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Article

Parametric Analysis of Nonlinear Oscillations of the “Rotor-Weakly Conductive Viscous Fluid-Foundation” System under the Action of a Magnetic Field

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Abstract: The generalized dynamic model of the rotor system, presented in the paper, is the first model that takes into account the interconnected oscillations of the “rotor - weakly conductive fluid – foundation” system under the action of such parameters as fluid and rotor motion, linear eccentricity, friction forces, foundation vibration and nonlinear characteristics of rolling bearings, as well as the action of a magnetic field on the fluid. Consistent equations of motion for the system “rotor - weakly conductive fluid – foundation” were derived and solved analytically. Forced and natural oscillations of the system were analyzed, and the distinctive features of the rotor system dynamics were revealed. The values of frequencies and amplitudes, which are one of the main factors determining the dynamic behavior of the system, were obtained and studied.

Keywords: rotor; moving foundation; nonlinear oscillations; critical frequency; natural oscillations; weakly conductive liquid; magnetic and electrohydrodynamics

1. Introduction

In many theoretical and practical studies on the dynamics of rotor systems containing liquid, only oscillations of the rotor with liquid are considered, without taking into account the electromagnetic properties of the liquid and the mobility of the foundation [1]. This assumption leads to certain errors in assessing the dynamic and kinematic characteristics of the rotor system [2]. Studies of such dynamic systems as rotary systems show the importance of taking into account the electromagnetic properties of the fluid, the nonlinear properties of the shaft supports, foundation vibration and the need to develop measures to reduce them [3]. The intensive development of magnetic and electrohydrodynamics (hereinafter referred to as MEHD) started in the 1960s by the Melcher group in the USA, in Europe - by the French and Spanish scientific centers [5] and others [6], where the issues of the possibility of using electrohydrodynamic effects in production and the importance of developing a general theory of MEHD were considered. In 1966, Taylor was the first to show how the application of a uniform electric field can deform the shape of a weakly conductive liquid depending on its electrohydrodynamic properties [7]. In the USSR, this direction was developed by such authors as V.V. Gogosov, I.E. Tarapov and others [19]. As a result, already in the 70-80s of the 20th century, the main provisions and systems of MEHD equations were developed, the conditions under which various MEHD models are valid were considered. Intensive MEHD studies using applied physics methods were carried out in [8], where gas-dynamic flows with similarly charged particles were studied, the general regularities of spatial MEHD flows were investigated, and new directions in MEHD were analyzed: MEHD turbulence, unsteady effects, new methods for diagnosing MEHD flows. Some MEHD problems on the stability of an inhomogeneously heated low-conductivity liquid were studied in [9]. A large amount of work on the study of electrophysical

processes in dielectric liquids initiated by high-intensity fields was carried out by the authors of [10]. The results of recent MEHD studies are presented in [11].

Oscillations of the free surface of a viscous fluid in a rotating cylindrical vessel and stability of rotational motions were first considered by Stewartson [33] and Kostandyan; further their studies were continued in the works of Bauer [37], Kimura [36], and Eidel. In these papers, linearized equations of a system acting on a vessel from the liquid side are studied, where it is assumed that the free surface of the liquid differs little from the unperturbed cylindrical shape. A similar formulation of the problem and a similar mathematical model was used in [35-38] where the oscillations of a rotating cylinder partially filled with an ideal and viscous fluid were studied.

This problem originates in the works of Kelvin (1877, 1880), Lamb (1945), Zhukovsky (1948, 1949) and Chetaev (1957), and was first solved in a general form by B.I. Rabinovich in 1951 [39]. In 1952, N.N. Moiseev, independent of [39], obtained similar equations of motion, which can be found in more detail in [40]. Among the works of modern authors devoted to various aspects of this direction, we should mention the following papers: in [41] the analytical and numerical results obtained in the study of the motion of a system consisting of a rigid body with a cavity filled with a viscous fluid are presented, in [42-44] chaotic motions of a rigid body and with a cavity filled with a liquid are studied, in [45] the experimental work was carried out to study the oscillations and displacements of the free surface of a liquid for the case when the cavity is filled with two different liquids, in [46] a quasi-analytical model of oscillations of magnetic fluids under low gravity under the action of external inhomogeneous magnetic fields is presented [47], the problem of free and forced oscillations is solved for models with axisymmetric geometries and loads in a linear formulation by the Ritz and finite element methods.

The analysis of the studies of rotor systems with a cavity partially filled with liquid shows that only in a few works the electromagnetic properties of the liquid were taken into account [48]. Therefore the study of the problem where the effect of a magnetic field on the oscillation of an oscillatory system is evaluated becomes especially important. The solution to this problem is complicated by the fact that the motion of a rotating rotor and the motion of a weakly conductive viscous fluid in its cavity are interconnected under the action of the electromagnetic field, which causes a change in the frequency of forced oscillations and instability [49], and the system being solved includes the equations of movement of a solid body, the equations of a continuous medium and boundary conditions for the liquid [50].

2. Materials and Methods

The rotor system rotates on rolling bearings (see Figure 1). In this case, elastic deformations in the rolling bearing occur in the radial and axial directions and are non-linear [1]. The radial compliance of the bearings is caused by the deformation of the rolling elements and roller ways at the points of contact [2]. Consider a symmetrical vertical rotor of mass m , having a cylindrical cavity of radius R , and a static imbalance e . A cylinder of height h is partially filled with a weakly conductive viscous liquid. The angular velocity of the rotor (shaft) $\Omega_0 = const$ is considered sufficiently large as it is beyond its critical velocity.

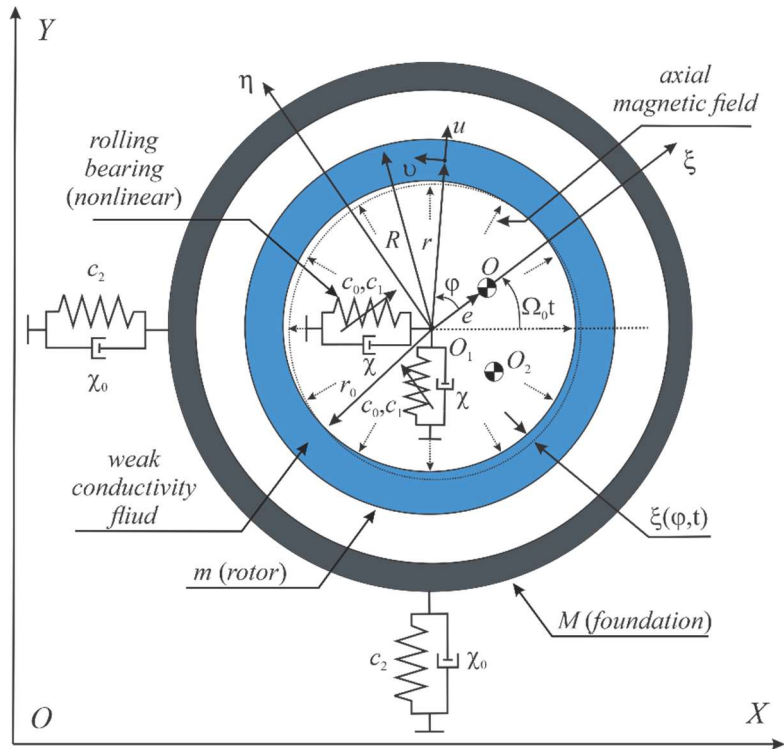


Figure 1. Scheme of a rotor partially filled with a weakly conductive liquid mounted on an elastic foundation.

The equations of the bearing static equilibrium are compiled in accordance with the Hertz theory [47-49]. The rolling bearing has a non-linear stiffness characteristic of the type:

$$F_C = c_0 \delta_r + c_1 \delta_r^3$$

where F_C is a component of the restoring force in the radial direction, δ_r is the deformation in the radial direction, c_0 and c_1 are stiffness coefficients for the linear and cubic terms.

The outer ring is rigidly connected to the foundation of mass M , which is mounted on an elastic support, with a linear stiffness coefficient c_2 . To compile the equation of motion of the system, a fixed coordinate system OXY is introduced. In an equilibrium state, the geometric center of the shaft (rotor) and the center of gravity of the foundation coincide with the origin of the fixed coordinate system [3]. The coordinates in the displaced position of the center of the shaft (rotor) O_1 are denoted by x_1 and y_1 , and the center of gravity of the rotor is denoted by x and y . The coordinates of the center of gravity of the foundation O_2 are denoted by x_2 and y_2 , c_0 and c_1 are the coefficients of rigidity of the rotor support (rigidity of the rolling bearing), χ and χ_0 are the coefficients of external friction [4-6]. It is assumed that the rotor performs a plane-parallel motion, and there is no rotation of the foundation around the coordinate axes. In this case, the equations of motion of the system are written as:

$$\left. \begin{aligned} m\ddot{x} + 2c_0(x - x_2) + 2c_1(x - x_2)^3 + \chi\dot{x} &= me\Omega_0^2 \cos \Omega_0 t + F_x \\ m\ddot{y} + 2c_0(y - y_2) + 2c_1(y - y_2)^3 + \chi\dot{y} &= me\Omega_0^2 \sin \Omega_0 t + F_y \\ m\ddot{x}_2 + 2c_2x_2 - 2c_0(x - x_2) - 2c_1(x - x_2)^3 + \chi_0\dot{x}_2 &= 0 \\ m\ddot{y}_2 + 2c_2y_2 - 2c_0(y - y_2) - 2c_1(y - y_2)^3 + \chi_0\dot{y}_2 &= 0 \end{aligned} \right\} \quad (1)$$

where F_x and F_y are components of the fluid reaction force:

$$F_x = Rh \int_0^{2\pi} \sigma_n|_{r=R} \cos(\Omega_0 t + \varphi) d\varphi \quad (2)$$

$$F_y = Rh \int_0^{2\pi} \sigma_n|_{r=R} \sin(\Omega_0 t + \varphi) d\varphi \quad (3)$$

where h - is the height of the rotor cavity, $\sigma_n|_{r=R}$ - is the normal pressure of the viscous fluid on the rotor wall.

The interaction of the azimuthal magnetic field induced in the liquid with radial and axial currents creates magnetic forces that deform the free surface of the liquid along the axis [7]. For a liquid with low conductivity, the magnetic Reynolds number is $Re_m \ll 1$ at $\sigma \ll 1$ and unchanged other scales. In this case, the induced magnetic field can be neglected in comparison with the applied external magnetic field, and the magnetic induction equation is not considered [8]. In the first approximation, the induced currents can be considered radial, and the influence of axial currents can be neglected.

Then the Lorentz force vector is written as:

$$\vec{F}_L = \sigma_F \mu_F H^2 \vec{V} \quad (4)$$

where σ_F , μ_F , H are electrical conductivity, magnetic permeability and intensity of a constant magnetic field, \vec{V} is the velocity vector of a fluid particle in a moving coordinate system.

Taking into account (4), the differential equations of motion of a weakly conducting viscous fluid in a polar coordinate system rotating together with the rotor are written as:

$$\begin{aligned} \frac{\partial u}{\partial t} - 2\Omega_0 v + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \varphi} - \frac{v^2}{r} - v \frac{\partial f}{r \partial \varphi} + b_m u &= -\frac{1}{\rho} \frac{\partial P}{\partial r} - \ddot{x} \cos(\Omega_0 t + \varphi) - \ddot{y} \sin(\Omega_0 t + \varphi) \\ \frac{\partial v}{\partial t} + 2\Omega_0 u + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \varphi} + \frac{uv}{r} + v \frac{\partial f}{\partial r} + b_m v &= -\frac{1}{\rho r} \frac{\partial P}{\partial \varphi} + \ddot{x} \sin(\Omega_0 t + \varphi) - \ddot{y} \cos(\Omega_0 t + \varphi) \end{aligned} \quad (5)$$

where P , u , v are the pressure, radial and tangential velocity components of a liquid particle in a polar coordinate system, $b_m = \frac{\sigma_F \mu_F^2 H^2}{\rho}$ is a parameter that determines the effect of a magnetic field on a

liquid particle; ρ , ν are the density and the coefficient of kinematic viscosity of the liquid; f is the Laplacian of the stream function [9-11], i.e.

$$f = \frac{1}{r} \frac{\partial u}{\partial \varphi} - \frac{\partial v}{\partial r} - \frac{v}{r} \quad (6)$$

The continuity equation for $q = \text{const}$:

$$\frac{\partial(ur)}{\partial r} + \frac{\partial v}{\partial \varphi} = 0 \quad (7)$$

The boundary conditions of the hydrodynamic problem have the following form on the wall of the rotor:

$$u|_{r=R} = 0 \quad (8)$$

$$v|_{r=R} = 0 \quad (9)$$

On the free surface of the liquid:

$$\nu \rho \left(\frac{1}{r} \frac{\partial u}{\partial \varphi} + \frac{\partial v}{\partial r} - \frac{v}{r} \right) \Big|_{r=r_0} = 0 \quad (10)$$

$$\left[-P + \frac{1}{2} \rho \Omega_0^2 (r^2 - r_0^2) + 2\nu \rho \frac{\partial u}{\partial r} \right] \Big|_{r=r_0 + \xi(\varphi, t)} = 0 \quad (11)$$

where $\xi(\varphi, t)$ - is the displacement of the free surface of a liquid from an equilibrium position:

$$\frac{\partial \xi(\varphi, t)}{\partial t} = u|_{r=r_0} = 0 \quad (12)$$

Excluding the unknown pressure P from system Eq. (5), taking into account Eq. (6) and Eq. (7), we obtain:

$$\frac{\partial f}{\partial t} + bf - \nu \Delta f = - \left(u \frac{\partial f}{\partial r} + \frac{v}{r} \frac{\partial f}{\partial \varphi} \right) \quad (13)$$

where Δ - is the Laplace operator in the polar coordinate system.

Equation (13) is solved by the method of successive approximations [12]. In the equilibrium position of the rotor and the foundation, i.e. in the absence of oscillations of the system [13], the viscous fluid rotates with the rotor as a single solid body. Then:

$$u_0 = 0, \quad v_0 = 0, \quad f_0 = 0 \quad (14)$$

Taking into account Eq. (14), equation (13) in the first approximation takes the form:

$$\frac{\partial f_1}{\partial t} + bf_1 - \nu \Delta f_1 = 0 \quad (15)$$

Further, for convenience of notation, we omit the index 1 in Eq. (15), as well as in the functions u_1, v_1, P_1 in the first approximation [14-15].

The continuity equation and the boundary conditions of the hydrodynamic problem in the first approximation are determined by Eqs. (7)-(12). Differential Eq. (1) and Eq. (15) with boundary conditions Eqs. (7)-(12) are consistent equations of motion of the rotor, foundation and weakly conductive viscous fluid [16]. To calculate the hydrodynamic force, the movement of the rotor and the foundation, it is convenient to represent them on the complex plane in the following form:

$$z = Ae^{i\Omega_0 t} + Be^{i\omega t} \quad (16)$$

$$z_2 = Ce^{i\Omega_0 t} + De^{i\omega t} \quad (17)$$

Taking Eq. (15) and Eq. (16) into account, the equations of motion of a viscous fluid Eq. (5) take the form:

$$\begin{aligned} \frac{\partial u}{\partial t} - 2\Omega_0 v + b_m u - \frac{\nu}{r} \frac{\partial f}{\partial \varphi} &= -\frac{1}{\rho} \frac{\partial P'}{\partial r} + \omega^2 B e^{i(\sigma-\varphi)} \\ \frac{\partial v}{\partial t} + 2\Omega_0 u + b_m v + \nu \frac{\partial f}{\partial r} &= -\frac{1}{\rho r} \frac{\partial P'}{\partial \varphi} - i\omega^2 B e^{i(\sigma-\varphi)} \end{aligned} \quad (18)$$

$$P' = P - A\Omega_0^2 r \rho e^{-i\varphi} \quad (19)$$

where $\sigma = \omega - \Omega_0$ - is the frequency of oscillations of the free surface of the liquid (the velocity of wave propagation on the free surface of the liquid in the forward direction), ω -- is the frequency of free oscillations (self-oscillations) of the system [17].

It is advisable to use special functions to solve equations (18) with account for Eq. (7) and boundary conditions Eqs. (8)-(12).

Taking Eq. (16), Eq. (17), Eq. (18) into account, we represent the velocity components of the liquid particle u and v , as well as the pressure P' and the function f in the form:

$$G(r, \varphi, t) = g(r) e^{i(\sigma-\varphi)} \quad (20)$$

where we consider forced oscillations of the fluid, as free fluctuations of the liquid quickly decay due to external friction and viscosity of the liquid [18].

Taking Eq. (20) into account, we can represent the function f as:

$$f(r, \varphi, t) = R(r) e^{i(\sigma-\varphi)} \quad (21)$$

After substituting Eq. (21) into Eq. (15), we will have the first-order Bessel equation with respect to $R(r)$, the solution to which is written as:

$$R(r) = z_1(\alpha r) = C_1 I_1(\alpha r) + C_2 N_1(\alpha r) \quad (22)$$

where

$$\alpha = \sqrt{-\frac{b_m + i\sigma}{\nu}} \quad (23)$$

where $I_1(\alpha r)$ and $N_1(\alpha r)$ are Bessel and Neumann functions of the first order; C_1 and C_2 are constants of integration, which are determined from the boundary conditions.

Using the continuity equation (7) from relation Eq. (6), taking into account Eq. (21) and Eq. (22), we obtain the inhomogeneous Euler equation, solving which we obtain an expression for the component u of the fluid particle velocity in the form:

$$u = \left[C_3 + \frac{C_4}{r^2} + \frac{i}{\alpha^2 r} z_1(\alpha r) \right] e^{i(\sigma-\varphi)} \quad (24)$$

Taking Eq. (24) into account, from the continuity equation, we find an expression for the component v of the fluid particle velocity:

$$u = \left[C_3 - \frac{C_4}{r^2} - \frac{i}{\alpha^2 r} z_1(\alpha r) + \frac{iz_0(\alpha r)}{\alpha} \right] e^{i(\sigma-\varphi)} \quad (25)$$

Now using Eq. (24), Eq. (25) and Eq. (20) from the second equation of system Eq. (18) we obtain an expression for P' :

$$P' = -i\rho r \left[(2\Omega_0 + \sigma - ib_m)C_3 + \frac{2\Omega_0 - \sigma + ib_m}{r^2} C_4 - \frac{(2\Omega_0 - \sigma + ib_m)\nu}{(\sigma - ib_m)r} z_1(\alpha r) - \frac{\nu}{r} z_1(\alpha r) + iB\omega^2 \right] e^{i(\alpha - \phi)} \quad (26)$$

The integration constants C_1, C_2, C_3 and C_4 are determined using the boundary conditions Eqs. (8)-(12). Taking into account Eq. (19), Eq. (25) and Eq. (26) from Eq. (2) and Eq. (3) we find the complex expression for the reaction force of a weakly conductive viscous fluid in the complex plane:

$$F_r = m_L \Omega_0^2 A e^{i\Omega_0 t} + m_L \omega^2 B \Phi e^{i\omega t} \quad (27)$$

where $F_r = F_x + iF_y$, $m_L = \pi\rho R^2 h$ - is the mass of fluid required to completely fill the rotor cavity:

$$\Phi = \frac{\sigma\nu}{\Delta_0} \left\{ f_{12} \left[\alpha I_0(\alpha R) - \frac{I_1(\alpha R)}{R} \right] - f_{11} \left[\alpha N_0(\alpha R) - \frac{N_1(\alpha R)}{R} \right] + 1 \right\} \quad (28)$$

where

$$\begin{aligned} \Delta_0 &= f_{11}f_{14} - f_{12}f_{13} \\ f_{11} &= \frac{2R^2\alpha}{r_0^2} I_0(\alpha R) - \frac{4R}{r_0^2} I_1(\alpha R) + \left(\frac{4}{r_0} + \frac{i(\sigma - ib_m)}{\nu} r_0 \right) I_1(\alpha r_0) - 2\alpha I_0(\alpha r_0) \\ f_{13} &= \frac{\nu\alpha}{2(\sigma - ib_m)} \left[\omega^2 - i\sigma b_m - q^2 \left(\Omega_0^2 - \sigma^2 + 2\Omega_0(\sigma - ib_m) + \frac{4\nu i\sigma}{r_0^2} \right) \right] I_0(\alpha R) + \\ &\quad + \frac{\nu R}{(\sigma - ib_m)r_0^2} \left(\Omega_0^2 + 2\Omega_0\sigma - \sigma^2 + i\sigma b_m + \frac{4\nu i\sigma}{r_0^2} \right) I_1(\alpha R) - \\ &\quad - \frac{\nu}{(\sigma - ib_m)r_0} \left(\Omega_0^2 + 2\Omega_0\sigma + \frac{4\nu i\sigma}{r_0^2} \right) I_1(\alpha r_0) - \frac{2\nu i\sigma}{\alpha r_0^2} I_0(\alpha r_0) \end{aligned} \quad (29)$$

Replacing the Bessel functions in the expressions for f_{11} and f_{13} by the Neumann functions of the same order, we obtain expressions for the functions f_{12} and f_{14} . The quantity $q = \frac{R}{r_0}$ characterizes the degree of filling of the rotor cavity with liquid.

Now, when the fluid reaction force F_r is known, we can determine the amplitude of forced oscillations and self-oscillations of the system [19-23].

Taking into account the identity of the first equation to the second, and the third to the fourth in system Eq. (1) we will further consider a system of the form:

$$\begin{aligned} m\ddot{x} + 2c_0(x - x_2) + 2c_1(x - x_2)^3 + \chi\dot{x} &= m\epsilon\Omega_0^2 \cos\Omega_0 t + F_x \\ M\ddot{x}_2 + 2c_2x_2 - 2c_0(x - x_2) - 2c_1(x - x_2)^3 + \chi_0\dot{x}_2 &= 0 \end{aligned} \quad (30)$$

Substituting the real parts from Eq. (16), Eq. (17) and Eq. (27) into Eq. (30), using the method of the imaging function, and making dimensionless each term of system Eq. (30), we obtain a system of nonlinear algebraic equations for unknown dimensionless amplitudes a, b, c and d . We get:

$$\begin{aligned} l_{11}a + l_{13}c &= \mu s^2 \\ l_{12}b + l_{14}d &= 0 \\ l_{13}c - (a - c) - \eta(a - c)[(a - c)^2 + 2(b - d)^2] &= 0 \\ l_{14}d - (b - d) - \eta(b - d)[(b - d)^2 + 2(a - c)^2] &= 0 \end{aligned} \quad (31)$$

where

$$l_{11} = -\mu_1 s^2 + ips; \quad l_{12} = -(\mu + \mu_L \Phi)\tau^2 + ip\tau; \quad l_{13} = (g_0 - s^2 + ip_0 s); \quad l_{14} = (g_0 - \tau^2 + ip_0 \tau).$$

$$\begin{aligned}
n_0^2 &= \frac{2c_0}{M}; \quad n_1 = \frac{2c_1}{M}; \quad n_2^2 = \frac{2c_2}{M}; \quad \mu = \frac{m}{M}; \quad \mu_L = \frac{m_L}{M}; \quad m_L = \pi\rho R^2 H; \quad \mu_1 = \mu + \mu_L; \\
p &= \frac{\chi}{Mn_0}; \quad p_0 = \frac{\chi_0}{Mn_0}; \quad g_0 = \frac{c_2}{c_0}; \quad \eta = \frac{3\mu n_1 e^2}{4n_0^2}; \quad a = \frac{A}{e}; \quad b = \frac{B}{e}; \\
c &= \frac{C}{e}; \quad d = \frac{D}{e}
\end{aligned} \tag{32}$$

where $\tau = \frac{\omega}{n_0}$ and $s = \frac{\Omega_0}{n_0}$ are dimensionless frequency of natural vibrations of the system and the angular velocity of the rotor, respectively.

From the formulas that determine the unknowns a , b , c and d , it is obvious that the amplitudes of the forced and natural oscillations of the system depend on the natural frequency of the system and they are interdependent [24]. This is one of the specific features of a nonlinear system. From formulas Eq. (16) and Eq. (17) it follows that one more frequency is superimposed on the forced oscillations of the system, i.e. the system makes oscillations consisting of the sum of two harmonic oscillations [25]. In this case, the system performs a processional motion that differs from a circular precession [26].

From the first and second equations of system Eq. (31) we have:

$$c = \beta_{13} + \beta_{11}a \tag{32a}$$

$$a - c = a(1 - \beta_{11}) - \beta_{13} \tag{32b}$$

$$d = -\frac{l_{12}}{l_{14}}b \tag{33a}$$

$$b - d = b\left(1 + \frac{l_{12}}{l_{14}}\right) \tag{33b}$$

$$b^2 = \lambda_2 a^2 + \lambda_1 a + \lambda_0 \tag{34}$$

where

$$\begin{aligned}
\beta_{11} &= -\frac{l_{11}}{l_{13}}; \quad \beta_{13} = \frac{\mu s^2}{l_{13}}; \quad \beta_{10} = -\frac{(l_{12}l_{14} + l_{12} + l_{14})}{\eta(l_{12} + l_{14})}; \quad \lambda_0 = -\frac{2l_{14}^2(1 - \beta_{11})^2}{(l_{12} + l_{14})^2}; \\
\lambda_1 &= -\frac{4l_{14}^2(1 - \beta_{11})\beta_{13}}{(l_{12} + l_{14})^2}; \quad \lambda_2 = (\beta_{10} - 2\beta_{13})\frac{l_{14}^2}{(l_{12} + l_{14})^2}.
\end{aligned}$$

Now, from the third equation of system Eq. (31), taking into account Eq. (32a), Eq. (33a), Eq. (34) we obtain an equation of the third power with respect to a :

$$a^3 + \delta_2 a^2 + \delta_1 a + \delta_0 = 0 \tag{35}$$

$$\begin{aligned}
\delta_2 &= -\frac{3\mu s^2}{\beta_{12}}; \quad \delta_1 = -\frac{l_{11}l_{13}^3 + \beta_{15}\mu s^2}{\beta_{12}\beta_{14}} + \frac{\beta_{16}}{\beta_{14}}; \quad \delta_0 = \frac{\mu s^2(l_{13}^3 - \beta_{16})}{\beta_{12}\beta_{14}}; \quad \beta_{12} = l_{11} + l_{13}; \\
\beta_{14} &= 3\eta\beta_{12}^2; \quad \beta_{15} = -6\eta\beta_{12}\mu s^2; \quad \beta_{16} = 3\eta\mu^2 s^4 - 2\eta\beta_{10}l_{13}^2 - l_{13}^2
\end{aligned} \tag{36}$$

The cubic equation (35) is solved analytically by the Cardano method and numerically by the Newton-Raphson method. The coefficients of equation (35) are, in the general case, complex (when friction forces are taken into account). If friction forces are neglected, i.e. at $p = p_0 = 0$, the coefficients of Eq. (35) become real [27]. In general, equation (35) has three real roots. Taking into account Eq. (36), from the solution of equation (35) the unknown dimensionless amplitude of forced oscillations of the rotor a is found, then from Eq. (32a), Eq. (34) and Eq. (33a) the dimensionless amplitude of forced oscillations of the foundation c is determined, as well as the dimensionless amplitudes of self-oscillations of the rotor b and the foundation d . As can be seen from Eq. (32)-(35), each value of the amplitude of the forced oscillations of the rotor a corresponds to three values of

the amplitude of the forced oscillations of the foundation c and self-oscillations of the rotor and foundation b and d .

Extreme values of the amplitude of forced oscillations depending on the angular velocity can be found from the following formulas:

$$\frac{\partial a}{\partial \Omega_0} = 0; \quad \frac{\partial b}{\partial \Omega_0} = 0; \quad \frac{\partial c}{\partial \Omega_0} = 0; \quad \frac{\partial d}{\partial \Omega_0} = 0.$$

From the solution of this equation we can find the values of the angular velocity of the rotor Ω_0 , at which the amplitudes of the forced and self-oscillations of the rotor and foundation will have maximum values [28].

From the obtained results it is obvious that, in contrast to a linear system, in a nonlinear system, the amplitudes of natural oscillations (self-oscillations) of the rotor and foundation depend on the natural oscillation frequency of the system ω .

Using smooth variations in ω at different values of Ω_0 in the interval of changes in the rotor, foundation, liquid and magnetic field parameters (rotor dimensions, rotor mass m and foundation mass M , coefficients of rigidity of rolling bearings and supports c_0, c_1, c_2 and external resistance coefficients χ, χ_0 , the extent of filling of the rotor q , liquid viscosity ν , magnetic field, etc.), it is possible to plot the dependences of the amplitude of forced oscillations and self-oscillations of the system, i.e. amplitude-frequency characteristics of the rotor and the foundation [29].

By varying the frequencies of natural oscillations of the system ω in a wide range at the most required operating modes of the rotor speeds Ω_0 (the angular speeds of rotor rotation required for the technological process) and other fixed parameters of the system using a PC, one can find the extreme values of the amplitudes of forced and self-oscillations of the rotor and the foundation (peaks of skeletal curves).

Based on the analysis of the results obtained, it is possible to find the optimal values of the system parameters, at which the amplitudes of forced oscillations and self-oscillations of the rotor and foundation will be minimal [30], i.e., so that they are significantly reduced over the entire range of the rotor system operation. In this case, the elastic foundation plays the role of a dynamic damper of rotor oscillations [31].

3. Special Cases

1) In the case of an ideal fluid, i.e. for $\nu=0$ and $b \neq 0$ from Eq. (27) and Eq. (28) we get the hydrodynamic force and the function Φ in the form:

$$F_r = F_x + iF_y = Am_L \Omega_0^2 \exp(i\Omega_0 t) + Bm_L \omega^2 \Phi \exp(i\omega t) \quad (37)$$

$$\Phi = \frac{(\sigma^2 - 2\Omega_0 \sigma - \Omega_0^2 + i\gamma b_m \sigma)}{(\gamma \sigma^2 - 2\Omega_0 \sigma - \Omega_0^2 + i b_m \sigma)} \quad (38)$$

or in the dimensionless form:

$$\frac{F_r}{eMn_0^2} = a\mu_L s^2 \exp(i\Omega_0 t) + b\mu_L \tau^2 \Phi \exp(i\omega t)$$

$$\Phi = \frac{[\tau^2 - 4\tau s + s^2 + i\gamma b_n (\tau - s)]}{[\gamma \tau^2 - 2(\gamma + 1)\tau s + (\gamma + 1)s^2 + i b_n (\tau - s)]}$$

where $\gamma = \frac{q^2 + 1}{q^2 - 1}$, $b_n = \frac{b_m}{n_0}$.

The case considered here, when the rotor cavity is partially filled with an ideal weakly conducting liquid, was chosen only for reasons of obtaining more or less simple formulas for an engineering assessment of the physical meaning of the process occurring in a nonlinear system [32].

2) For synchronous precession of the rotor when $\omega = \Omega_0$, $\sigma = 0$, we have $\Phi = 1$, $F_r = (A + B)m_L \Omega_0^2 \exp(i\Omega_0 t)$, i.e. the foundation and the centrifuge perform circular

synchronous precession. The main feature of synchronous precession is that the fluid is motionless with respect to the rotor cavity [33]. There is no wave motion of the fluid. In this case, the rotor and its foundation make forced movements caused by the unbalance of the rotor, and its cavity behaves as if it is completely filled with liquid. In this case, the magnetic field does not affect the motion of the system [34].

3) For $v = 0$, $b_m = 0$ and for $\tau \rightarrow s \left(1 + \frac{1}{\gamma} (1 \pm \sqrt{\gamma + 1}) \right)$ the fluid reaction force tends to infinity,

which will entail an unlimited increase in the amplitude of forced and natural oscillations of the rotor and its foundation [35].

4) For $v = 0$, $b_m \rightarrow \infty$ and the reaction force of the liquid will take the form $F_r = Am_L \Omega_0^2 \exp(i\Omega_0 t) + Bm_L \omega^2 \gamma \exp(i\omega t)$. In the case $\gamma = 1$ ((when the rotor cavity is completely filled with liquid) we get exactly the same picture as described in subparagraph 1. When $q \rightarrow \infty$ or $\gamma \rightarrow \infty$ (the amount of liquid in the cavity rapidly decreases), the amplitude of natural oscillations tends to infinity.

4. Free oscillations of system

Let us consider a rotor system under the assumption that the rotor is balanced, with no imbalance, and solve the problem of oscillations of the rotor and the foundation [36], when the cavity of the first is partially filled with a weakly conductive viscous liquid. Using the equations of motion of system Eq. (30) without imbalance we get their solution in the form:

$$x = b_1 \cos \omega t \quad (39)$$

$$x_2 = d_1 \cos \omega t \quad (40)$$

Substituting Eq. (39) and Eq. (40) into system Eq. (30) and using the mapping function method, we obtain:

$$l_{12}b_1 + l_{14}d_1 = 0$$

$$l_{14}d_1 - (b_1 - d_1) - \eta_0(b_1 - d_1)^3 = 0 \quad (41)$$

where, the coefficients of system Eq. (41) have the same form, but the amplitudes of natural oscillations b_1 and d_1 have a linear dimension and the parameter $\eta = \eta_0 = \frac{3\mu n_1}{4n_0^2}$.

From the first system of equation (41) we find:

$$d_1 = -\frac{l_{12}}{l_{14}}b_1 \quad (42)$$

Substituting Eq. (42) into the second equation of system Eq. (41), we obtain expressions for the amplitude of natural oscillations of the rotor and foundation:

$$b_1 = \left[-\frac{l_{14}^2(l_{14}l_{12} + l_{12} + l_{14})}{\eta_0(l_{12} + l_{14})^3} \right]^{-2} \quad (43)$$

$$B = \left[(\operatorname{Re} b_1)^2 + (\operatorname{Im} b_1)^2 \right]^{-2} \quad (44)$$

$$D = \left[(\operatorname{Re} d_1)^2 + (\operatorname{Im} d_1)^2 \right]^{-2} \quad (45)$$

From the earlier and the last formulas [37], it is obvious that the amplitudes of natural oscillations of the rotor B and foundation D depend on the natural frequency ω of the nonlinear system [38-40]. For various fixed values of the angular speed of the rotor Ω_0 , by smoothly changing the value of the natural frequency ω , it is possible to construct backbone curves, i.e. the dependence of the amplitudes b_1 and d_1 on ω and Ω_0 .

5. Results and Discussion

To evaluate the dependence of the damping and shift of frequencies of natural oscillations on the magnetic field, the operation of the rotor system with different values of parameters b_m and γ at the maximum amplitudes of the rotor system, which are observed at the main resonance, i.e. at $s = 1$, was considered (see Figures 2–11).

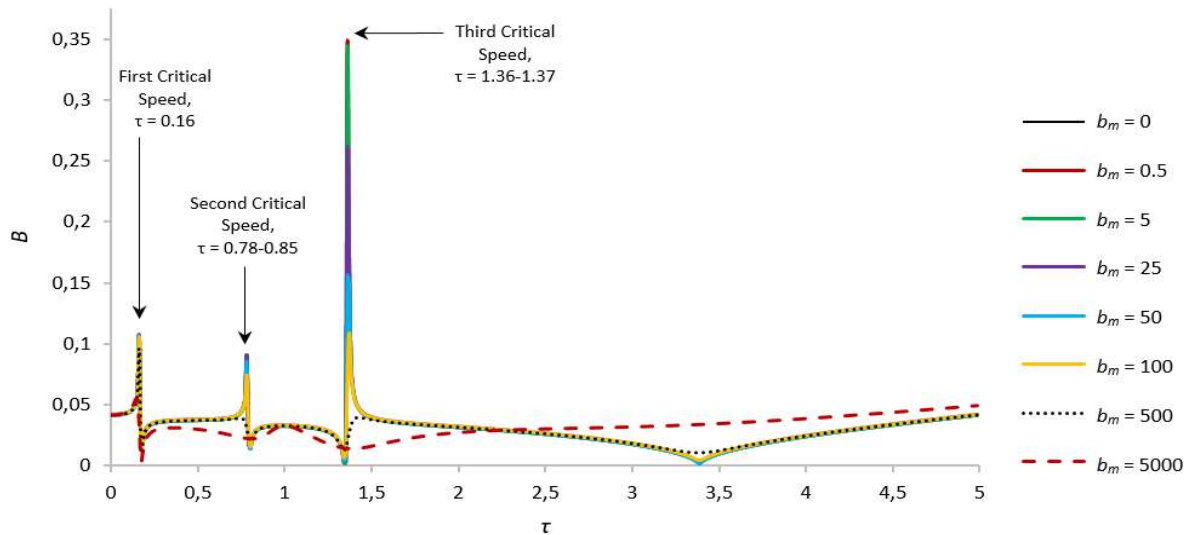


Figure 2. Amplitude-frequency characteristics of natural oscillations of the rotor for $s = 1$, $\gamma = 13.8$.

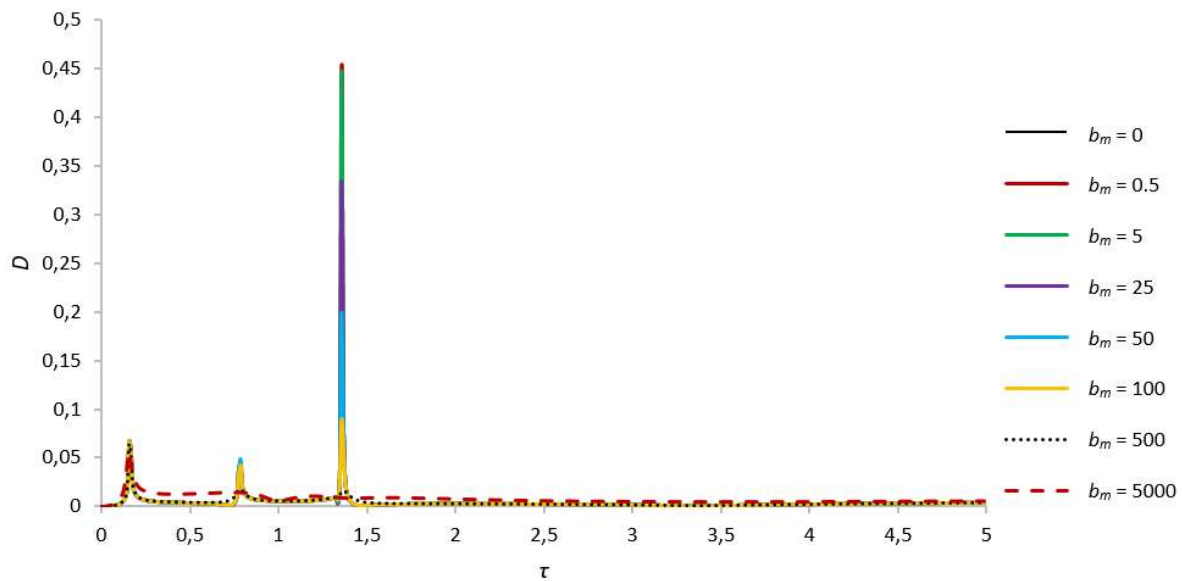


Figure 3. Amplitude-frequency characteristics of natural oscillations of the foundation for $s = 1$, $\gamma = 13.8$.

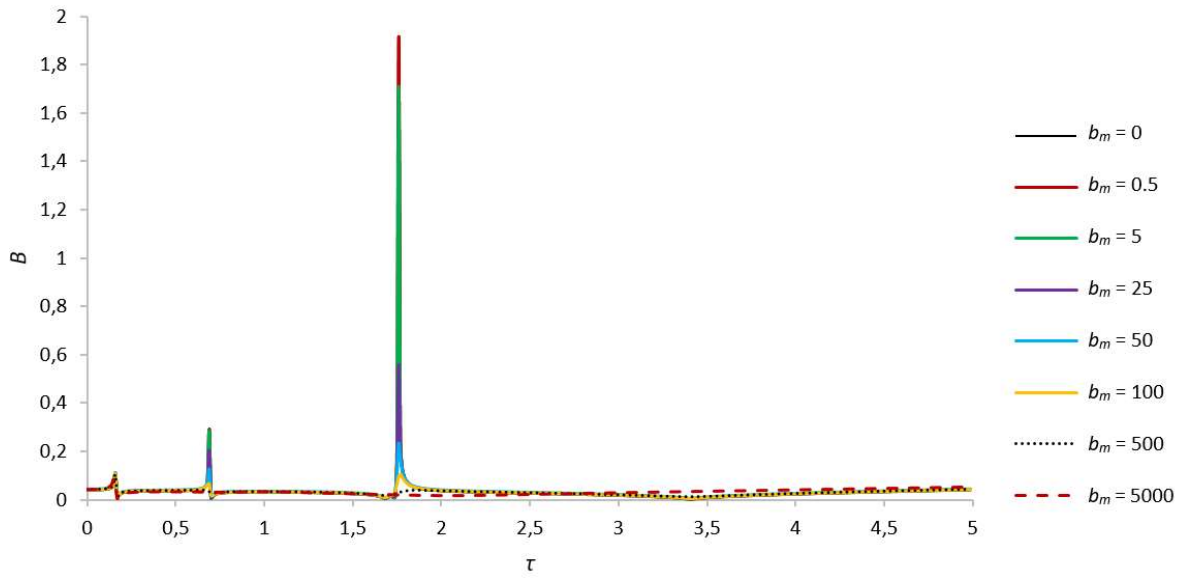


Figure 4. Amplitude-frequency characteristics of natural oscillations of the rotor for $\gamma = 4.56$, $s = 1$.

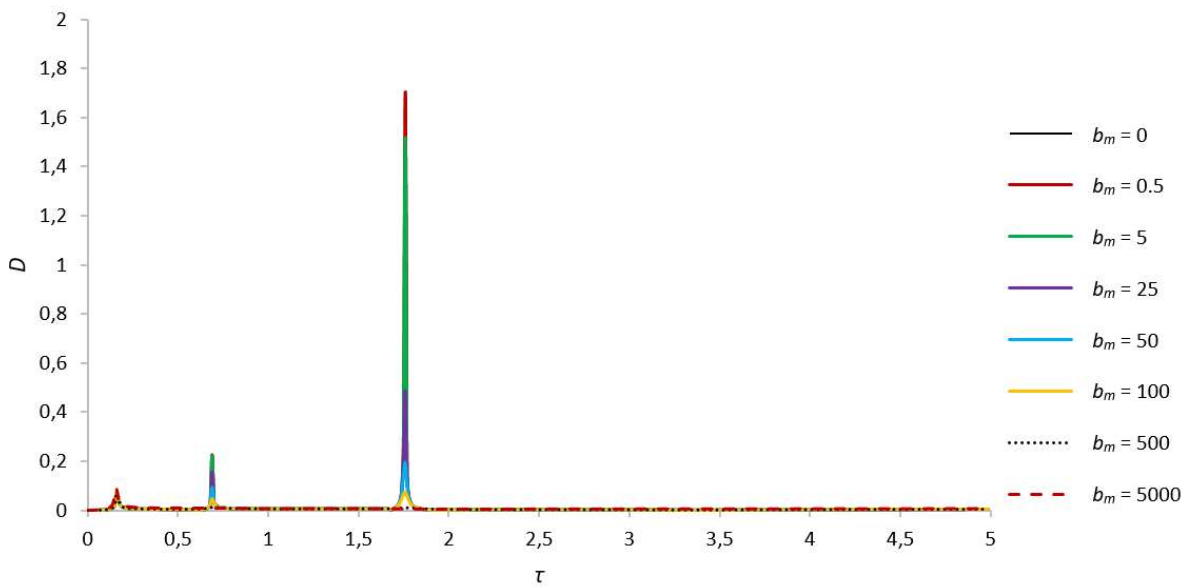


Figure 5. Amplitude-frequency characteristics of natural oscillations of the foundation for $\gamma = 4.56$, $s = 1$.

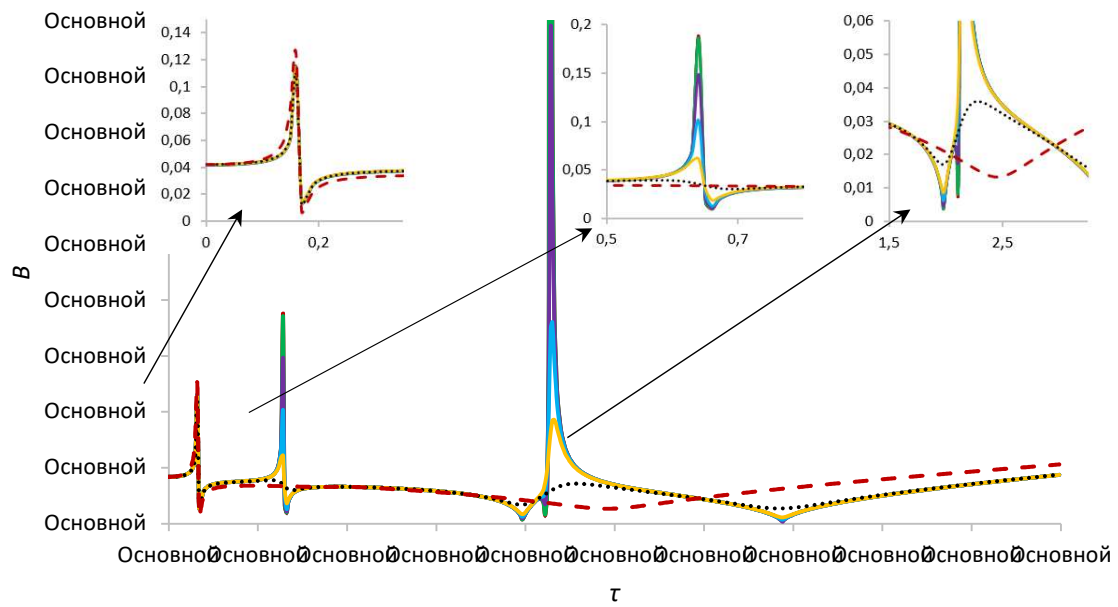
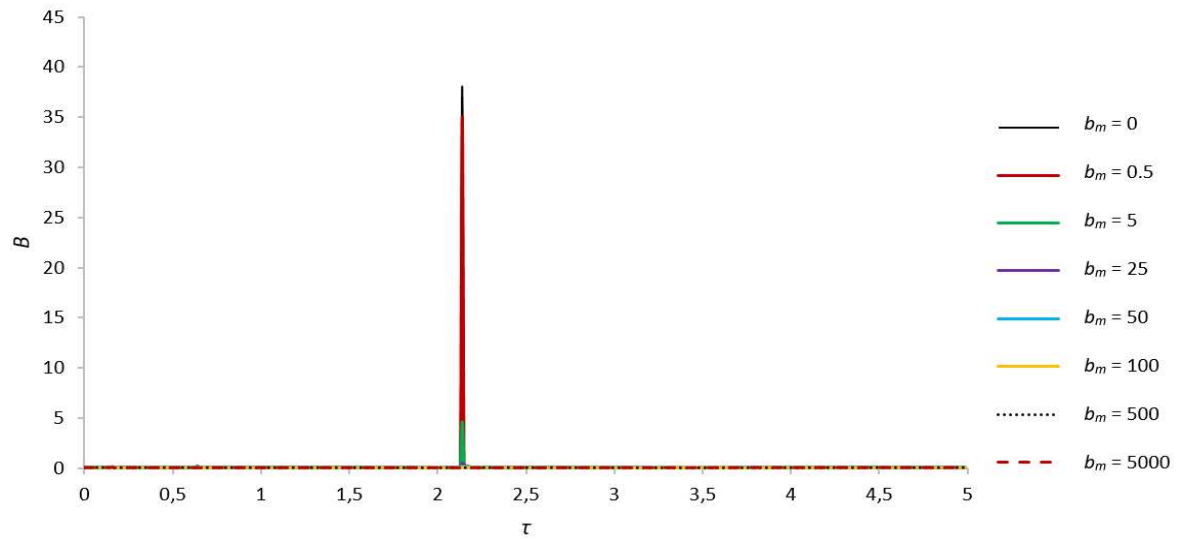
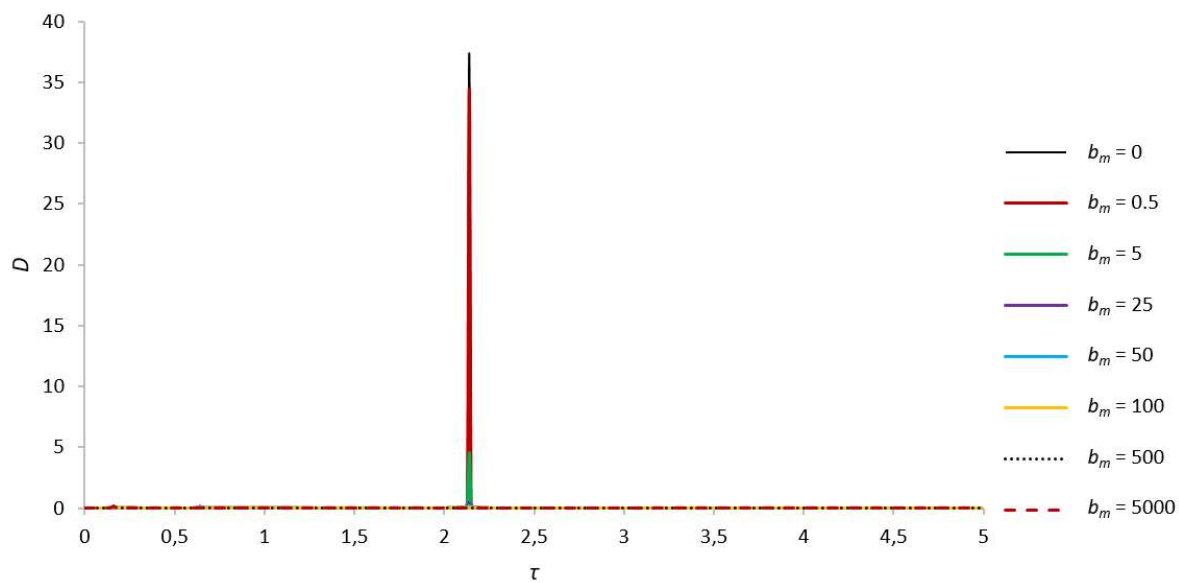
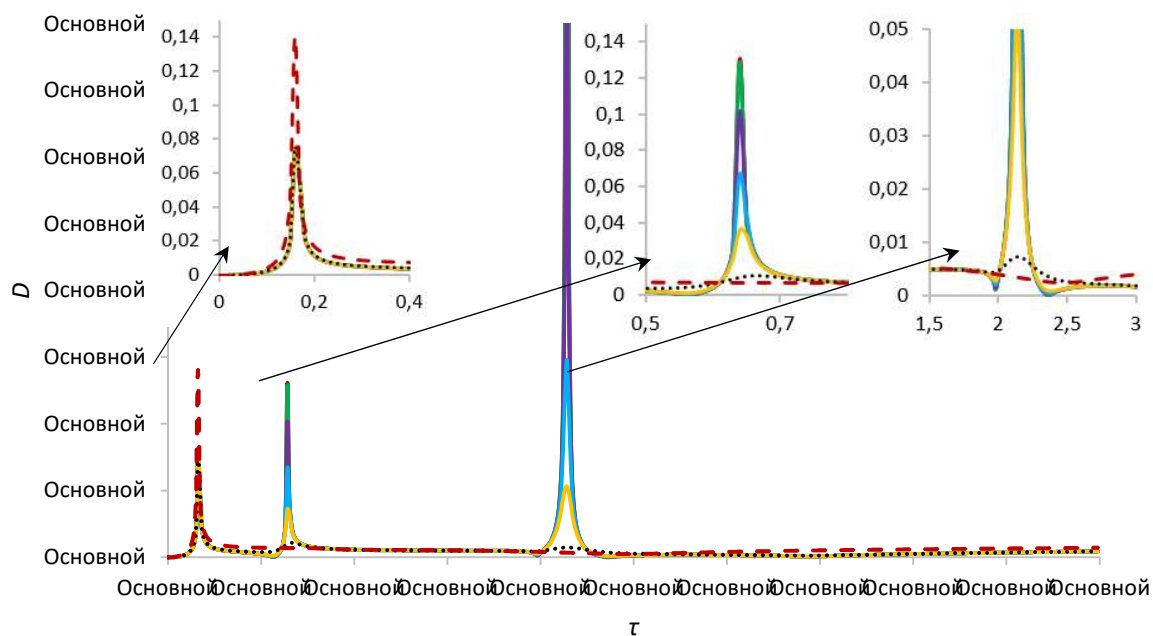


Figure 6. Amplitude-frequency characteristic of natural oscillations of the rotor for $s = 1$, $\gamma = 2.6$; a) general view, b) enlarged view.



a) General view



b) enlarged view

Figure 7. Amplitude-frequency characteristic of natural oscillations of the foundation for $s = 1$, $\gamma = 2.6$; a) general view, b) enlarged view.

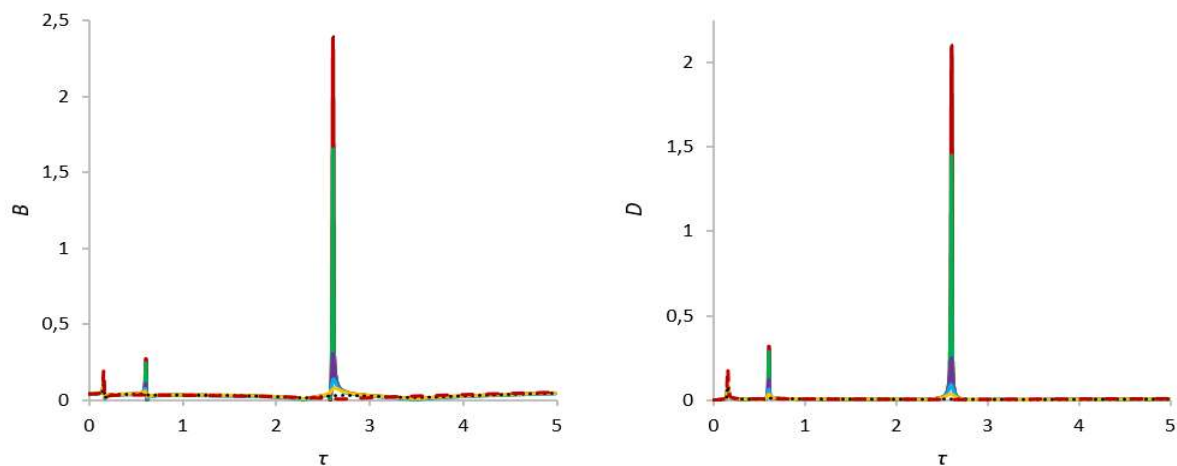


Figure 8. Amplitude-frequency characteristic of natural oscillations of the rotor and foundation for $s = 1, \gamma = 1.67$.

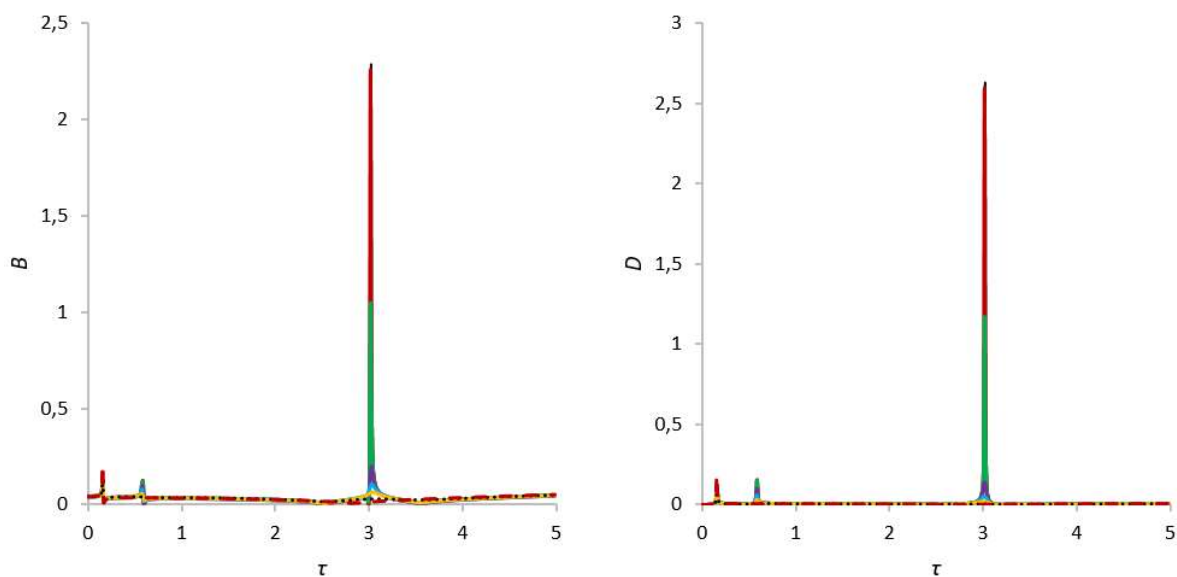


Figure 9. Amplitude-frequency characteristic of natural oscillations of the rotor and foundation for $s = 1, \gamma = 1.25$.

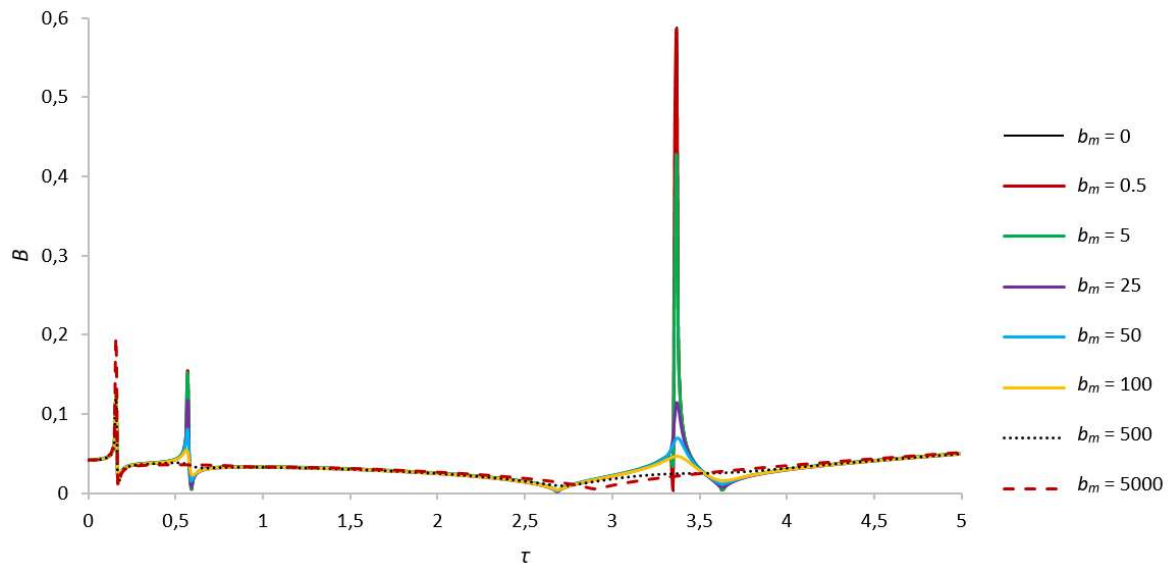


Figure 10. Amplitude-frequency characteristic of natural oscillations of the rotor for $s = 1$, $\gamma = 1.03$.

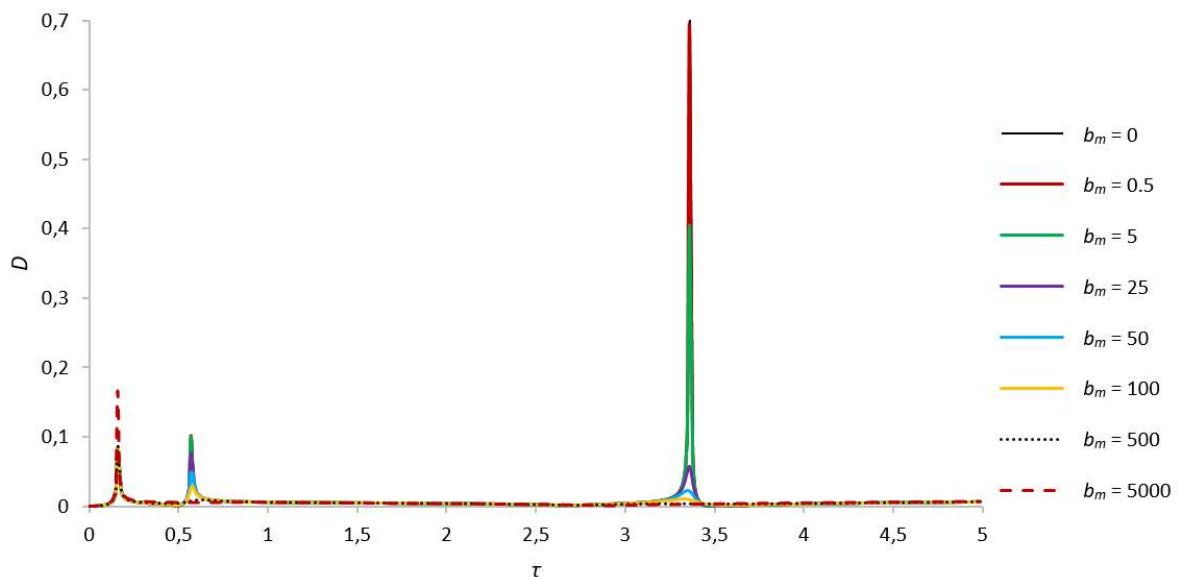


Figure 11. Amplitude-frequency characteristic of natural oscillations of the foundation for $s = 1$, $\gamma = 1.03$.

In the general case, three critical frequencies are observed in the system (see Figures 2 and 3). It means that due to the presence of liquid even in a small amount in the rotor cavity, two additional critical frequencies are superimposed with amplitudes, the values of which increase until the cavity is filled by one third [41]. A further increase in the amount of liquid in the cavity leads to the complete suppression of the second critical frequency, thus, at $\gamma = 1$ (complete filling of the cavity with liquid), two critical frequencies will be observed in the system.

An increase in the value of the parameter that characterizes the effect of a magnetic field on a liquid particle, in general, positively affects the dynamics of the system. When varying the b_m parameter, a significant influence is observed for the amplitudes at the third critical frequency [42]. With an increase in b_m , the amplitudes of the second critical frequency are damped more weakly, and the amplitudes of the first critical frequency practically do not change, as they are also present in the absence of liquid in the rotor cavity. It should be noted that at a sufficiently high magnetic field strength, i.e. at $b_m = 5000$, the oscillations of the system are practically similar to the case of an empty

rotor, as the liquid in this case, as if “solidifies” and behaves like a solid body [43]. No shift of critical frequencies is observed in all cases, except for $b_m = 5000$. For clarity and convenience of application of the results in engineering practice, the authors plotted the dependencies of critical amplitudes for different values of the parameters b_m and γ (see Figures 2 and 3).

With a small amount of liquid in the rotor cavity, for example, at $\gamma = 13.8$ ($r_0 = 0.93R$), three critical frequencies are observed at $\tau = 0.16$, $\tau = 0.78$, and $\tau = 1.36$, and three zones of self-oscillations, the maximum amplitudes of the rotor and foundation are observed at the third critical speed in cases $b_m = 0.100$. Due to the small amount of liquid, the second critical frequency has smaller amplitudes compared to the first. With an increase in the b_m parameter to 500, a slight shift of the second and third critical frequencies is observed towards an increase in the angular velocities of the rotor rotation [45]. In the case of a sufficiently high magnetic field strength, the second critical frequency shifts in the direction of increasing dimensionless frequency up to $\tau = 1$, whereas the third critical frequency of the rotor and foundation is completely damped. The numerical values of the amplitudes for this case are presented in Table 1. Dashes in the table indicate the absence of critical frequencies or the complete damping of the natural oscillation amplitudes in this interval [46].

Table 1. Critical Speeds.

$B, \gamma = 13.8$	First Critical Speed Amplitude Value	Second Critical Speed Amplitude Value	Third Critical Speed Amplitude Value
$b_m = 0$	0.1068045741, $\tau = 0.16$	0.090720226, $\tau = 0.78$	0.345717672, $\tau = 1.36$
$b_m = 0.5$	0.106796702, $\tau = 0.16$	0.090718, $\tau = 0.78$	0.345709, $\tau = 1.36$
$b_m = 5$	0.106725474, $\tau = 0.16$	0.09065, $\tau = 0.78$	0.341007, $\tau = 1.36$
$b_m = 25$	0.10640083, $\tau = 0.16$	0.089259, $\tau = 0.78$	0.256448, $\tau = 1.36$
$b_m = 50$	0.105976946, $\tau = 0.16$	0.085366, $\tau = 0.78$	0.155548, $\tau = 1.36$
$b_m = 100$	0.105071989, $\tau = 0.16$	0.073742909, $\tau = 0.78$	0.109144682, $\tau = 1.36$
$b_m = 500$	0.095853985, $\tau = 0.16$	0.039189072, $\tau = 0.79$	0.03976382, $\tau = 1.43$
$b_m = 5000$	0.054819732, $\tau = 0.15$	0.032983197, $\tau = 1$	–
$D, \gamma = 13.8$	First Critical Speed Amplitude Value	Second Critical Speed Amplitude Value	Third Critical Speed Amplitude Value
$b_m = 0$	0.066385927, $\tau = 0.16$	0.040110122, $\tau = 0.78$	0.453737506, $\tau = 1.36$
$b_m = 0.5$	0.066382727, $\tau = 0.16$	0.040109872, $\tau = 0.78$	0.45372321, $\tau = 1.36$
$b_m = 5$	0.066353881, $\tau = 0.16$	0.040087202, $\tau = 0.78$	0.447400839, $\tau = 1.36$
$b_m = 25$	0.066224805, $\tau = 0.16$	0.045069061, $\tau = 0.79$	0.334005995, $\tau = 1.36$
$b_m = 50$	0.066061617, $\tau = 0.16$	0.048328381, $\tau = 0.79$	0.198268343, $\tau = 1.36$
$b_m = 100$	0.065730061, $\tau = 0.16$	0.041947113, $\tau = 0.79$	0.088761642, $\tau = 1.36$
$b_m = 500$	0.063037959, $\tau = 0.16$	0.01798351, $\tau = 0.79$	0.016219871, $\tau = 1.37$
$b_m = 5000$	0.054819732, $\tau = 0.16$	0.014450226, $\tau = 0.85$	–

As the extent of filling increases, as mentioned above, the amplitudes of the second and third critical frequencies increase (see Figures 4 and 5). For example, at $\gamma = 4.56$ ($r_0 = 0.8R$) and $b_m = 0$, the maximum values of the rotor amplitudes at the second and third critical frequencies are 3.3 times and 5.4 times greater than the values for the same case at $\gamma = 13.8$, and are 2.64 times and 17.45 times greater than the amplitudes at the first critical frequency, i.e. at $\tau = 0.16$. Moreover, an increase in the amount of liquid in the rotor cavity affects the displacement of the second and third critical frequencies [47]. For example, in this case, the second critical frequency occurs a little earlier, compared with the case of $\gamma = 13.8$, at $\tau = 0.69$, whereas the third critical frequency appears a little later, at $\tau = 1.76$. With an increase in the parameter b_m , for example, at $b_m = 500$ and $b_m = 5000$, the oscillation amplitudes at the second and third critical speeds of the rotor and foundation, as expected, are damped up to their complete damping [48], which also indicates a positive effect of the high-frequency magnetic field on the system. The numerical values of the amplitudes for this case are presented in Table 2.

Table 2. Critical Speeds.

$B, \gamma = 4.56$	First Critical Speed	Critical Speed	Second Critical Speed	Critical Speed	Third Critical Speed
	Amplitude Value	Amplitude Value	Amplitude Value	Amplitude Value	Amplitude Value
$b_m = 0$	0.110805396, $\tau = 0.16$	0.292896271, $\tau = 0.69$	1.917205238, $\tau = 1.76$		
$b_m = 0.5$	0.110797785, $\tau = 0.16$	0.292762234, $\tau = 0.69$	1.915895568, $\tau = 1.76$		
$b_m = 5$	0.110729214, $\tau = 0.16$	0.286890823, $\tau = 0.69$	1.710435466, $\tau = 1.76$		
$b_m = 25$	0.110422857, $\tau = 0.16$	0.205546804, $\tau = 0.69$	0.558152699, $\tau = 1.76$		
$b_m = 50$	0.110036396, $\tau = 0.16$	0.124228936, $\tau = 0.69$	0.231023687, $\tau = 1.76$		
$b_m = 100$	0.10925273, $\tau = 0.16$	0.066849372, $\tau = 0.69$	0.104944945, $\tau = 1.76$		
$b_m = 500$	0.102684806, $\tau = 0.16$	–	–		
$b_m = 5000$	0.08448214, $\tau = 0.15$	–	–		
$D, \gamma = 4.56$	First Critical Speed	Critical Speed	Second Critical Speed	Critical Speed	Third Critical Speed
	Amplitude Value	Amplitude Value	Amplitude Value	Amplitude Value	Amplitude Value
$b_m = 0$	0.07047227, $\tau = 0.16$	0.226388549, $\tau = 0.69$	1.704994351, $\tau = 1.76$		
$b_m = 0.5$	0.070468917, $\tau = 0.16$	0.226287308, $\tau = 0.69$	1.70381006, $\tau = 1.76$		
$b_m = 5$	0.07043893, $\tau = 0.16$	0.221699183, $\tau = 0.69$	1.520112352, $\tau = 1.76$		
$b_m = 25$	0.070309814, $\tau = 0.16$	0.157597686, $\tau = 0.69$	0.488944743, $\tau = 1.76$		
$b_m = 50$	0.070157935, $\tau = 0.16$	0.034896767, $\tau = 0.7$	0.194103909, $\tau = 1.76$		
$b_m = 100$	0.069885617, $\tau = 0.16$	0.045941501, $\tau = 0.69$	0.072065559, $\tau = 1.76$		
$b_m = 500$	0.069093807, $\tau = 0.16$	0.012824952, $\tau = 0.7$	0.010085603, $\tau = 1.77$		
$b_m = 5000$	0.089252733, $\tau = 0.16$	–	–		

When the rotor cavity is filled with liquid by one third, i.e. at $\gamma = 2.6$, the system still has three critical frequencies ($\tau = 0.16$, $\tau = 0.64$ and $\tau = 2.14$, see Figures 6 and 7). In this case, the maximum values of the rotor amplitudes at the third critical frequency are 19.8 times greater than the amplitudes in the similar case at $\gamma = 4.56$ ($b_m = 0$), whereas the amplitudes of the second critical frequency are comparable with the previous case. An increase in the parameter that determines the influence of the magnetic field on a fluid particle from 0 to 0.5 leads to damping of the rotor and foundation amplitudes corresponding to the third critical frequency [49]. A significant decrease in the amplitudes of the second critical frequency occurs with an increase in b_m to 25. Further, as b_m increases to 500, the amplitudes of the second and third critical frequencies are almost completely suppressed. A change in the parameter that determines the influence of the magnetic field on a liquid particle has practically no effect on the amplitudes of the first critical frequency, except for large values of the magnetic field strength. It should be noted that in this case the third critical frequency shifts quite strongly to the right, in the direction of increasing natural frequencies, from 1.76 to 2.14, which also imposes certain restrictions on the choice of operating speed range in the absence of magnetic field influence [50]. The numerical values of the amplitudes for this case are presented in Table 3.

Table 3. Critical Speeds.

$B, \gamma = 2.6$	First Critical Speed	Critical Speed	Second Critical Speed	Critical Speed	Third Critical Speed
	Amplitude Value	Amplitude Value	Amplitude Value	Amplitude Value	Amplitude Value
$b_m = 0$		0.188180917, $\tau = 0.64$	38.0983794, $\tau = 2.14$		
$b_m = 0.5$		0.188128512, $\tau = 0.64$	35.05368501, $\tau = 2.14$		
$b_m = 5$	0.114550189, $\tau = 0.16$	0.185743424, $\tau = 0.64$	4.620198303, $\tau = 2.14$		
$b_m = 25$	0.114543039, $\tau = 0.16$	0.148131171, $\tau = 0.64$	0.442357104, $\tau = 2.14$		
$b_m = 50$	0.113852547, $\tau = 0.16$	0.100918717, $\tau = 0.64$	0.180448011, $\tau = 2.15$		
$b_m = 100$	0.113189756, $\tau = 0.16$	0.06088884, $\tau = 0.64$	0.093096022, $\tau = 2.16$		
$b_m = 500$	0.109089699, $\tau = 0.16$	–	–		
$b_m = 5000$	0.124710542, $\tau = 0.16$	–	–		
$D, \gamma = 2.6$	First Critical Speed	Critical Speed	Second Critical Speed	Critical Speed	Third Critical Speed
	Amplitude Value	Amplitude Value	Amplitude Value	Amplitude Value	Amplitude Value

$b_m = 0$	0.074299992, $\tau = 0.16$	0.130906778, $\tau = 0.64$	37.42981166, $\tau = 2.14$
$b_m = 0.5$	0.074296638, $\tau = 0.16$	0.130872119, $\tau = 0.64$	34.43775859, $\tau = 2.14$
$b_m = 5$	0.074266809, $\tau = 0.16$	0.129169584, $\tau = 0.64$	4.531230535, $\tau = 2.14$
$b_m = 25$	0.074142119, $\tau = 0.16$	0.101854993, $\tau = 0.64$	0.417323892, $\tau = 2.14$
$b_m = 50$	0.074004213, $\tau = 0.16$	0.067028512, $\tau = 0.64$	0.148186179, $\tau = 2.14$
$b_m = 100$	0.073787317, $\tau = 0.16$	0.036021818, $\tau = 0.64$	0.053425977, $\tau = 2.14$
$b_m = 500$	0.074595252, $\tau = 0.16$	0.01092513, $\tau = 0.66$	0.007393946, $\tau = 2.14$
$b_m = 5000$	0.140446071, $\tau = 0.16$	–	–

When the rotor cavity is half and two-thirds full, i.e. at $\gamma = 1.67$ and $\gamma = 1.25$, the character of changes in the amplitude and frequency of natural oscillations of the system is similar to the case when the rotor cavity is one-third filled. In this case, the system oscillates with smaller amplitudes by almost an order of magnitude lower than when the rotor cavity is one-third full (see Figure 8 and Figure 9). In the case of half filling, the second and third critical frequencies are still greater than the first critical frequency [51]. Accordingly, a stronger shift of critical frequencies is observed compared to those listed above, for example, the second critical frequency appears already at $\tau = 0.61$, while the third already at $\tau = 2.61$.

When the cavity is filled by two-thirds, the amplitudes of the first and second critical frequencies are commensurate with each other and with the above case. The second critical frequency shifts in the direction of decreasing angular velocities and appears at $\tau = 0.59$, whereas the third critical frequency shifts to the left in the direction of its increase and appears at $\tau = 3.02$. With an increase in the parameter b_m , the oscillation amplitudes at the second and third critical speeds of the rotor and foundation, as before, are damped until their full damping. The numerical values of the amplitudes for this case are presented in Tables 4 and Table 5.

Table 4. Critical Speeds.

$B, \gamma = 1.67$	First Critical Speed Amplitude Value	Second Critical Speed Amplitude Value	Third Critical Speed Amplitude Value
$b_m = 0$	0.118574461, $\tau = 0.16$	0.27197385, $\tau = 0.61$	2.395991343, $\tau = 2.61$
$b_m = 0.5$	0.11856806, $\tau = 0.16$	0.271326026, $\tau = 0.61$	2.386092738, $\tau = 2.61$
$b_m = 5$	0.118510785, $\tau = 0.16$	0.244571454, $\tau = 0.61$	1.657566825, $\tau = 2.61$
$b_m = 25$	0.118263621, $\tau = 0.16$	0.108761599, $\tau = 0.61$	0.307975554, $\tau = 2.61$
$b_m = 50$	0.117971542, $\tau = 0.16$	0.079987206, $\tau = 0.61$	0.141437807, $\tau = 2.61$
$b_m = 100$	0.117443017, $\tau = 0.16$	0.058212949, $\tau = 0.6$	0.078056973, $\tau = 2.61$
$b_m = 500$	0.115713077, $\tau = 0.16$	–	–
$b_m = 5000$	0.200140464, $\tau = 0.16$	–	–
$D, \gamma = 1.67$	First Critical Speed Amplitude Value	Second Critical Speed Amplitude Value	Third Critical Speed Amplitude Value
$b_m = 0$	0.078416443, $\tau = 0.16$	0.319661082, $\tau = 0.61$	2.101480075, $\tau = 2.61$
$b_m = 0.5$	0.078413261, $\tau = 0.16$	0.318899815, $\tau = 0.61$	2.092707266, $\tau = 2.61$
$b_m = 5$	0.078385116, $\tau = 0.16$	0.286969287, $\tau = 0.61$	1.448573522, $\tau = 2.61$
$b_m = 25$	0.078270549, $\tau = 0.16$	0.122332756, $\tau = 0.61$	0.248941102, $\tau = 2.61$
$b_m = 50$	0.078151336, $\tau = 0.16$	0.064906783, $\tau = 0.61$	0.091711735, $\tau = 2.61$
$b_m = 100$	0.077991713, $\tau = 0.16$	0.03343016, $\tau = 0.6$	0.034567932, $\tau = 2.61$
$b_m = 500$	0.080131667, $\tau = 0.16$	–	–
$b_m = 5000$	0.193818388, $\tau = 0.16$	–	–

Table 5. Critical Speeds.

$B, \gamma = 1.25$	First Critical Speed Amplitude Value	Second Critical Speed Amplitude Value	Third Critical Speed Amplitude Value
$b_m = 0$	0.121638282, $\tau = 0.16$	0.122951082, $\tau = 0.59$	2.281429046, $\tau = 3.02$

$b_m = 0.5$	0.121632639, $\tau = 0.16$	0.122901957, $\tau = 0.59$	2.25145796, $\tau = 3.02$
$b_m = 5$	0.121582276, $\tau = 0.16$	0.121641647, $\tau = 0.59$	1.035804071, $\tau = 3.02$
$b_m = 25$	0.121367604, $\tau = 0.16$	0.098499341, $\tau = 0.58$	0.166361948, $\tau = 3.02$
$b_m = 50$	0.121120227, $\tau = 0.16$	0.078839058, $\tau = 0.58$	0.103151902, $\tau = 3.02$
$b_m = 100$	0.120694759, $\tau = 0.16$	0.055532351, $\tau = 0.58$	0.06308127, $\tau = 3.02$
$b_m = 500$	0.120406103, $\tau = 0.16$	–	–
$b_m = 5000$	0.210144507, $\tau = 0.16$	–	–
$D, \gamma = 1.25$	First Critical Speed	Second Critical Speed	Third Critical Speed
	Amplitude Value	Amplitude Value	Amplitude Value
$b_m = 0$	0.081552542, $\tau = 0.16$	0.154403646, $\tau = 0.59$	2.6271788, $\tau = 3.02$
$b_m = 0.5$	0.081549629, $\tau = 0.16$	0.154341266, $\tau = 0.59$	2.59220114, $\tau = 3.02$
$b_m = 5$	0.081523936, $\tau = 0.16$	0.152395469, $\tau = 0.59$	1.173065248, $\tau = 3.02$
$b_m = 25$	0.081421162, $\tau = 0.16$	0.098951316, $\tau = 0.58$	0.14148849, $\tau = 3.02$
$b_m = 50$	0.081318754, $\tau = 0.16$	0.03067025, $\tau = 0.59$	0.051515196, $\tau = 3.02$
$b_m = 100$	0.081199692, $\tau = 0.16$	0.055532351, $\tau = 0.59$	0.020073614, $\tau = 3.02$
$b_m = 500$	0.083967196, $\tau = 0.16$	–	–
$b_m = 5000$	0.186578729, $\tau = 0.16$	–	–

When the rotor cavity is almost completely filled with liquid, for example, at $\gamma = 1.03$ ($r_0 = 0.125R$), the amplitudes at the second critical frequency are smaller than the amplitudes at the first critical frequency, whereas the amplitudes at the third critical frequency are several times smaller than the values observed in the case when the cavity is filled with liquid by half and by two thirds (see Figure 10 and Figure 11). The shift of critical frequencies due to an increase in the amount of liquid in the cavity in this case is even more pronounced. For example, the second critical frequency in this case appears already at $\tau = 0.57$ and the third already at $\tau = 3.36$. It means that with an increase in filling, the second critical frequency shifts to the first critical frequency until it merges with it, whereas the third critical frequency shifts in the direction of increasing frequency to the right and is almost completely damped with an increase in the amount of fluid in the rotor cavity. As before, with an increase in the parameter characterizing the influence of the magnetic field on a liquid particle b_m , the amplitudes of the second and third critical frequencies are damped up to their complete dampening. The numerical values of the amplitudes for this case are presented in Table 6.

Table 6. Critical Speeds.

$B, \gamma = 1.03$	First Critical Speed	Second Critical Speed	Third Critical Speed
	Amplitude Value	Amplitude Value	Amplitude Value
$b_m = 0$	0.123836133, $\tau = 0.16$	0.154082698, $\tau = 0.57$	0.582653298, $\tau = 3.37$
$b_m = 0.5$	0.123831142, $\tau = 0.16$	0.15403417, $\tau = 0.57$	0.580389919, $\tau = 3.37$
$b_m = 5$	0.123786664, $\tau = 0.16$	0.151690752, $\tau = 0.57$	0.425847453, $\tau = 3.37$
$b_m = 25$	0.123598669, $\tau = 0.16$	0.117230408, $\tau = 0.57$	0.114353556, $\tau = 3.37$
$b_m = 50$	0.123385864, $\tau = 0.16$	0.080130589, $\tau = 0.57$	0.069492128, $\tau = 3.38$
$b_m = 100$	0.123033788, $\tau = 0.16$	0.054080999, $\tau = 0.56$	0.046679793, $\tau = 3.37$
$b_m = 500$	0.123518243, $\tau = 0.16$	–	–
$b_m = 5000$	0.196425395, $\tau = 0.16$	–	–
$D, \gamma = 1.03$	First Critical Speed	Second Critical Speed	Third Critical Speed
	Amplitude Value	Amplitude Value	Amplitude Value
$b_m = 0$	0.083803356, $\tau = 0.16$	0.101565041, $\tau = 0.57$	0.124078552, $\tau = 3.35$
$b_m = 0.5$	0.083800714, $\tau = 0.16$	0.101535013, $\tau = 0.57$	0.123990942, $\tau = 3.35$
$b_m = 5$	0.08377746, $\tau = 0.16$	0.099920685, $\tau = 0.57$	0.115971317, $\tau = 3.35$
$b_m = 25$	0.083685484, $\tau = 0.16$	0.075582465, $\tau = 0.57$	0.050630183, $\tau = 3.35$
$b_m = 50$	0.08359651, $\tau = 0.16$	0.048524802, $\tau = 0.57$	0.022746045, $\tau = 3.35$
$b_m = 100$	0.083504258, $\tau = 0.16$	0.028624914, $\tau = 0.58$	0.010481636, $\tau = 3.33$

$b_m = 500$	0.08647026, $\tau = 0.16$	0.009075681, $\tau = 0.62$	-
$b_m = 5000$	0.165670988, $\tau = 0.16$	-	-

The electrical conductivity of the liquid is directly proportional to the length of the liquid volume in the rotor cavity and inversely proportional to the cross-sectional area of this volume and its electrical resistance. The intensity of a uniform axial magnetic field is proportional to the current strength, length and number of turns of the solenoid. Thus, due to sufficient damping of the amplitudes of the second and third critical frequencies, as well as due to its simpler implementation from the technical point of view, for further studying of the effect of a high-frequency magnetic field on nonlinear oscillations it was decided to use the value of the parameter $b_m = 100$. The shift of critical frequencies depends not only on the amount of liquid being filled but also on the frequency of the disturbing force, in our case, on the angular velocity of the rotor. To assess the shift of critical frequencies, the diagrams were constructed where the abscissa axes correspond to the dimensionless angular velocity of rotation of the rotor - s and the ordinate axes - to the dimensionless natural frequency τ .

6. Conclusion

The analysis of sources in this field of research shows that this work is the first where an analytically generalized dynamic model of the “rotor-weakly conductive fluid-foundation” system, which takes into account the electromagnetic properties of a viscous fluid and its vibrations, the nonlinear stiffness properties of bearing supports and vibrations of the foundation, was developed and solved. Due to the presence of liquid in the rotor cavity, several additional critical frequencies are imposed on the system, whose amplitudes in some cases exceed the amplitudes of the empty rotor by several orders of magnitude, which also imposes certain restrictions on the choice of operating speeds. An increase in the value of the parameter that characterizes the influence of the magnetic field on a fluid particle leads to damping of the amplitudes of the natural oscillations of the rotor and the foundation, which generally has a positive effect on the dynamics of the system. In this case, a change in the value of the magnetic field strength has practically no effect on the shifts of critical frequencies. In case of synchronous precession, the fluid is motionless with respect to the rotor cavity. There is no wave motion of the liquid. The motion caused by the rotor unbalance is similar to the case when the cavity is completely filled with liquid.

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