Article

An Entropic Approach for Pair Trading in PSX

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ABSTRACT

The perception in pair trading is to recognize two stocks that move together, and their prices will converge to a mean value in future. However, finding the mean-reverted point at which the value of the pair will converge, and optimal boundaries of the trade is not easy. As uncertainty and model misspecifications may lead to losses. To cater for the problems, this study employs the novel entropic approach that utilizes entropy as penalty function for the misspecification of the model. The use of entropy as a measure of risk in pair trading is a nascent idea and this study utilizes daily data for 64 companies listed on PSX for the years 2017, 2018, and 2019, respectively to compute the returns based on the entropic approach. These companies cover the major sectors including Cement, Chemical, Automobile Assembler, Food and Personal Care Products, Oil and Gas Marketing Companies, Oil and Gas Exploration Companies Ltd, Power Generation and Distribution, Refinery and Pharmaceuticals. The returns to these stocks are then evaluated and compared with the Buy and Hold strategy. The results show positive and significant returns from pair trading using an entropic approach.

Keywords: Pair Trading; Model uncertainty; Model risk; Optimal boundary; PSX

1. INTRODUCTION

According to quantitative models, pairs trading involves a driving mechanism for mean reversions using a statistical arbitrage strategy. The perception is to recognize two stocks that move together, and their prices will converge to a mean value in the future (Ramos-Requena et al., 2017). When the prices vary, a trader can just take a short position with the over-priced stock by selling and a long position with the underpriced one by buying, and as effect of mean reversion, wait for the prices to unite in the future. When they do, the broker clears the positions and makes a profit (Narayan & Smyth, 2007). Pair trading has been extended

 doi:10.20944/preprints202212.0375.v1

from two stocks to the formation of flexible portfolios and it is an efficient method for the formation of portfolios (Guerra Cavalcanti et al., 2021). However, this market-neutral trading strategy is neither risk-neutral nor risk-free. As can be expected, the risks are different from those linked with market directional trading (Habibi & Pakizeh, 2017).

Pair trading was a highly profitable strategy when it was introduced however, it seems to have disappeared when the risk involved in this strategy shot up back in the days (Vidyamurthy, 2004). Later, several researchers contributed to the revival of the pair trading method. Krauss, (2017) categorizes pairs trading strategies into distance method, cointegration method, time series method, stochastic control method etc. Gatev et al., (2006) proposed GGR pairs trading process by using six-month trading cycle from 1962 to 1997 on a substantial sample of the U.S. equities. After assessing the effectiveness of various trading guidelines, they noted that their approach yielded annualized excess returns of up to eleven percent at minimal exposure to methodical resources of risk. Do & Faff, (2012) extended the GGR method, evaluating the test data over various years and distinct industries and confirming that the decreasing profitability in pairs trading is due to an increasing share of non-converging pairs. One tentative result also shows that more technically matched portfolios yield more significant profits than portfolios selected from the entire market. They thus lessen the convergence failure of the preferred stock portfolio. The literature shows that pairs trading is an efficient trading strategy used by financiers to yield profits with near risk-free earnings (Habibi & Pakizeh, 2017; Keshavarz Haddad & Talebi, 2021) from various market situations in a variety of financial fields. Pair trading methodologies were keenly evaluated by Carrasco Blázquez et al., (2018), their purpose was to compare the different techniques and determine the most suitable for pair selection, the results indicated that though all these techniques can determine the accurate pairs, the most efficient is cointegration approach for structuring the pairs trading. After the settlement on how to find the accurate pairs, the problem arose of how to find the mean-reverted point and how to identify the boundaries for when exactly the investors can buy or sell any asset. The solution to this optimal boundary problem was conducted by Chen & Lin, (2017) through a statistical method where they uncovered arbitrage opportunities via the daily return spreads of 12 stock pairs in the U.S. marketplaces and then report the performance of pair trading for two out-of-sample periods. The pragmatic results suggest that merging the minimum squared distance method and nonparametric one-sided tolerance limits generates positive excess returns, relative to the underlying stocks.

One applicant method of tackling the above problem is proposing fuzzy logic, which is built on the concept of fuzzy sets. The standing characteristic of the fuzzy set is the integration of the idea of part membership.

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This feature of fuzzy sets makes it possible to distinguish elements with borderline importance that involve roughness and uncertainty. Thus, the presentation of fuzzy logic in originating an optimal strategy may lead to a complicated transaction cost, such as a strong sell where in stock trading investment analysts have said that these stocks underperform when compared with the average market return. It is an emphatic negative comment on a stock's prospects (Bayram & Akat, 2019).

An alternative to this is to introduce entropy as a penalty function for the misspecification of the model. Entropy has a wide application in finance as well (Bekiros, 2014; Bowden, 2011; Yin, 2019). Yoshikawa, (2017) derived the entropy-based optimal boundary points for pair trading using Tokyo Stock Exchange 2015 data. The proposed approach for optimal stopping problem is motivated by the work of Ekström et al., (2011) and Suzuki, (2016). This method is based on maximizing profit via pair trading and minimizing the relative entropy (risk). This is a robust method as it directly tackles the model misspecification (Krätschmer et al., 2018) and provide a more persuasive solution. The choice of pairs is made through cointegration that is the most effective way to identify stocks that move together (Tokat & Hayrullahoğlu, 2022).

In context of Pakistan, there are handful of studies conducted on pair trading (Qazi et al., 2015; Sohail et al., 2020) and interestingly no one has considered the optimal stopping problem using stocks listed on Pakistan Stock Exchange (PSX). This study employs the novel entropic approach to explore the optimal boundary points that yield maximum profit for 64 companies listed on Pakistan Stock Exchange (PSX) for the period 2017-2019. The concept of maximizing the profit in pair trading based on relative entropy is a nascent idea in literature and this study is the first attempt in context of Pakistan. The performance of this entropic approach is contrasted with the buy & hold strategy in terms of returns.

2. DATA & METHODOLOGY

As mentioned in the last section, this study utilizes the daily data for 64 companies listed on PSX for the years 2017, 2018, and 2019, respectively. These companies cover the major sectors including Cement, Chemical, Automobile Assembler, Food and Personal Care Products, Oil and Gas Marketing Companies, Oil and Gas Exploration Companies Ltd, Power Generation and Distribution, Refinery and Pharmaceuticals. The firm's selection criterion is based on year-wise price earnings ratios (PER)- a firm with PER lower than the sample median value is selected in the sample. The underlying idea is that the stock below median PER is undervalued and signifies potential for higher returns (Chutka & Kramarova, 2020; de Lima Amorim & de Camargos, 2021). The choice of pairs is made through Johansen cointegration that is the most effective

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way to identify stocks that move together (Tokat & Hayrullahoğlu, 2022). In each year, we formulated all pairs $((n^2-n)/2)$ of the selected stocks and assessed each pair for cointegration.

Keeping in view the potential jumps/structural breaks in high-frequency financial data (Laurent & Shi, 2022), following breakpoint unit root test proposed by Bai & Perron, (1998) is employed.

$$\Delta y_t = \alpha_0 + \alpha_1 t + \delta y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \mu_t \ldots (2)$$

Where, μ_t is white noise.

Ornstein-Uhlenbeck (OU) Process

Pair trading utilizes the mean reversion of the composite process of two stocks. Following (Yoshikawa, 2017), we consider the Ornstein-Uhlenbeck (OU) process X_t such that

$$dX_t = -\mu(X_t - \alpha)dt + \sigma dB_t, \quad X_0 = \alpha \ldots (3)$$

where, μ, σ are the positive constants and α is the mean-reversion point and B_t is the p-Brownian motion. Let $X_t - \alpha = \check{X}_t$ then, equation (3) implies

$$d\check{X}_t = -\mu \check{X}_t dt + \sigma dB_t, \quad \check{X}_0 = 0 \ldots (4)$$

Optimal stopping problem at time t for the process, $\check{X}_t$, is defined as follows

$$v^0(t,x) = \underset{\tau \epsilon \Im}{SUP} E_{\check{x}}^S \left[e^{-\rho(\tau - t)} \check{X}_\tau\right] \ldots (5)$$

where, $\Im$ is the set of all stopping points of B and ρ is the discount rate. The solution of equation (5) gives us the trading strategy: we short pair X when it attains the highest value and liquidate it when X attains zero value. These values are specified by the above equation. Alternatively, we take long position for X for zero value and liquidate for highest value. The superscript, S, in the equation (5) is the solution to the following

$$Inf_{s \epsilon \Im} \left\{E_{\check{x}}^S \left[e^{-\rho(\tau - t)} \check{X}_\tau\right] + \lambda e^{-\rho(\tau - t)} H_{\check{x}} [S|P]\right\} \ldots (6)$$

λ is a positive constant and H(.) is a relative entropy defined as follows:

$$H_{\check{x}} = \begin{cases} E_{\check{x}}^S [\ln\left(\frac{dS}{dP}\right), & S \epsilon \Im \\ \infty, & otherwise \end{cases} \ldots (7)$$

Thus, the optimal boundary b(t) for eq. 5 is given as

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$$\ln\big(b(t)\big) + \frac{1}{\sigma^2}\frac{\rho}{\rho-\mu}(g(t)-b(t))^2 = \ln(b^*) + \frac{1}{\sigma^2}\frac{\rho}{\rho-\mu}(b^*)^2 \dots (8)$$

Where, $g(t) = -\frac{\sigma^2}{\lambda}te^{-\mu t}$ & $b(0) = b^*$. Any investor holding pair X liquidate when X touches b(t) and if not holding X should short position when X touches b(t) and liquidate it when it reverts to mean zero.

3. RESULTS & DISCUSSION

Having selected 64 companies for the years 2017, 2018, and 2019 respectively based on the PER values, unit root test is applied to the time series data of these stocks to find the order of integration. All the time series are integrated of order one. This led us to find the cointegrated pairs using Johansen cointegration test at 0.05 level of significance. We found 74 cointegrated pairs out of 2016=(64!/62!*2!) pairs of the selected stocks. Having found the pairs, we applied maximum likelihood method to find the parameters of Ornstein–Uhlenbeck processes, $\mu, \alpha, \& \sigma$ as given in equation (3). MATLAB R2021b is used for coding and estimation of these parameters. However, to compute the optimal boundary points, we need to find the parameters, ρ & λ as well. The parameter ρ is the discount rate and the parameter λ represents the level of confidence, lower the value of λ, lower is the confidence and vice-versa. We used $\rho = 0.08978, 0.1315, \& 0.1440$ as per the annual report of State Bank of Pakistan for the respective years

Table 1: Ornstein-Uhlenbeck Process parameters estimation

Sr	Pair Name	μ	α	σ
			2017	
1	Pak State Oil (PSO) & Maple Leaf Cement LTD (MPLF)	0.04	60.29	84.91
2	Thata Cement (THAT) & Gharibwal Cement (GHAR)	0.02	101.3	39.25
3	Pak Oil Fields (PKOL) & Ittehad Chemicals LTD (ITHD)	3.74	1361.7	474.9
4	Pioneer Cement (PION) & Lalipir Power LTD (LPLP)	10.52	14.54	7.70
5	Engro Polymer and Chemical (EPCL) & Lalipir Power (LPLP)	2.29	15.71	8.47
			2018	
6	Engro Power Generation Qadirpur LTD & Thata Cement LTD	6.59	69.44	16.43
7	Gharibwal Cement LTD & Dewan Cement LTD	0.04	79.3	31.87
8	Pakistan State Oil Company LTD & Best Way Cement LTD	0.02	90.7	50.78
9	Pakistan State Oil Company LTD & Byco Petroleum Pak LTD	5.37	9.26	5.25
10	Pioneer Cement LTD &Dewan Cement LTD	0.13	40.14	19.14
			2019	
11	Nishat Chunnian Power LTD &Engro Polymer and Chemical LTD	0.01	91.05	35.11
12	Nishat Chunnian Power LTD & Maple Leaf Cement Factory	0.004	94.76	39.29
13	Thata Cement LTD & Pakistan State Oil Company LTD	1.85	23.69	10.38
14	Thata Cement LTD & Pakistan Oilfields LTD	0.01	42.82	14.44
15	Pioneer Cement LTD &Al Shaheer Corporation LTD	0.06	29.87	13.36

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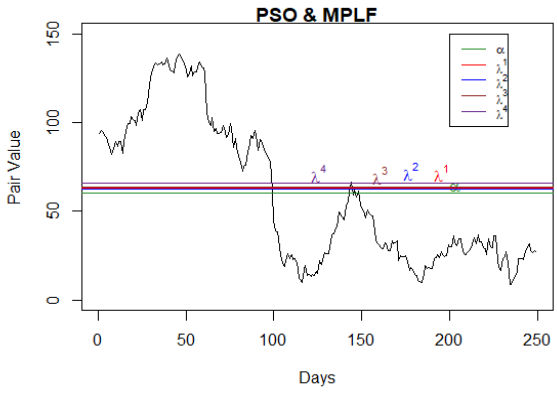
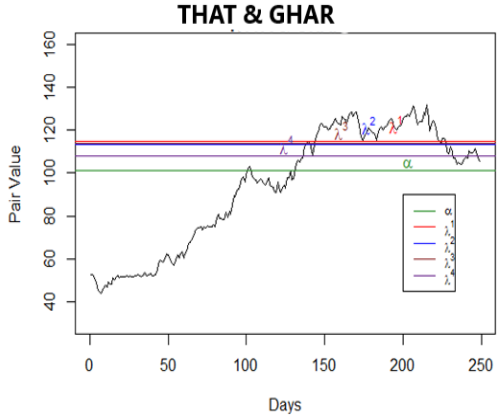
Table 2: Rate of returns for different values of λ

Pair Name	λ = 0.001	λ = 0.01	λ = 0.1	λ = +∞
			2017	
PSO & MPLF	0.042	0.037	0.051	0.087
THAT & GHAR	0.132	0.12	0.122	0.065
PKOL & ITHD	0.252	0.246	0.206	0.023
PION & LPLP	0.186	0.18	0.114	0.013
EPCL & LPLP	0.002	0.006	0.208	0.08
			2018	
ENGP & THAT	0.055	0.054	0.04	0.004
GHAR & DECE	0.05	0.038	0.064	0.006
PSO & BEST	0.177	0.192	0.175	0.103
PSO & BYCO	0.187	0.177	0.14	0.022
PION & DECE	0.187	0.195	0.178	0.131
			2019	
NCPL & ENGRO	0.094	0.087	0.091	0.041
NCPL & MPLF	0.131	0.157	0.135	0.089
THAT & PSO	0.131	0.132	0.121	0.039
THAT & PKOIL	0.024	0.015	0.03	0.111
PION & ALSHAHEER	0.034	0.049	0.038	0.089

Fig. 1: Pair values, boundaries, and mean values for the pairs (2017)





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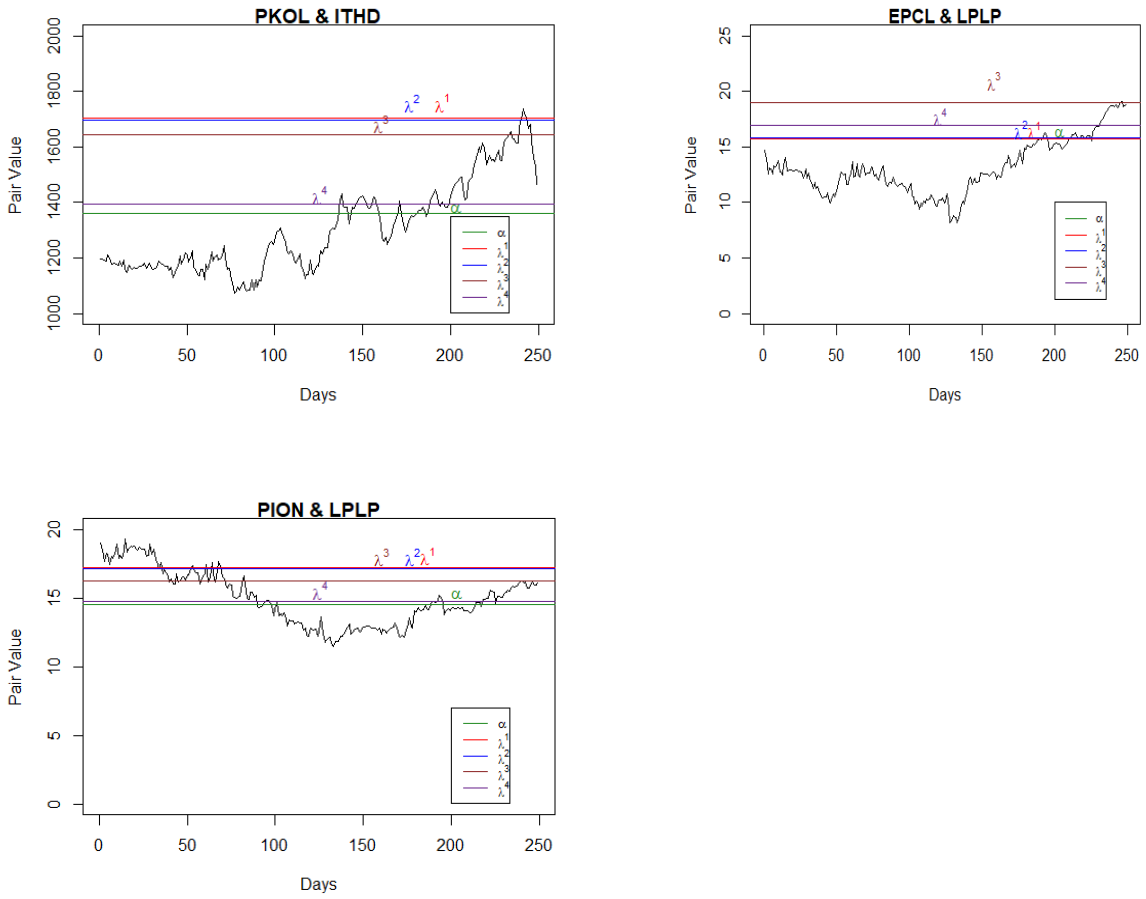

Fig. 2: Pair values, boundaries, and mean values for the pairs (2018)

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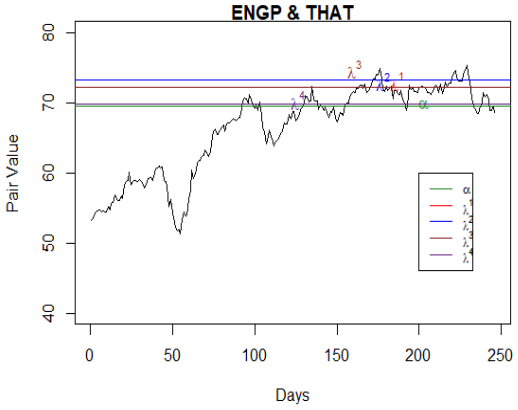

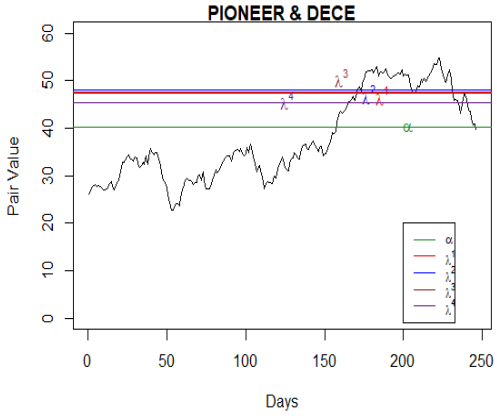

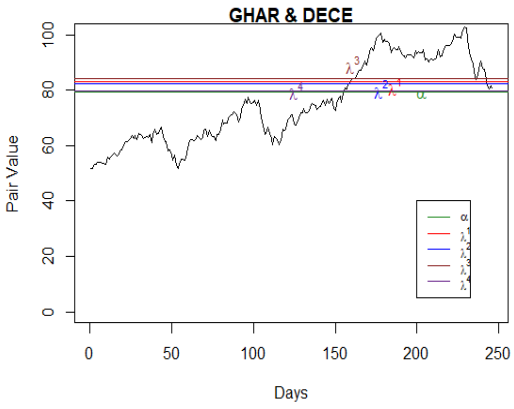

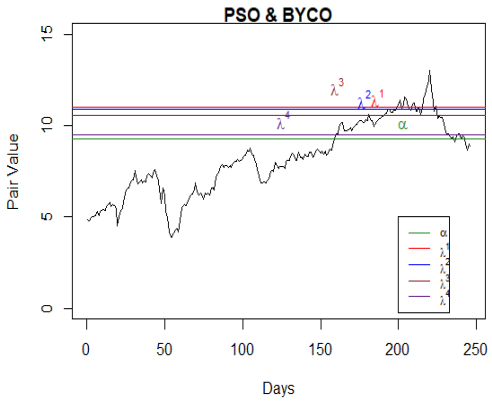

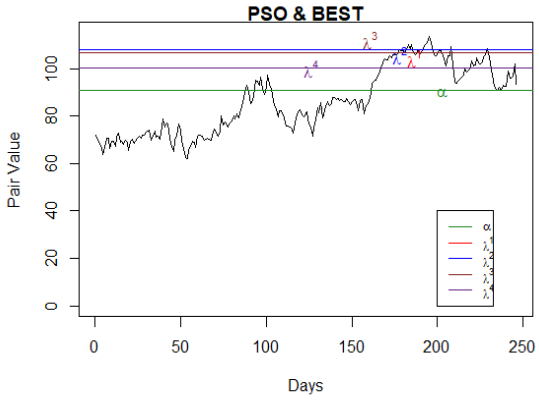

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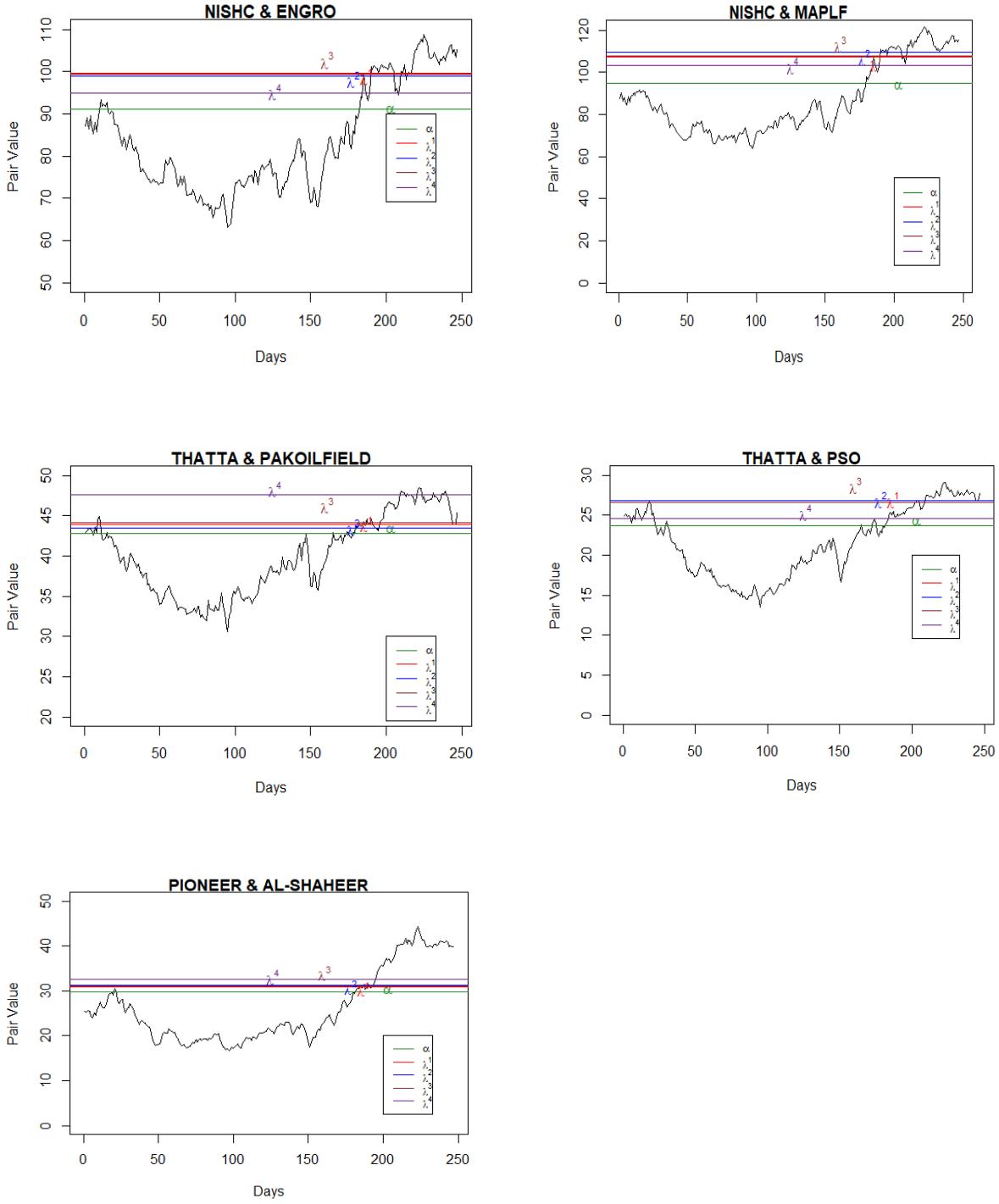
Fig. 3: Pair values, boundaries, and mean values for the pairs (2019)



and by following Yoshikawa (2017), four cases for the parameter, $\lambda = 0.001, 0.01, 0.1, \& \infty$, are considered. Table 1 & 2 presents the results for only five pairs of stocks in each year involving the top listed companies (see appendix A, table 1A-7A for results of other companies). After computing the values of $\mu, \alpha, \& \sigma$ as

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furnished in table 1, we estimated the rate of returns for different values of λ for the selected 64 companies (table 2). All the estimated parameter values are presented in figure 1, 2, and 3 for the respective years. From these figures, it is evident that the values of the mean reversion parameter differ when the stocks in the pair are selected within the sector in comparison when the stocks are selected across the sectors.

For the real data sets, the pair trading strategy is to set the position when the pair value touches either the mean reverted point or the boundary. For example, in figure 1 (pair: PSO & MPLF), the mean reversion point is 60.29 where we set the position and we liquidate the position when the pair value touches the boundary b(t). If the position is set when the pair value touches the boundary then it is liquidated when it touches the mean reversion point, α. In figure 2 (pair: PSO & BYCO), if we set our position when the pair value touches the boundary then we would liquidate at the mean reversion point, α=9.26. Next position is set when the pair value touches either the boundary b(t) or mean reversion point, α, and liquidated following the same rule and so forth.

According to this trading strategy, we estimated the rate of returns for the 64 companies for the years 2017, 2018, and 2019, respectively. Gatev et al., (2006) highlighted the transaction fee as an obstacle in trading. Per transaction cost in Pakistan Stock Exchange is 0.15 percent and we are dealing with pair trading, so we discounted our return values by 0.3 percent. Table 2 provides these return values for 5 pairs from each year. The return values range from 0.2 to 25.2 percent for the year 2017, 0.4 to 19.5 percent for the year 2018, and 1.5 to 15.7 percent for the year 2019. The rationale behind pairs trading is to profit from mean-reversion forces that eliminate short-term price deviations in favor of long-term historical pricing relationships. All positive returns are confirming the profits which is line with the findings in literature (Ramos-Requena et al., 2020; Yoshikawa, 2017).

Table 3: Rate of returns from the Buy & Hold strategy

Company Name	**Return**		
	2017	**2018**	**2019**
Pak State Oil	-20.88	-8.56	-2.38
Thata Cement LTD	-44.89	-37.78	-17.75
Pioneer Cement LTD	-55.61	-33.67	-30.80
Nishat Chunnian Power	-43.19	-28.53	-19.02
Gharibwal Cement LTD	-53.49	-36.16	-14.02

Further, to evaluate our results, we contrasted our results against the buy & hold strategy. The buy & hold strategy simply requires buying stocks on the first of January and selling on the last day of December each year. The rate of returns for the alternative strategy is summarized in table 3. All the top performing stocks make a loss for this strategy whereas table 2 provides stable profits based on pair trading. The buy & hold

 doi:10.20944/preprints202212.0375.v1

strategy has a considerable risk of human error and pressure of all the wrong choices one can make (Hui Ling et al., 2014). The optimization of the boundaries backed by Ornstein-Uhlenbeck Process allows us to incorporate all the risks, improves the profitability of pair trading, and gives maximum positive returns (Lee & Leung, 2020). Therefore, we suggest the pair trading strategy while taking model uncertainty into account.

4. CONCLUSION

This study employs the novel entropic approach to explore the optimal boundary points that yield maximum profit for 64 companies listed on Pakistan Stock Exchange (PSX) for the period 2017-2019. The concept of maximizing the profit in pair trading based on relative entropy is a nascent idea in literature and this study is the first attempt in context of Pakistan. The performance of this entropic approach is contrasted with the buy & hold strategy in terms of returns.

The rationale behind pairs trading is to profit from mean-reversion forces that eliminate short-term price deviations in favor of long-term historical pricing relationships. All positive returns are confirming the profits which is line with the findings in literature (Ramos-Requena et al., 2020; Yoshikawa, 2017) whereas the returns for the selected companies from the buy & hold strategy are negative except for few cases implying losses. Therefore, we suggest the pair trading strategy while taking model uncertainty into account.

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Appendix A

Table A1: 2017 Ornstein–Uhlenbeck process Parameters

Pair Name	μ	α	σ
Fauji Food (FAUJ) & Pak State Oil (PSO)	3.7	32.48	15.19
Fauji Food (FAUJ) & Gharibwal Cement (GHAR)	3.84	33.94	15.34
Fauji Food (FAUJ) & National Refinery (NATR)	3.11	24.63	14.63
Fauji Food (FAUJ) & Engro Power Qadirpur LTD (ENGP)	3.5	4.31	13.72
Fauji Food (FAUJ) & Bestway Cement (BEST)	3.29	41.38	15.74
Fauji Food (FAUJ) & Dewan Cement LTD(DECE)	3.8	36.58	15.89
Fauji Food (FAUJ) & Ghani Automobile Industries LTD (GAIL)	3.21	21.26	14.4
Fauji Food (FAUJ) & Ittehad Chemicals LTD (ITHD)	10.9	6.27	16.69
Fauji Food (FAUJ) & Ghandhara Industries LTD (GHIN)	4.06	32.23	15.1
Fauji Food (FAUJ) & Power Cement LTD (POWE)	3.99	30.8	14.84
Fauji Food (FAUJ) & Pakistan Petroleum LTD (PPL)	0.056	61.45	18.81
Fauji Food (FAUJ) & Lalpir Power LTD (LPLP)	3.6	56.36	18.76
Pak State Oil (PSO) & Maple Leaf Cement LTD (MPLF)	0.04	60.29	84.91
Thata Cement (THAT) &Gharibwal Cement (GHAR)	0.02	101.34	39.25
Pak Oil Fields (PKOL) & Ittehad Chemicals LTD (ITHD)	3.74	1361.72	474.89
Gharibwal Cement & Ghandhara Industries LTD(GHIN)	0.19	35.94	21.6
Gharibwal Cement & Power Cement LTD (POWE)	2.99	34.93	13.56
National Refinery (NATR)& Dewan Cement LTD(DECE)	3.22	72.22	24.43
National Refinery (NATR) & Ittehad Chemicals LTD (ITHD)	0.05	42.95	16.08
National Refinery (NATR) & Lalipir Power LTD (LPLP)	9.37	9.4	8.3
Pioneer Cement (PION) & Lalipir Power LTD (LPLP)	10.52	14.54	7.7
Dewan Cement LTD (DECE) & Ittehad Chemicals LTD (ITHD)	0.88	152.64	34.55
Dewan Cement LTD (DECE) & Power Cement LTD (POWE)	0	13.77	12.48
Ittehad Chemicals LTD (ITHD) & Ghandhara Industries (GHIN)	0.04	43.54	16.8
Ittehad Chemicals LTD(ITHD) & Power Cement LTD (POWE)	2.82	39.49	19.11
Ittehad Chemicals LTD (ITHD) & Pakistan Petroleum (PPL)	0.02	313.08	95.34
Ittehad Chemicals LTD (ITHD) & Lalipir Power LTD (LPLP)	0.01	55.73	18.22
Engro Polymer and Chemical (EPCL) & Lalipir Power (LPLP)	2.29	15.71	8.47

Table A2: 2017 Rate of return of at different λ

Pair Name	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
FAUJ & PSO	0.060	0.059	0.048	0.007
FAUJ & GHAR	0.061	0.059	0.05	0.006
FAUJ & NATR	0.080	0.075	0.068	0.010
FAUJ & ENGP	1.522	1.525	1.294	0.358
FAUJ & BEST	0.001	0.001	0.000	0.010

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Pair Name	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
FAUJ & DECE	0.055	0.055	0.043	0.004
FAUJ & GAIL	0.120	0.125	0.101	0.022
FAUJ & ITHD	1.046	0.910	0.630	0.075
FAUJ & GHIN	0.085	0.078	0.064	0.010
FAUJ & POWE	0.083	0.082	0.066	0.011
FAUJ & PPL	0.059	0.060	0.060	0.030
FAUJ & LPLP	0.051	0.047	0.041	0.006
PSO & MPLF	0.042	0.037	0.051	0.087
THAT & GHAR	0.132	0.12	0.122	0.065
PKOL & ITHD	0.252	0.246	0.206	0.023
GHAR & GHIN	0.044	0.045	0.031	0.048
GHAR & POWE	0.041	0.036	0.031	0.003
NATR & DECE	0.029	0.027	0.023	1.650
NATR & ITHD	0.216	0.218	0.201	0.172
NATR & LPLP	0.327	0.317	0.214	0.028
PION & LPLP	0.186	0.18	0.114	0.013
DECE & ITHD	0.041	0.04	0.043	0.011
DECE & POWE	0.077	0.086	0.081	0.020
ITHD & GHIN	0.216	0.234	0.232	0.191
ITHD & POWE	0.220	0.216	0.191	0.060
ITHD & PPL	0.051	0.044	0.034	0.032
ITHD & LPLP	0.079	0.095	0.077	0.065
EPCL & LPLP	0.002	0.006	0.208	0.080

Table A3: 2018 Ornstein–Uhlenbeck process Parameters

Pair Name	μ	α	σ
Nishat Chunnian Power LTD (NCPL)&Nishat Power LTD (NISH)	0.060	62.870	19.110
Nishat Chunnian Power LTD &Lotte Chemicals Pak LTD	14.340	38.550	13.150
Nishat Chunnian Power LTD &Dewan Cement LTD	0.045	71.740	24.950
Nishat Chunnian Power LTD &Byco Petroleum Pak LTD	4.080	36.460	10.500
Nishat Power LTD &Dewan Cement LTD	0.020	67.290	53.350
Nishat Power LTD &Byco Petroleum Pak LTD	38.400	31.070	21.260
Engro Power Generation QadirPur LTD (ENGP) &Thata Cement LTD	6.590	69.440	16.430
Engro Power Generation QadirPur LTD & Dewan Cement LTD	0.020	60.270	18.600
Attock Cement Pak LTD &Dewan Cement LTD	3.620	45.500	17.340
Honda Atlas Cars Pak LTD & Fauji Cement Company LTD	1.950	827.500	312.410
KOT Addu Power Company LTD & Bestway Cement LTD	7.920	100.470	26.610
KOT Addu Power Company LTD & Dewan Cement LTD	11.980	33.310	17.050
KOT Addu Power Company LTD &Byco Petroleum Pak LTD	6.850	22.350	7.080
Gharibwal Cement LTD & Dewan Cement LTD	0.040	79.300	31.870

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Pair Name	μ	α	σ
Gharibwal Cement LTD& Fauji Cement Company LTD	0.010	66.230	24.540
Gharibwal Cement LTD & Byco Petroleum Pak LTD	5.030	26.280	8.920
Gharibwal Cement LTD & Quice Food Industries LTD	5.700	16.540	6.240
Ghandhara Nissan LTD&FAUJI Food LTD	0.010	60.720	30.090
Ghandhara Nissan LTD &Byco Petroleum Pak LTD	5.720	16.810	6.670
Pakistan State Oil Company LTD & Bestway Cement LTD	0.020	90.700	50.780
Pakistan State Oil Company LTD & Dewan Cement LTD	2.950	24.090	14.430
Pakistan State Oil Company LTD & Byco Petroleum Pak LTD	5.370	9.260	5.250
DYNEA Pak LTD & Dewan Cement LTD	14.920	39.560	53.730
Lotte Chemicals Pak LTD & Dewan Cement LTD	5.910	21.390	11.330
Lotte Chemicals Pak LTD &Byco Petroleum Pak LTD	0.120	32.490	13.350
Pioneer Cement LTD &Dewan Cement LTD	0.130	40.140	19.140
Millat Tractors LTD & Byco Petroleum Pak LTD	6.190	29.740	9.060
Dewan Cement LTD &Ghandhara Industries LTD	6.360	14.350	17.310
Ghandhara Industries LTD & Byco Petroleum Pak LTD	6.360	19.660	7.390

Table A4: 2018 Rate of return of at different λ

PAIR NAME	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
NCPL & NISH	0.058	0.049	0.062	0.028
NCPL & LOTTE	0.161	0.153	0.085	0.008
NCPL & DECE	0.128	0.128	0.125	0.097
NCPL & BYCO	0.014	0.013	0.010	0.006
NISH & DECE	0.090	0.094	0.080	0.065
NISH & BYCO	0.228	0.193	0.057	0.004
ENGP & that	0.055	0.054	0.040	0.004
ENGP & DECE	0.016	0.007	0.005	0.033
ATTOC & DECE	0.024	0.020	0.015	0.008
HONDA & FAUJ	0.273	0.274	0.244	0.051
KOT & BEST	0.077	0.074	0.052	0.005
KOT & DECE	0.177	0.167	0.102	0.009
KOT & BYCO	0.092	0.087	0.065	0.008
GHAR & DECE	0.050	0.038	0.064	0.006
GHAR & FAUJ	0.095	0.055	0.048	0.013
GHAR & BYCO	0.068	0.065	0.051	0.006
GHAR & QUICE	0.103	0.098	0.075	0.010
GHAN & FAUJ	0.010	0.109	0.010	0.036
GHAN & BYCO	0.111	0.106	0.079	0.011
PSO & BEST	0.177	0.192	0.175	0.103
PSO & DECE	0.072	0.064	0.057	0.003
PSO & BYCO	0.187	0.177	0.140	0.022

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PAIR NAME	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
DYNEA & DECE	0.477	0.446	0.245	0.020
LOTTE & DECE	0.347	0.341	0.264	0.049
LOTTE & BYCO	0.084	0.055	0.059	0.027
PION & DECE	0.187	0.195	0.178	0.131
MILLAT & BYCO	0.078	0.074	0.055	0.007
DECE & GHAN	0.454	0.438	0.335	0.049
GHAN & BYCO	0.110	0.105	0.079	0.010

Table A5: 2019 Ornstein–Uhlenbeck process Parameters

Pair Name	μ	α	σ
Pakistan Refinery LTD & Oil & Gas Development CO LTD	0.020	96.400	42.740
Pakistan Refinery LTD & Ghani Automobile Industries LTD	6.840	41.610	39.760
National Refinery LTD & Pakistan Oilfields LTD	0.040	670.990	208.610
Nishat Chunnian Power LTD & Engro Polymer and Chemical LTD	0.010	91.050	35.110
Nishat Chunnian Power LTD & Pioneer Cement LTD	0.780	26.860	11.540
Nishat Chunnian Power LTD & Maple Leaf Cement Factory	0.004	94.760	39.290
Attock Refinery LTD & Attock Petroleum LTD	0.030	644.640	233.200
Dewan Farooque LTD & Descon Oxychem LTD	0.001	36.770	21.570
Dewan Farooque LTD & Cherat Cement Company LTD	0.040	100.360	61.410
Ittehad Chemicals LTD & Pak Suzuki Motors Company LTD	0.110	40.210	19.010
Thata Cement LTD & Pakistan State Oil Company LTD	1.850	23.690	10.380
Thata Cement LTD & Pakistan Oilfields LTD	0.010	42.820	14.440
Thata Cement LTD & Ghani Automobile Industries LTD	0.006	24.430	8.140
Descon Oxychem LTD & Pakistan Oilfields LTD	0.010	588.420	191.100
Cherat Cement Company LTD & Hi-Tech Lubricants LTD	0.008	68.540	29.140
Mari Petroleum Company LTD & Fauji Cement Company LTD	0.240	29.100	10.630
K Electric LTD & Fauji Cement Company LTD	1.320	15.310	5.080
Pakistan State Oil Company LTD & Pakistan Oilfields LTD	0.009	580.660	192.900
Pakistan Oilfields LTD &Honda Atlas Cars Pak LTD	4.310	787.640	261.200
Pakistan Oilfields LTD & Ghani Automobile Industries LTD	0.070	629.090	165.800
Fauji Cement Company LTD & Ghani Automobile Industries LTD	0.260	28.410	10.400
Pioneer Cement LTD &Al Shaheer Corporation LTD	0.060	29.870	13.360
Maple Leaf Cement Factory & Al Shaheer Corporation LTD	0.050	29.550	13.300

Table A6: 2019 Rate of return of at different λ

PAIR NAME	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
PAKR & OG	0.082	0.087	0.095	0.025
PAKR & Ghani	0.343	0.333	0.241	0.029
NATR & PKOIL	0.084	0.077	0.096	0.065

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PAIR NAME	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = +\infty$
NCPL & ENGRO	0.094	0.087	0.091	0.041
NCPL & PION	0.207	0.202	0.199	0.115
NCPL & MPLF	0.131	0.157	0.135	0.089
ATTOCR & ATTOCP	0.211	0.187	0.202	0.027
DEWAN & DESCON	0.186	0.192	0.197	0.124
DEWAN & CHERAT	0.257	0.232	0.222	0.117
ITHD & PAK SUZUKI	0.147	0.155	0.135	0.097
THAT & PSO	0.131	0.132	0.121	0.039
THAT & PKOIL	0.024	0.015	0.03	0.111
THAT & GHANI	0.037	0.045	0.041	0.071
DESCON & PKOIL	0.023	0.01	0.016	0.064
CHERAT & HITECH	0.068	0.031	0.041	0.008
MARI & FAUJ	0.027	0.029	0.015	0.123
KELEC & FAUJ	0.009	0.007	0.007	0.009
PSO & PKOIL	0.103	0.025	0.037	0.103
PKOIL & HONDA	0.073	0.072	0.056	0.002
PKOIL & GHANI	0.086	0.087	0.1	0.009
FAUJ & GHANI	0.194	0.189	0.193	0.147
PION & ALSHAHEER	0.034	0.049	0.038	0.089
MPLF & ALSHAHEER	0.042	0.036	0.035	0.104

Table A7: Returns based on Buy & Hold Strategy

2017		2018		2019	
Company Names	Returns	Company Names	Returns	Company Names	Returns
Attock Cement	-44.90	Attock Cement Pak LTD	-26.50	Al Shaheer Corporation LTD	-40.43
Attock Petroleum LTD	-24.60	Attock Petroleum LTD	-1.74	Attock Cement Pak LTD	-9.49
Attock Refinery LTD	-45.33	BestWay Cement LTD	-17.93	Attock Petroleum LTD	-16.85
Bestway cement	-52.19	Byco Petroleum Pak LTD	-33.59	Attock Refinery LTD	-23.76
Cherat Cement Company LTD	-40.97	Cherat Cement Company LTD	-35.48	BestWay Cement LTD	-3.36
Dera Ghazi khan Cement	-39.93	Dera Ghazi Khan Cement LTD	-41.21	Cherat Cement Company LTD	-20.13
Descon Oxychem LTD	-24.34	Descon Oxychem LTD	119.92	Descon Oxychem LTD	-21.96
Dewan Cement LTD	-56.42	Dewan Cement LTD	-35.56	Dewan Farooque LTD	-56.18
DYNEA Pak LTD	65.35	DYNEA Pak LTD	-12.50	DYNEA Pak LTD	20.03

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2017		2018		2019	
Company Names	Returns	Company Names	Returns	Company Names	Returns
Engro Polymer and Chemical	54.77	Engro Polymer and Chemical LTD	48.95	Engro Polymer and Chemical LTD	-14.18
Engro Power Qadirpur	-5.91	Engro Power Generation QadirPur LTD	-15.31	Engro Power Generation QadirPur LTD	-11.56
Fauji Food LTD	-47.45	Fauji Cement Company LTD	-16.35	Fauji Cement Company LTD	-27.47
Ghandhara Industries LTD	-27.15	FAUJI Food LTD	83.63	FAUJI Food LTD	-54.14
Ghani Automobile Industries	7.79	Ghadhara Nissan LTD	-32.34	Ghani Automobile Industries LTD	-34.46
Gharibwal Cement	-53.49	Ghandhara Industries LTD	0.33	Gharibwal Cement LTD	-14.02
Indus Motor Company LTD	2.60	Gharibwal Cement LTD	-36.16	Hi Tech Lubricants LTD	-52.84
Ittehad Chemicals LTD	-31.67	Honda Atlas Cars Pak LTD	-64.27	Honda Atlas Cars Pak LTD	19.05
Kohat cement	-52.29	Indus Motor Comapany LTD	-29.46	Indus Motor Company LTD	-4.61
KOT ADDU Power	-31.49	Ittehad Chemicals LTD	15.57	Ittehad Chemicals LTD	-16.52
Lalipir Power LTD	-5.82	Kohat Cement LTD	-25.78	K Electric LTD	-27.48
Maple Leaf Cement Factory	-52.16	KOT Addu Power Company LTD	-11.16	Kohat Cement LTD	-7.11
National Refinery	-24.81	Lalipur Power LTD	-22.73	KOT Addu Power Company LTD	-36.57
Nishat Chunnian Power	-43.19	Lotte Chemicals Pak LTD	129.48	Lalipir Power LTD	-9.66
Nishat Power LTD	-45.95	Maple Leaf Cement Factory	-40.21	Lotte Chemicals Pak LTD	-20.80
Pak Oilfields	12.81	Mari Petroleum Company LTD	-5.47	Maple Leaf Cement Factory	-35.88
Pak State Oil	-20.88	Millat Tractors LTD	-28.84	Mari Petroleum Company LTD	13.55
Pakistan Petroleum LTD	10.17	Nishat Chunnian Power LTD	-28.53	Millat Tractors LTD	-4.01
Pakistan Refinery LTD	-9.83	Nishat Power LTD	-19.11	National Refinery LTD	-49.66
Pioneer Cement	-55.61	Oil & Gas Development CO LTD	-21.11	Nishat Chunnian Power LTD	-19.02
Power Cement LTD	-22.03	Pakistan Petroleum LTD	-16.89	Nishat Power LTD	3.42

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2017		2018		2019	
Company Names	Returns	Company Names	Returns	Company Names	Returns
Shell Pakistan LTD	-41.53	Pakistan State Oil Company LTD	-8.56	Oil & Gas Development CO LTD	5.80
Sitara Peroxide LTD	-55.80	Pioneer Cement LTD	-33.67	Pak Suzuki Motors Company LTD	27.62
Thata Cement LTD	-44.89	Quice Food Industries LTD	-8.66	Pakistan Oilfields LTD	0.13
		Thata Cement LTD	-37.78	Pakistan Petroleum LTD	5.57
				Pakistan Refinery LTD	-9.83
				Pakistan State Oil Company LTD	-2.38
				Pioneer Cement LTD	-30.80
				Quice Food Industries LTD	-19.56
				Sitara Peroxide LTD	-29.36
				Thata Cement LTD	-17.75