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Application of Fuzzy Simple Additive Weighting Method in Group Decision Making for Capital Investment

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Abstract. Investment management is a common process and practice used for achieving a desirable investment goal or outcome. Investment assessment should be carried out at various stages of project realization in accordance with capital investment volume. Investment risk management implies the effective control of all procedures and monitoring of risks in all phases of the investment project. Because of the reason that a single indicator in probability calculation of achieving optimal return from the investment does not exist, performing sufficiently reliable estimates of the quality of investments becomes a tedious task. There are many indicators, factors, and criteria required for consideration to reach the effective solution of the investment problem. Unfortunately, the systematic variation of economic situations in the marketplace stipulates the continuous and frequent changes of investment conditions and environment in which the investor should act and operate. Hence, the rules required for providing a reasonable quality of investment projects can be based only on investor's management strategy and rely on investor's intuition and practice. The importance of classification in investment management and decision making process is undeniable. The objects to be classified are described using assessments in accordance with various criteria which can be both quantitative and qualitative. With a competent formulation of the investment process, both methods are used in parallel. There exist various decision making approaches for the investment management, and simple additive weighting (SAW) is one of the well-known multi-criteria decision making (MCDM) methods aiming to provide an optimal decision for decision maker when solving various real-life problems, and particularly, investment problem. In this paper, fuzzy simple additive weighting (FSAW) method in group decision making is applied to undertake the capital investment expenditure for purchasing cars with the purpose of renting them to the public. The development of existing FSAW method is accomplished and this process involves the sensibility of outcomes to changes in the rate of fuzziness represented with decisions taken. Eventually, the degree of fuzziness involved in an analysis that directly attempts to model the immanent vagueness and imperfectness in particular precedency judgments made, is determined. A numerical example illustrates the importance and effectiveness of the suggested approach with the aim of ranking alternatives and hence, determining the most preferred alternative in MCDM problem.

Keywords: investment management; multi-criteria decision making (MCDM); fuzzy number; fuzzy simple additive weighting; fuzzy group decision making

1. Introduction

A holistic theory of investment management in business and economics does not yet exist. However, studying the experience of entrepreneurship in various countries, the first theoretical developments in risk assessment and management help us outline different ways to solve investment problems. Currently, there is enough information present to solve problems associated with possible manifestations of investment risk [1,2,3]. The determination of sufficient and reliable quantitative and qualitative assessments of the effectiveness of investments is a complex and difficult task to perform.

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In most cases, every investment project has several performance indicators to be considered in the process of selecting the best objects among the set of available ones.

In decision making process, the role of risk assessment becomes incredibly significant. In solving the problems of multi-purpose selection of efficient resource-saving investment decisions from a certain set of possible options, various decision making methods are used (TOPSIS, SAW, COPRAS, AHP, etc.). The information that an investor or project manager (decision maker) possesses when solving problems of preparing a construction investment project and performing construction processes, is distinguished by its structure and level of certainty. When solving most problems, cardinal (quantitative) information is used. However, in practice, there are tasks that require information of the ordinal (qualitative) nature or information of both types simultaneously. The practical tasks of a construction investment project are solved in the presence or absence of information about the significance of performance indicators. The main problem that decision makers face in situations which are difficult to analyze is the presence of information handled by fuzzy sets [4,5,6,7,8]. The decision makers hereby deal with situations that might be uncertain and vague in nature. The uncertainty may include imperfect information, insufficient understanding, and undifferentiated alternatives [9,10,11,12]. In the case of investments, the level of vagueness gets higher due to the difficulty of assessing the impact of unexpected changes in opinions of public relations (PR). The focus is on obtaining subjective judgments from the decision makers which may be uncertain or imperfect, to choose the best option from a set of accessible alternatives. FSAW method is based on the weighted average, also known as weighted linear combination [13,14,15]. The basic principle of SAW method in group decision making is getting a weighted sum of the performance ratings for each alternative under all criteria and opinions of decision makers.

In this paper, FSAW method is considered that enables determining the investment risk by choosing the most effective capital investment option in terms of purchasing cars with the purpose of hiring them to the public. Investment efficiency is carried out in respect to ranking the available alternatives from the most preferred to the least preferred.

The maintenance content introduced in this paper is structured as follows. Section 2 presents the preliminaries required to understand the main steps of FSAW method in group decision making. In section 3, we consider and solve the problem for capital investment to illustrate the efficiency of the suggested approach, and conclusions are given in section 4.

2. Preliminaries

Definition 1. *If the priority and weight of each expert are emphasized, then the fuzzy weights* $\tilde{\omega}_t,\ t=1,2,\cdots,k$ *of experts are appointed consequently to the importance defined by interviewing the final expert. Eventually, the rate of importance* I_t *is determined as follows:*

$$I_t=\frac{d\left(\tilde{\omega}_t\right)}{\sum_{t=1}^{k} d\left(\tilde{\omega}_t\right)},\ t=1,2,\cdots,k \qquad (1)$$

where $d\left(\tilde{\omega}_t\right)$ is the defuzzified value of the fuzzy weight by applying the oriented distance.

Definition 2. *Let* $\tilde{W}_{jt}=\left(a_{jt},b_{jt},c_{jt},d_{jt}\right),\ j=1,2,\cdots,n,\ t=1,2,\cdots,k$ *be the linguistic weight of subjective criteria -* $C_1,C_2,\cdots,C_h$*, and objective criteria -* $C_{h+1},C_{h+2},\cdots,C_n$ *provided by experts* D_t*. The aggregated fuzzy criteria weight* $\tilde{W}_j=\left(a_j,b_j,c_j,d_j\right),\ j=1,2,\cdots,n$ *of criteria* C_j *assessed by group of k decision makers is determined in the following form:*

$$\tilde{W}_j=\left(I_1\otimes\tilde{W}_{j1}\right)\oplus\left(I_2\otimes\tilde{W}_{j2}\right)\oplus\cdots\oplus\left(I_k\otimes\tilde{W}_{jk}\right) \qquad (2)$$

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where $a_j = \sum_{t=1}^{k} I_t a_{jt}$, $b_j = \sum_{t=1}^{k} I_t b_{jt}$, $c_j = \sum_{t=1}^{k} I_t c_{jt}$, $d_j = \sum_{t=1}^{k} I_t d_{jt}$.

Definition 3. *The fuzzy rating matrix $\tilde{M}$ can be represented as follows:*

$$\tilde{M} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \quad (3)$$

where $\tilde{x}_{ij}, \forall i, j$ is the aggregated fuzzy rating of alternative $A_i, i = 1, 2, \cdots, m$ with appropriate criteria C_j.

Definition 4. *Weighted fuzzy matrix is determined by multiplying the fuzzy rating matrix $\tilde{M}$ by the weight vector W, i.e.,*

$$\tilde{F} = \tilde{M} \otimes W = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} \otimes \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} = \begin{bmatrix} \tilde{x}_{11} \otimes W_1 & \tilde{x}_{12} \otimes W_2 & \cdots & \tilde{x}_{1n} \otimes W_n \\ \tilde{x}_{21} \otimes W_1 & \tilde{x}_{22} \otimes W_2 & \cdots & \tilde{x}_{2n} \otimes W_n \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} \otimes W_1 & \tilde{x}_{m2} \otimes W_2 & \cdots & \tilde{x}_{mn} \otimes W_n \end{bmatrix} == \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} = \left[\tilde{f}_i \right]_{m*1} \quad (4)$$

where $\tilde{f}_i = (r_i, s_i, t_i, u_i), i = 1, 2, \cdots, m$.

3. Statement and solution of the problem

The case study in this paper describes the capital investment regarding cars purchase by a small company with the purpose of hiring them to the public. A company spent £100,000 at once on purchasing cars. The cars purchase was held in UK, and assume that four decision makers (denoted as D_1, D_2, D_3, and D_4) were involved in the process of expressing opinions to determine car models which are most commonly used for rental purposes. Finally, the following three alternatives (car models) were chosen by decision makers: Proton Persona, Vauxhall Merit, and Daewoo Lanos, denoted as A_1, A_2, and A_3, respectively. In choosing the above-mentioned alternatives, the following four criteria were taken into consideration by decision makers: equipment quality (C_1), comfort (C_2), car parts and components reliability (C_3), and safety (C_4) [16].

The following steps of FSAW method are used to rank alternatives, resulting in determination the best (winning) alternative.

Step 1: Create a group of decision makers. Select criteria and determine promising alternate feature investment. A group of decision makers is responsible for determining the most suitable alternative. In this paper, we use four subjective criteria as mentioned above in the case study part.

Step 2: Define the rate of importance of decision makers. The above-mentioned four group decision makers D_1, D_2, D_3, D_4 are accountable for evaluating three alternatives A_1, A_2, A_3 under each of the four criteria C_1, C_2, C_3, C_4 as well as the significance of criteria. The degrees of importance of decision makers are I_1, I_2, I_3, I_4 ($I_t, t = 1, 2, \cdots, k$), where $I_t \in [0,1]$ and $\sum_{t=1}^{k} I_t = 1$). The group of decision makers is called a homogeneous group if $I_1 = I_2 = \cdots = I_k = \frac{1}{k}$, otherwise, the group of decision makers is called a heterogeneous group. Assume that group of decision makers is homogeneous group: $I_1 = I_2 = I_3 = I_4 = \frac{1}{4} = 0.25$.

Step 3: Input linguistic weighting terms so that decision makers can evaluate the importance of criteria and calculate aggregated fuzzy weights for individual criteria. Linguistic terms and fuzzy numbers for importance weights are represented as follows:

Very low (VL) - (0,0,0,3)
Low (L) - (1,3,3,5)
Medium (M) - (2,5,5,8)
High (H) - (5,7,7,10)
Very high (VH) - (7,10,10,10)

We use linguistic weighting terms and their respective fuzzy numbers to assess the significance weights for each criterion. Using formula (2), we determine the aggregated fuzzy weight for each criterion (Table 1). For example, aggregated fuzzy weight for C_1 is calculated as follows:

$$C_1 = ((2+5+5+2)/4, (5+7+7+5)/4, (5+7+7+5)/4, (8+10+10+8)/4) = (3.5, 6, 6, 9)$$

Table 1. Aggregated fuzzy weight for each criterion.

Criteria	Decision makers				
	D_1	D_2	D_3	D_4	Aggregated fuzzy weight
C_1	(2,5,5,8)	(5,7,7,10)	(5,7,7,10)	(2,5,5,8)	(3.5,6,6,9)
C_2	(2,5,5,8)	(5,7,7,10)	(2,5,5,8)	(1,3,3,5)	(2.5,5,5,7.75)
C_3	(2,5,5,8)	(2,5,5,8)	(2,5,5,8)	(7,10,10,10)	(3.25,6.25,6.25, 8.5)
C_4	(7,10,10,10)	(2,5,5,8)	(1,3,3,5)	(2,5,5,8)	(3,5.75,5.75,7.7 5)

Step 4: Defuzzify the fuzzy weights of individual criteria to determine the normalized weights and to state the weight vector. For defuzzification weights of fuzzy criteria, the oriented distance is determined. The defuzzification of $\tilde{W}_j$, denoted as $d(\tilde{W}_j)$, is given by $d(\tilde{W}_j) = \frac{1}{4}(a_j + b_j + c_j + d_j)$, $j = 1, 2, \cdots, n$. For different criteria, the calculations will be as follows:

$$C_1 = (3.5 + 6 + 6 + 9)/4 = 6.125$$

$$C_2 = (2.5 + 5 + 5 + 7.75)/4 = 5.0625$$

$$C_3 = (3.25 + 6.25 + 6.25 + 8.5)/4 = 6.0625$$

$$C_4 = (3 + 5.75 + 5.75 + 7.75)/4 = 5.5625$$

Then the defuzzified value of the normalized weight for criteria C_j, denoted as W_j, is calculated in the following form:

$$W_j = \frac{d(\tilde{W}_j)}{\sum_{j=1}^{n} d(\tilde{W}_j)}, \; j = 1, 2, \cdots, n$$

where $\sum_{j=1}^{n} W_j = 1$. The weight vector $W = [W_1, W_2, \cdots, W_n]$ is consequently determined. For example, the normalized weight for the first criterion is

$C_1 = 6.125/(6.125+5.0625+6.0625+5.5625) = 0.27$. The defuzzified values of the aggregated fuzzy weight and normalized weights are depicted in Table 2.

Table 2. The defuzzified values of the aggregated fuzzy weight and normalized weights.

Technique	Criteria			
	C_1	C_2	C_3	C_4
Defuzzified values	6.125	5.0625	6.0625	5.5625
Normalized weights	0.27	0.22	0.27	0.24

$$W = [0.27, 0.22, 0.27, 0.24]$$

Step 5: Apply linguistic terms for decision makers to evaluate fuzzy ratings of alternatives with respect to individual subjective criteria, and then combine them to get aggregated fuzzy rates. Assume that $\tilde{x}_{ijt} = (o_{ijt}, p_{ijt}, q_{ijt}, s_{ijt})$, $i = 1,2,\cdots,m,$ $j = 1,2,\cdots,h, t = 1,2,\cdots,k$ is the linguistic conformity rate appointed to alternative A_i for subjective criteria C_j by decision maker D_t. $\tilde{x}_{ij}$ is determined as the aggregated fuzzy rating of alternative A_i for subjective criteria C_j, such that

$$\tilde{x}_{ij} = (I_1 \otimes \tilde{x}_{ij1}) \oplus (I_2 \otimes \tilde{x}_{ij2}) \oplus \cdots \oplus (I_1 \otimes \tilde{x}_{ijk})$$

which can subsequently be represented and defined as

$$\tilde{x}_{ij} = (o_{ij}, p_{ij}, q_{ij}, s_{ij}), \quad i = 1,2,\cdots,m, \quad j = 1,2,\cdots,h$$

where $o_{ij} = \sum_{t=1}^{k} I_t o_{ijt}$, $p_{ij} = \sum_{t=1}^{k} I_t p_{ijt}$, $q_{ij} = \sum_{t=1}^{k} I_t q_{ijt}$, $s_{ij} = \sum_{t=1}^{k} I_t s_{ijt}$

We need to evaluate the fuzzy rates of three alternatives by applying the linguistic terms and their respective fuzzy numbers by considering each subjective criterion and then determine an aggregated fuzzy rate for each alternative-criterion combination by using $\tilde{x}_{ij} = (I_1 \otimes \tilde{x}_{ij1}) \oplus (I_2 \otimes \tilde{x}_{ij2}) \oplus \cdots \oplus (I_1 \otimes \tilde{x}_{ijk})$ (Table 3).

The linguistic terms for the ratings are represented below:

Very poor (VP) - (0,0,0,20)
Poor (P) - (0,0,20,40)
Slightly poor (SP) - (0,20,20,40)
Very fair (V F) - (0,20,50,70)
Fair (F) - (30,50,50,70)
Slightly fair (SF) - (30,50,80,100)
Slightly good - (SG) (60,80,80,100)
Good (G) - (60,80,100,100)
Very good - (VG) (80,100,100,100)

Table 3. Estimation of subjective criteria of decision makers.

Criteria	Alternative	D_1	D_2	D_3	D_4	Aggregated ratings
C_1	A_1	(30,50,80,100)	(60,80,80,100)	(80,100,100,100)	(30, 50, 50, 70)	(50,70,77.5,92.5)
	A_2	(30,50,50,70)	(60,80,80,100)	(30,50,80,100)	(60,80,80,100)	(45,65,72.5,92.5)
	A_3	(60,80,80,100)	(30,50,80,100)	(60,80,100,100)	(30,50,50,70)	(45,65,77.5,92.5)
C_2	A_1	(60,80,80,100)	(60,80,100,100)	(30,50,50,70)	(30,50,50,70)	(45,65,70,85)
	A_2	(30,50,50,70)	(60,80,100,100)	(30,50,80,100)	(60,80,80,100)	(45,65,77.5,92.5)
	A_3	(30,50,50,70)	(80,100,100,100)	(60,80,100,100)	(30,50,80,100)	(50,70,82.5,92.5)
C_3	A_1	(60,80,100,100)	(30,50,50,70)	(60,80,80,100)	(30,50,50,70)	(45,65,70,85)
	A_2	(30,50,80,100)	(60,80,100,100)	(60,80,80,100)	(60,80,100,100)	(52.5,72.5,90,100)

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	A_3	(60,80,80,100)	(30,50,50,70)	(60,80,80,100)	(30,50,80,100)	(45,65,72.5,92.5)
C_4	A_1	(60,80,100,100)	(30,50,50,70)	(60,80,80,100)	(80,100,100,100)	(57.5,77.5,82.5,92.5)
	A_2	(30,50,80,100)	(60,80,80,100)	(30,50,50,70)	(60,80,100,100)	(45,65,77.5,92.5)
	A_3	(60,80,80,100)	(60,80,100,100)	(30,50,80,100)	(60,80,80,100)	(52.5,72.5,85,100)

Step 6: Group decision makers evaluate the fuzzy costs or benefits related with different alternatives against objective criteria and then determine fuzzy rates of alternatives with considering individual objective criteria. The objective criteria are defined in different items and must be transformed into dimensionless indexes to provide reconcilability with the linguistic rates of subjective criteria. The alternatives with the minimum cost or maximum benefit should have the largest rating. Let $\tilde{r}_{ij}=\left(a_{ij},b_{ij},c_{ij},d_{ij}\right)$, $i=1,2,\cdots,m$, $j=q,q+1,\cdots,n$, $q=h+1$ be the fuzzy cost or benefit represented with different alternatives A_i for objective criteria C_j. The transforming objective criteria are defined as follows:

$$\tilde{x}_{ij}=\left\{\tilde{r}_{ij}\big/\max\left\{d_{ij}\right\}\right\}\otimes 100,\ i=1,2,\cdots,m,\ j=q,q+1,\cdots,n$$

where $\max\left\{d_{ij}\right\}>0$, $\tilde{x}_{ij}$ represents the transformed fuzzy rating of fuzzy benefit $\tilde{r}_{ij}$. $\tilde{x}_{ij}$ can also be represented by fuzzy number $\tilde{x}_{ij}=\left(o_{ij},p_{ij},q_{ij},s_{ij}\right)$, $i=1,2,\cdots,m,\ \ j=q,\,q+1,\cdots,n$. In addition, $\tilde{x}_{ij}$ gets larger as $\tilde{r}_{ij}$ gets larger:

$$\tilde{x}_{ij}=\left\{\min\left\{a_{ij}\right\}\big/\tilde{r}_{ij}\right\}\otimes 100,\ i=1,2,\cdots,m,\ j=q,\,q+1,\cdots,n$$

where $\min\left\{a_{ij}\right\}>0$, $\tilde{x}_{ij}$ represents the transformed fuzzy rating of fuzzy cost $\tilde{r}_{ij}$. $\tilde{x}_{ij}$ can also be represented by fuzzy number $\tilde{x}_{ij}=\left(o_{ij},p_{ij},q_{ij},s_{ij}\right)$, $i=1,2,\cdots,m,\ \ j=q,\,q+1,\cdots,n$, $q=h+1$, but $\tilde{x}_{ij}$ gets smaller as $\tilde{r}_{ij}$ gets larger. Our criteria are subjective, and we will state the fuzzy matrix based on fuzzy ratings.

Step 7: Define the fuzzy matrix on the base of fuzzy ratings. The fuzzy rating matrix $\tilde{M}$ is represented as in formula (3).

Using aggregated ratings, the fuzzy rating matrix is structured as given in Table 4.

Table 4. The fuzzy rating matrix.

A_i	Criteria			
	C_1	C_2	C_3	C_4
	$w_1=0.27$	$w_2=0.22$	$w_3=0.27$	$w_4=0.24$
A_1	(50,70,77.5,92.5)	(45,65,70,85)	(45,65,70,85)	(57.5,77.5,82.5,92.5)
A_2	(45,65,72.5,92.5)	(45,65,77.5,92.5)	(52.5,72.5,90,100)	(45,65,77.5,92.5)
A_3	(45,65,77.5,92.5)	(50,70,82.5,92.5)	(45,65,72.5,92.5)	(52.5,72.5,85,100)

Step 8: Normalize decision matrix by using the formula given below:

$$\tilde{R}=\left[\tilde{r}_{ij}\right]_{m\times n},\ i=1,2,3,\cdots,m;\ \ j=1,2,3,\cdots,n$$

$$\tilde{r}_{ij}=\left(\frac{a_{ij}}{c_j^*},\frac{b_{ij}}{c_j^*},\frac{c_{ij}}{c_j^*}\right),\quad c_j^*=\max c_{ij}\quad \text{(Benefit Criteria)}$$

$$\tilde{r}_{ij}=\left(\frac{a_j^-}{c_{ij}},\frac{b_j^-}{c_{ij}},\frac{c_j^-}{c_{ij}}\right),\quad a_j^-=\min a_{ij}\quad \text{(Cost Criteria)}$$

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Thus, we have the normalized fuzzy decision matrix as depicted in Table 5.

Table 5. The normalized fuzzy decision matrix.

A_i	Criteria			
	C_1	C_2	C_3	C_4
	$w_1 = 0.27$	$w_2 = 0.22$	$w_3 = 0.27$	$w_4 = 0.24$
A_1	(0.541,0.757,0.838,1)	(0.529,0.765,0.824,1)	(0.529,0.765,0.824,1)	(0.622,0.838,0.892,1)
A_2	(0.486,0.703,0.784,1)	(0.486,0.703,0.838,1)	(0.525,0.725,0.9,1)	(0.486,0.703,0.838,1)
A_3	(0.486,0.703,0.838,1)	(0.541,0.757,0.892,1)	(0.486,0.703,0.784,1)	(0.525,0.725,0.85,1)

Step 9: Total fuzzy estimation for individual alternatives is determined by multiplying the fuzzy rating matrix by its respective weight vector $W = [0.27, 0.22, 0.27, 0.24]$. Using formula (4), we get the determined total fuzzy estimation vector by multiplying the fuzzy rating matrix $\tilde{M}$ by the corresponding weight vector W.

The weighted fuzzy rating matrix is represented in Table 6.

Table 6. The weighted fuzzy rating matrix.

A_i	Criteria			
	C_1	C_2	C_3	C_4
A_1	(0.146,0.204,0.226,0.27)	(0.116,0.168,0.181,0.22)	(0.143,0.207,0.222,0.27)	(0.15,0.201,0.214,0.24)
A_2	(0.131,0.19,0.212,0.27)	(0.107,0.155,0.184,0.22)	(0.142,0.196,0.243,0.27)	(0.117,0.169,0.201,0.24)
A_3	(0.131,0.19,0.226,0.27)	(0.119,0.167,0.196,0.22)	(0.131,0.19,0.212,0.27)	(0.126,0.174,0.204,0.24)

Step 10: Determine the defuzzified (crisp) value for each total score by applying defuzzification technique and choose the alternative with the highest total score. Order total fuzzy scores $\tilde{f}_1, \tilde{f}_2, \cdots, \tilde{f}_m$ by the oriented distance to define the best alternative. Define crisp total scores of individual locations using the defuzzification formula represented below:

$$d(f_i) = \frac{1}{4}(r_i + s_i + t_i + u_i),\ i = 1, 2, \cdots, m$$

where $d(f_i)$ is the crisp value of the total fuzzy score of alternatives A_i by using the oriented distance. The ordering of the alternatives can then be forerun with the above crisp value of the total scores for individual alternatives.

The fuzzy numbers and defuzzified (crisp) values of total fuzzy scores of alternatives are represented in Table 7.

Table 7. The fuzzy numbers and defuzzified (crisp) values of total fuzzy scores of alternatives.

A_i		
A_1	Fuzzy numbers	(0.139,0.195,0.211,0.25)
	Defuzzified value (crisp)	0.199
A_2	Fuzzy numbers	(0.124,0.178,0.21,0.25)
	Defuzzified value (crisp)	0.191
A_3	Fuzzy numbers	(0.127,0.18,0.21,0.25)
	Defuzzified value (crisp)	0.192

The comparison of defuzzified values depicted in Table 7 can yield the result in the ranking form of alternatives as $A_1 > A_3 > A_2$. So, the best alternative is determined as A_1.

4. Conclusions

Fuzzy evaluations defined by linguistic variables are intuitive and effective for decision makers in the estimation process. In this paper, FSAW method is applied to solve the capital investment decision making problem for cars purchase to be rented to the public, where the significant weights of all criteria and the ranking of various alternatives with respect to subjective criteria are evaluated in linguistic terms defined by fuzzy numbers. As a result of the ranking process, the alternatives are ordered in respect to their defuzzified values and, consequently, the optimal alternative is defined.

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