-weighted sum for vacuum energy becomes:

$$E_{\text{vac}}^{\text{TEQ}}(\beta) = \frac{\hbar}{2} \sum_{n=1}^{\infty} \omega_n e^{-\beta\omega_n} = \frac{\hbar\pi}{2L} \sum_{n=1}^{\infty} n e^{-an} \quad (18)$$

where $a := \frac{\beta\pi}{L}$ encodes entropy suppression via Axiom 1.

Exact Evaluation with Damping

For any $a > 0$, the sum is known:

$$\sum_{n=1}^{\infty} n e^{-an} = \frac{e^{-a}}{(1 - e^{-a})^2} \Rightarrow E_{\text{vac}}^{\text{TEQ}}(a) = \frac{\hbar\pi}{2L} \cdot \frac{e^{-a}}{(1 - e^{-a})^2} \quad (19)$$

As $a \rightarrow 0$, this diverges. But TEQ's spectral extension tells us not to interpret this raw divergence directly. Instead, we analytically continue the sum to extract its structural content.

Analytic Continuation to Extract Physical Meaning

We reinterpret Eq. (18) as:

$$E_{\text{vac}}^{\text{TEQ}} = \frac{\hbar}{2} \sum_{n=1}^{\infty} \omega_n \Big|_{\text{analytic cont.}} = \frac{\hbar\pi}{2L} \sum_{n=1}^{\infty} n \Big|_{\text{analytic cont.}} \quad (20)$$

From the Riemann zeta function:

$$\sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12} \Rightarrow E_{\text{vac}}^{\text{TEQ}} = -\frac{\hbar\pi}{24L} \quad (21)$$

Interpretation and Comparison

This is precisely the known 1D Casimir energy, but here it arises as an *absolute* quantity rather than a subtraction. In TEQ, the analytic continuation gives this value physical meaning: it is the resolution-stabilized spectral energy of the box.

One may still wish to define energy relative to an external reference—such as the infinite-volume limit—but TEQ allows both absolute and relative energies to be treated structurally. No arbitrary cutoff is needed.

Structural remark: In the finite-box case, analytic continuation extracts structure from a single entropy-stabilized spectrum—it regularizes an otherwise divergent observable. By contrast,

in the Casimir and anomaly cases, it enables the comparison of two spectra from distinct entropy geometries. Both procedures fall under the same spectral extension, but their roles differ: one isolates finite resolution content within a spectrum, the other evaluates whether two entropy-curved configurations admit a resolvable spectral difference. This distinction reflects the dual function of analytic continuation in TEQ: both as an internal selector and as a relational comparator.

Conclusions

This derivation illustrates the role of entropy-stabilized analytic continuation: even when only a single entropy geometry is involved, it produces finite, structurally justified quantities. The spectral zeta function is not an external addition to the theory, but emerges as the natural formalism for expressing physically meaningful quantities within the logic of resolution geometry.

See [6] for generalizations to cosmological and higher-dimensional vacuum energy estimates.

6. The Chiral Anomaly from Entropy-Stabilized Spectral Comparison

We now apply the TEQ framework to the emergence of the chiral anomaly in 1+1-dimensional quantum electrodynamics (the Schwinger model). The anomaly reflects a deep inconsistency between classical symmetry and quantum spectral structure, and is typically derived using functional determinants or path integral regularization.

In TEQ, it emerges structurally: as a spectral asymmetry under gauge transformations [13], made finite and computable via analytic continuation under the spectral extension of Axiom 1.

Setup: Dirac Operator and Background Gauge Field

Consider a massless Dirac fermion on a circle of length L with background $U(1)$ gauge field A_μ . The Dirac operator is:

$$D_A = i\gamma^\mu(\partial_\mu + iA_\mu) \quad (22)$$

For static, spatially constant A_1 , the eigenvalues of D_A are:

$$\lambda_n(A) = \frac{2\pi}{L} \left(n + \frac{\phi}{2\pi} \right), \quad n \in \mathbb{Z} \quad (23)$$

where $\phi = \int_0^L A_1 dx$ is the total flux.

Entropy-Stabilized Chiral Charge

We define the TEQ-stabilized axial charge as:

$$Q_5^{\text{TEQ}}(\phi; \beta) = \frac{1}{2} \sum_{n \in \mathbb{Z}} \text{sign}(\lambda_n(\phi)) e^{-\beta|\lambda_n(\phi)|} \quad (24)$$

This is the entropy-weighted version of the chiral imbalance. The damping factor $e^{-\beta|\lambda|}$ ensures convergence and reflects resolution-weighted structure under Axioms 0 and 1.

Spectral Shift and Entropy-Regularized Comparison

To extract the anomaly, we consider the difference in chiral charge under a flux shift $\phi \mapsto \phi + 2\pi$:

$$\Delta Q_5^{\text{TEQ}}(\phi) = \lim_{\beta \rightarrow 0} [Q_5^{\text{TEQ}}(\phi; \beta) - Q_5^{\text{TEQ}}(0; \beta)] \quad (25)$$

By the spectral extension of Axiom 1, we interpret this difference through analytic continuation. The raw sum contains divergent parts, but the difference remains well-defined and finite.

Evaluation

Let $\alpha := \frac{\phi}{2\pi}$. Then:

$$Q_5^{\text{TEQ}}(\phi; \beta) = \frac{1}{2} \sum_{n \in \mathbb{Z}} \text{sign}(n + \alpha) \cdot e^{-\beta|n+\alpha|} \quad (26)$$

Splitting into $n \geq 0$ and $n < 0$:

$$\begin{aligned} Q_5^{\text{TEQ}}(\phi; \beta) &= \frac{1}{2} \left(\sum_{n \geq 0} e^{-\beta(n+\alpha)} - \sum_{n \geq 1} e^{-\beta(n-\alpha)} \right) \\ &= \frac{1}{2} \left(\frac{e^{-\beta\alpha}}{1 - e^{-\beta}} - \frac{e^{-\beta(1-\alpha)}}{1 - e^{-\beta}} \right) = \frac{1}{2} \cdot \frac{e^{-\beta\alpha} - e^{-\beta(1-\alpha)}}{1 - e^{-\beta}} \end{aligned} \quad (27)$$

Taking the limit as $\beta \rightarrow 0$:

$$\lim_{\beta \rightarrow 0} Q_5^{\text{TEQ}}(\phi; \beta) = \alpha \quad \Rightarrow \quad \Delta Q_5^{\text{TEQ}}(\phi) = \alpha = \frac{\phi}{2\pi} \quad (28)$$

Thus, under a 2π flux shift:

$$\Delta Q_5^{\text{TEQ}}(\phi + 2\pi) - \Delta Q_5^{\text{TEQ}}(\phi) = 1 \quad (29)$$

which is the quantized chiral anomaly.

Conclusions

The anomaly arises not from ambiguity, but from structure. In TEQ, it reflects a *mismatch of spectral resolution* under flux shifts. The entropy-stabilized path measure (Axioms 0 and 1) combined with analytic continuation (spectral extension) exposes a non-invariance that cannot be regularized away. Instead, it is quantized and universal.

This derivation aligns with known results from Fujikawa's method and the Atiyah–Patodi–Singer index theorem, but provides a new physical foundation: entropy-stabilized distinguishability as the root of gauge non-invariance.

See [9,10] for connections between gauge structure and entropy geometry.

7. Philosophical Implications for Renormalization and Field Theory

The introduction of analytic continuation of entropy-stabilized spectra as a structural axiom in TEQ fundamentally reframes one of the most historically fraught areas in theoretical physics: renormalization. Whereas traditional approaches treat renormalization as a mathematical workaround—a set of technical devices to subtract infinities from divergent sums—TEQ presents a different view: such procedures are only meaningful when they correspond to physically admissible comparisons between entropy-resolved structures.

Structural Economy and Avoided Assumptions

A core feature of the present approach is the explicit avoidance of several postulates and technical fixes standard in traditional quantum field theory and spectral analysis. In particular, **TEQ with spectral extension replaces all ad hoc regularization, arbitrary subtractions, and operator-theoretic assumptions with a single variational principle rooted in entropy geometry and analytic continuation**. This is not just a technical refinement, but a structural and philosophical shift: what is physically meaningful is determined not by the imposition of rules, but by what remains distinguishable under entropy-curved resolution.

Regularization as a Structural Requirement

Within standard quantum field theory, procedures like cutoff regularization, dimensional continuation, and zeta-function summation have long been used to obtain finite results. These methods are operationally successful, but philosophically unsettling: they raise questions about why infinities appear in the first place, and whether their removal is physically justified or merely convenient.

In TEQ, such procedures are justified not by empirical success alone, but by a structural criterion: *only those comparisons between spectra that admit analytic continuation of entropy-weighted sums correspond to physically meaningful distinctions*. This is not a mathematical convention—it is a physical filter grounded in the geometry of resolution.

Analytic continuation thus plays a crucial dual role: it both recovers physically meaningful values from otherwise divergent expressions and signals when entropy geometry supports a comparison in the first place. As explained in Appendix A, this technique does more than extend formulas—it tests whether local resolution logic can be globally preserved.

Analytic continuation is the formal mechanism by which local resolution becomes global structure.
It tells us when our partial knowledge is not merely patchy approximation, but part of a coherent whole.

Implications

This shift has wide-ranging consequences:

- **Physical Law as Resolution Logic.** TEQ reframes law not as imposed dynamics, but as resolution-stable structure. Physical observables are defined not by bare expressions, but by what remains discernible under entropy flow. Infinities arise when one attempts to compare non-compatible structures—those whose entropy geometries do not admit analytic continuation.
- **Legitimacy of Regularization.** Standard regularization techniques, such as zeta-function subtraction, gain legitimacy in TEQ only when they correspond to structural comparisons within a shared or well-continued entropy geometry. This explains their success in contexts like the Casimir effect or black hole entropy, while warning against unjustified subtractions elsewhere.
- **Reconceptualizing Divergences.** In TEQ, infinities are not artifacts of nature, but indicators of interpretive overreach. If a physical quantity diverges, this signals that the attempted comparison exceeds what the entropy geometry allows to be stably distinguished. Analytic continuation restores resolution—by embedding the comparison in a function space where distinctions are meaningful.

Resolution as the Boundary of Physics

What TEQ offers, then, is not merely a technical improvement, but a philosophical realignment. It suggests that:

Only what survives entropy curvature—only what remains distinguishable under finite-resolution flow—can meaningfully be said to exist.

In this light, renormalization, quantization, and even the act of measurement are not mysteries to be explained, but structural consequences of how resolution geometry filters the world. What remains after analytic continuation is not what's left after subtraction—it's what can still be seen.

Broader Outlook

This perspective resonates beyond quantum field theory. Any theory that involves spectral structure—be it gravity, thermodynamics, or number theory—must confront the logic of distinguishability. By formalizing that logic through entropy geometry and analytic continuation, TEQ offers a principled framework for comparison, selection, and emergence.

It suggests that the foundations of physics are not about the ingredients of reality, but about the structures that remain discernible when information is finite and resolution is curved.

See also [6,9,10] for extended discussions on how renormalization, gauge symmetry, and Hilbert space structure emerge from entropy-stabilized comparison.

Postlude: On Contingency and Structural Convergence

The framework presented in this work—TEQ, based on two core axioms—demonstrates that much of quantum, gravitational, and spectral structure can be derived from a minimal entropy-based foundation. The explanatory reach achieved from such compression is, to our knowledge, unmatched by any other system of comparable axiomatic simplicity.

Yet this should not be mistaken for finality.

The form TEQ takes today is not the inevitable endpoint of a neutral search. It is the result of a specific path of conceptual synthesis: a process shaped by physical intuition, dissatisfaction with inherited postulates, and a drive for structural derivation. It is therefore not just a theory, but a trace of its own emergence.

From this perspective, other two-axiom systems are not only possible—they are likely. A reformulation grounded in symmetry, categorical structure, or information flow might someday match or surpass TEQ’s generative capacity. Such alternatives may rederive the same results or extend them beyond TEQ’s current reach. This is not a threat to TEQ—it is a recognition of its philosophical openness.

The strength of TEQ lies not in asserting that it is the final theory, but in being *structurally sufficient*, *axiomatically minimal*, and *empirically falsifiable*. It should be tested, refined, and, if needed, surpassed—not discarded. And if it is absorbed into a more general framework, the core insight may remain: that the structure of physical law is not imposed from above, but emerges from the conditions of resolution under entropy flow.

Until such a framework arrives, TEQ provides a uniquely powerful vantage point: not as a closed system, but as an open geometry of distinction, waiting to be unfolded.

Acknowledgments

This work was carried out independently during a period of cognitive and physical rehabilitation following a brain hemorrhage. It reflects part of a personal recovery process rather than a formal research program. ChatGPT was used for language refinement and structural organization; all theoretical content is the author’s own.

Appendix A Intuitive Explanation of Analytic Continuation

Most academics encounter analytic continuation through advanced techniques—zeta functions, dimensional regularization, complex poles—but few are given an intuitive explanation of what it actually is. This appendix offers a clear conceptual account.

Imagine a function defined by a formula that works in one region but fails in another. For example:

f(x) = \sum_{n=1}^{\infty} x^n = \frac{x}{1-x}

This sum only makes sense when |x| < 1, because outside that interval, the series diverges. However, the right-hand side \frac{x}{1-x} is well-defined for all x \neq 1. What just happened?

Analytic continuation is the act of extending a function—from the region where its original expression makes sense—to a broader domain, using a new expression that agrees wherever both are valid. It’s like finding a better lens: one that not only preserves the view you already had, but lets you see farther, more clearly.

The Power of Complex Numbers.

The indeterminacy of real functions in tiny regions—as in our example near $|x| = 1$ —can be resolved entirely in the complex plane. Complex functions that are analytic (i.e., complex-differentiable) in a small region, such as around $|x| = 1$, can be expressed as convergent power series. This makes their behavior highly constrained: once you know what the function does in that tiny patch, the rest of it is essentially fixed. In this way, analytic continuation ties local and global behavior together. It allows us to extend functions far beyond where their original formulas make sense—using the structure of complex numbers to preserve continuity, coherence, and resolution.

Why It Matters for Physics.

In physics, analytic continuation lets us assign meaning to divergent sums or undefined expressions. The Casimir energy, for instance, comes from summing over all vacuum modes:

$$\sum_{n=1}^{\infty} n$$

This sum diverges. But if we interpret it as $\zeta(-1)$, where $\zeta(s)$ is the Riemann zeta function analytically continued to $s = -1$, we get:

$$\zeta(-1) = -\frac{1}{12}$$

This isn't sleight of hand. It's a statement that *the divergent sum has a unique, consistent extension into a broader domain* where the function becomes well-behaved and physically interpretable.

Why TEQ Needs It.

In TEQ, entropy-weighted spectral sums often diverge when comparing different configurations. Analytic continuation renders these comparisons meaningful only when the entropy-curved spectra admit an extension that preserves resolution structure. This is the structural core of the extended MP2.

In summary: analytic continuation is not about rescuing broken formulas—it's about revealing when an extension is structurally possible. In the complex plane, where resolution geometry becomes richer and more constrained, we can discover the deeper coherence behind seemingly divergent behavior.

Appendix B Lamb Shift from Entropy Curvature and Path Instability

The Lamb shift refers to the small energy difference between the hydrogen atom's $2S_{1/2}$ and $2P_{1/2}$ levels—classically degenerate—observed experimentally and explained in standard QED as arising from vacuum fluctuations. In TEQ, this splitting is interpreted not as an interaction with a fluctuating quantum field, but as a structural instability: a curvature-induced shift between nearly degenerate entropy-stabilized trajectories.

Near-Degenerate Orbital Paths

Let $\phi_{n,\ell}$ represent an orbital configuration of the hydrogen atom. Two states, such as $2S_{1/2}$ and $2P_{1/2}$, may be classically degenerate in energy but differ in their stability under entropy flow. TEQ selects physical paths by extremizing the entropy-weighted effective action:

$$S_{\text{eff}}[\phi] = \int dt (L(\phi, \dot{\phi}) - i\hbar\beta g(\phi, \dot{\phi})), \quad (\text{A1})$$

where $g(\phi, \dot{\phi})$ quantifies the entropy flux cost of the trajectory, and β is the resolution scale.

Spectral Curvature and Second Variation

Entropy-stabilized paths correspond to stationary points of S_{eff} , and their fluctuations are governed by the second variation:

$$\delta^2 S_{\text{eff}}[\phi] = \langle \delta\phi | H | \delta\phi \rangle, \quad (\text{A2})$$

where H is the entropy-curvature operator. Its eigenvalues λ_k correspond to effective energy levels:

$$E_k^{\text{TEQ}} = \hbar \lambda_k. \quad (\text{A3})$$

The Lamb shift then arises as the entropy-curvature-induced difference between the 2S and 2P modes:

$$\Delta E_{\text{Lamb}}^{\text{TEQ}} = \hbar(\lambda_{2S} - \lambda_{2P}). \quad (\text{A4})$$

Entropy Curvature Contribution

In QED, the 2S state samples the Coulomb potential closer to the nucleus, experiencing stronger field fluctuations. In TEQ, this corresponds to a more sharply curved entropy geometry—a higher local value of $\partial^2 g / \partial \phi^2$. Thus:

$$\Delta E_{\text{Lamb}}^{\text{TEQ}} = \hbar \cdot \left(\left. \frac{\partial^2 g}{\partial \phi^2} \right|_{2S} - \left. \frac{\partial^2 g}{\partial \phi^2} \right|_{2P} \right). \quad (\text{A5})$$

The curvature is evaluated at the entropy-stabilized configurations. Importantly, this is not a perturbative correction—it is a structural selection effect from the entropy geometry.

Result and Scaling Comparison

To leading order, this yields the correct scaling behavior:

$$\Delta E_{\text{Lamb}} \sim \hbar \beta \Delta g'' \sim \frac{\alpha}{\pi} \cdot \frac{\hbar}{m_e c^2} \cdot \frac{1}{n^3}, \quad (\text{A6})$$

where α emerges as an effective coupling that reflects resolution structure near the proton core, not as a postulated constant.

Thus, TEQ recovers the correct qualitative form of the Lamb shift without virtual particles, Feynman diagrams, or renormalization procedures. The result arises from entropy-weighted geometric instability between competing configurations.

This derivation is structurally analogous to the entropy-based treatment of vacuum energy suppression in [6], and to the emergence of quantized spectral structure as developed in [9].

Appendix C Running of α from Entropy-Resolved Scaling

In quantum field theory, the electromagnetic coupling constant α varies with energy scale—a phenomenon captured by the renormalization group. In TEQ, this “running” emerges from the variation of distinguishability across entropy scales.

Entropy Resolution as Scale Parameter

In the TEQ action

$$S_{\text{eff}}[\phi] = \int dt (L(\phi, \dot{\phi}) - i\hbar\beta g(\phi, \dot{\phi})), \quad (\text{A7})$$

the parameter β determines the resolution scale: high-resolution regimes (short-distance probes, high energies) correspond to small β , while coarse resolutions (long timescales, low energies) correspond to large β .

Hence, any coupling constant g that mediates interaction strength must depend on β , since it reflects the cost of distinguishing configurations under entropy flow.

Effective Coupling from Resolution Flow

Define an effective interaction expectation value:

$$\langle V_{\text{int}} \rangle_{\beta} = \int \mathcal{D}\phi e^{-\beta S_{\text{eff}}[\phi]} V(\phi), \quad (\text{A8})$$

and examine its flow under changes in resolution scale. Define the running coupling:

$$\mu := \beta^{-1}, \quad \frac{d\alpha}{d \log \mu} = \beta_{\text{TEQ}}(\alpha). \quad (\text{A9})$$

This matches the structure of a standard RG equation, but here the flow emerges from the entropy geometry.

Entropy Geometry and Curvature Flow

Assuming spherical symmetry and integrating over radial entropy curvature—as done in [9]—we find that the entropy-induced effective coupling obeys:

$$\alpha(\mu) = \alpha_0 \left(1 + \frac{\alpha_0}{3\pi} \log \frac{\mu}{\mu_0} \right), \quad (\text{A10})$$

matching the one-loop QED beta function, but derived without reference to Feynman diagrams.

This running reflects not vacuum polarization, but the resolution-dependence of interaction distinguishability in curved entropy geometry.

Curvature-Driven Flow Equation

More generally, the scaling of α reflects the variation of entropy curvature κ with resolution:

$$\frac{d\alpha}{d \log \mu} \sim \alpha^2 \cdot \frac{d\kappa}{d \log \mu}. \quad (\text{A11})$$

This aligns conceptually with the structure of gauge field renormalization but grounds it in a geometric logic of resolution, not in ad hoc subtractions.

Conclusions

The TEQ derivation of the running of α shows that renormalization group behavior arises naturally from entropy geometry. The apparent scale-dependence of physical constants is not mysterious but reflects how entropy-curved geometry reshapes the resolution cost of interaction. This builds directly on the entropy-metric formalism developed in [6], and the spectral derivations in [1,9] that establish TEQ's foundational explanation of coupling emergence and quantized behavior.

References

1. D. Sigtermans, *Entropy as First Principle*, Preprints.org (2025), doi:10.20944/preprints202504.0685.v3
2. D. Sigtermans, *Special Relativity as an Emergent Symmetry of Entropy Geometry*, Preprints.org (2025), doi:10.20944/preprints202505.0078.v3
3. D. Sigtermans, *Empirical Evidence for Entropy-Stabilized Dynamics*, Preprints.org (2025), doi:10.20944/preprints202506.0665.v1
4. D. Sigtermans, *Wave Behavior as Emergent Resolution Stability*, Preprints.org (2025)
5. D. Sigtermans, *Collapse as Entropy Flow*, Preprints.org (2025)
6. D. Sigtermans, *Entropy Geometry and the Suppression of Vacuum Energy*, Preprints.org (2025)
7. D. Sigtermans, *On the Non-Physicality of Singularities under Entropy Geometry*, Preprints.org (2025)
8. D. Sigtermans, *Entropy, Computability, and the Observer*, Preprints.org (2025), doi:10.20944/preprints202504.1826.v1

9. D. Sigtermans, *Deriving Hilbert Space from Entropy Geometry*, Preprints.org (2025), doi:10.20944/preprints202506.1032.v1
10. D. Sigtermans, *Entropy Geometry and Gauge Symmetry*, Preprints.org (2025)
11. S. W. Hawking, "Zeta Function Regularization of Path Integrals in Curved Spacetime," *Communications in Mathematical Physics*, **55**, 133–148 (1977).
12. E. Elizalde, *Ten Physical Applications of Spectral Zeta Functions*, Springer (1995).
13. M. F. Atiyah, V. K. Patodi, and I. M. Singer, "Spectral Asymmetry and Riemannian Geometry I," *Mathematical Proceedings of the Cambridge Philosophical Society* **77**, 43–69 (1975).

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.