Article

Not peer-reviewed version

The Time Interval Only Set, Part 1

Harmen H. Hollestelle *

Posted Date: 19 March 2024

doi: 10.20944/preprints202209.0151.v5

Keywords: time interval; (a-)symmetry; commutation relations; simultaneity; radiation; gravitation; Curie's principle; Noether charges; structure constants; group-velocity; light velocity; time interval only set properties

Article

The Time Interval Only Set, Part 1

Harmen H. Hollestelle

Independent researcher, Bandung 40132, West-Java, Indonesia; harmhollestelle@yahoo.com

Abstract: A time interval only description necessarily needs its own independent definitions, without one moment time coordinates to it. In this paper derived are time interval only set properties, including commutation relations, addition, multiplication and derivatives. Time interval only set commutation relations, consistent with one moment time set properties, are introduced by defining correspondence relations for these sets. Time development and equilibrium within the time interval description depend on the well-known 'mean velocity theorem' and the time interval version of the Legendre transform. In contrast, usually applied is only the traditional vector and one moment time coordinate kinematic approach from Newton's laws. The main results of this paper are correspondences and time interval only set commutation relations. All other properties are derived from these results, while for the time interval only set, different from the one moment time set the canonical property is not valid. The time interval only description can be applied assuming spherical symmetric star-source radiation propagation, with finite time intervals for realistic measurement events. From time development with (a-)symmetry of time intervals, and from Curie's principle, change can be described with a set theoretic approach. Time interval only set properties include the time interval only version of Noether charges and structure constants. A time interval only description for wave propagation, including group-velocity, for zero and non-zero mass wave particles, relates to the properties of the time interval only set. A second result in terms of the time interval only description is propagation wave energy equivalence with gravitation energy, derived from time interval only set commutation relations. A time interval approach, however including one moment time description dependence, was introduced in earlier publications of this author. In part 1 investigated are the time interval only set and its properties, a forthcoming part 2 to application of the time interval only description to specific groups of stars as emission sources.

Keywords: time interval; (a-)symmetry; commutation relations; simultaneity; radiation; gravitation; Curie's principle; Noether charges; structure constants; group-velocity; light velocity; time interval only set properties

---

1. Introduction

1.1. Introduction and Results

The time interval only description is a new approach to time development, and differs from the Newtonian kinematic and one moment time coordinate description, due to the application of only time intervals and of 'mean velocity' theorem (TE) equilibrium which differs from Lagrange equilibrium. With finite events, like realistic measurement events, in this paper, meant is introducing a relevant event time interval Δt. This is a finite time interval, even without interference or interaction during the event. Traditionally, rather than interval related, with an event is meant a one moment time coordinate space-time event, with one-value event coordinates and derivatives in affine 4-dim. universe (Arnold, 1974).

For a time interval only description including star-source radiation assumed are spherical symmetric star-like emission sources, and finite wave average group-velocity to be able to introduce propagation surfaces and a propagation surface related relevant event time interval Δt. Remaining within the time interval description, quantities can be t-quantity, variable with one moment time coordinate t during time interval Δt, or Δt-quantity, with Δt the variable, evidently invariant for Δt and independent of one moment time t.

For symmetric time intervals the results for the time interval description are the same as for the usual one moment time description, from Newtonian kinematics and with Hamiltonian H time independent, e. g. by ref. (Goldstein, 1950; Arnold, 1974). Time interval Δt symmetry implies it has infinite measure, asymmetric Δt implies a finite measure. Several of the arguments to develop a time interval only description besides the existing one moment time description have been discussed extensively in earlier papers, (Hollestelle, 2020, 2021). This paper, part 1, with forthcoming part 2, is a new version for preprint (Hollestelle, 2023). Of the traditional approaches within the one moment time description some are summarized to clear the way, even when they are commonly known.

Within the time interval only description one moment time concept 'now' is not introduced, it is not to be regarded the same as the average for t during time interval Δt. However, one can consider Δt to be the finite time interval version and 'reverse' to the tangent, one parameter group, infinitesimal limit process within the one moment time description. This limit process has been problematic for traditional calculus and for derivatives, where the limit means approximation of the one moment time 'addition zero', (Newton, 1686 a; Goldstein, 1950; Arnold, 1974). Within general relativity theory (GR) a secure space-time measure, including simultaneity and realistic signaling with finite velocity depending on the Lorentz metric, is applied necessarily to give meaning to the limit process and finite realistic measurement events, i. e. to recover space-time with secure measures for any place within overall non-linear space-time, (Einstein, 1923). A step-by-step addition process within overall non-linear space-time, approaching measures beyond a secure measure, was probed in (Hollestelle, 2018). With a place is meant a connected and finite part of space-time where a linear metric can be recovered. The time interval only set does not include vectors, the covariant approach in space-time with the traditional vector calculus is not expected to be well-defined.

The main result of this paper is the definition of correspondence of the time interval only set with the one moment time coordinate set, a correspondence which has value by itself. The time interval only set properties all depend on correspondence.

The time interval only set can add to the one moment time description.

A secondary result is the time interval only description suggests by itself an upper limit to, i. e. a finite, e.m. group-velocity *c*. This result is independent of the assumption of a finite and non-zero group-velocity c included in the definition of covariant 4-dim. one moment space time coordinates, which was introduced to allow 4-dim space time to be discussed as one space time. A finite group-velocity assures consistency with experimental e.m. phenomena. Other secondary results remain within the discussion comments A to I. ***Inferred is, from*** addition and scalar multiplication being similar within the time interval only set, gauge transformations dependent on addition and multiplication can be similar within the time interval only description.

Time development from time interval equilibrium, with time intervals being asymmetric or symmetric, can be described with Curie's principle (Curie, 1894), since for these time intervals Hamiltonian H is time dependent or time independent resp., which is a measurable and observable difference. Equilibrium equations for a set of closed surfaces with (a-)symmetric surface geometry energy, however within the one moment time description, were derived in (Hollestelle, 2017), an example of surface asymmetry and its development in time. The surface geometry energy equals the e.m. field energy for this surface interpreted as propagation surface.

Within Curie's principle (a-)symmetry can imply the existence of a causal relation towards resultants (Curie: symétrie des causes, effets produits). The (a-)symmetries are not restricted to 3-space, rather occur in phase space. It is not meant causes are unknown's or effects are observables. The 'causes' and 'effects' need evaluation, if only due to the absence of the concept 'now'. The traditional description of time development with one moment time coordinates is 1-dim. which allows the assumption a cause precedes, in time, the effect. Philosophy, of change or time development or causality, however is not part of this paper.

For the one moment time coordinate set, with the canonical set property, infinitesimal tangent transformations define derivative tangents, that are linearly additive, and commutation relations in phase space, while Lagrange equilibrium remains valid and H time independent, (Goldstein, 1950; Arnold, 1974).

For covariant space-time or phase-space this remains applicable for possible (a-)symmetries and for the description of time development. It is assumed traditionally Curie's principle implies a

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

description of events remaining in Lagrange equilibrium, with H time independent, except for some of events of spontaneous symmetry breaking. Within the time interval description including TE equilibrium, the time interval only set is a 1-dim. set, however H is allowed to be time dependent, while time development from tangent transformations, i. e. from canonical Hamilton equations, remains valid, although the canonical set property and usual representation theory does not apply, (Serre, 1971; Hollestelle, 2020). Even so, time interval only Noether charges and structure constants can be defined, comment G.

The time interval only description for asymmetric Δt does not allow time reversal (Hollestelle, 2020). Plane waves in the time interval description depend on energy rather than coordinates, and energy remains invariant with reversal of kinetics. Time reversal for GR depends on Lorentz invariance.

An alternative to traditional Lorenz invariance and Lorentz transformations is discussed in (Hollestelle, 2021) and par. 4, below. Time development measurements within the time interval description depend on counting forward, not counting backwards, (Hollestelle, 2018, 2020). Time reversal in this paper means the transformation of a time interval to its time interval 'addition inverse' which is non-trivial, different from the one moment time 'addition inverse', and different from multiplication of time interval boundaries with a scalar, e. g. -1. It is found, par. 3, the time interval only addition inverse for Δt, the relevant event time interval, is the time interval only set 'addition zero' which is equal to Δt itself, for asymmetric Δt.

Uncertainty relations are part of QM if only because they relate QM quantities, like elementary particle or radiation field properties, to Newtonian experimental subject properties or quantities, accessible from sequences of interactions or measurements, (Sakurai, 1967; Roos, 1978). The philosophical interpretation of QM is not aimed at in this paper however, while measurement event description is part of it, if only due to relevant event time interval Δt being assigned to a relevant, i. e. measurement, event. Within the time interval description, one finds uncertainty relations different from the usual uncertainty relations, with time dependent 'variable quantities' h+ and h- instead of Planck's constant h when H time dependent, however recovering h when H time independent (Hollestelle, 2020), comment G below, and part 2 of this paper.

Time development for vector fields in QM field theory usually is introduced with time operators, and the QM version of Lagrange equilibrium, commutation relations and derivatives within the one moment time description. For operators the concept 'working to the right' defines meaning in terms of measurement results: operators are 'working to the right' on vector fields, deriving the expectation value of the operator related quantity, (Roos, 1978). Within the time interval only description time development relates similarly to commutation relations and derivatives, however with operators reduced to time interval multiplication and addition. Time interval addition is non-commutative, even so, for two time-intervals being the same, the result of usual commutation brackets is time interval 'addition zero'. This might support a specific time interval particle-oriented description, e. g. with Boson particles, which however is not followed in this paper.

1.2. Time Interval Only Set Properties

The definition of the time interval only set and correspondences are the main results of this paper. This is an accomplishment of its own. The value of the time interval only set is, it is new and alternative to existing time development descriptions and correspondence relates the time interval only description to existing physics, which is within the one moment time description only.

Two different correspondences are defined, applying the definition of time interval only 'addition units' and 'multiplication units', par. 3, or the definition of time interval commutation relations, par. 4. From the correspondences, the commutation relations and derivatives for the time interval only set are derived, par. 5. For the validity of these, two new theorems for the time interval only set are proven: the multiplication closure theorem, and the multiplication linearity theorem, comment B and D resp. The time interval only set derivative includes a non-linear part, included in the time interval derivative 'set-rule', comment C.

1.3. The 'Infinite Regress' Problem

The time interval and TE equilibrium description include one moment time quantities, since time intervals and derivatives to time intervals within its results means boundaries are implied. To find a time interval only description, one can let time interval quantities relate to one moment time quantities due to a transformation being onto however not one-to-one. In this way a correspondence relation can resolve the 'infinite regress' problem quoted in (Hollestelle, 2020), where equilibrium is defined, including the Hamiltonian, being the time interval version of Lagrange equilibrium and Legendre transformations. With one moment time quantities, like time interval boundaries, equated one-to-one with time intervals this would mean, due to the derivative within the time interval version of the Legendre transform, time interval boundaries return within the definition for time interval equilibrium causing the regress. This problem is most relevant when introducing the time interval set as a time interval only set. The term correspondence otherwise sometimes is applied for the relation of QM concepts corresponding with experimental phenomena (Roos, 1978).

1.4. Star-Source Radiation, Radiation and Gravitation

Propagation surfaces are a space-like limit for propagation plane waves, assuming a finite group-velocity and a finite relevant event time interval. A star-like source implies sphere symmetric radiation, and rather the term propagation than 'ray' is applied. Usually, in contrast, for radiation applied are parallel plane waves, meaning parallel rays, and a one moment time description for QM. Zero or non-zero mass wave-particle momentum complementarity is one of few relations of QM to remain within the QM time interval only description (Hollestelle, 2020). With star-source is meant radiation- or photon-production and emission, not meant is a source for a specific field-interaction like usual, e. g. the source density for screened inverse distance, or screened gravitational, interaction, (De Wit, Smith, 1986). The introduction of mass density and gravitation within the time interval only description is discussed in forthcoming part 2 of this paper.

The equivalence principle implies the gravitation energy and kinematic energy references of action to be equivalent where it cannot be decided upon which one is due, and within GR this is expressed with gravitation energy and space-time geometry in terms of measures and Lorentz transformations (Newton, 1686; Einstein, 1923). A result within the time interval only description is: equivalence of reference to gravitation energy and radiation field energy, related to radiation propagation surfaces, which implies a relation to exist for time interval equilibrium and time interval derivatives with geometry, i. e. with space-time.

Star-source outward wave propagation and star-source gravitation are found opposite inclined. Gravitation, an interaction towards its sources, resembles in some respect the reverse of outward propagation, i. e. a phenomenon related to density of which some implications in cosmological setting are discussed in (Hollestelle, 2018). Equilibrium maintenance, including density considerations, derived for sets of (a-)symmetric geometric surfaces, without a priori assumptions, is at the origin of the theorem of continuation of constraints, closely related to Liouville's theorem (Hollestelle, 2017, 2021).

Newton's second law relates kinematics with source related action, applying vectors and spacetime coordinates within a one moment time description, similar to above equivalences. However, following (Arnold, 1974), due to differential eq. considerations, Newton's equations are valid only within a compact part of the time coordinate set where for simplicity often is assumed a complete infinite part. Within the time interval only description, equilibrium is due to energy rather than action, and this allows to discard the question of inertial forces. Within the time interval only description, the equivalence of star-source wave propagation with gravitational interaction, and the introduction of source mass to mean a reference to gravitation energy, originating from (Hollestelle, 2021), is discussed in part 2 of this paper.

2. Commutation Relations: Derivatives and Operators 'Working to the Right'

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

The commutation relations for two one moment time quantities A1 and A2 with the same one moment time parameter t can be written with usual commutation brackets and subscript u, [A1, A2]u = [A1(t). A2(t) - A2(t). A1(t)], for Newtonian quantities and derivatives with reference to (Goldstein, 1950), for operators, (Jauch, Rohrlich, 1976). For the 1-dim. one moment time set assumed are two invariant commutation quantities cn and cn′ within two inter-dependent relations, eq. 1. For higher dimensional sets, one assumes different numbers of quantities and relations. Writing the ordinary multiplication . and with the usual + and –, these relations are:

1 t. cn′ = cn. t

1/t. cn = cn′. 1/t

2 [t, cn]u = t. (cn – cn′)

[1/t, cn]u = (cn′ – cn). 1/t

The cn and cn′ can be different from each other. Scalar multiplication with -1, is part of one moment time commutation relations. One can instead choose for the addition inverse, discussed in comment I.

For the time interval only set, representation theory is not applicable since the canonical property is not valid, i. e. the linear derivative assumption for time development is not valid, however time interval only commutation bracket results similar to eq. 2., are derived in par. 5.

From the commutation relations, one relates derivatives to t, d/dt, to parameter t itself. Within the one moment time description the usual derivative d/dt [A1. A2], with A1. A2 meaning A1 'multiplication' A2, is 'working to the right' on the square brackets, i. e. on every part of the multiplication, the traditional first order differential calculus, originating from (Newton, 1686 a). Assuming the symmetric second associative property AP2 for derivatives one writes:

3 d/dt [A1. A2] = d/dt [A1]. A2 + A1. d/dt [A2] = 2. A1. d/dt [A2] + Rest

Rest = (d/dt [A1]. A2 - A1. d/dt [A2])

Essentially, in the second part of this eq. the multiplication disappears and d/dt is 'working to the right' on A1 or A2 only. The interpretation of eq. 3 is within the third part. Eq. 3 includes only one step: d/dt 'moving to the right', this leaves to the left quantity A1, itself 'moving to the left' of d/dt, including multiplication with a scalar, in this case scalar 2, and to the right side a factor Rest. Two successive steps require an extra one moment time quantity on the right side, e. g. a one moment time set 'multiplication unit'.

There is similarity with one moment time t or 1/t 'moving to the right' and cn or cn′ respectively in eq. 1. Together the properties of cn and cn′ define d/dt. Time interval only set derivatives are discussed in par. 5 and comment C, with a nonlinearity factor Rest, which can differ for each set.

Within the time interval description, t-quantities and Δt-quantities can occur, depending on coordinate t resp. time interval Δt. Assumed is the described event is 'remote' from H time independent, meaning time interval Δt = [tb, ta] is asymmetric and finite such that |tb| << |ta|. Then t-quantities cn and cn′ remain invariant during Δt and this allows to introduce a correspondence with time interval commutation Δt-quantities cn(Δt) and cn′(Δt), Δt-quantities being not t dependent, linear or otherwise. Multiplication M and addition A within the time interval set, applied in par. 2 to 5 for evidently simple combinations, are defined independently in comments B and C. The indications A and M for addition and multiplication are not applied for one moment time parameters like t or 1/t.

Time Interval Derivatives and Commutation Relations

The time interval derivative to one moment time parameter t, D* [A1], of t-quantity A1(t), equals the usual commutation bracket [1/t, A1]u, the first of eq. 4. This is due to time interval equilibrium: any t-quantity A1(t) is linear in t, since D* [A1] = +/- A1. 1/t remains invariant during Δt, the + or - depending on A1 increasing or decreasing with t or A1 with positive or negative sign, (Hollestelle, 2020). This is the t-quantity perspective.

Within Δt-quantity perspective, the time interval derivative to time interval Δt, D*||Δt [A1], of t-quantity A1(t), is derived to be the second of eq. 4, this result defines time interval commutation bracket [1/t, A1]||Δt for Δt = [tb, ta].

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

4 D* [A1] = 1/2 A [1/t. A1(t), –1. A1(t). 1/t] = [1/t, A1]u

D*||Δt [A1] = 1/2 A [1/tb. A1(tb), –1. A1(ta). 1/ta] = [1/t, A1]||Δt

3. Correspondence C1: Time Interval Only Set Units

Introducing time interval Δt averages < A1 >||Δt for t-quantities A1, and assuming < A1. 1/t >||Δt = < A1 >||Δt. 1/< t >||Δt, within t-quantity perspective this means:

5 A1. 1/t = < A1 >||Δt. 1/< t >||Δt

This relation is valid for any variable t-quantity A1 and follows from D* [A1] = +/- A1. 1/t remaining invariant within the time interval description and TE equilibrium. Average < A1 >||Δt is invariant for Δt, and can be considered invariant t-quantity or Δt-quantity.

A correspondence can be introduced from the definition of units for the one moment time set and for the time interval only set. The average of one moment time parameter t for time interval Δt, < t >||Δt, corresponds with the (a-)symmetry of Δt, (Hollestelle, 2020). For asymmetric Δt, < t >||Δt = 1/2(ta + tb) = t0, with t0 the 'multiplication unit' for one moment time parameters during Δt, meaning t0. t = t. t0 = t. The set 'multiplication unit' is an indication for the equilibrium of a set, TE equilibrium for the time interval only set. The time interval only set 'multiplication unit' ΔU is equal to relevant event time interval Δt, comment 2.

Indicated with ~, a t-quantity corresponds with a Δt-quantity due to the resp. multiplication units. Eq. 5 is written from t-quantity perspective, eq. 6 from Δt-quantity perspective. Subscript i indicates the multiplication inverse for Δt-quantities, where ΔUi = ΔU. For scalar multiplication M [a, Δt] = Δt3, the result Δt3 is a time interval since Δt is a time interval, due to the multiplication closure theorem derived independently in comment B.

6 A1.1/t ~ M [Δt3, ΔU] = M [M [a, Δt], ΔU]

Correspondence C1 and Time Interval Only Set Units

For H time dependent and Δt asymmetric, the average for Δt of one moment time t equals the one moment time 'multiplication unit', < t >||Δt equals t0, however t0 ≠ 0. t0, where 0. t0 is 'multiplication zero' and 'addition zero'. Per definition and for simplicity t0 corresponds with Δt0 as time interval set term in any case. For Δt = [tb, ta] asymmetric with |tb| < |ta|, Δt0 is equal to time interval set 'multiplication unit' ΔU, different from time interval set 'multiplication zero' ΔU0, and there is: < t >||Δt = t0 ~ Δt0 = ΔU for asymmetric Δt.

For Δt = [tb, ta] symmetric with |tb| = |ta| there is < t >||Δt = t0 = 0. t0, and it follows t0 = 0. t0 equals t + -1.t = 0. t and t0. t = t. t0 = 0. t, meaning t0 is equal to both one moment time 'addition zero' and 'multiplication zero'. Per definition 0. t0 = t0 ~ Δt0, however in this case Δt0 equals time interval only set 'addition zero' and 'multiplication zero' ΔU0 and there is: < t >||Δt = t0 = 0. t0 ~ Δt0 = ΔU0 for symmetric Δt.

<u>*Comment 1.*</u> With average < t >||Δt = t0 ~ Δt0, time interval Δt0 equal to 'multiplication unit' or 'multiplication zero' depends on Δt (a-)symmetric or H time dependence. This difference for average < t >||Δt = t0 ~ Δt0 for quantities and properties while remaining within physics is interesting by itself. Interpretation is needed for a time development, where H time dependent reaches a time independent limit.

<u>*Comment 2.*</u> The defining property for 'multiplication unit' ΔU is: M [ΔU, Δt1] = M [Δt1, ΔU] = Δt1, for the multiplication inverse Δt1i the defining property is: M [Δt1, Δt1i] = M [Δt1i, Δt1] = ΔU, obvious relations for any time interval Δt1. One can argue, disregarding uniqueness, ΔU can be identified with relevant event time interval Δt, from M [ΔU, Δt] = Δt while also M [Δt, Δt] = Δt. This last eq. is independently derived to be valid for any Δt1, addition A [Δt1, Δt1] = Δt1 in comment A, multiplication M [Δt1, Δt1] = Δt1 in comment B. It follows Δt is a solution for ΔU. The discussion from comment A and B, and the definition of the derivative of these multiplications, comment C, implies some uniqueness properties for Δt, while it is proven Δt0 is equal to the time interval 'addition zero', even while Δt0 = ΔU the time interval 'multiplication unit' corresponding with t0.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

When M [ΔU, Δt] = Δt and M [Δti, Δt] = ΔU = Δt, Δti is well-defined with solution Δt: M [Δt, Δt] = ΔU.

Comment 3. For addition A one defines A [Δt1, Δt0] = A [Δt0, Δt1] = Δt1 and A [Δt1, Δt1iv] = A [Δt1iv, Δt1] = Δt0, obvious relations for Δt0 time interval set 'addition zero' and Δt1iv with subscript iv the addition inverse for any time interval Δt1. Recall Δt0 equals Δt = ΔU for Δt asymmetric, comment A. Multiplication with scalar -1 of time interval Δt1, M [-1, Δt1], is not necessarily the addition inverse Δt1iv for Δt1, and is not a well-defined time interval when Δt asymmetric.

4. Correspondence C2: Reciprocal Pairs of Commutation Quantities, Noether Charges

The pair of commutation quantities cn and cn' are assumed to be invariant within the time interval only description, cn = cn(t) and cn' = cn'(t) remain invariant t-quantities for variable t within their resp. domain. Dependence on Δt is described with the positive average scalar 'domain' density D(cn, Δt). Time interval Δt integration I* to one moment time t and 'domain' density D, applying time interval averages and comment 2, is introduced within:

< cn(t) >||Δt = M [∫||Δt dt [cn], Δti] = M [I* [cn], Δti] = N. D(cn, Δt)

D(cn, Δt) = |cn- domain|. 1/|Δt- domain|, is an invariant scalar quantity, N is an invariant t-quantity defined below.

Applied is the time interval integral to one moment time t: e. g. I* [2] = ∫||Δt dt [2] = 2. Δt and I* [2] = M [< 2 >||Δt, Δt] and < 2 >||Δt = N. D(2, Δt) = N. |2- domain|. 1/|Δt| = 2, where the domain for scalar 2, the 2- domain, is equal to Δt while density D(2, Δt) = |Δt|/|Δt| = 1. One finds 2(t) = 2 and N(2) = 2. Other ways are possible however in this case one assumes the average to acquire the same dimension as the averaged quantity and the integral to acquire the dimension of the multiplication of averaged quantity and integration interval, i. e. time interval Δt. This agrees with the definitions for I* and average <   >||Δt in (Hollestelle, 2020).

Not always necessarily D(cn, Δt) = 1, rather due to a specific equilibrium requirement for cn and cn', D(cn, Δt) + D(cn', Δt) = 1, both for the same Δt, where similar to D(cn, Δt) for cn introduced is D(cn', Δt) for cn'. Within t-quantity perspective, with cn and cn' the same sign, and applying the usual one moment time addition properties, one defines the equilibrium requirement with N:

7 cn + cn' = N. D(cn, Δt) + N. D(cn', Δt) = N

The t-quantities cn(t) and cn'(t) are termed 'reciprocal'. Since cn, cn' and N, are not ordinary constants, rather physical quantities, a relation due to equilibrium to exist is expected. Some arguments for the requirement are discussed in comment 5.

Δt-quantities cn(Δt) and cn'(Δt) are introduced from correspondence C1 with t-quantities cn and cn' and the equilibrium requirement implies A [cn(Δt), cn'(Δt)] = ΔN, including < N >||Δt = N ~ ΔN. Domain measure zero is not considered for cn(Δt) or cn'(Δt), since corresponding t-quantities cn or cn' would equal one moment time 'multiplication zero', interfering with eq. 1.

Correspondence for the asymmetric situation means < t >||Δt ~ Δt0 = ΔU: in the specific case when < D(cn, Δt) >||Δt ~ ΔU and cn(Δt) = M [ΔN, ΔU] = ΔN, it follows < D(cn', Δt) >||Δt ~ ΔU0 and cn'(Δt) = M [ΔN, ΔU0] = ΔU0, and the other way around, while in terms of measures invariant Δt-quantity ΔN remains the upper limit for cn(Δt) or cn'(Δt). For invariants within the one moment time description, e. g. the usual one moment time Noether charges NC, referred is to (Noether, 1918; De Wit, Smith, 1986). The equilibrium requirement in Δt-quantity perspective while Δt remains the variable is:

8 A [cn(Δt), cn'(Δt)] = A [M [ΔN, ΔU], M [ΔN, ΔU0)]] = A [ΔN, ΔU0] = ΔN

Time interval only set addition A can be applied when including ΔU or ΔU0, time interval units discussed in par. 3. These additions are 'simple enough' to be defined from comment 3, the complete time interval only set addition is introduced independently in comment C.

In support for the equilibrium requirement, one argument includes a theorem for time and space averages (Arnold, 1974). Overall time averages are interpreted to be overall time interval set averages and Δt-quantities. ΔN is equal to the overall time interval set average < cn(Δt) >||set, itself by theorem equal to the overall space average < cn(Δt) >||space, defining cn'(Δt) = cn(Δt') with the specific case upper limits at Δt and Δt'.

 doi:10.20944/preprints202209.0151.v5

ΔN remains an invariant for the overall time interval set, with derivative equation D*||Δt [ΔN] = ΔN, from eq. 8. From the similarity for A and M for time intervals, derived in comment D, it follows ΔN = A [cn(Δt), cn′(Δt)] = M [cn(Δt), cn′(Δt)]. The derivative of Noether charge NC is an invariant, and one can define the time interval only set Noether charge NCset linear with ΔN, while ΔN ~ N, and NCset fulfills the derivative eq. above, with NCset ~ NC.

For any time-interval Δt = [tb, ta], boundary ta defines, however not regarding causality, boundary tb due to eq. 9, (Hollestelle, 2020).

9 tb = -1. (1 – (c. q(t))^2)^(-1). ta

Comment 4. The invariance of quantities cn and cn′ is required at least during relevant event time interval Δt for the validity of commutation relations eq. 1, and is decisive for dispersion-free measurement event results to relate to a Noether charge.

Comment 5. The quantities cn(Δt) and cn′(Δt) are in principle independent of each other. However, assuming equilibrium, i. e. time interval equilibrium, this means a relation for cn(Δt) and cn′(Δt) = cn(Δt′) applies, the requirement to maintain equilibrium during time development. An equilibrium requirement is inferred to be universally existing for any equilibrium description, e. g. total energy H0 = T + V remains invariant when T and V resp. kinematic energy and interaction energy can vary while maintaining Lagrange equilibrium within the one moment time description.

Comment 6. The Hamiltonian H is an invariant for transformations TA, TB and TB(-1), TA(t) with TA(ta) = tb, its opposite TB(t) with TB(ta) = -1. tb and its reverse TB(-1). Transformation TA defines time interval [tb, ta] with [TA(ta), ta] from eq. 9, and this decides all possible boundaries tb and ta for any time interval. A series of n transformations TB, with TB([tb = TA(ta), ta]) = [TA(TB(ta)), TB(ta)] and TB(-1) with TB(-1)(tn) = t(n-1), defines [tbn, tan], transforming Δt to another well-defined Δt′, which takes care (c. q(ta))^2 << 1 for transformation TA to remain valid with always tb = TA(ta) and ta with negative or positive sign resp., i. e. to belong to the future or past. These relations originate from (Hollestelle, 2020).

Defined is a specific infinitesimal equilibrium transformation *TS*, where space- and time coordinates q and t transform to resp. q* = q + qL and t* = t + tL, with qL and tL invariant and infinitesimal. *TS* is equal to transformation TB(-1), with *TS*(t) = t* = t′, where one moment time t and t′ correspond with time interval Δt = [tb, ta] and Δt′ = [tb′, ta′], and where q′ = q(ta′) remains well-defined. From *TS*, specific non-infinitesimal equilibrium transformations TS can be found which leave Hamiltonian H invariant.

Any infinitesimal equilibrium transformation *TL*, being origin of the usual non-infinitesimal Lorentz transformation TL, should leave metric distances unchanged. It is argued, (Hollestelle, 2021), to introduce new Lorentz transformations that leave metric surface measures unchanged. These Lorentz transformations agree with radiation time development, with star-source radiation energy remaining invariant, an energy proportional to propagation surface measure. The specific transformation TS is a metric surface measure preserving new Lorentz transformation.

Comment 7. Constant c with dimension similar to the multiplication inverse of q, assures scalar product (c. q) remains dimensionless. The factor (c. q0) for q(t) = q0 at t = t0 is a scalar invariant, and |(c. q0)| = |M [c, q0]| within the time interval set, where both c and q0 remain invariant during Δt. Place q0 is a coordinate in 3-dim. space, invariant during Δt however not equal to space coordinate origin qc. Constant c is not the group-velocity for e.m. radiation propagation, velocity *c*.

Comment 8. Two different solutions exist: q(t) = +/- q0. 1/t0. t and q(t) = +/- q0. t0. 1/t. Due to TE equilibrium with D*[q(t)] = +/- q(t). 1/t and D* [q(t)] invariant with t during Δt, q(t) necessarily is linear in t and the solution q(t) linear in 1/t can exist only when it is the same solution, i. e. also linear in t.

With ti = 1/t the multiplication inverse for t, where ti not necessarily belongs to the one moment time set, one finds: ti. t = t. ti = t0, with t0 the ‘multiplication unit’, which leaves different solutions for ti possible. To give meaning to ti required is ti belongs to the one moment time set, and also it is assumed when t1 and t2 belong to the one moment time set, multiplication t1. t2 = t3 does similarly. These and other one moment time coordinate and time interval properties are discussed in (Hollestelle, 2020). For Δt boundaries ta and tb there is: ta + tb = 2. t0 and ta. tb = 1/2. (ta^2 + tb^2) approaching to ta. tb = 1/2. ta^2 for |tb| << |ta|, valid for situations with H time dependent and with

 doi:10.20944/preprints202209.0151.v5

Δt asymmetric. These one moment time coordinate properties for tb and ta resemble somehow the relevant event time interval Δt properties M [Δti, Δt] = M [Δt, Δt] = Δt = ΔU, comment 2.

Correspondence C2 and Time Interval Commutation Pairs

One defines the time interval Δt-quantities cn(Δt) and cn′(Δt) = cn(TS(Δt)) from cn′(Δt) = cn(Δt′) and TS(Δt) = Δt′, from the discussion above. It follows correspondence C2: commutation pair t-quantities cn and cn′, from eq. 1, corresponds with commutation pair Δt-quantities cn(Δt) and cn′(Δt). Correspondence C1 defined in par. 3 and correspondence C2 result in the same relation (~). In eq. 10, the one moment time parameter t-quantities on the left side correspond with time interval Δt-quantities on the right side of the equation- or correspondence sign.

10 | ta |. 1/| tb | = | cn′(Δt) |. 1/| cn(Δt) |

ta. tbi ~ M [cn′(Δt), cni(Δt)]

5. Time Interval Only Commutation Relations, and the 'Infinite Regress' Problem

Time interval only description I*| | Δt and D*| | Δt mean integration and differentiation to time interval Δt. By applying correspondence C1, eq. 6, and correspondence C2, eq. 10, and from eq. 4 for t-quantity A1, one derives for Δt-quantity cn(Δt) the time interval description relations:

11a I*| | Δt [cn(Δt)] = M [Δt, A [cn(Δt), –1. cn′(Δt)]] = A [cn(Δt), – 1. cn′(Δt)] = [Δt, cn(Δt)] | | Δt

D*| | Δt [cn(Δt)] = M [A [cn′(Δt), –1. cn(Δt)], Δti] = A [cn′(Δt), – 1. cn(Δt)] = [Δti, cn(Δt)] | | Δt

Eq. 11a, the right side for both I*| | Δt and D*| | Δt, includes the interpretation 'working to the right' of resp. Δt and Δti towards cn(Δt), the time interval version for commutation relations. In particular the occurrence of Δt and Δti recovers linearity in Δt for I*| | Δt and linearity in Δti for D*| | Δt like it should for time interval equilibrium. For cn′(Δt) similar equations can be derived.

The validity of eq. 11b for any Δt-quantity ΔA1 requires at least one of two theorems. Within the time interval description and TE equilibrium all Δt-quantities are linear in Δt and can be reduced to time intervals, due to the multiplication closure theorem, comment B, or the multiplication linearity theorem, comment D. Due to these theorems, implying cn(Δt) is linear in Δt, any Δt-quantity ΔA1 can be represented linear in cn(Δt), and eqs. exactly the same like eq. 11a are valid for any time interval or Δt-quantity ΔA1.

With eq. 11b the 'infinite regress' problem is resolved. These eq. are time interval only set relations, and do not require one moment time set parameters. Transformation ΔA1 to ΔA1′ = TS(ΔA1) and correspondence C2 include multiplication with Δt, from eq. 10 or alternatively from the multiplication closure theorem.

11b I*| | Δt [ΔA1] = [Δt, ΔA1] | | Δt = M [Δt, A [ΔA1, –1. ΔA1′]] = A [ΔA1, – 1. ΔA1′]

D*| | Δt [ΔA1] = [Δti, ΔA1] | | Δt = M [A [ΔA1′, –1. ΔA1], Δti] = A [ΔA1′, – 1. ΔA1]

From eq. 11a one finds I*| | Δt [cn(Δt)] and D*| | Δt [cn(Δt)] have similar results, in agreement with Δti = Δt and comment 8, except for the order of cn(Δt) and cn′(Δt). From correspondence t0 ~ ΔU = ΔUi it follows I*| | Δt [cn(Δt)] = D*| | Δt [cn(Δt)], i. e. including the order difference the results in terms of time intervals for integration and differentiation are equal. Within the time interval only description this is valid for all Δt-quantities due to at least one of the two above theorems. In terms of corresponding t-quantities, the two results for eq. 11b have opposite sign.

Discussion

Comment A. The Time Interval Only Set, Time Development in Terms of Addition

Overall or cosmological time in this paper means there is the same 'time' for the physical universe together and no part is late or early in reference to this 'time': anything is simultaneous with any other thing. One can suggest overall time can be measured depending on some agreement of universal with 'local' properties, e. g. by measuring 'local' temperature or radiation properties to agree with or indicate overall cosmological time, and assuming place- or measurement independent overall simultaneity.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

However, time within the time interval only description needs to be independent of one moment time set properties. Not is meant time is a zero-dimensional concept without development and not is meant reference can be variable or chosen. Neither is meant a one moment time description parameter for the whole universe, introducing uncertainties from QM or simultaneity or signaling from GR.

Simultaneity is meant to be measurement event Δt dependent, and for the time interval only description is defined with the introduction of star-source radiation waves, with finite group-velocity and emitted and 'on the way' during Δt, (Hollestelle, 2020), while Δt being simply measurable or not, a term discussed in comment F.

For one moment time coordinates, derivatives depend on linear addition and the method of tangents. Continuous and invariant time development within the one moment time set means at any 'time-lapse' a similar 'time-lapse' is added. From this one can introduce one moment time invariant quantities, for instance Noether charges.

One moment time equilibrium including the principle of least action implies application of one moment time coordinate variation, with invariant transit space end-coordinates and variable transit time, i. e. Δ* variation, not to be confused with time interval indication Δ or Δt, (Hollestelle, 2020). For symmetric and infinite Δt, time interval set results are the same as those for one moment time set results, and time interval derivatives are similarly linear addition related. For the time interval only description and TE equilibrium, derivatives imply non-trivial, i. e. non-linear, time interval addition.

The 'addition zero' Δt0 for the time interval set at least can be relevant event time interval Δt itself, comment 2 and 3, there is A [Δt1, Δt] = Δt1. This depends on the premise that there is only 'one' 'time' for the cosmological universe together and all is at the same 'time' and nothing is late or early.

A time interval only set addition property then follows to be: time intervals do not exist outside themselves, do not add time from outside to themselves and remain only with themselves. Otherwise said, this property defines addition for sets of finite time intervals where the finite time interval 'addition' the same finite time interval, remains the same finite time interval, i. e. invariant.

12a Δt1 'addition' Δt1, to the same time interval, leaves any time interval Δt1 invariant: A [Δt1, Δt1] = Δt1

This is valid for any time interval Δt1 within the time interval only set. This confirms the interpretation of time interval only set addition with domain addition where addition of two identical domains results in the same domain.

Within the time interval only description events and addition properties depend on the relevant event time interval Δt, equal to 'addition zero' Δt0 for asymmetric Δt, with A [Δt, Δt] = Δt and A [Δt1, Δt] = Δt1. The 'addition inverse' Δtiv for the finite relevant event time interval Δt equals Δt, meaning A [Δtiv, Δt] = A [Δt, Δt] = Δt.

Multiplication -1. Δt does not always equal Δtiv and its definition from eq. 9 fails, meaning it is not a well-defined time interval. One can argue Δt 'addition' -1. Δt equals -1. Δt from the above discussion where 'addition zero' Δt0 equals Δt. Notice that Δt0 is not a zero-measure time interval in the sense of domain measure equal to zero. For infinitesimal Δt0 with zero-measure, eq. 12a is not valid, however Δt0 is not well-defined. The order within addition matters. Addition for any two time-intervals means Δt1 'addition' Δt2, writing A [Δt1, Δt2]. It is possible to introduce the 'addition inverse' within time interval commutation relations, comment I, while in par. 2 and par. 5 applied is multiplication with scalar -1.

Comment B. Time interval only multiplication and the multiplication closure theorem

One defines multiplication M [Δt1, Δt2] for any two time-intervals Δt1 and Δt2 from the introduction of time interval 'multiplication unit' ΔU, while the relevant event time interval remains Δt, comment 2. Like with addition it is not clear immediately what multiplication means for the time interval only set. Within the time interval only set addition includes added domain, multiplication includes shared domain. Several properties for M are discussed in the following, where some obvious ones are already applied in par. 3.

Multiplication closure theorem. The result of multiplication belongs to the time interval only set: M [Δt1, Δt2] = Δt3, implying Δt3 is a time interval when Δt1 and Δt2 time intervals. This is supported

 doi:10.20944/preprints202209.0151.v5

by the time interval set ‘multiplication unit’ ΔU relation $M [\Delta U, \Delta t1] = \Delta t1$. The multiplication closure theorem is proven below.

Time development. From similar arguments for the validity of eq. 12a there is,

12b $\quad M [\Delta t1, \Delta t1] = \Delta t1$ is valid for any time interval $\Delta t1$

Associative property. The symmetric first associative, ‘series’, property for multiplication M: $M [M [\Delta t1, \Delta t2], \Delta t3] = M [\Delta t1, M [\Delta t2, \Delta t3]]$ is assumed valid, the symmetric second associative, ‘parallel’ or distributive, property for M: $M [\Delta t1, M [\Delta t2, \Delta t3]] = M [M [\Delta t1, \Delta t2], M [\Delta t1, \Delta t3]]$ is not necessarily valid, for Δti any three time-intervals. The first and second associative property both valid can be contradictory. The usual derivative d/dt to one moment time coordinate t of a multiplication includes the second associative property, eq. 3. The time interval description derivative D* to a t-quantity or Δt-quantity includes one moment time t or time interval Δt commutation brackets resp., eq. 4. Associative properties for ordinary or usual variable sets are studied within group theory, (Jacobson, 1974). For the first and second associative property can be written resp. AP1 and AP2 whether symmetric or not.

Preliminary Definition for Multiplication

For time interval only set multiplication the following definition, eq. 13, is feasible however preliminary, and depends on shared domain. The time interval description integral $I^*||\Delta t2$ to time interval $\Delta t2$ is introduced, just like the time interval description integral I* to one moment time t, with the integral domain equal to the $\Delta t2$-domain:

13 $\quad M [\Delta t1, \Delta t2] = M [M [< \Delta t1 >||\Delta t, \Delta t], \Delta t2] = M [< \Delta t1 >||\Delta t2, \Delta t2] = I^*||\Delta t2 [\Delta t1]$

This equation agrees with time development property eq. 12b, while M is $\Delta t1$, $\Delta t2$ order dependent. For $I^*||\Delta t2 [\Delta t1]$ symmetric, meaning $\Delta t1$ equals $\Delta t2$, eq. 13 is equal to eq. 12b. Preliminary eq. 13 follows from assuming time interval set properties I to III. These are not definitions; they depend on the interpretation of multiplication with shared domain. When time interval $\Delta t1$ and $\Delta t2$ do not extend to outside Δt, i. e. the common domain for $\Delta t1$ or $\Delta t2$ with Δt is equal to $\Delta t1$ or $\Delta t2$, and with the domain for Δt being continuous, these properties are evident.

I: Since time interval $\Delta t1$ remains invariant during Δt there is: $< \Delta t1 >||\Delta t = \Delta t1$.

II: Averages $< \Delta t1 >||\Delta t2$ remain independent of time interval $\Delta t2$: $< \Delta t1 >||\Delta t2 = < \Delta t1 >||\Delta t$ for all $\Delta t2$.

III: The integral to $\Delta t2$ and the time interval average to $\Delta t2$ are related due to $I^*||\Delta t2 [\Delta U] = M [< \Delta U >||\Delta t2, \Delta t2] = M [\Delta U, \Delta t2] = \Delta t2$, and preliminary eq. 13 implies ΔU generalized to any $\Delta t1$, multiplication M equal and including $I^*||\Delta t2 [\Delta t1]$.

There is $M [I^*||\Delta t3 [\Delta t1], \Delta t3i] = M [I^*||\Delta t2 [\Delta t1], \Delta t2i]$. This depends on the following: $I^*||\Delta t [\Delta t1] = M [\Delta t1, \Delta t] = \Delta t1$ for time interval $\Delta t1$ with $< \Delta t1 >||\Delta t = \Delta t1$ (property I). Then preliminary eq. 13 implies Δt generalized to any $\Delta t2$, multiplication M similarly equal and including $I^*||\Delta t2 [\Delta t1]$.

Eq. 13 without generalizations is valid for $\Delta t1 = cn(\Delta t)$ or $cn'(\Delta t)$ with $\Delta t2 = \Delta t$ due to the results from par. 5. The equations for both $cn(\Delta t)$ and $cn'(\Delta t)$ can be generalized due to properties I to III. From the preliminary definition and comment 2 follows eq. 14.

14 $\quad M [\Delta t1, \Delta t] = < \Delta t1 >||\Delta t = I^*||\Delta t [\Delta t1] = \Delta t1$

$\quad M [\Delta U, \Delta t] = I^*||\Delta t [\Delta U] = \Delta t$

$\quad M [\Delta U, \Delta ti] = I^*||\Delta ti [\Delta U] = \Delta ti$

<u>*Comment 9.*</u> The multiplication closure theorem implies $M [\Delta t1, \Delta t2] = \Delta t3$ belongs to the time interval only set and validity of $\Delta t3 = a. \Delta ta$, scalar a depending on time interval Δta, implies the existence of solution Δta being a time interval. Not all scalar arguments a can be allowed when $M [\Delta t1, \Delta t2] = a. \Delta ta = \Delta t3$ is a proper time interval, where for all asymmetric time intervals the future domain part measure exceeds the past domain part measure by definition. When Δta is a well-defined time interval, $-1. \Delta ta$ is not, when H time dependent with Δt and Δta asymmetric, due to eq. 9. Transformation TA is the defining and necessary relation for the one moment time parameter boundaries for any time interval.

Multiplication Closure Theorem I

Time interval only set multiplication for any two time-intervals Δt1 and Δt2, M [Δt1, Δt2] = Δt3, implies result Δt3 is a time interval and can be rewritten with a scalar multiplication, Δt3 = a. Δta = M [ca, Δta], where a is scalar and ca and Δta time-interval.

Uniqueness property for linear dependence on Δt: when Δt3 depends linear on Δt, there is no other Δt' such that Δt3 depends also linear on Δt' independent from Δt. Similar to t-quantities being linear with parameter t, par. 2, Δt-quantities are linear with Δt, both due to time interval equilibrium. Then the multiplication closure theorem includes: when Δt1 and Δt2 are Δt-quantities, Δt3 is a Δt-quantity, and is a time interval and linear with Δt.

Finding some time interval Δta and some scalar a, implies Δt3 = a. Δta is a time interval, with reference to constraint comment 9.

Applying the symmetric first associative property AP1 for any time interval ca, there is ca = M [ca, Δt] = M [ca, M [Δta, Δtai]] = M [M [ca, Δta], Δtai], i. e. there is ca = < ca >||Δta, an invariant.

One finds from Δta = Δt an existing and well-defined multiplication time interval Δt3 = a. Δta = M [ca, Δta], for any scalar a, with reference to constraint comment 9. Solutions ca = a. Δt and Δt3 = a. Δt both belong to the time interval only set. Linearity of Δt3 in Δta and thus linearity in Δt is assured, discussed in comment H. Once one finds one solution to be a time interval linear in Δt, all solutions are time interval and linear in Δt.

Other solutions for M [Δt1, Δt2] can be found directly from a. Δta = M [ca, Δta] and from the time development property eq. 12b, M [Δta, Δta] = Δta. These solutions for Δt3, with any Δta, not only Δta = Δt, are already known to exist since the time interval set is derived to be uniquely linear in Δt, comment H. All these solutions are linear in Δt, with ca = a. Δt and Δta = a1. Δt, and result Δt3 = a. a1. Δt.

For the specific scalar a = 1 the solution Δt3 = Δta = M [Δt1, Δt2] is well-defined and a time interval: Δta = M [ca, Δta] implies ca = Δt or ca = Δta. Scalar a = 1 and Δt3 = Δta then can be the resultant from any of the two ca, with reference to comment 9.

Multiplication Closure Theorem II

An alternative approach is the following. The relation Δt3 = a. Δta = M [ca, Δta] depends on scalar a having 'positive' sign. For scalar a with 'negative' sign the scalar multiplication does not refer to constraint comment 9. From the first associative property for M [Δt1, Δt2] with Δt1 = a. Δt, and from the definition of D*||Δt, par. 2, one finds Δt3 = a. Δta = a. M [Δt, Δta] = M [a. Δt, Δta] while Δt3 = M [ca, Δta] with solution ca = a. Δt, linear in Δt. With ca = M [ca, Δta], from the first associative, series, property, it follows Δta = Δt and ca = Δt3, or it follows a = 1 and ca = Δt. For both results one finds Δt3 = ca = a. Δt, i. e. Δt3 is itself a time interval and linear in Δt, and it follows validity for the multiplication closure theorem for any scalar a, with reference to comment H and constraint comment 9. *This completes the derivation for the multiplication closure theorem within the time interval only set.*

<u>*Comment 10.*</u> The first associative 'series' property for the derivative, of a multiplication of t-quantities, seems valid if only due to the preserved order of the Δti in the series. More precisely, this property is related to the following commutation relation for t-quantities A(t) that are linear in parameter t following TE equilibrium. A(t) depends on invariant t-quantities cn and cn' due to relation D* [A(t)] = cn, and applying commutation relations eq. 1 for multiplications including one moment time parameters t and 1/t there is:

15 t|1. A(t)|2. cn' = A(t)|1. t|2

1/t|1. A(t)|2 = A(t)|1. 1/t|2. cn

Indications |1 or |2 mean the t-quantity at this place depends on parameter t with the specific value t = t1 or t = t2. Eq. 15 are not definitions; they are derived from the relation D* [A(t)] = cn. One finds from commutation relations eq. 1, the order of |1 and |2 is preserved within eq. 15, when changing the order, due to commutation, of parameter t or 1/t with quantity A(t), meaning the 'series' of |1 and |2 remains unchanged. These relations can be inferred to be valid for the corresponding time interval quantities. The cn and cn' correspond with invariant Δt-quantities cn(Δt) and cn'(Δt),

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

par. 4., linear dependent on Δt due to TE equilibrium. Inferred is the validity of the first associative series property for D*||Δt [ΔA], while ΔA is Δt-quantity, and a multiplication, where any Δt-quantity can be written with a multiplication of Δt-quantities, i. e. the multiplication closure theorem.

Comment C. Time interval only set addition, and derivatives and the derivative 'set rule'

The One Moment Time Derivative 'Set-Rule'

For the one moment time derivative of multiplication A1. A2, with A1 and A2 arbitrary scalar quantities, one can apply the asymmetric second associative property AP2 arguments c1 and c2: d/dt [A1. A2] = c1. d/dt [A1]. A2 + c2. A1. d/dt [A2], similar to eq. 3 where scalars c1 = c2 = 1. The symmetric AP2 for multiplication, is discussed in comment B.

Multiplication a. A1 means invariant scalar a 'multiplication' scalar quantity A1. The one moment time set derivative 'set-rule' emerges when d/dt is 'moving to the right' of scalar a: d/dt [a. A1] = c1. d/dt [a]. A1 + c2. a. d/dt [A1] = c2. a. d/dt [A1] for d/dt [a] = 0, and equal to d/dt [a. A1] = 2. a. d/dt [A1] + Rest = a. d/dt [A1] for c2 = 1. In this case scalar a is 'moving to the left', and factor Rest = -1. a. d/dt [A1]. Scalar multiplication and the derivative 'set rule' is applied to the time interval only set, existing of linear subsets comment H.

The Time Interval Only Derivative and Associativity Arguments c1 and c2

The perspective is changed to Δt-quantities and the time interval only set. The properties: A [a1. A1, a2. A1] = (a1 + a2). A1, with A [1. A1, 1. A1] = A [A1, A1] = 2. A1, where a1 and a2 scalars and A1 one moment time quantity, usually are assumed valid for the one moment time set depending on vector addition. They are not valid for the time interval set, where property A [Δt1, Δt1] = Δt1, comment A, is essentially different and depends on domain addition. For the time interval derivative and addition, eq. 16 and 17, the existence of undecided arguments c1 and c2 reduces its value for application. The relation A [M] and D*||Δt [M] for Δt2 = Δt depends on the results of par. 5, see below for eq. 18.

16 D*||Δt [M [Δt1, Δt2]] = A [c1. M [D*||Δt [Δt1], Δt2], c2. M [Δt1, D*||Δt [Δt2]]]

17 A [a. Δta, Δt] = D*||Δt [a. Δta] = A [c1. M [D*||Δt [a], Δta], c2. M [a, D*||Δt [Δta]]]

Similar with the definition for M, eq. 13, proposed is a preliminary definition for A [Δt1, Δt2], eq. 18, that includes the derivative D*||Δt2 to time interval Δt2, and multiplication M [Δt1, Δt2].

<u>*Comment 11.*</u> The definitions for M and A depend on the existence of a real, i. e. measurement, event. Only with Δt2 equal to Δt the properties for Δt0 and ΔU make sense in the way of averaging, where averaging and measurement both relate to relevant event time interval Δt.

Preliminary Definition for Addition

18 A [Δt1, Δt2] = D*||Δt2 [M [Δt1, Δt2]]

A [Δt1, Δt] = D*||Δt [M [Δt1, Δt]] = Δt1

A [Δt0, Δt] = D*||Δt [M [Δt0, Δt]] = Δt0

Eq.18 is valid for Δt1 identified with Δt-quantities cn(Δt) or cn'(Δt) and Δt2 with Δt. Since cn(Δt) is linear in Δt due to TE equilibrium generalization to the overall time interval only set and Δt-quantities is supported. The multiplication closure theorem, comment B, or the multiplication linearity theorem, comment D, are necessary for this generalization. Linear subsets are discussed in comment H. First associative property AP1 is applied and canceling terms are left out. Terms cancel due to A [Δt1, Δt] = Δt1.

The time interval derivative 'set rule', eq. 19 and 20, is derived from preliminary definition eq. 18. Applied is, 'addition zero' Δt0 equals Δt and 'multiplication unit' ΔU similarly equals Δt, comment 2. For multiplication with scalar a, brackets are left out, e. g. M [a. Δt, Δt1] = a. Δt1. The time interval derivative 'set-rule' is such, that D* and D*||Δt both are 'moving to the right' of scalar a, while scalar a is 'moving to the left' out of the derivative brackets while to the right remains factor Rest. This 'set rule' is non-trivial since for invariant scalar a, D*||Δt [a] not necessarily equals ΔU0 and does not

 doi:10.20944/preprints202209.0151.v5

correspond with d/dt [a], equal to both the one moment time set 'addition zero' and 'multiplication zero'. Differentiation is consistent with correspondence for Δt-quantities with t-quantities only when introducing a non-linearity factor Rest with the time interval only derivative 'set rule'.

Eq. 19 and 20 are due to evaluation of A and M itself, applying derivatives D* [A1] = [1/t, A1]u and D*||Δt [Δt1] = [1/t, Δt1]||Δt, for any time interval Δt1, with Δt = [tb, ta], and A1 any one moment time quantity. D*||Δt [a. Δt1] does not have to be linear in scalar a. The result includes Rest(a) and Rest(a)||Δt on the right side, and remains without arguments c1 and c2. One finds the multiplication within brackets is resolved by the derivative 'moving to the right' property specific for any 'set rule'.

The Time Interval Only Derivative 'Set Rule'

19 D* [a. Δt1] = A [a. D* [Δt1], Rest(a)]

Rest(a) = M [[1/tb, a]u. Δt] = M [D* [a], Δt]

20 D*||Δt [a. Δt1] = A [a. D*||Δt [Δt1], Rest(a)||Δt]

Rest(a)||Δt = M [D*||Δt [a], Δt]

With the time interval only derivative 'set rule', scalar a is 'moving to the left' and included is a non-linearity factor Rest(a). This agrees with the interpretation of D*||Δt 'moving to the right', leaving a commutation quantity to the left like in eq. 1 and eq. 11. Factor Rest(a), invariant during Δt, can be considered either from t-quantity or Δt-quantity perspective. Eq. 19 includes a multiplication of a t-quantity and a Δt-quantity which will be discussed below. Rest(a)||Δt is derived by applying correspondence C1, from time interval units.

The usual commutation bracket [1/tb, a]u has to be evaluated carefully. For a = 1 there is [1/tb, 1]u = tbi. 1 – 1. tbi, with tbi = 1/tb, indeed part of the one moment time set. Since the specific scalar 1 is 'multiplication unit' for the scalar set, tbi. 1 – 1. tbi = tbi + (tbi)iv is valid only, i. e. equal to the one moment time 'addition zero', when the one moment time addition inverse tbiv equals -1. tb for all tb within the one moment time set. The usual commutation bracket can be resolved by rewriting it corresponding to a time interval commutation bracket equal to a time interval derivative, with commutation value 'addition zero' due to TE equilibrium, par. 2. Applied is scalar a is an invariant for time interval description operator Δt*, i. e. a is invariant for time interval Δt, and the derivation includes a transformation of scalar a by multiplication with Δt.

Within eq. 19, Rest(a) equals a multiplication of a t-quantity A1 with Δt, M [A1, Δt] = M [A1, ΔU], a non-trivial equation since, being a t-quantity, A1 arguably has time interval domain 'zero'. The operator Δt* is defined with Δt* [A1] = M [D*||Δt [M [Δt, A1]], Δt] = M [A [ΔU, A1], ΔU], for any t-quantity A1, including for A1 = a, (Hollestelle, 2021). This confirms for any scalar a: correspondence D*||Δt [a] ~ [1/tb, a]u and Rest(a)||Δt ~ Rest(a).

<u>*Comment 12.*</u> One moment time average < t >||Δt = t0, eq. 11, and D* [a] equals < a >||Δt. 1/< t >||Δt = a. t0i, for scalar a positive and invariant with t. Assuming H time dependent, the one moment time 'multiplication unit' corresponds with the time interval 'multiplication unit': t0 ~ ΔU. Recall ΔUi = ΔU = Δt0 and Δti = Δt, comment 2. It follows D*||Δt [a] and Rest(a)||Δt both equal a. ΔU. This means Rest(a)||Δt is a Δt-quantity, implying eq. 20 is well-defined and a time interval only derivative 'set rule'.

Rest(a) = M [a. t0i, Δt] = a. M [ΔU, Δt] = a. ΔU for any scalar a. Similarly, Rest(a)||Δt = M [a. ΔU, Δt] = a. ΔU, equal to ΔU for a = 1. In this case, for a = 1, one finds D* [a. Δt1] = a. D* [Δt1] and D*||Δt [a. Δt1] = a. D*||Δt [Δt1] as it should be.

<u>*Comment 13.*</u> Rest(a) = M [D* [a], Δt], the last part in eq. 19, can be derived independently, without considering the second part of eq. 19: M [[a, 1/tb]u, Δt]. This confirms one moment time derivative D* [A1] = [1/t, A1]u, eq. 4, for A1 equal to scalar a from the correspondences above.

<u>*Comment 14.*</u> M [ΔU0, Δt1] = ΔU0, with ΔU0 time interval set 'multiplication zero', corresponding with one moment time set 'multiplication zero'. The domain measure for ΔU0 is zero and ΔU0 = 0. Δt1 for any finite Δt1 including ΔU0 = 0. ΔU. With zero domain measure, ΔU0 is not a well-defined time interval, and resembles a one moment time parameter. Time intervals with zero domain measure are not considered in the time interval set in this paper.

 doi:10.20944/preprints202209.0151.v5

Time Interval Only Derivatives, Asymmetric Arguments c1 and c2, and Time Development

With arguments c1 and c2 defining asymmetric second associative property AP2 for derivative D*||Δt [M [Δt1, Δt2]] with eq. 16, and with D*||Δt2 [Δt1] = Δt1 for Δt2 equal to Δt, eq. 18, one finds for a = 1:

21 D*||Δt [M [Δt1, Δt]] = A [c1. M [Δt1, Δt], c2. M [Δt1, Δt]]

Within the last part, M [Δt1, Δt] = Δt1, due to Δt = ΔU and Δt = Δt0 for the time interval only set.

22 A [Δt1, Δt] = D*||Δt [M [Δt1, Δt]] = A [c1. Δt1, c2. Δt1] = Δt1

This is the time interval only set relation equivalent to one moment time set relation eq. 3 where usually c1 = c2 = 1. When both c1 and c2 equal scalar 1 it follows: A [Δt1, Δt1] = Δt1. The same result is derived from time development interpretation only, considering domain addition, comment A. The c1 and c2 are not any scalars, they are AP2 arguments for time interval only derivative D*||Δt, and possibly asymmetric and unequal. However, eq. 19 and eq. 20 introduce Rest(a) and Rest(a)||Δt, without applying any arguments c1 or c2. These are two interpretations for the time interval only derivative, one from the associative property and one from the 'moving to the right' property, par. 2. When c1 = c2 = 1 and Δt1 = Δt, the time interval only set relation, D*||Δt [M [Δt, Δt]] = Δt, seems similar to the usual one moment time set relation: d/dt [t. t] = c1. (1. t) + c2. (t. 1) = 2. t, however it is not. The time interval only set differs from the usual one moment time set also in this way.

Comment D. Multiplication Linearity theorem

Within the time interval only description, preliminary definitions for integration and derivative in terms of time interval only set multiplication M and addition A can be eq. 13 and 18. Due to the results of par. 5, the following eq. 23 are valid at least for Δt1 equal to resp. cn(Δt) or cn'(Δt) while Δt2 = Δt. They are valid for any two Δt-quantities Δt1 and Δt2 within the time interval only set due to the multiplication linearity theorem discussed below. This theorem is equivalent with the multiplication closure theorem, comment 8.

23 I*||Δt2 [Δt1] = M [Δt1, Δt2]

D*||Δt2 [M [Δt1, Δt2]] = A [Δt1, Δt2]

Comment 15. In the time interval description operators are indicated with *, depending on an 'operator quantity' for their interpretation. Operator quantities are indicated with ||operator*||, for example the time interval description operator Δt* relates to ||Δt*|| = Δt. Such operator quantities are not defined for I* and D* and I*||Δt2 and D*||Δt2, even while 'working to the right' like operators. The time interval description operator working on t-quantity A1 is Δt* [A1] = M [D*||Δt [M [Δt, A1]], Δt] = M [A1, Δt], comment C, in agreement with time development addition, comment A, and the multiplication linearity theorem below. Time interval description operators are introduced in (Hollestelle, 2021).

Since I*||Δt2 and D*||Δt2 give equal results, from eq. 11, and M and A give equal results, at least for Δt1 equal to cn(Δt) or cn'(Δt) and for Δt2 = Δt. From eq. 13 and 18 and the discussion following time interval only derivative 'set rule' eq. 19 and 20, with Δt2 = Δt, one finds I*||Δt and D*||Δt for cn(Δt) and cn'(Δt) correspond exactly with I* and D* for cn and cn'. Eq. 23 is a time interval only relation.

M and A depend on time interval properties resembling shared domain resp. domain addition. The difference, due to eq. 11 including I*||Δt [cn(Δt)] and D*||Δt [cn(Δt)], depends on the order for cn(Δt) and cn'(Δt) and does not return in the result itself in terms of time intervals. I*||Δt and D*||Δt and M and A can have the same results since this follows from D* [M [Δt1, Δt]] = A [Δt1, Δt] and D* [M [Δt1, Δt]] = M [Δt1, Δt], from properties A [Δt1, Δt] = Δt1 and M [Δt1, Δt] = Δt1 which are valid for any Δt1, due to relations Δt = Δt0 and Δt = ΔU.

Multiplication Linearity Theorem

Equations I*||Δt2 [Δt1] = M [Δt1, Δt2] and D*||Δt2 [M [Δt1, Δt2]] = A [Δt1, Δt2] are valid for any two Δt-quantities, including any two time-intervals Δt1 and Δt2 from the time interval only set.

The theorem means generalization of eq. 23 from commutation quantities to Δt-quantities and time intervals. Applied are TE equilibrium and time interval derivatives. It is valid for any two Δt-quantities, due to comment H for linear subsets, when proven for any two Δt1, Δt2 time intervals.

The specific Lorentz transformation TS defines Δt′ from Δt with cn′(Δt) = cn(Δt′), par. 4. Transformation TS and the specific change Δt to Δt′ such that Hamiltonian H remains time independent is introduced in (Hollestelle, 2021). Even when H time independent should imply Δt = Δt′ both infinite, it is assumed existence of case I, Δt ≠ Δt′. For transformation TS, with Δt transformed to Δt′, the multiplication linearity theorem is derived for Δt1 = Δt and Δt2 = Δt′, below in case I. For other transformations TN, Δt1 to Δt2, both time intervals are finite and there is H time dependent. With nonspecific TN, the linearity of Δt2 with Δt1 is assured, even when the linearity constants do not agree with any specific transformation TS, and this is included in Case I. Case II considers the no-reverse case.

Case I. Define cn′(Δt) = cn(Δt′), correspondence C2. Since due to TE equilibrium cn(Δt) and cn(Δt′) are linear in relevant event time interval, resp. Δt and Δt′ before and after transformation TS. From time interval equilibrium follows the linear transformation Δt to Δt′, and its reverse transformation. The reverse transformation Δt′ is linear in cn(Δt) and thus also Δt′ is linear in Δt. This means the theorem is trivially valid for these Δt1 = Δt and Δt2 = Δt′. TS can be reversed unless it is a ‘constant’ transformation, not linear, and derivative D*||Δt for cn(Δt) or cn′(Δt) equals the time interval set ‘multiplication zero’ ΔU0, this is the no-reverse case.

When no specific transformation TS, with Δt1 to Δt2 = (Δt1)′, exists, a nonspecific TN transforms commutation constant cn(Δt1) to cn(Δt2) and Δt1 to Δt2, however Δt2 ≠ (Δt1)′, and in terms of proof this is the same as for TS, while time dependence for H is not decisive. This means for both TN or TS the theorem is valid for any time interval Δt1 and Δt2, and for any Δt-quantities being linear in Δt, except for the no-reverse case.

An interpretation of change, time development, for Δt is necessary. For changing Δt the validity of the following is to be ensured: M′ [Δt1, Δt′] = M′′ [Δt1, Δt′′] = Δt1, i. e. relevant event time interval Δt′ changes to Δt′′ while specific properties for M and A do not change and are defined with the relevant event time interval indication Δt, one of these properties is Δt = ΔU. It depends on what is meant to change: before change Δt = Δt′, after change Δt = Δt′′, where events with Δt′ or Δt′′ share properties, however not all properties, to assure change.

This makes sense since it agrees with the description of star source radiation propagation, including wave packet reduction during a measurement event, relevant event time interval Δt, where a requirement is the metric surface measure remains invariant and equal to the propagation surface radiation energy, independent of relevant event time interval Δt time development, (Hollestelle, 2021).

Case II. The no-reverse case

In this case a transformation TS changes Δt1 to Δt2 however the time interval commutation quantities remain invariant: cn(Δt1) = cn(Δt2), cn′ = cn, and this cannot be reversed one to one. Recall cn′ relates to cn with the specific transformation TS. Consider change of Δt from Δt1 to Δt2 with distinct Δt1 ≠ Δt2, with Δt = Δt1 before change. Time development for invariant cn(Δt) implies A [cn(Δt2), cn(Δt1)iv] = M [+/-1. D*||Δt [cn(Δt)], A [Δt2, Δt1iv]] = Δt0, par. 2.

With D*||Δt [cn(Δt)] = ΔU0 for variable Δt at Δt1, one finds A [cn(Δt2), cn(Δt1)iv] = ΔU0. Then ΔU0 = Δt0, i. e. ‘multiplication zero’ equals ‘addition zero’ for the time interval only set. Before change, cn(Δt1)iv = cn(Δt)iv = Δtiv, and since ΔU0 has time interval domain measure zero, it follows cn(Δt2) = ΔU0 for any Δt2, and is not well-defined, comment 14.

With D*||Δt [cn(Δt)] ≠ ΔU0 for variable Δt at Δt1, where A [Δt2, Δt1] = A [Δt1, Δt2] = Δt2 and M [Δt1, Δt2] = Δt2. Started is from Δt = Δt1, with Δt = Δt2 after change.

It follows M [+/-1. D*||Δt [cn(Δt)], Δt2] = Δt0, time interval ‘multiplication unit’, for any Δt2. One finds +/-1. D*||Δt [cn(Δt)] = +/-1. cn(Δt) = Δt2i.

From relation M [cn(Δt), Δt2] = ΔU, where cn(Δt) is linear in Δt due to TE equilibrium, with some scalar constant a, and one finds cn(Δt) = a. Δt, which implies a = 1. Solution for M [Δt, Δt2] = ΔU before

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

change is Δt = Δt2, disproving the above requirement for change of Δt from Δt1 to Δt2 with Δt1 ≠ Δt2. *This completes the derivation for the multiplication linearity theorem.*

The validity of eq. 23 is assured for Δt1 = cn(Δt) or cn'(Δt) from par. 5. From eq. 23, including the multiplication linearity theorem follow eq. 24 to 27.

24 I*||Δt [cn(Δt)] = A [cn(Δt), A [ΔU, cn(Δt)]] = A [cn(Δt), cn(Δt)]

I*||Δt [cn'(Δt)] = M [cn'(Δt), Δt] = A [cn'(Δt), A [ΔU, cn'(Δt)]] = A [cn'(Δt), cn'(Δt)]

A [a. Δt1, Δt] = D*||Δt [M [a. Δt1, Δt]] = D*||Δt [a. I*||Δt [Δt1]] = A [a. D*||Δt [I*||Δt [Δt1]], Rest(a)||Δt]

Not necessarily cn(Δt) follows the requirement of time interval asymmetry like Δt itself for situations when H time dependent, and – 1. cn(Δt) can be valid however -1. Δt is not. Scalar a is 'moving to the left' from I*||Δt without a non-zero factor Rest, however from D*||Δt only with 'set rule' eq. 20, including Rest(a)||Δt. From eq. 11, one can find I*||Δt [cn(Δt)] = A [cn(Δt), -1. cn'(Δt)].

Time development is equal for cn(Δt) and cn'(Δt), supported by the definition cn'(Δt) = cn(Δt') and linearity of cn(Δt) in Δt. For Δt1 = cn(Δt) and Δt2 = Δt, there is Rest(a)||Δt = a. ΔU is non zero and for scalar a = 1 this is equal to ΔU = Δt0, comment 12. Due to property A [Δt1, ΔU] = Δt1 for any time interval Δt1, and the second associative property for A, factor Rest(a)||Δt can be left out and the results for D*||Δt, and I*||Δt are exactly the same. For any scalar a and Δt-quantity cn(Δt) linear with Δt, one finds eq. 25 to 27.

25 a. cn(Δt) = D*||Δt [a. I*||Δt [cn(Δt)]]

26 a. cn(Δt) = A [a. D*||Δt [I*||Δt [cn(Δt)]], Rest(a)||Δt]

27 cn(Δt) = D*||Δt [I*||Δt [cn(Δt)]]

Comment E. Time interval only wave equations and structure constants

The time interval only version of a plane wave depends on wave energy rather than one moment time quantities. I*||Δt and D*||Δt, and M and A, have the same results, due to results from comment D, and eq. 26 and 27 can be interpreted as second order derivative equations, since the multiplication linearity theorem is valid for any Δt-quantity. The time interval equation sign is positive on both sides.

For the time interval version of QM field theory, where the wave momentum relation k = *h*. p, together with wave energy E = *h*. ν, remain valid, referred is to (Hollestelle, 2021). The above equations can only be a time interval only 'wave' equation version, when derived from the time interval field.

From combinations of commutation brackets, equal to scalar multiplications within the generator set of a group, structure constants depend on second derivatives, (Veltman, 1974; De Wit, Smith, 1986). Structure constants for the time interval set depend on D*||Δt [D*||Δt [M]], for some multiplication combination M, and on time development property, eq. 12b, M [Δt1, Δt1] = Δt1. From the discussion eq. 8, ΔN equals both addition and multiplication of the reciprocal pair of commutation quantities. Derivative D*||Δt [ΔN] = A [cn(Δt), cn'(Δt)] = ΔN, and similarly the second derivative D*||Δt [D*||Δt [ΔN]] = ΔN, and eq. 25 to 27 apply to ΔN. These eq. are exactly the requirements for time interval only structure constants.

Originating from multiplication within the time interval set, M = a1. ΔN = a1. M [cn(Δt), cn'(Δt)], including scalar a1, is solution for the time interval only wave equation eq. 25 and 27, and equals time interval only structure constant NCset = a1. ΔN. Combination M = a1. ΔN is a possible solution due to the D*||Δt derivative 'set rule' with Rest(a1)||Δt = a1. ΔU, for any scalar a1.

Second derivative invariants are specific Lorentz transformation TS invariants, since TS is a surface measure preserving transformation, unlike the usual Lorentz transformation TL. NCset = a1. ΔN is an invariant for TS. Both eq. 26 and 27, and Lorentz transformation TS, can be interpreted to be a 'time interval only' structure constant requirement.

The commutation constants form an n-pair and the existence of an overall equilibrium invariant, in this case ΔN = M [cn(Δt), cn'(Δt)], implies properties for structure constants NCset.

Comment 16. The dimension of the set decides the number of quantities within the n-pair for this set.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

Comment 17. The n-pair, a number of n quantities, for the time interval only set is a reciprocal 2-pair, an ordinary pair of two quantities, since time development and the time interval only set are 1-dim. Generalization to different n is possible. There is n = 2 for 2 reciprocal time interval set commutation quantities cn(Δt) and cn'(Δt), eq. 11a, and the time interval only set is linear in Δt, due to TE equilibrium, i. e. the multiplication linearity theorem. The equilibrium requirement, par. 4, implies a reduction for the number of structure constants from n to n - 1, and one independent time interval only structure constant remains, linear with ΔN = M [cn(Δt), cn'(Δt)].

Comment F. Relevant event time interval, simply measurable, simultaneity

One can define one moment time t, time interval Δt = [tb, ta] and boundaries tb and ta within a 1-dim. time development, such that they agree with how time can be measured or counted. A time measurement is proposed to include counting towards ta in the future, while counting from tb in the past. The application of counting, a method of measuring, can relate quantities like one moment time coordinates with measurements. One can proof time intervals can be measured from within to beyond relevant event time interval Δt, i. e. from 'timely' to 'non-timely' time intervals until a certain limit. Being 'timely', infinitesimal or 'local' in 3-space terms, means the time interval measure is additive, where 'non-timely', until a certain limit, means measurable however non-additive, non-linear. The definition of 'timely' and counting or measuring time intervals and one moment time coordinates tb and ta, is discussed in (Hollestelle, 2018, 2020).

Within the time interval description simultaneity is introduced for radiation emitted and 'on the way' during finite relevant event time interval Δt, however only when Δt is 'simply measurable', meaning measurable within one measurement. The radiation group-velocity is considered finite. It is argued the relevant event time interval Δt is always simply measurable. Applied are results from (Hollestelle, 2020).

Assumed is a measurement event including the reduction of one wave packet. This is a non-stationary event, and change in wave energy E = T, kinetic wave particle energy, is argued to be Δ* E = Δ*T = -1. E = -1. T, (Merzbacher, 1970). The Δ^* means energy change, not variation. For complete reduction of the wave packet introduced is the indication _, e. g. wave packet reduction time interval is Δt_, with corresponding one moment time coordinate t_, and |t_| = |Δt_|. For |Δt_| or one moment time measure |t_| = |Δt_| < |Δt| this means Δt is not simply measurable, i. e. not measurable within one measurement and one wave packet reduction.

Due to TE equilibrium, at Δt = Δt_, time interval derivative | D*||Δt [E] |= |E|. |Δt_i | = |T|. |t_i | = | D* [T] | at t = t_. By definition M [M [T, Δt_i], Δt_] = T = -1. Δ*T, and change in energy T is -1. |T|. |Δt_i|] = Δ*T and -1. |Δ*T|. |Δt| = Δ*T by definition of Δt = Δti, with solution |Δt_i| = |Δt|. One finds |t_| = |Δt| meaning Δt is simply measurable and this confirms Δt is the relevant event time interval, for any measurement event and wave packet reduction event. Δt not necessarily is infinitesimal or 'timely'. The solution |Δt_| = |Δt|, simply measurable requirement, is always fulfilled during any measurement event, i. e. wave packet reduction event, and relevant event time interval Δt always is simply measurable.

Comment G. Noether charges and structure constants

According to set theory, structure constants are independent of set representation. Within the time interval only set, the multiplication linearity theorem implies linearity for multiplications, e. g. M [Δt1, Δt2] = Δt3 = a. Δta, providing linearity constants, structure constants, without introducing the generator set or representations. The canonical property is not valid due to existence of non-zero, time interval only, derivative 'set-rule' factors Rest(a)|t and Rest(a)||Δt, eq. 19 and 20.

For the time interval only set, structure constants are independent of the number of necessary and different Δta, i. e. whether the collection of time interval only set subsets of different Δta is reducible or irreducible, comment D and H. Similarly, the collection of Δta subsets is not completely determined by the structure constants. The time interval only set being 1-dim. means there is one independent subset, and one independent structure constant. Structure constants can relate to time development, e. g. dispersion during measurements, traditionally they are applied for particle scattering measurements. With quantities time interval Δt, radiation wave energy *h*. ν, and group-velocity c(Δt), introduced is spherical symmetric wave propagation, within the time interval description in this paper, part I and part II. In the following linear multiplication scalars will be omitted.

Within the 1-dim. one moment time description, Noether charge NC = ∫||overall space dq [N0] and 3-dim. Noether current NU are related with a continuity equation integrated for overall space: ∫||overall space dq [d/dt. N0 + d/dq. NU] equals one moment time 'addition zero', (De Wit, Smith, 1986). The N0 and NU together are the Noether current in 4-dim. Then d/dt [NC] = ∫||overall space dq [d/dt. N0] equals one moment time 'addition zero' for any scalar multiplication, since for any Noether current NU always, including for scalar multiplication, the overall space integral equals < NU >||overall space equals one moment time 'addition zero'.

Within the time interval only description, due to similarity of multiplication M and addition A, scalar multiplication invariance can relate, due to inferred similarity with the above argument, in general to a 'gauge' transformation, meaning usually within the one moment time description a time interval addition invariance. When scalar multiplication invariance can be interpreted with 'gauge' invariance, there is the possibility of 'addition terms' for the Noether current in 4-dim. This is supported by time interval only set properties eq. 12a and 12b, A [Δt1, Δt1] = Δt1, and M [Δt1, Δt1] = Δt1.

Within the time interval only set, from correspondence can be derived the time interval Noether charge and Noether current, NC equals N0 ~ NCset. Gauge transformations by itself are not the subject of this paper. For the time interval only set, in the following applied is; any Noether charge or structure constant multiplication scalar is disregarded, i. e. chosen equal to the time interval 'multiplication unit' ΔU.

A1. Measuring plane wave group-velocity c(Δt), the relevant event time interval is Δt, and similarly Δt is relevant event time interval for measuring Δq, with c(Δt) and Δq being Δt-quantities linear in Δt. There is c(Δt) = M [Δq, Δti] = D*||Δt [Δq], invariant with t during Δt. Relation c(Δt) = M [c(Δt), Δti] = M [M [c(Δt), Δti], Δti] provides c(Δt) to be the second derivative linearity constant, i. e. NCset = c(Δt) is time interval only set structure constant. Similar arguments are valid for ΔN with result NCset = a1. ΔN, comment E. However, c(Δt) is not dimensionless.

There is, M [c(Δt), NCsetiv] = A [c(Δt), NCsetiv] = NCset, and it follows c(Δt) = ΔU and c(Δt) = NCset.

A2. This can be resolved by introducing a variable t-quantity *h*+ = 1/2 (Δ*p. Δ*q + Δ*q. Δ*p), with variations Δ*, a possibly variable alternative for the constant of Planck *h*. Any constant, like *h* is, is both t-quantity and Δt-quantity within the time interval description. For H time independent and Δt symmetric and of infinite measure one recovers *h*+ = *h*. While considering non-interacting, dispersion free, star-source radiation propagation, it can however be claimed variations equal interval measures |Δ*q| = |Δq| and |Δ*p| = |Δp|, and *h*+ = M [E, Δt] is a t-quantity and Δt-quantity, linear in Δt, for E equal to the invariant e.m. propagation surface field energy. Recall the time interval for E can differ from the relevant event time interval for c(Δt). The variations Δ*, variable *h*+ and field energy E are introduced in (Hollestelle, 2020, 2021).

The linearity relation, for c(Δt) from A1, is similar to E = M [*h*+, Δti], invariant during time development without interaction. Both *h*+ and c(Δt) are linear in Δt, they are Δt-quantities: E = M [*h*+, Δti], and c(Δt) = M [c(Δt), Δti].

Since trivially E = M [E, Δti] = M [M [E, Δti], Δti], both c(Δt) and E = M [E, Δti] can be considered linearity constants and time interval structure constants except for scalar multiplication. Related to different radiation properties, E or c(Δt) have different dimension and value, however, there is one independent structure constant, comment 16 and 17. Interaction, from t-quantity perspective, with Δ*q = Δ*q(t) or with *h*+ = *h*+(t), implies E and c(Δt) are one moment time quantities.

A3. The time interval invariants c_ = M [c(*Δ*t)i, |c(*Δ*t)|] for group-velocity and *h*. ν_ = M [(*h*. ν)i, |*h*. ν|] for radiation energy, result in dimensionless time interval structure constants DC and DN: DC = |M [c(Δt), c_]| = |c(Δt)|, and DN = |M [E, *h*. ν_]| = |*h*. ν|. There is an ambiguity in the sign of DC and DN. Only one of DC and DN can be independent, however not necessarily DC = DN.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 March 2024 doi:10.20944/preprints202209.0151.v5

A4. The usual one moment time Noether charge NC equals in value (a2. |*h*. ν|)^2, with a2 scalar, a result from (Hollestelle, 2021). Within the time interval only description a Δt-quantity squared through multiplication is linear in the Δt-quantity itself. Noether charges depend on derivative differences, and, comment E, this is an argument to expect any Noether charge is proportional with some structure constant.

A5. It is inferred the overall time interval and space interval averages for ΔN = A [cn(Δt), cn'(Δt)] are the same, par. 4. The average domain densities with Δt the variable within the overall time interval only set, < D(cn, Δt) >| |Δt, and similar for cn', are assumed to acquire lower and upper limits ΔU0 and ΔU resp., meaning cn(Δt) and cn'(Δt) acquire the same limits, par. 4. The limits follow from solving the wave equation for M, comment E, with the solution time interval structure constant M = NCset = a1. ΔN, for any scalar a1, including a1 = 1. The time interval wave equation is related to plane wave energy solutions |E| = |t0|, where t0 ~ ΔU, for ΔN.

A6. From eq. 8 there is, ΔN = M [ΔN, A [< D(cn, Δt) >| |set, < D(cn, Δt') >| |set]] = M [ΔN, M [Dset, A [ΔU, ΔU0]]], with solution ΔN = M [Dset, A [ΔU, ΔU0]] = Dset. Invariant Dset is an approximation for the domain density averages. The averages are overall time interval set averages.

For one moment time Noether charge NC = a2. |*h*. ν| = a2. DN from (A3) and (A4), there is correspondence NC = N ~ ΔN = Dset. There is time interval structure constant NCset = a1. ΔN = M [|c(Δt)|, ΔU] from (A1) and (A5), with a1 scalar, and where NCset and ΔN belong to the linear subset for Δt.

A7. Time interval Noether charge NCset = a1. ΔN ~ a2. DN, from (A3), (A4) and (A5), still including linearity constant a1, since there is only one ΔN linear subset.

A8. With dimensionless DC = M [c(Δt), c_] = |c(Δt)| equal to M [|c(Δt)|, ΔU], and from (A1) the time interval Noether charge DC = M [|c(Δt)|, ΔU] ~ NCset, and the group-velocity follows exactly from the commutation quantities, c(Δt) ~ a1. ΔN, for e.m. radiation, photons, there is c(Δt) equals light-velocity *c*. For all multiplication scalars equal to one, suggested above, c(Δt) ~ M [ΔN, ΔU] = ΔN. From (A6) ΔN depends on Dset. In this case c(Δt) ~ M [Dset, ΔU] = Dset.

Dset depends on lower and upper limit of the domain densities, (A5). Disregarding the lower limit to be zero, this suggests there is an upper limit |Dset| < |ΔU| for the group-velocity, i. e. a finite group-velocity, which obviously relates to relevant time interval Δt to be finite. For photons, zero mass wave particles, a finite velocity *c* is expected. This result is independently derived within the time interval only description. However, an infinite velocity *c* is not consistent with observed e.m. phenomena from traditional experiments, even while however these usually are not discussed within the time interval only description.

A9. A new variable is complementary wave particle mass m, which can be zero or non-zero. Recall non-linearity factor Rest| |Δt depends on time interval c(Δt) including on mass m. From Curie's principle alone one can expect the difference, zero or non-zero mass, to be measurable or distinguishable from observables, depending on Noether charges and structure constants, e. g. measurable quantities energy or group-velocity.

Forthcoming part 2 includes a discussion of relevant mass m and mass density within a time interval only description of star-source radiation energy.

Comment H. Linear subsets

Multiplication result M [Δt1, Δt2] = Δt3 belongs to the time interval set due to the multiplication closure theorem, comment B. One assumes, TE equilibrium being maintained, any time interval including Δt3 is linear in Δt. This assumption remains to be verified for the validity of the multiplication closure theorem.

I. In (Hollestelle, 2020), and summarized in comment 6, defined are several transformations including scalar multiplication, with H remaining invariant. Due to the results in comment B including time interval multiplication interpreted with shared domain, with definition a. Δta = Δt3 = M [Δt1, Δt2], the multiplication a. Δta is a well-defined time interval.

II. Choose Δt3' = Δta, and one finds, when Δta' = Δt, the specific time interval ca' and scalar a', where Δta = M [ca', Δta'] = M [ca', Δt] = a'. Δt. The Δta' define linear subsets, i. e. 1-dim. subsets a'. Δt, for Δt.

III. Choose newly Δt3 = a. Δta = M [ca, Δta] and a. Δta = a. M [ΔU, Δta] = M [a. ΔU, Δta] and this means multiplication with time interval 'multiplication unit' ΔU or with time interval ca both leave linear subsets defined by a. Δta for Δta, invariant, where linear subsets are defined by Δta for Δt, step II. Applied is the symmetric first associative property for M, comment B. Indeed, it follows M [Δt1, Δt2] = Δt3 is linear in Δt.

This confirms, in terms of linear subsets, time interval multiplication closure and the closure theorems from comment B.

Comment 23. Within the time interval description Δt-quantity Noether charge ΔN = M [cn(Δt), cn'(Δt)] depends on multiplication and TE equilibrium, t-quantity one moment time coordinate t depends on addition and Lagrange equilibrium. Time interval description transformations 'working to the right' D* or D*||Δt can include multiplication with t or ti from usual commutation brackets, or can be non-linear due to the time interval derivative 'set rule' including factor Rest, eq. 19 and 20. Within the time interval description this means, non-linear events and linear events, in terms of H; time (in-)dependent events, or in terms of time intervals only; Δt (a-)symmetric events, can be related by applying these transformations. For a transformation from H time dependent to H time independent or time interval Δt asymmetric to symmetric reversible or not can be an issue.

Comment I. Addition commutation quantities and addition inverse

The time interval relations A [ΔU0, Δt0] = Δt0 and ΔU0 = (1 + (- 1)). Δt0 = A [Δt0, -1. Δt0] seem contradictory. The confusion exists in applying -1. Δt0 to mean Δt0iv, the addition inverse, within time interval only addition equations like ΔU0 = A [Δt0, -1. Δt0], and interpreting time interval only 'addition zero' ΔU0 to mean a difference including scalar -1. This originates from the one moment time description, with (1 + (- 1)). t0 = A [t0, -1. t0] to mean A [t0, t0iv], this description depending on vectors. Differently, in the time interval only description for any time interval Δt2 the 'addition zero' depends on the addition inverse, defined with A [Δt2, Δt2iv] = ΔU0.

To resolve this, one can consider ΔU0 has zero domain, comment 14, or consider there is extra degrees of freedom when quantities are subject to 'moving to the other side of the equation-sign', e. g. including scalar multiplication with -1. a, with scalar a, i. e. meaning introduction of addition commutation quantities. These can be joined to the time interval only multiplication commutation quantities, and similarly to Noether charges and structure constants, and this changes their number of degrees of freedom. The result with respect to the side of the equation-sign matters and can acquire different value, with dependence on the order of the involved time intervals.

To define addition commutation quantities, necessary is consistent correspondence with the time interval only set, including $< t >||\Delta t \sim \Delta U$. However, correspondence can itself be subject to commutation quantities. A similar freedom seems to reside within rewriting Newton's laws and equilibrium definitions depending on equation-sign side in a similar way, where equilibrium is defined from differences being equal to addition 'zero', (Hollestelle, 2020).

References

1. Arnold, V. I. (1974), Mathematical Methods of Classical Mechanics, English translation 1989, Vogtmann K. & Weinstein A., Springer
2. Curie, P. (1894), Sur la symétrie dans les phénomènes physiques, symétrie d'un champ électrique et d'un champ magnétique, J. de Phys. Theor. Appl. 3 (1), Hal archives-ouvertes 10.1051.fr, Curie's principle (quotation): *"Lorsque certaines causes produisent certains effets, les éléments de symétrie des causes doivent se retrouver dans les effets produits. Lorsque certains effets révèlent une certaine dissymétrie, cette dissymétrie doit se retrouver dans les causes qui lui ont donne naissance."*
3. Einstein, A. (1923), in The Principle of Relativity, Lorentz, H., Einstein, A., Minkovski, H. and H. Weyl, Methuen and Company, reprint, Perrett, W. and G. Jeffery, 1952, Dover, Includes: The Foundation of the General Theory of Relativity
4. Goldstein, H. (1978), Classical Mechanics, Addison-Wesley
5. Hocking, G., Young, S. (1961), Topology, Addison-Wesley
6. Hollestelle, H. (2017), Thermodynamics of a Set of Closed Curved Surfaces and Noether's Theorem, Int. Jrnl. of Science & Tech. 5(1)
7. Hollestelle, H. (2018), Gravitation: Immediate Action at a distance or Close up Events? Int. Jrnl. of Science & Tech. 6(12)

doi:10.20944/preprints202209.0151.v5

8. Hollestelle, H. (2020), Time, Equilibrium, and General Relativity, Int. Jrnl. of Science & Tech. 8(5)
9. Hollestelle, H. (2021), Some Results for a Time Interval Approach to Field Theory and Gravitation, preprints202106.0684.v3
10. Hollestelle, H. (2023), Some Comments on a Time Interval Only Description/A Time Interval Only description and Radiation, preprints202209.0151.v3/202209.0151.v4
11. Jacobson, N. (1974), Basic Algebra I, Freeman
12. Jauch, J. M., Rohrlich, F. (1976), The Theory of Photons and Electrons, Springer
13. Merzbacher, E. (1970): Quantum Mechanics, Wiley
14. Noether, E. (1918), Invariant Variation Problems, Transport Theory and Statistical Physics 1(3), English translation 1971, Tavel, M. A., arXiv:physics/0503066v3
15. Newton, I. (1686), The Mathematical Principles of Natural Philosophy, English translation 1729, A. Motte, Motte, Book I, Includes: (a) the principles of derivatives by infinitesimal limit, (b) the axioms or laws of motion, (c) the definition of the quantity of matter, related to acceleration, proportional to known by its weight
16. Roos, W. J. (1978), Grondslagen van de quantum mechanica (Dutch), (Foundations of Quantum Mechanics), Utrecht Univ. postgraduate-course
17. Sakurai, J. J. (1978), Advanced Quantum Mechanics, Addison-Wesley
18. Serre, J.-P. (1971), Représentations Linéaires Des Groupes Finis, Hermann
19. Veltman, M. (1974), Representations of Groups and Applications to Elementary Particle Physics, Utrecht Univ. postgraduate-course
20. De Wit, B., Smith, J. (1986), Field Theory in Particle Physics, Vol. I, North Holland Physics