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Let's All Dance and Play Mathematics Innovative Teaching of Mathematics

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Article

Let's All Dance and Play Mathematics Innovative Teaching of Mathematics

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Abstract: Influential topics in innovative teaching practice of mathematics are covered in this work, namely employing dancing and physical education (PE) as powerful and favourable dynamics to lead innovative mathematics' teaching. So, by default, the current work briges a unified approach to teach mathemtatics via two interdisciplinaries, namely, dancing and physical education. Funadmentally, this paper includes a variety of resources appropriate for scholars, researchers, and students studying mathematics. On another strong note, a plethora of influential open problems are introduced to provide food for thought to the research community to steer ahead a new era of contemporary teaching of mathematics.

Keywords: Dancing; Physical Education(PE); Mathematics

1. Introduction

In everyday life, we can experience the three interconnected pieces of human knowledge, namely, the art of dancing, physical eduction(PE), and language of mathematics, which, up to a certain belief is widely spoken, almost everywhere.



Dancing

The work of [1] emphasizes that in dance, the focus has expanded beyond just the physical body to include the influence of technology, which has significantly advanced both dance education and performance. While digital tools are important and necessary in today's world, they should enhance rather than replace traditional teaching methods. Effective dance pedagogy involves understanding each student's unique needs and adapting teaching approaches accordingly, as there is no one-size-fits-all method for teaching dance.

Figure 1(c.f., [1]) refers to the cover page of a book titled "Improvisation Technologies: A Tool for the Analytical Dance Eye," written by William Forsythe and published in 1999. This book explores the intersection of dance and technology, providing insights into how improvisation can be analyzed

and understood through a technological lens. The copyright information indicates that the image is owned by the ZKM | Center for Art and Media in Karlsruhe, which is known for its focus on art and digital media.

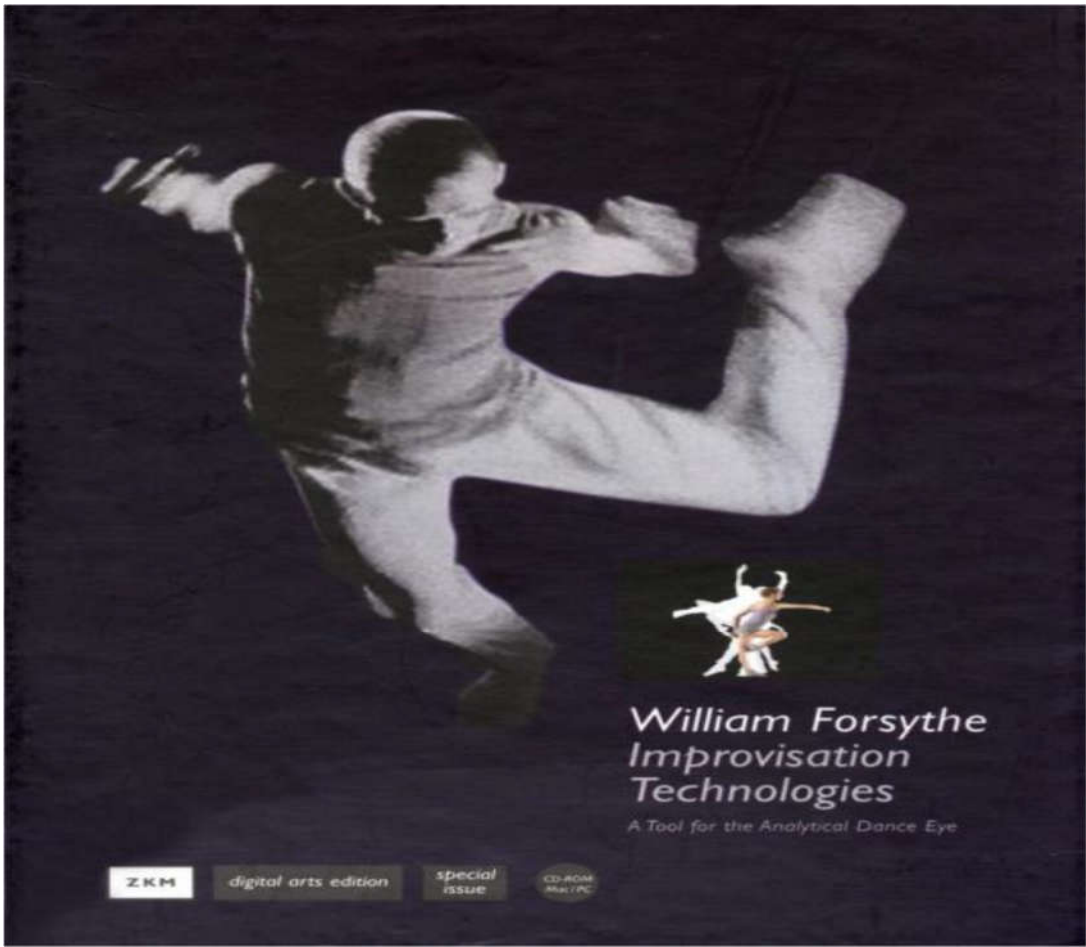


Figure 1.

The phrase "Do as I Do" motion [2] transfer refers to a technique that allows a graduate student to mimic the movements of a ballerina shown in a YouTube video. By using this method, the student's performance can be enhanced by directly transferring the ballerina's dance motions onto them, effectively teaching the student how to perform those specific movements. This approach highlights how digital technology can be used in dance education to improve learning and performance through visual examples. This can be visualized by Figure 2 (c.f., [2]).



Figure 2.

The method [2] [2]described involves two main steps to create videos from poses detected in video frames. First, it identifies the specific body positions or poses of a person in the video (Video to Pose). Then, it uses that information to generate new images or video frames of the same person

based on those poses (Pose to Video). This approach allows for the creation of dynamic visual content by translating physical movements into digital representations, as illustrated by Figure 3 (c.f., [2]).

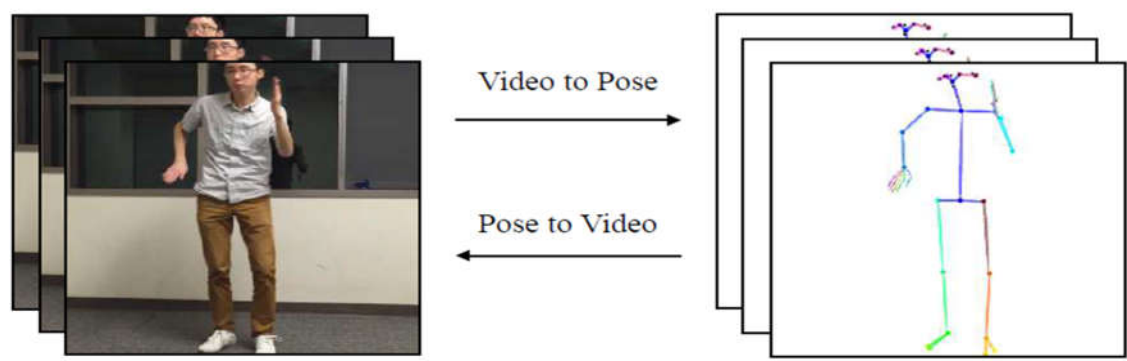


Figure 3

Multi-subject synchronized dancing refers to a performance technique where several dancers use the same initial movement or motion as a starting point. By applying this shared source motion, the dancers can create the illusion that they are moving in perfect harmony, executing synchronized dance moves together. This approach enhances the visual impact of the performance and emphasizes the connection between the dancers, making it a compelling aspect of choreographic practice in genres like Screendance, as showcased by Figure 4 (c.f., [2]).



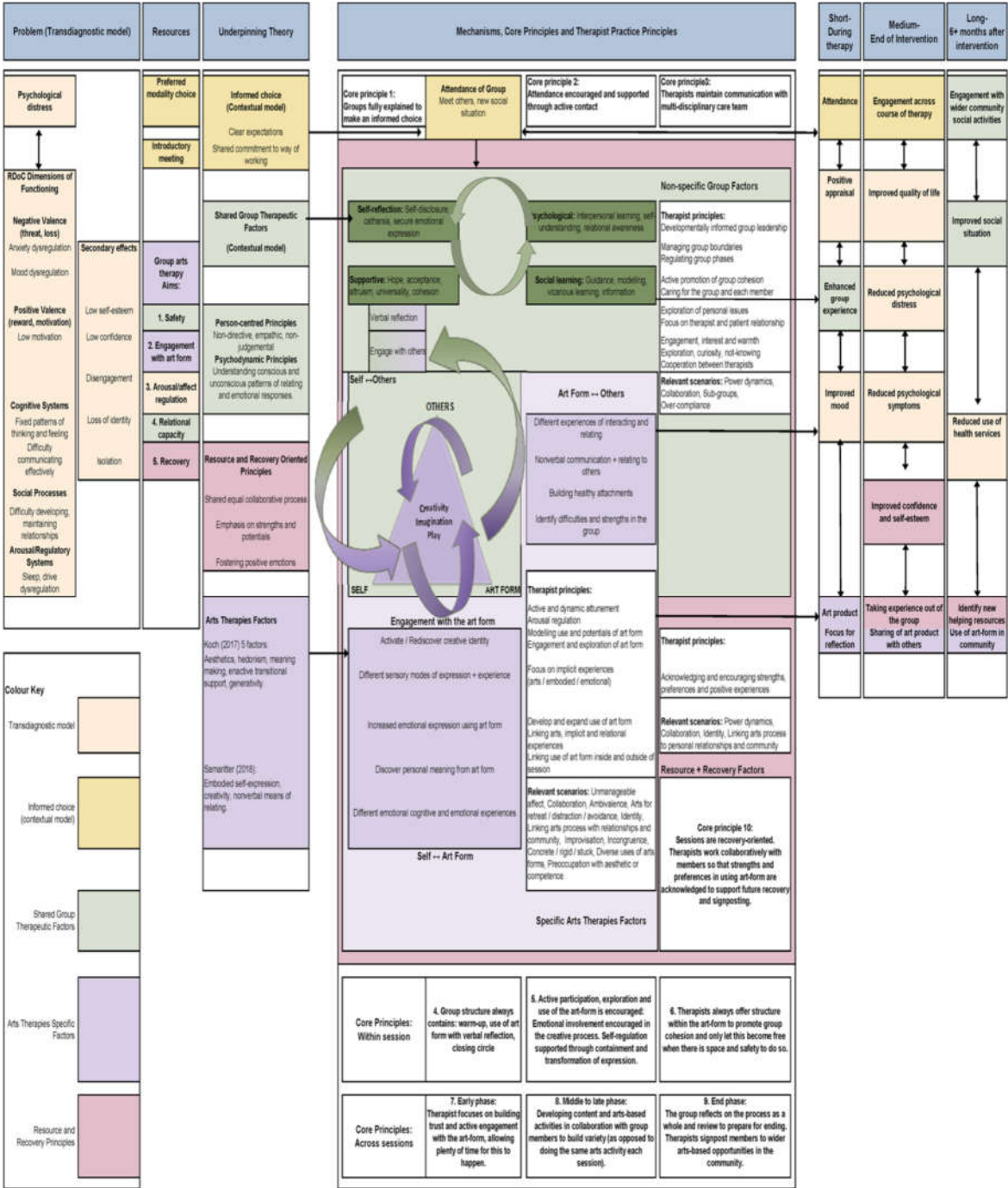
Figure 4

The study[3] aimed to explore the factors that influence the availability of arts education courses, like visual art, music, dance, and theater, in U.S. high schools. By analyzing data from 940 high schools, the researchers found that larger schools were more likely to offer arts courses, with traditional public schools having the highest availability, while public charter schools had the least. Additionally, schools with a higher percentage of students eligible for free or reduced-price lunch tended to offer fewer arts courses, and factors like location or region did not significantly affect arts availability.

The authors [4] have emphasized the significance of dance as a vital part of Uzbekistan's cultural and spiritual heritage, preserved by ancestors for future generations. This highlights the government's commitment to promoting and developing national dance[4], especially after the country's independence, by creating schools and fostering respect for this art form. The importance

of dance is seen not only in its artistic expression but also in its role in educating the younger generation about national values and traditions.

In the model presented, as shown by Figure 5 (c.f., [5]) , the art form used in therapy is important because it attracts patients based on their preferences and encourages them to actively participate and engage emotionally. This creative involvement helps foster interactions among group members[5], which can enhance the therapeutic experience. The model also shows that while these arts-based processes are valuable, they work alongside other therapeutic methods, like talking therapies, and benefit from the diverse backgrounds and needs of the group members.



Creative arts therapists[6], including art, dance/movement, and music therapists, work with military families dealing with traumatic brain injury (TBI) and posttraumatic stress disorder (PTSD) to support their overall well-being. They use specific methods to address the emotional and psychological needs of these families, helping them to bond, identify challenges, and recognize their

strengths. The therapy sessions lead to improved family interactions and resilience, making creative arts therapies an important part of the care for military families facing these challenges.

Dance/movement therapy (DMT) uses the connection between the mind and body to help people heal[6], recognizing that movement can express feelings and improve communication. Originating from programs for veterans after World War II, DMT today incorporates ideas from polyvagal theory to address trauma by focusing on how the nervous system responds to movement. This therapy helps families bond and understand each other better by allowing them to share experiences and emotions through dance, which can be especially beneficial when verbal communication is challenging due to trauma.

In the context of their therapy[6], Susan and Linda engaged in an exercise where they observed and copied each other's physical movements and emotional expressions, a technique known as mirroring(See Figure 6 (c.f.,[6])). This activity allowed them to explore their dynamic as a couple, where they typically fell into a pattern of Susan leading and Linda following. By practicing leading and following together, they aimed to foster empathy and better understand their roles in the relationship, which could help them address their conflicts.



Figure 6

The undertaken research in [7] has examined how dance can help reduce human suffering and reconnect us with our humanity by linking it to the ideas of French philosopher Maurice Merleau-Ponty. Merleau-Ponty emphasizes that our experiences and interactions are shaped by our physical bodies[7], which allows us to see dance as a healing art that helps us connect with ourselves and others. By using his philosophical ideas[7], it has been argued that dance can be a powerful and positive experience that enhances well-being and self-care.

The process of dancing with someone involves a unique form of communication that happens without words[7], relying instead on physical closeness and body movements. Dancers develop a deep awareness of each other's bodies and intentions[7], allowing them to respond instinctively to one another, which creates a strong emotional and mental connection. This connection enhances their understanding of each other as individuals[7], fostering a sense of shared experience and mutual openness during the dance.

Intercorporeality and the transfer of corporeal schema in dance refer to how dancers physically connect and understand each other through movement[7], leading to emotional and mental closeness. This connection allows dancers to sense each other's feelings and states of being without needing to communicate verbally[7], creating a deep sense of empathy and shared experience. As dancers engage with one another, they can let down their emotional barriers, fostering a genuine connection that highlights their shared humanity.

There is growing evidence that participating in the arts[8], especially performing arts like dance, can have significant health benefits for people of all ages. A review of various studies found that even short sessions of performing arts[8], such as drumming or dance, can promote health across multiple areas. However[8], the exact health effects and the best ways to engage in these activities are still not fully understood, indicating a need for more research in this field.

A PRISMA diagram, as portrayed by Figure 7 (c.f., [8]) is a visual tool used to summarize the results of a systematic review, showing how many studies were included or excluded in the research process. In this context[8], the diagram details the findings of an umbrella review, which looks at multiple studies on the health impacts of participating in music and dance. The specific reasons for excluding certain reviews or studies are provided in an appendix, allowing readers to understand the selection criteria used in the research.

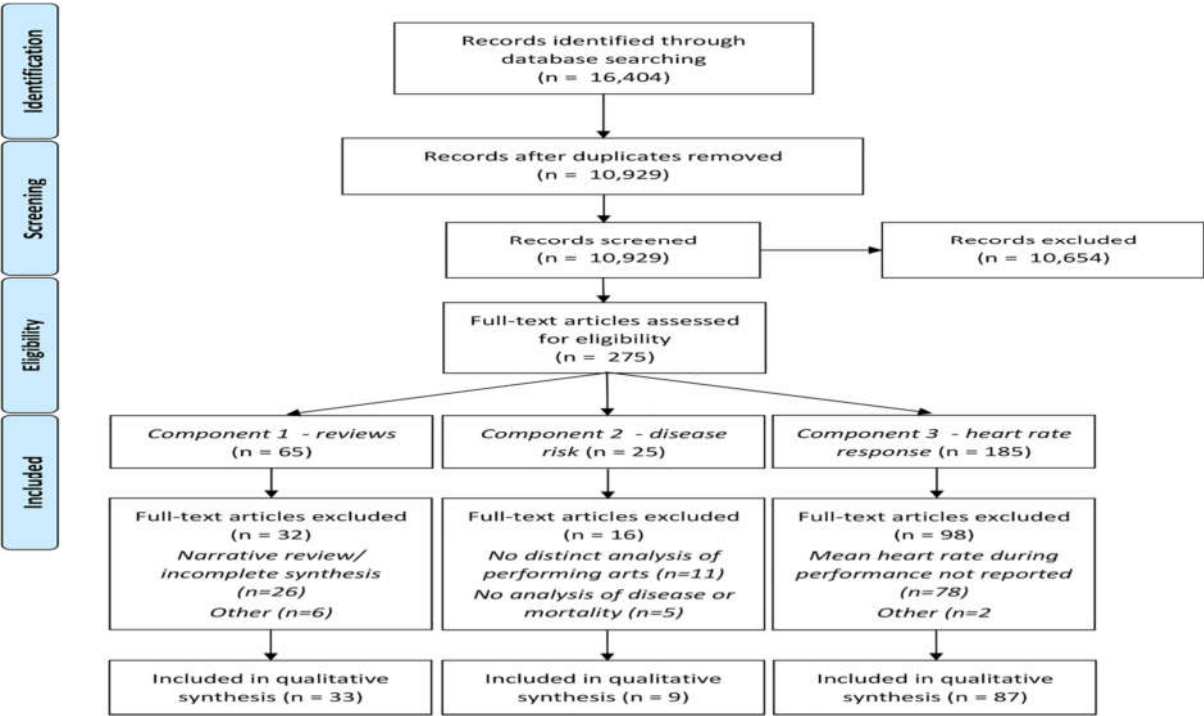


Figure 7



The authors of [9] have thoroughly investigated how traditional physical education (PE) programs, which focus mainly on various sports, can limit students' engagement and learning opportunities. The study [9] aimed to explore how a new educational approach to PE can enhance student participation by considering the school as a learning community and examining different ways students learn about movement. By analyzing data from observations and student focus

groups, the researchers [9] found that this broader approach encourages more students to engage in PE, depending on their roles within the school community.

In the educational physical education (PE) program[9], the goal was to connect PE to the overall school environment, making it more relevant for students. Some students[9], especially those who were academically inclined, found this connection meaningful and felt they could contribute more to PE activities. However[9], there were also students who resisted this approach, viewing PE as separate from school and preferring the informal nature of leisure sports, which created challenges for the teacher in bridging these different perspectives and fostering engagement.

The aim of [10] was to examine the lessons learned in Physical Education (PE) during the COVID-19 pandemic across three countries: Argentina, Spain, and Sweden. Through semi-structured interviews, the researchers found differences in class content, resources used, teachers' emotions, and physical contact, all influenced by local regulations and available resources. The pandemic disrupted traditional PE practices[10], challenging both teachers' and students' habits, while teachers adapted their teaching methods to address new challenges and uncertainties brought about by the situation.

The participating teachers from Argentina[10], Spain, and Sweden emphasized the importance of physical contact in physical education (PE) for building relationships and providing emotional support to students. It is noted that during the COVID-19 pandemic[10], the shift to online classes led to a loss of this essential interaction, making teaching feel "dehumanized" and "disembodied." As schools returned to face-to-face learning[10], teachers expressed relief and comfort in re-establishing physical connections with their students, which they believe enhances the learning experience and allows them to better assess student progress.

The authors [10] emphasized that the teaching of Physical Education (PE) varies greatly depending on the resources and guidelines available in different countries, such as Argentina, Sweden, and Spain. Additionally[10], it is argued that there is no single "best practice" for teaching PE that can be applied universally, as each context has unique challenges and needs.

An exploratory research was undertaken [11] to investigate how decision-making and learning are connected, particularly in the context of Physical Education (PE). This was aimed[11] to create a methodology that combines theoretical ideas from John Dewey's educational theories with practical research methods, allowing researchers to better understand how individuals make decisions while learning in PE. By doing this[11], the authors believed they can uncover important aspects of learning that are often overlooked in current research methods.

This would evidence[11] the importance of understanding how students make decisions during their learning in physical education (PE), but it also acknowledges that it doesn't focus enough on the teacher's role in this process. The authors [11] suggested creating a framework that explores how teachers can support different types of student decision-making, which could include guiding students in reflecting on their choices to enhance their learning. They [11] also noted that by concentrating on student decision-making, other important aspects of student learning may be overlooked.

On another different note, the critical review [12] highlighted that remote teaching and learning in Physical Education (PE) and Physical Education Teacher Education (PETE) during the COVID-19 pandemic was often viewed negatively, with many describing it as disengaging and lacking motivation. Surprisingly[12], there was little pushback from educators against these negative views, leading to a reliance on traditional teaching methods rather than exploring innovative approaches in the online environment. The authors [12] argued that future online PE should focus on creating more engaging and inclusive learning experiences by breaking down traditional hierarchies and fostering a sense of care and connection among teachers and students.

The authors [12] suggested principles for improving online learning environments in Physical Education (PE) and Physical Education Teacher Education (PETE) based on bell hooks' engaged pedagogy. These principles [12] emphasize the importance of creating democratic and inclusive spaces that recognize diverse truths and cultural experiences, especially those of marginalized

groups. Additionally[12], they advocate for challenging discriminatory norms and promoting cultural pluralism by sharing personal stories and discussing our shared social realities, which can enhance the learning experience for all students.

The purpose of [13] to ensure that physical education for students with disabilities is taught by qualified instructors. To achieve this[13], the project aims to develop national standards for adapted physical education and create a certification exam to assess knowledge of these standards. The first two years focus on establishing these standards[13], which will then be used to create the certification exam in the following years, with ongoing input and evaluation from various stakeholders to refine the process.

In recent years[14], the criteria used to define an "able" physical education (PE) student have been questioned, as traditional assessments often focus on skills in sports, fitness, and competitiveness. Despite previous calls for more research on this topic[14], there has been little new empirical work, and most existing studies rely on interviews rather than direct observations of teaching practices. The study [14] aimed to explore how students are categorized as "able" or "less able" in PE classes, using discourse theory to analyze observations from 92 classes in Oslo, revealing that specific physical test scores and skills in traditional games are key factors in these assessments, while also highlighting gender biases in evaluations.

The concept of "ability" in Norwegian physical education (PE) has changed over time[14], shifting from a focus on military training to promoting health and fitness based on scientific principles. After World War II, sports, especially English ball games like football[14], became more central, emphasizing the skills needed for these activities to define an "able" student. Additionally[14], influences from gymnastics and dance led to a more individualized approach in PE, particularly for girls, highlighting creativity, cooperation, and expressive movement alongside traditional sports skills.

The Norwegian educational system for physical education (PE) is decentralized[14], meaning that individual PE teachers have significant control over what sports and activities are taught. The curriculum focuses on students achieving specific competence aims related to practicing movements and understanding physical activity[14], rather than how well they perform specific skills. Additionally[14], unlike many other countries, Norwegian policy emphasizes that students' effort should influence their grades[14], rewarding those who put in significant effort even if their skill level is low.

The research in [14] investigated how students are categorized as 'able' or 'less able' in physical education (PE) based on their sports skills and physical fitness. It was found that [14] that traditional ideas of masculinity and sports performance heavily influence these classifications, often favoring boys and those with prior sports experience. The author [14] argued that this approach is unfair and does not promote equitable learning opportunities for all students, suggesting a need to rethink how 'ability' is defined in PE to support social equity.

The study [15] have deeply identified that while students often focus on sports and techniques, there is an opportunity to explore other areas, like "Movement Composition," which encourages creativity and collaboration. The study [15] aimed to investigate how this creative content is implemented in school PE, highlighting the importance of diverse learning experiences in teacher education.

On another strong note[15], Physical Education Teacher Education (PETE) students often prioritizes practical skills, like teaching sports, over theoretical knowledge in their training. This focus is influenced by what is assessed in their courses, leading them to value physical skills more than critical perspectives on physical education. The study [15] suggested that while this emphasis can limit the scope of teaching, it also presents an opportunity to incorporate creative and collaborative content areas, such as Movement Composition, into school physical education, promoting a more well-rounded approach to teaching.

Over the past twenty years[16], using models in physical education (PE) has been recognized as a promising approach, and collaboration between teachers and researchers is crucial for successfully

implementing these models. However[16], the actual practice of collaboration has not been thoroughly studied, and it varies widely. The undertaken research in [16] has emphasized the importance of understanding different collaborative practices and their effects on how teachers implement models, showcasing three examples that illustrate the evolution from a "researcher knows best" mindset to one where teachers have more autonomy and can shape their own learning experiences.

Researchers can challenge teachers' thinking and help them improve their practices by suggesting overall objectives. Additionally[16], there is a strong supporting evidence that even after a formal collaboration ends, teachers can continue to apply and develop the ideas they learned, indicating that the collaboration can lead to ongoing improvements in their teaching methods. Therefore[16], it has been evidenced that effective collaboration requires flexibility and planning, allowing teachers to become active participants in their own practices rather than just following researchers' instructions.

Peer assessment is a process where students evaluate each other's work[17], and research shows that it enhances learning for both the person giving feedback (the observer) and the one receiving it (the observed). In the context of physical education teacher education (PETE), this study explores how Swedish preservice teachers perceive peer assessment, highlighting four key aspects[17]: building social relationships, clarifying learning objectives, providing accurate feedback, and managing sensitive or gender-related comments. The authors also discuss how social and physical factors influence the effectiveness of peer assessment in educational settings.

Building social relationships is crucial in peer assessment during Physical Education Teacher Education (PETE) placements, as it fosters trust between the teacher and students[17], as well as among the students themselves. For peer assessment to be effective[17], students need to feel comfortable with each other, which can help alleviate pressure during evaluations. Additionally[17], establishing these relationships early on prepares students for future collaborative environments, emphasizing the importance of social dynamics in educational settings.

The meaningful physical education approach [18] provides a framework for teaching physical education but does not specifically guide teacher educators on how to teach future teachers (pre-service teachers) to use this approach. To fill this gap[18], the Learning about Meaningful Physical Education (LAMPE) method was developed to help teacher educators effectively teach PSTs about meaningful physical education. The research [18] primary focus was on understanding how to implement one of LAMPE's principles, which emphasizes using features of meaningful participation in teaching, and involves a self-study where the researcher collaborates with PSTs to create a shared understanding of these concepts.

The phrase "Features of meaningful physical education – a co-constructed shared language" refers to the idea that effective physical education involves collaboration among teachers(See Figure 8 (c.f., [18]), students, and the community to create a common understanding of what is important in physical education. This shared language[18] helps everyone involved to communicate their goals, values, and expectations, making the learning experience more relevant and engaging. By co-constructing this language[18], participants can better appreciate the significance of physical

education in promoting health, teamwork, and personal development.

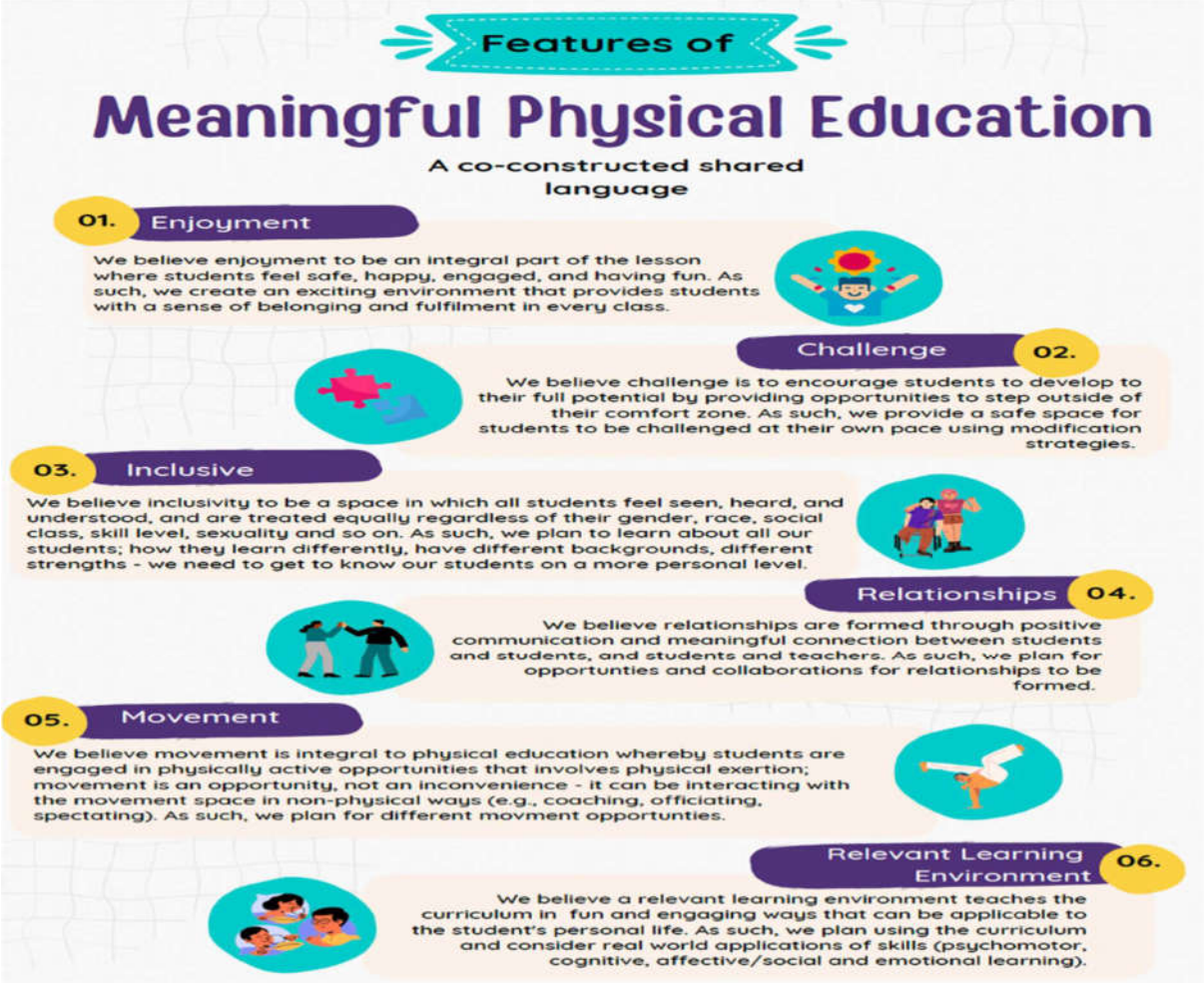


Figure 8

In discussion of the process of developing a shared language for meaningful physical education (MPE) among prospective teachers (PSTs) through a cyclical approach of identification[18], exploration, experience, and reflection, this method allows PSTs to define and refine what "meaningful" means to them, while also balancing their perspectives as both learners and future teachers. The challenges include defining "meaningful" and navigating the established features of MPE, emphasizing the importance of dialogue and co-construction in understanding and teaching physical education.

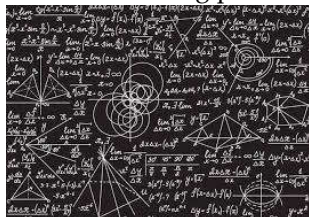
The S-STEP [18] research focuses on a teacher educator's experience in teaching meaningful physical education within Physical Education Teacher Education (PETE) by applying the fourth principle of LAMPE. The educators and their students worked together to create a common understanding or "shared language" around what meaningful physical education means[18], and they shared their findings from this collaborative process.

The concept of "weak education" refers to a teaching approach that embraces uncertainty and risk[18–20], allowing for a more flexible and creative learning environment. In contrast to "strong education," which follows a structured and predictable path[18–20], weak education encourages teacher-student collaboration and exploration, giving students more control over their learning process. Dylan's experience[18–20] illustrates how this approach can lead to deeper understanding and engagement, even though it may involve challenges and moments of vulnerability for the teacher.

The LAMPE principles are interconnected ideas that guide teacher education[18], and in this research, Dylan [18] focused on principle four while also using all five principles in his teaching approach. By emphasizing principle four, he was able to incorporate aspects of the other principles,

such as promoting democratic practices and encouraging continuous reflection among prospective teachers. The authors [18] suggested that these principles should be refined to better support teacher educators in teaching meaningful physical education, and they propose adding a "principle zero" to help educators understand what meaningfulness means before applying the other principles.

The research findings [18] suggested that educators should embrace the idea of "weak education," which involves taking risks in teaching methods while having support from a "critical friend," someone who provides constructive feedback. Although this wasn't the main focus of the research, the authors found that having supportive communities helps teachers like Dylan take these risks effectively. Overall, the research promotes the idea of incorporating "weak practice" into teacher education and proposes adding a new foundational principle to the existing LAMPE principles to enhance teaching practices.



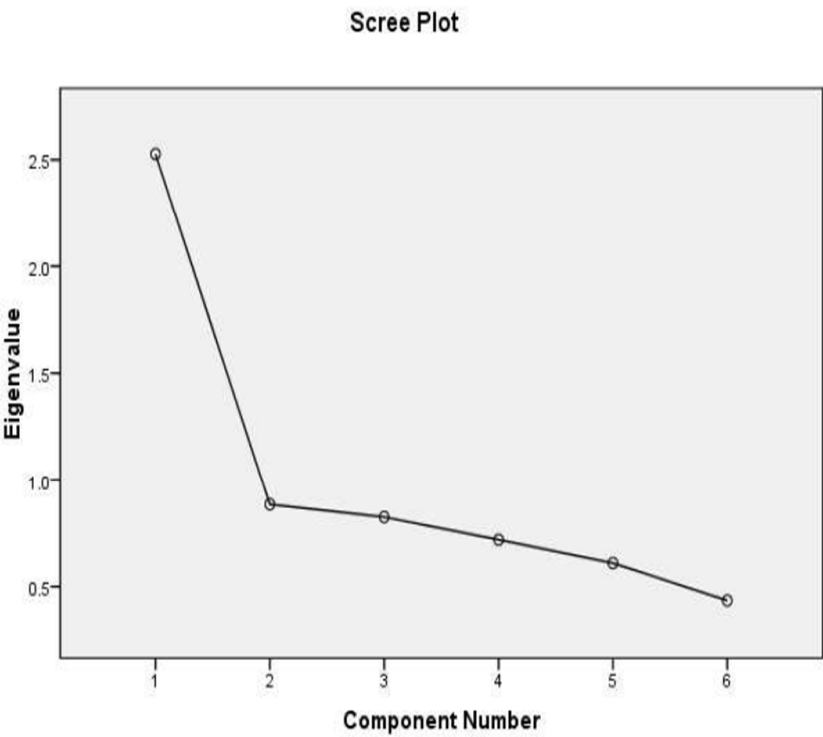
Mathematics

The authors [21] have explored how well mathematics teachers can relate mathematical concepts to real-life situations and how this affects students' interest in the subject. By surveying over 1,200 students from ten high schools [21], the researchers used statistical methods to analyze the data and found that teachers' ability to make these connections significantly influences students' interest, accounting for 57.4% of the variation in their engagement with mathematics. The findings [21] suggested that when teachers spend quality time on practical exercises and link mathematics to other subjects, students are more likely to develop a greater interest in mathematics.

The teaching of mathematics [21] becomes more engaging for students when teachers connect mathematical concepts to real-life problems and experiences. This approach [21] helps students see the relevance of math in their daily lives and enhances their understanding of how different mathematical ideas relate to each other. Research [21] shows that when teachers effectively make these connections, it can improve students' interest and performance in mathematics, which is crucial for their academic success.

The study [21] used statistical methods like exploratory factor analysis and multiple linear regression to evaluate how well mathematics teachers connect math concepts to real-life problems and other subjects. It found that [21] a significant majority of participants believed that teachers' ability to make these connections positively influences students' interest in mathematics. The results [22] also highlighted the importance of providing examples and coordinating classwork with assignments to enhance student engagement and understanding of mathematics, as depicted by scree plot 9 (c.f., [21]).

A scree plot is a graphical tool used in statistics to help determine the number of significant components or factors in a dataset after performing a factor analysis. It displays the eigenvalues (which represent the amount of variance explained by each component) on the y-axis against the component number on the x-axis. The "rotated structure component" refers to the process of adjusting the axes of the plot to make the patterns in the data clearer, helping researchers identify which components are meaningful and should be retained for further analysis.



Scree plot 9

Figure 10 (c.f., [22]) visualizes a part of a study called Life Cycle Sustainability Assessment (LCSA) or Life Cycle Assessment (LCA), which involves gathering data about the environmental impact of products throughout their entire life cycle. A significant challenge in this process is extracting life cycle inventory data[22], which is essential for understanding the effects of pollutants. The paper aims to present a new web-based tool called TESARREC™, which will help automate and simplify these complex calculations, making the data more accessible and useful for researchers and engineers.

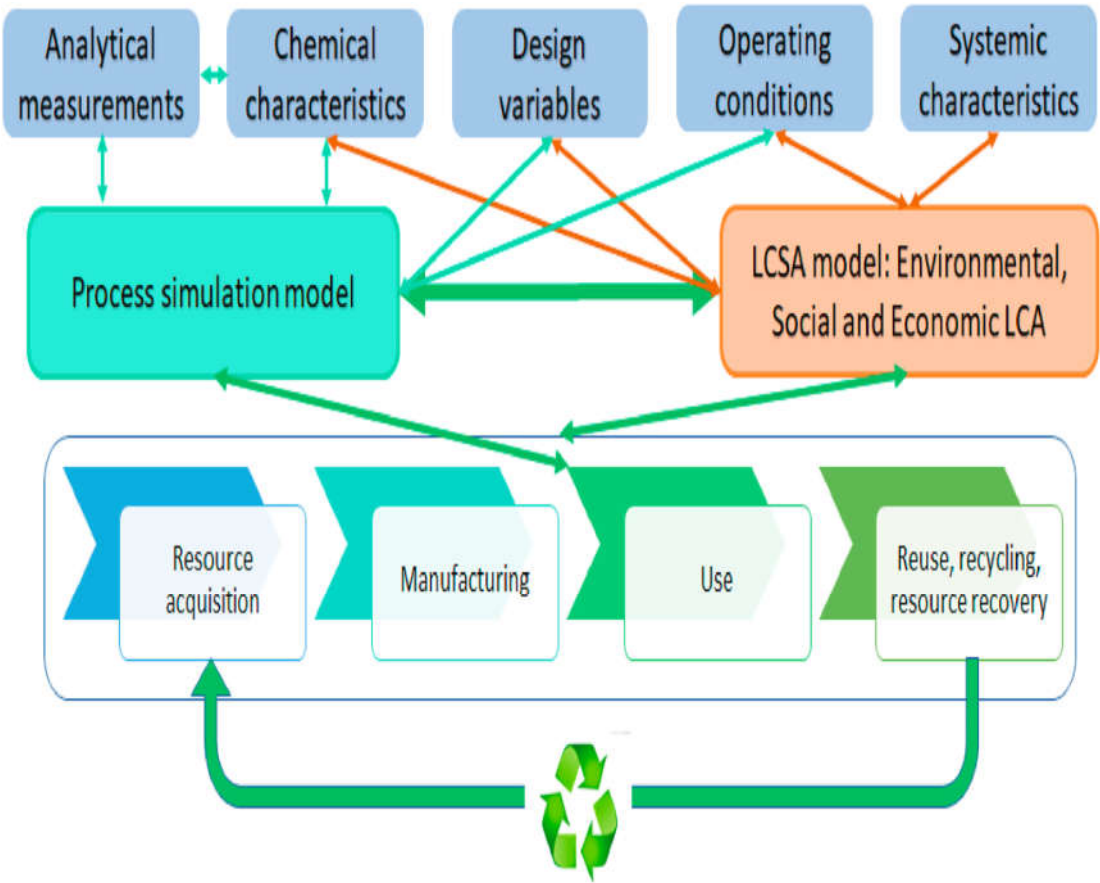


Figure 10

The supply chain [22]for electricity import to the UK involves the process of bringing electricity from other countries into the UK market. The "country of origin approach" in Sustainable Life Cycle Assessment (SLCA) focuses on evaluating the environmental impacts based on where the electricity is generated[22], while the "life cycle approach" considers the entire journey of electricity, from production to consumption, assessing its environmental effects at each stage. Both approaches help in understanding the sustainability of electricity imports and their implications for the UK’s energy policies. This is illustrated by Figure 11 (c.f., [22]).

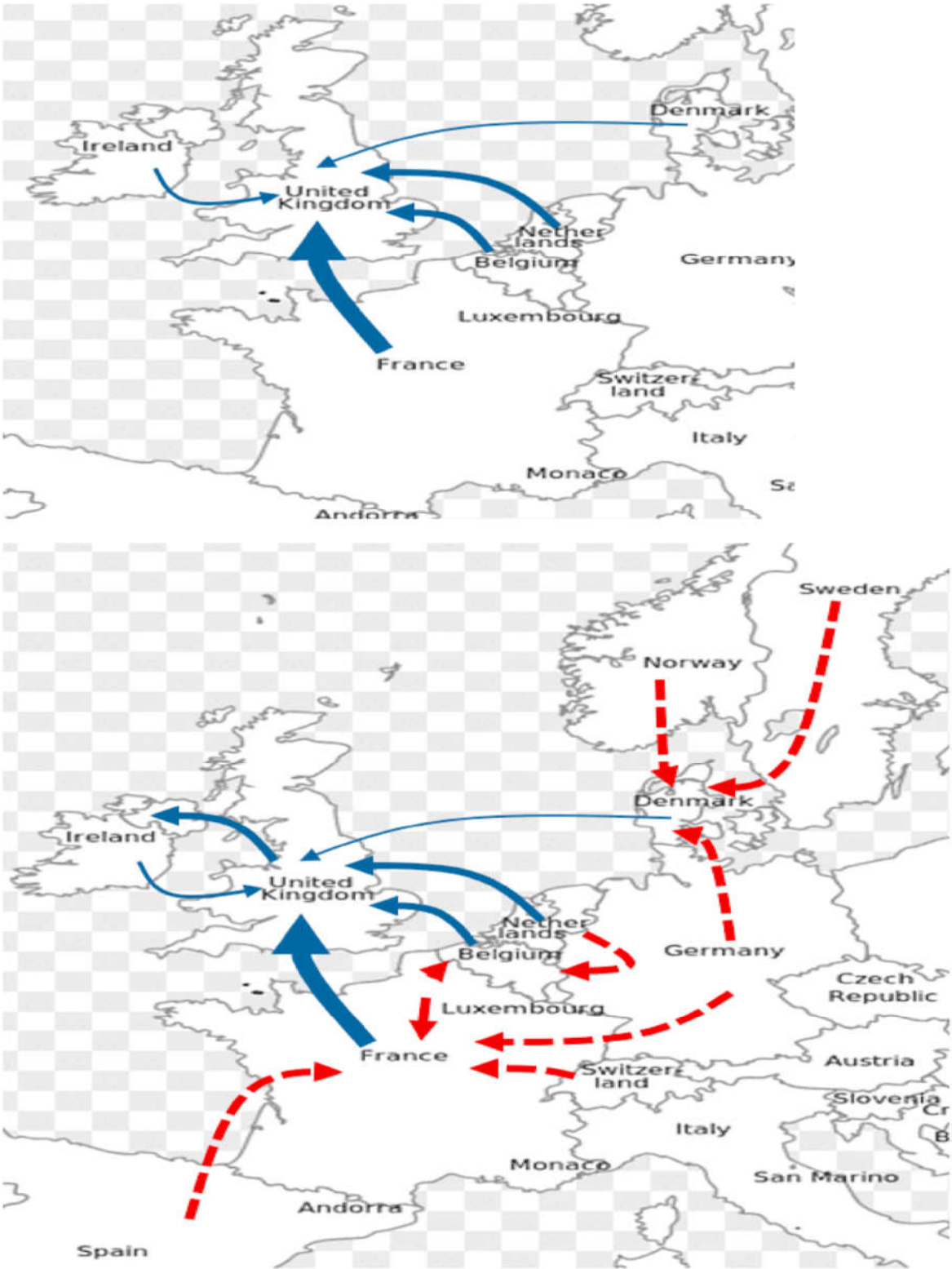


Figure 11

On the furthest remit of the spectrum[22], the "hierarchy of weighted risk calculations" refers to a structured approach for assessing risks by breaking them down into smaller, more specific issues (granular), as depicted by Figure 123 (c.f., [22]). These issues are then grouped into broader themes, which are further organized into categories that reflect their overall importance or impact. This method allows for a more systematic evaluation of risks, helping decision-makers prioritize and address them effectively.

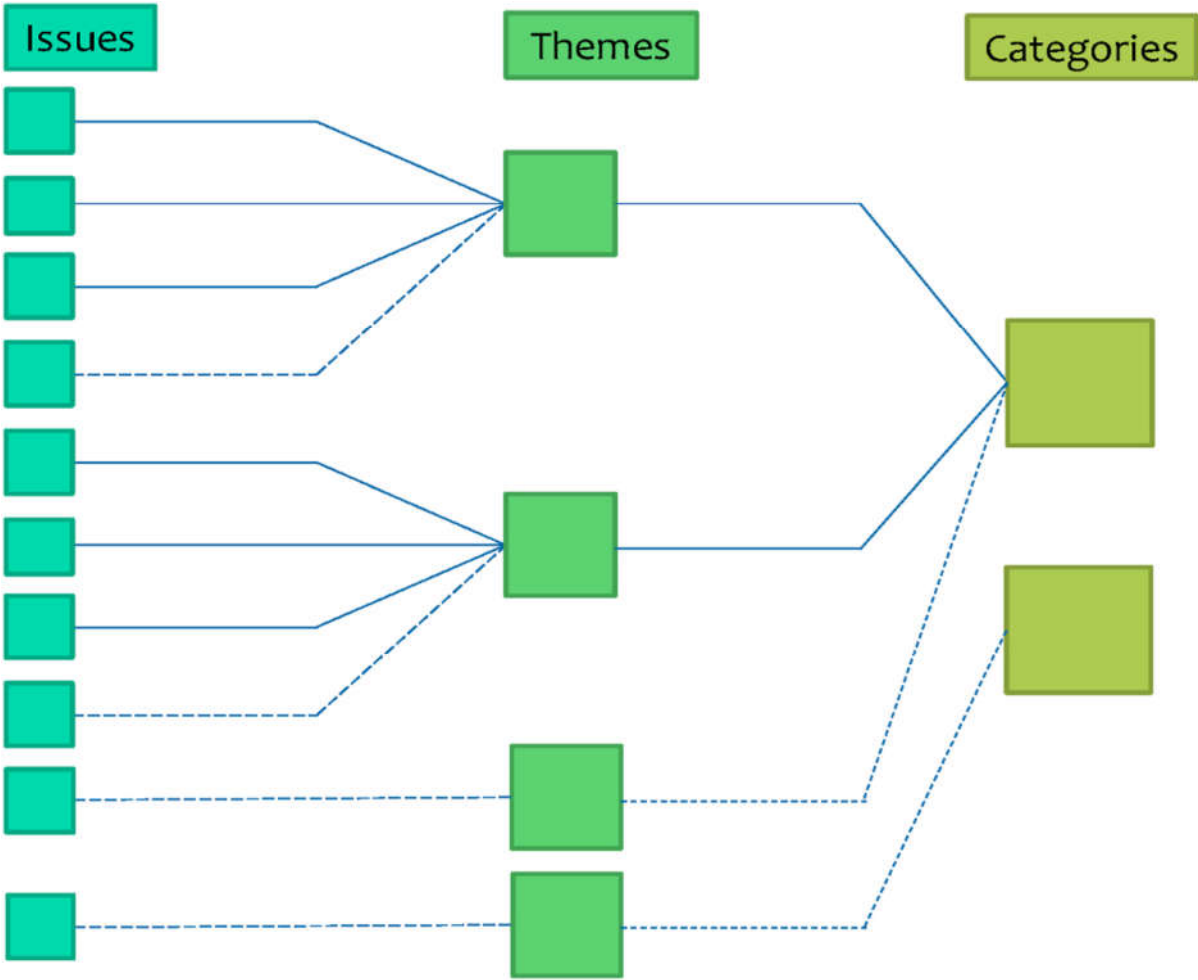


Figure 12

Figure 13 (c.f., [22]) describes two different energy generation systems: Combined Heat and Power (CHP) and Microbial Fuel Cells (MFC). It lists various input variables that are important for techno-economic models of these systems, such as biomass moisture content, efficiency, and costs associated with different components. These variables help evaluate the performance and economic feasibility of generating electricity from biomass in both configurations.

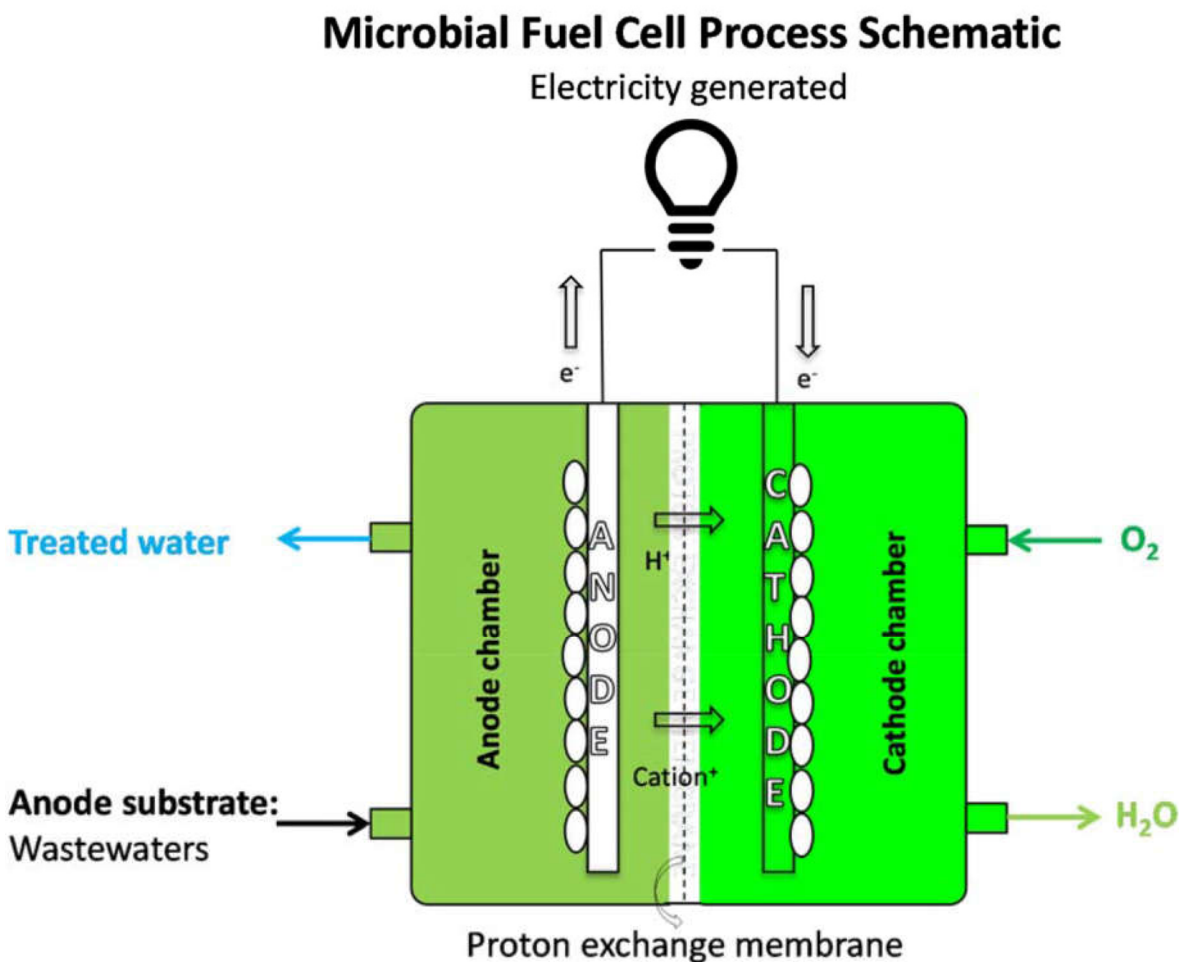
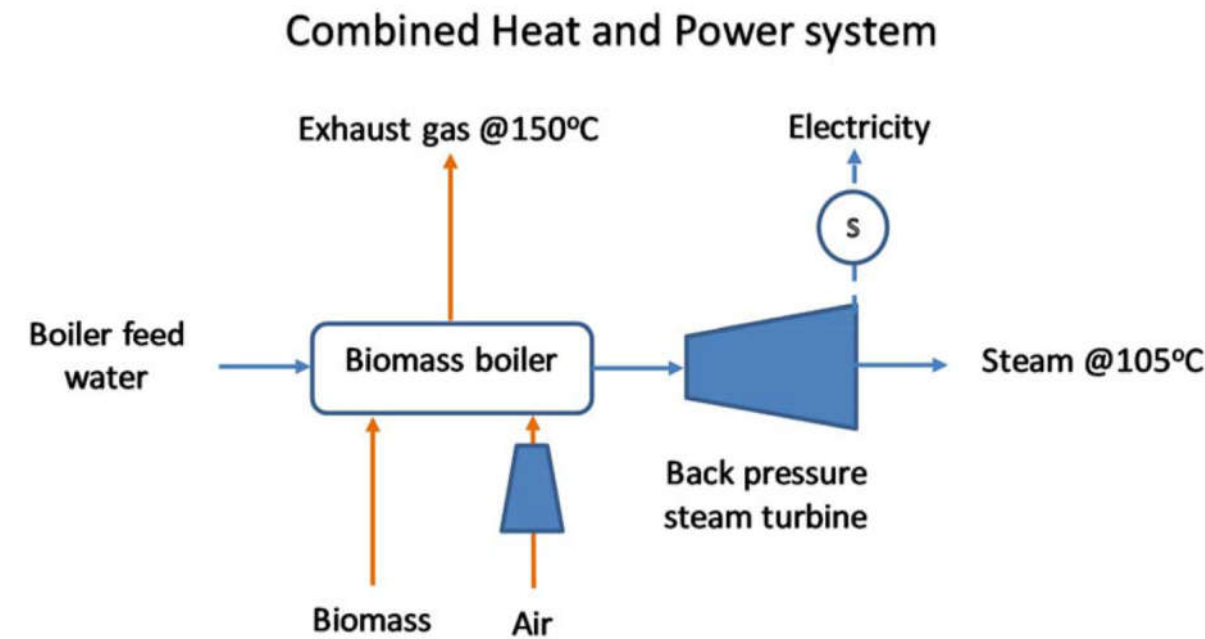


Figure 13

The work of [23] has reviewed the key concepts of systematic reviews and meta-analyses, which are methods used to combine results from multiple studies on the same topic. It has discussed how to calculate the "effect size," which measures the strength of the results, using different approaches for various types of data, such as continuous or categorical outcomes. Researchers[23] can use fixed-

effect or random-effect models to average these effect sizes and visualize the findings with a forest plot, helping them understand and conduct their own meta-analyses effectively.

The Cochrane Database of Systematic Reviews, established in 1994[24], marked a significant shift in how research reviews are conducted. Before this[23], many reviews were based on personal opinions and narratives, but now they focus on systematic reviews that rely on objective data and rigorous methods. This change aligns with the principles of evidence-based medicine, which prioritizes accurate and reliable information to guide healthcare decisions, as in Figure 14 (c.f., [23]).

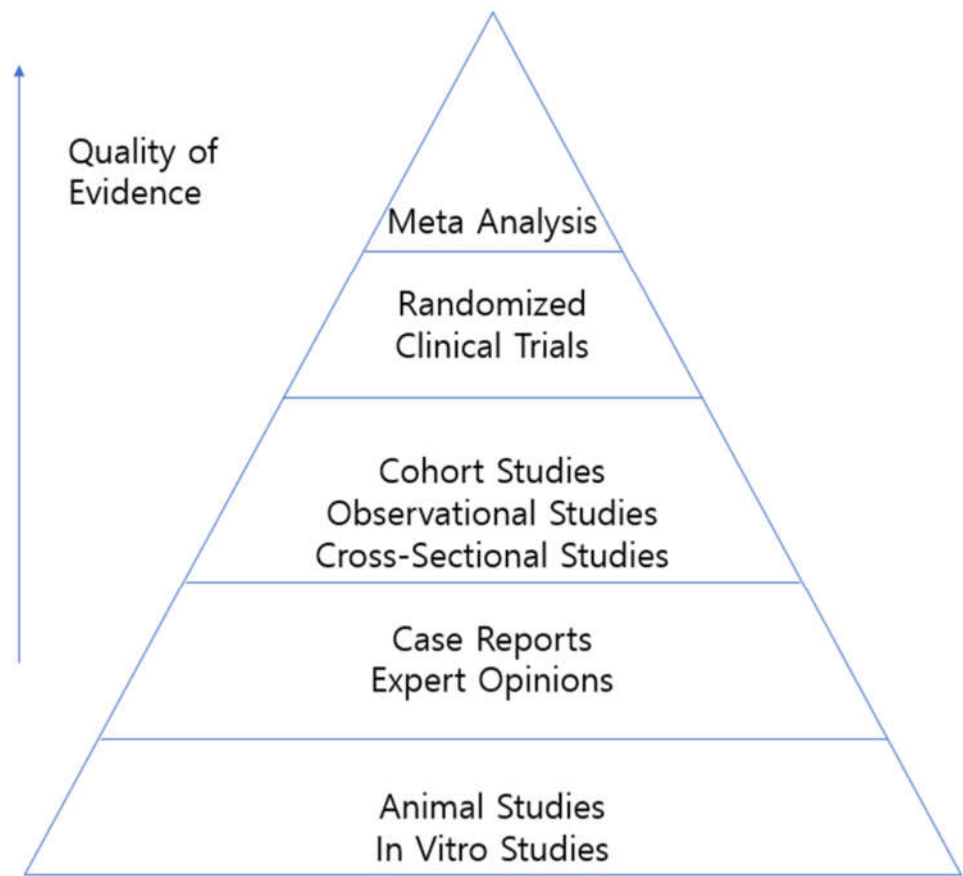


Figure 14

In research[23], a forest plot is a visual tool used to display the results of multiple studies, particularly in meta-analysis. Some researchers prefer to use original values[23], like the odds ratio (OR), instead of effect sizes (ES) because it makes the data easier for readers to interpret, especially in studies with two possible outcomes (dichotomous outcomes). Typically, the log odds ratio is calculated as the effect size, but presenting the original odds ratio helps clarify the findings for a broader audience(Figure 15 (c.f., [23])).

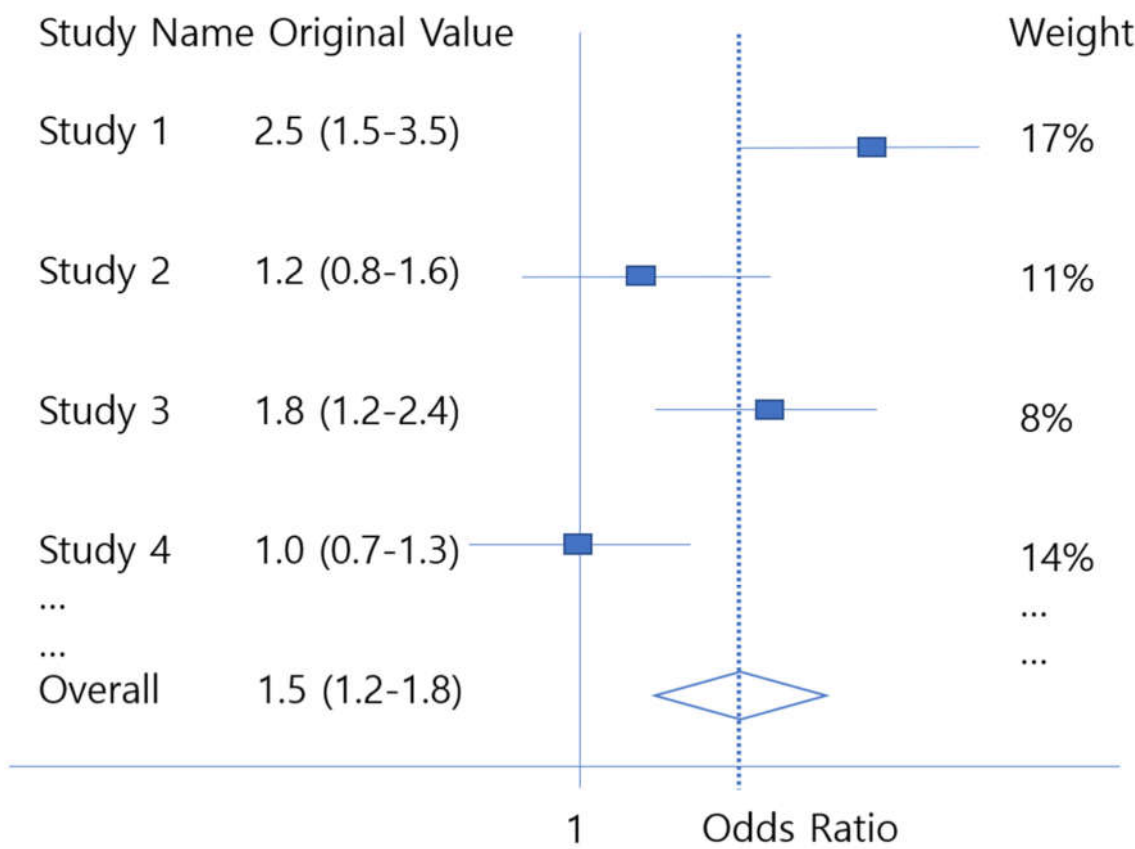


Figure 15

[25] discussed the dual nature of mathematics, highlighting that while it is often seen as a positive and essential subject in education and society, it can also have negative effects. Some students may feel discouraged or rejected by mathematics, leading to a sense of failure that impacts their future opportunities. The author[25] argued for the importance of teaching the ethical implications of mathematics, suggesting that incorporating discussions about its social responsibilities can help mitigate these negative outcomes.

The value of mathematics [25] can be observed in two main ways: intrinsic and extrinsic. Intrinsic value refers to the appreciation of mathematics as a discipline and an important part of human culture[25], while extrinsic value highlights its practical applications in society, such as in science and technology. Learning mathematics [25] not only helps individuals gain skills for various opportunities but also boosts their confidence and sense of self-worth.

In discussion of the key components of mathematics that contribute to its value[25], highlighting numbers and calculation as the most important aspects. Calculation has been central to mathematics since its origins around 3000 BCE, with proof only becoming significant later. Additionally[25], algebra emerged during the Middle Ages, providing a more abstract language for mathematics that allows for general rules and procedures, rather than focusing solely on specific numerical values.

It is an ever-recurring task to explore how mathematics is expressed through a specific language that often uses commands or instructions[25], known as the imperative mood, which directs actions like "add" or "solve." This strict adherence to rules in mathematics means that students must learn to follow these rules precisely[25], as any mistakes can disrupt their calculations. Additionally[25], the early and continuous teaching of mathematics instills a mindset in individuals to quantify and measure everything in their lives, reflecting the importance of numerical values in society.

The teaching and learning of mathematics in schools[25], starting from a young age, requires children to adopt a specific language focused on objects and processes. They learn to perform mathematical operations without necessarily understanding their meaning or value[25], leading to a detachment from their real-life experiences. This process involves replacing their natural understanding with simplified models and prioritizing the results of mathematical work over personal feelings or connections[25], which can impact their development of mathematical identity.

Some students develop negative feelings towards mathematics[25], such as low confidence and anxiety, which can lead to avoiding math altogether. Accordingly [25], people are motivated to avoid risks that could harm their self-esteem, especially in activities that are socially valued, like math. This avoidance results in fewer learning opportunities and repeated failures, which further reinforces their negative attitudes[25], creating a cycle that makes it increasingly difficult for them to succeed in mathematics (Figure 16 (c.f., [26])).

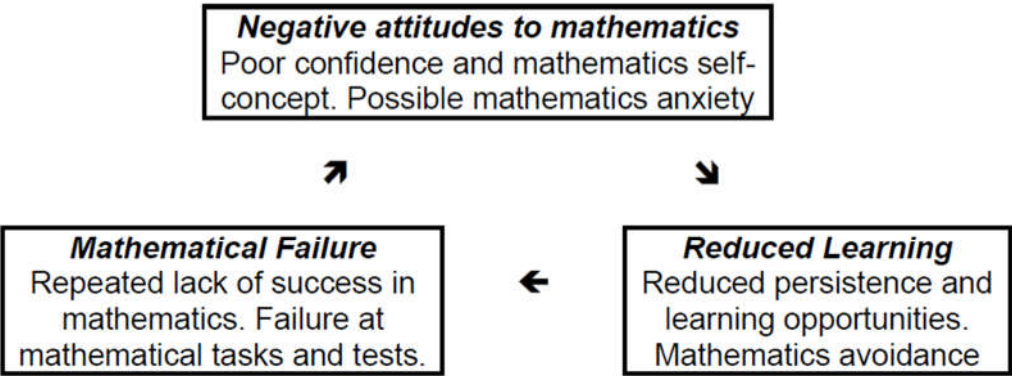


Figure 16

Positive student attitudes towards mathematics[25], such as confidence and motivation, encourage them to put in more effort and tackle more challenging tasks. This is because they find intrinsic rewards[25], like intellectual satisfaction and enjoyment from succeeding. As students engage more and experience success[25], their learning improves, reinforcing their positive attitudes and creating a cycle of continued success in mathematics(Figure 17 (c.f., [26])).

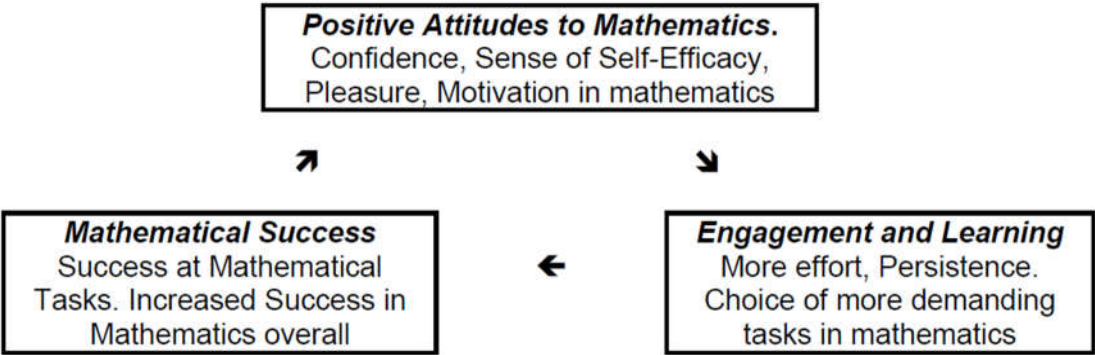


Figure 17

The author[25] has arguably stated that mathematics has both positive and negative aspects, describing it as having "two faces." On one hand, mathematics is a powerful tool that can promote human creativity and has many beneficial applications in society. On the other hand, it can lead to

harmful outcomes, such as reinforcing social inequalities and fostering a mindset that neglects ethical considerations. To address these issues, the author suggests incorporating the philosophy and ethics of mathematics into education at all levels, helping students understand its broader implications and responsibilities.

The inclusion of philosophy and ethics in mathematics education at both school and university levels has been argued[25]. This suggests that students should learn about the nature of mathematical knowledge[25], its limitations, and the ethical implications of its applications in society. By understanding these concepts[25], students can better recognize the social responsibilities of mathematics and its potential negative impacts, fostering a more critical and informed approach to its use in real-world situations.

The negative effects of placing too much importance on mathematics in education and society were deeply discussed[25], which can harm students' confidence, especially among females and those who struggle with math. This overvaluation [25] can lead to labeling students as failures, reducing their opportunities in life. It is argued that [25] while mathematics itself isn't harmful, the way it is taught and applied can lead to ethical issues, and therefore, it's essential to include discussions about the philosophy and ethics of mathematics in education to ensure it contributes positively to society.

The research work [27] outlined three ways mathematics relates to crises. First, mathematics can represent a crisis by creating models that depict real situations, like a pandemic. Second, it can be part of the crisis itself, as seen in the 2008 economic crisis, where mathematical systems influenced the dynamics of the situation. Lastly, mathematics can shape responses to a crisis, which can sometimes lead to ineffective or harmful actions, particularly in the context of climate change, highlighting the complex role of mathematics in understanding and managing crises.

This shines on [27] how human activities are leading to more frequent and severe crises, such as environmental disasters, resource shortages, and social inequalities. It also emphasizes the connection between large-scale crises[27], like epidemics and financial downturns, and personal crises that individuals may face as a result. Additionally[27], This explores three ways mathematics relates to these crises: it can represent a crisis, be a part of a crisis, or influence how we respond to a crisis, highlighting the complex role of mathematics in understanding and managing critical situations.

In the Tractatus[28], Ludwig Wittgenstein describes language as a way to represent or "picture" reality, and he specifically refers to mathematics as this language. He[28] argued that mathematics can model real-world situations, like epidemics, by using mathematical concepts to create representations of how things behave. This means that mathematical models can help us understand and predict the dynamics of an epidemic[28], such as how the number of infected people grows over time, and how interventions like social distancing can influence that growth.

When we shop at a supermarket[27], the process of selecting items and paying involves complex mathematical algorithms that operate behind the scenes. These algorithms[27] handle tasks like identifying credit cards, processing payments, and ensuring security, making the transaction seamless for us. This illustrates how mathematics is not just a tool for understanding transactions but is actively involved in shaping our daily experiences and can also contribute to larger economic issues[27], such as financial crises.

Advanced mathematical modeling [27] is essential for making weather forecasts and predicting long-term climate changes, as it relies on various mathematical techniques rather than laboratory experiments. A climate model typically includes components that represent the atmosphere[27], ocean, land, and ice, and it uses equations to connect different parameters, like temperature and salinity. The process of creating these models involves making decisions about which parameters to include[27], and the resulting predictions can influence how we understand and respond to climate change, highlighting the role of mathematics in shaping our actions and perceptions regarding environmental issues.

The key idea [27] is that mathematics shapes how we understand and respond to climate change. Mathematical models don't just explain or predict climate situations; they influence our perceptions

and actions[27], often leading us to view climate change as something we can easily manage through technology. However[27], these models are not neutral; they reflect specific viewpoints that may align with certain political or economic interests, which can affect how we approach climate-related issues.

This demonstrates how mathematical models[27], like those used during the COVID-19 pandemic, not only describe situations but also influence our actions in response to crises. These models can prioritize certain groups of people based on factors like age[27], education, and health, potentially leading to ethical concerns about valuing some lives more than others. This raises important questions about how society makes decisions during critical situations, balancing the costs of saving lives against economic considerations.

The author [27] expressed concern that during future crises, decisions about saving lives versus saving money may rely on mathematical models that assign economic value to human lives. These models [27] could lead to biased conclusions, prioritizing certain groups of people over others based on factors like age, education, or nationality. This reliance on mathematical reasoning might create a false sense of rationality[27], potentially resulting in unethical decisions that devalue human life in critical situations.

The author [29] discussed recent reforms in the education system of Uzbekistan, particularly focusing on the mathematics curriculum, highlighting the transition from an old curriculum, which emphasized theoretical knowledge and lacked practical application, to a new national curriculum designed to develop students' skills for real-life situations. This new approach[29] included interactive textbooks that encourage independent thinking, practical exercises, and interdisciplinary integration, aiming to improve the overall quality of mathematics education and better prepare students for future challenges.

Another study [30] has focused on creating math problems based on the PISA (Programme for International Student Assessment) standards to help students become more familiar with these types of questions. The researchers [30] used a design research method to develop valid and practical math problems that relate to the Tanjung Kalian Lighthouse in Bangka, which also assess students' basic math skills. The results showed [30] that students improved in areas such as communication, representation, reasoning, and problem-solving strategies through these context-based questions.

Based on the results from the PISA assessments [30] in 2012 and 2015, Indonesian students have been performing below their neighboring countries, particularly in areas requiring problem-solving, reasoning, and effective communication in mathematics. The study[30] highlighted the importance of mathematical literacy, which involves the ability to analyze and apply math in real-life contexts, and suggests that traditional teaching methods may not adequately prepare students for these types of problems. To address this, researchers [30] have developed PISA-style math questions using local contexts to improve students' mathematical skills and better prepare them for future assessments.

The validators assess the developed math problems based on their content[30], structure, and language to ensure they align with PISA standards, which focus on mathematical literacy. The study [30] involved testing these problems with students to see if they could understand and effectively solve them, revealing that while many students showed good reasoning and communication skills, some struggled due to a lack of familiarity with creative and contextual problem-solving. Overall[30], the research successfully created 11 valid and practical PISA-type math questions that helped enhance students' mathematical abilities.

More interestingly, [31] focussed on creating a mathematics learning method that uses the local market in Telukdalam as a teaching tool. By engaging students in real-life scenarios involving price calculations and money management[31], the research aimed to improve their mathematical skills while also connecting these concepts to their everyday experiences. The findings suggested that this approach not only boosts students' motivation and understanding of math but also helps them appreciate their local culture and environment, potentially leading to a more relevant mathematics curriculum.

The research on contextual mathematics learning at Telukdalam Market shows that involving students in real-life situations[31], like shopping, has significantly improved their math skills, motivation, and social abilities. Students learned to apply basic arithmetic in practical scenarios, which made math more relevant and engaging for them [31]. The study suggested that educators [31] should incorporate more hands-on learning experiences and collaborate with local businesses to enhance students' understanding and appreciation of mathematics in their everyday lives.

Cell signaling mechanisms, which are essential for understanding physiology[32], can be traced back to the earliest forms of life were discussed, suggesting that cells are interconnected with the universe, almost like a Mobius Strip, meaning they don't have a clear inside or outside and are continuously linked to the cosmos. This idea connects the concepts of physiology[32], consciousness, and mathematics, proposing that our understanding of these areas can be unified in a new way.

Figure 18 (c.f., [32]) illustrates a new Soft Coordinate System that visually represents mathematical concepts, particularly soft numbers, in a geometric space. This system features a zero axis and two one axes, allowing for the visualization of processes and relationships between different types of numbers. The Möbius function is used to illustrate how seemingly opposing points can be unified, reflecting deeper ideas about consciousness and the coexistence of duality and non-duality.

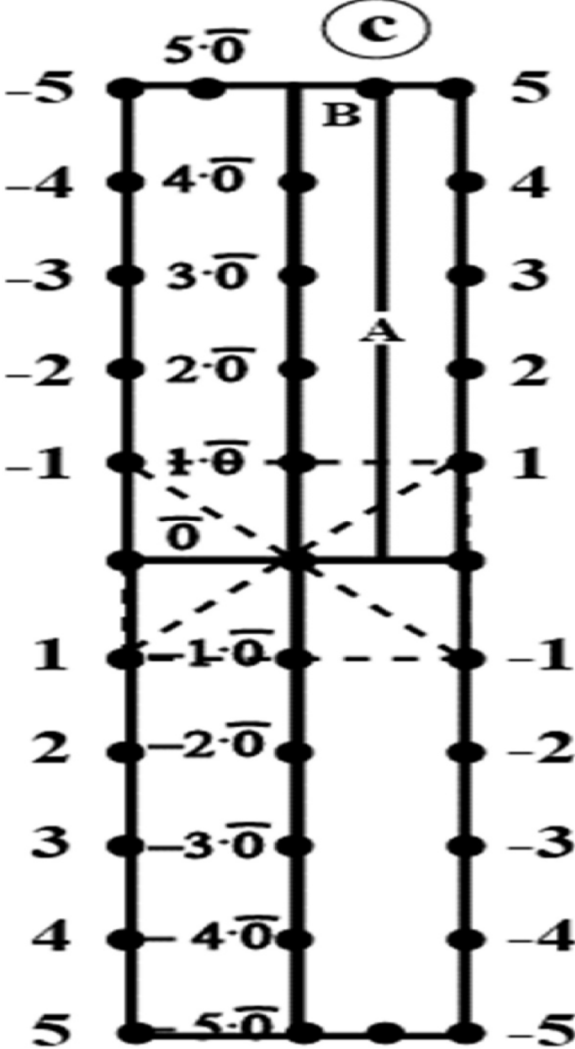


Figure 18

The Soft Möbius Map (Figure 19 (c.f., [32])) is a mathematical tool that converts points from a flat plane into a special type of number called "soft numbers," which are then represented on a Möbius strip. This transformation allows for a unique way to visualize and understand these soft numbers, as different areas or regions on the flat plane correspond to specific areas on the Möbius strip.

Essentially, it helps to illustrate complex relationships in a more intuitive format by using the properties of the Möbius strip.

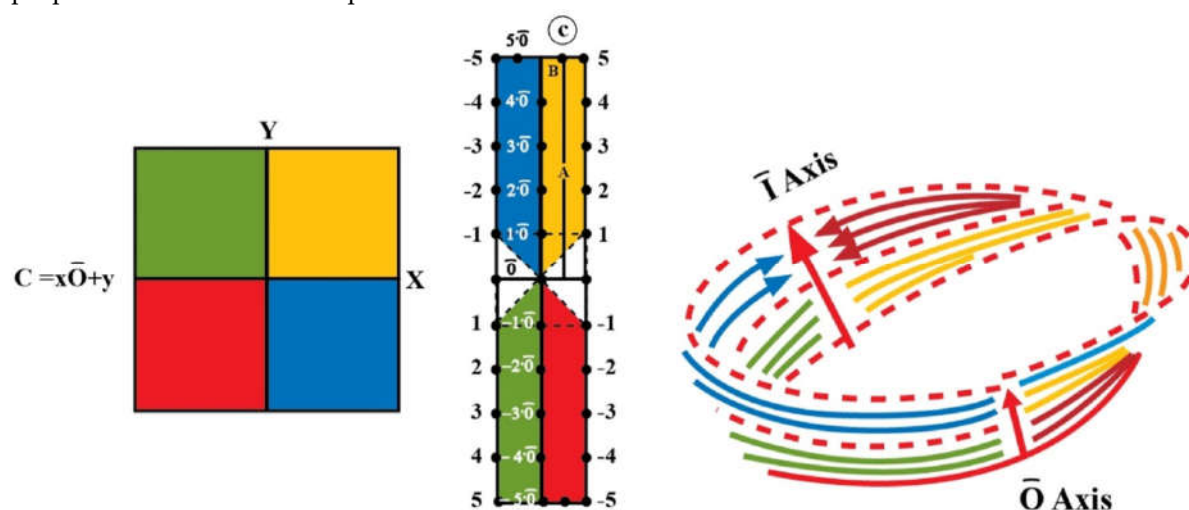


Figure 19

The idea of Soft Logic[32], which is a flexible way of reasoning, and its potential role in epigenetic inheritance. Epigenetic inheritance refers to how organisms can adapt to changes in their environment and pass those adaptations to future generations without altering their DNA. The authors [32] suggested that if everything in existence can be thought of as balancing out to zero, Soft Logic might help explain how these environmental changes are recognized and processed by living organisms to facilitate evolution.

Mathematics plays a crucial role in enhancing human thinking skills[33], including rational, logical, and creative thinking, as well as problem-solving and communication abilities. The structure of school mathematics is designed to help students develop these skills progressively[33], but its abstract nature can make it challenging for some students to grasp. To address these difficulties[33], teachers should create effective learning designs and use approaches like Realistic Mathematics Education (RME), which connects mathematical concepts to real-life situations, making it easier for students to understand and engage with the material.

Several studies[33] have shown that implementing Realistic Mathematics Education (RME) in classrooms can lead to improved learning outcomes compared to traditional teaching methods. However[33], these findings have not yet been explored in schools located on islands, such as those in the Buton Islands region. Therefore[33], further research is needed to assess the effectiveness of RME in enhancing mathematics learning outcomes for primary school students in that area.

This research began with a pre-test to assess students' prior knowledge of fractions before implementing RME. The results [33] showed that students had some foundational understanding, which helped in applying RME effectively. After using RME[33], both student participation and learning outcomes improved significantly, indicating that this teaching approach enhances students' engagement and understanding of mathematics in real-life contexts.

Teaching mathematics at the primary level is a complex task that requires teachers to understand both mathematical concepts and the specific language used in mathematics[34], known as the mathematics register. This importance of helping students develop their mathematical understanding while also mastering the language of mathematics through problem-solving and communication with peers, was strongly evidenced and advocated by [34]. By engaging in meaningful discussions and tackling challenging problems[34], students can build their knowledge and improve their ability to express mathematical ideas effectively.

The academic language register [34] is a specialized way of communicating that differs from everyday conversation, tailored for academic settings like classrooms and research. Each academic discipline[34], such as mathematics or science, has its own unique language features and ways of

expressing ideas, which students need to learn to succeed in that field. Understanding these specific language patterns requires students to develop metacognitive skills[34], allowing them to think about and analyze their own use of language and discourse.

Eventually[34], the complexity of academic language, particularly in disciplines like mathematics, where specialized vocabulary and structures are essential for effective communication and understanding, would predominately indicate that students must learn to navigate this complex language, which includes precise terms and syntactic constructions, to demonstrate their knowledge and reasoning in mathematics. Additionally[34], it emphasizes the importance of sociocultural contexts in learning, suggesting that proficiency in mathematics requires integrating linguistic, cognitive, and social skills.

Learning mathematics is a complex process that requires both students and teachers to use various types of knowledge[34], including different ways to express mathematical ideas. Effective instruction involves creating opportunities for students to interact with each other and their teacher, allowing them to discuss and justify their solutions[34], which helps reveal their understanding. In contrast[34], assessment contexts focus on individual work, where students must demonstrate their knowledge without support, highlighting the importance of mastering language skills in both spoken and written forms for standardized tests.

The differences between everyday language and academic language[34], particularly in the context of learning mathematics, were highlighted by a case study of a student named Ariel, who, despite being fluent in everyday English, struggled with the specialized language of mathematics, which includes technical vocabulary and complex grammatical structures. The author[34] claimed that teaching should focus on helping students develop both their mathematical understanding and their ability to communicate mathematically, especially in preparation for standardized tests that require comprehension of complex language.

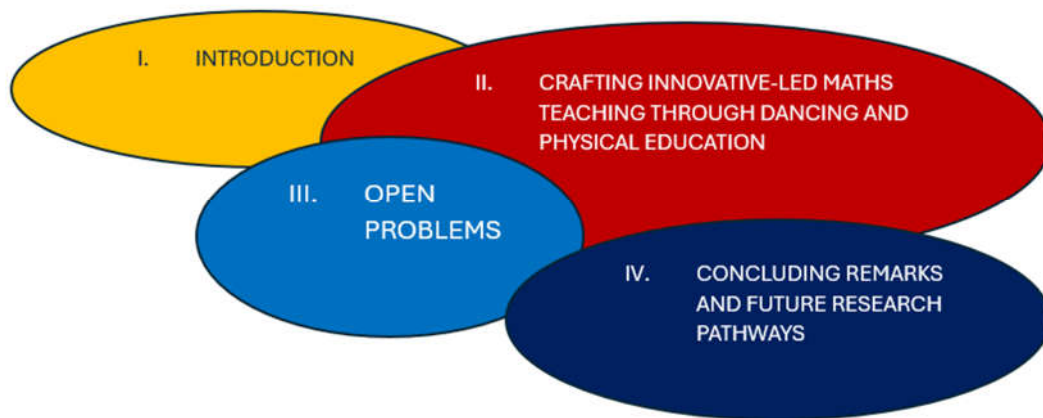
Undertaking an exploratory investigation[34] of the language challenges students face when taking high-stakes tests, such as the Kenya Certificate of Secondary Education Mathematics Examination (KCSE) and the Program for International Student Assessment (PISA), has strongly identified how complex vocabulary and grammatical structures in test questions can make it difficult for students to understand and solve problems, as they need both mathematical knowledge and proficiency in academic language. This complexity[34] can hinder their ability to demonstrate their understanding effectively, emphasizing the importance of clear language in educational assessments.

The main takeaway from [34] is that mathematics teachers need to understand and teach the "mathematics register," which is the specific language and symbols used in math. This requires[34] teachers to change their teaching methods to help students engage in discussions and tackle challenging problems, using both everyday language and formal mathematical language. By doing so[34], teachers can better support students in understanding math concepts and improve their overall learning experience.

The current paper contributes to:

- Putting in both theory and practice how dancing and physical education are employed to teach mathematics
- The provision of open problems to enrich our way of thinking on how to think beyond classical frameworks of teaching mathematics

The paper is portrayed by the following schematic:



2. Crafting Innovative-Led Maths Teaching Through Dancing and PE

The current part provides exceptional taste of teaching mathematics through dancing and PE, as metacognitive journey to a phenomenal practice of teaching mathematics.

A Dancing-Based Approach to Teach Mathematics

[35] has introduced a thought experiment where you are asked to extend your hand and see how far you can rotate it without moving your feet, exploring concepts of movement and rotation. It has also clarified that while dance has mathematical elements[35], such as patterns and arrangements, the focus of the column is not on teaching or illustrating math through dance, but rather on the mathematical pleasures that can be found in dance itself. The author aimed to highlight intriguing mathematical aspects that stimulate curiosity and exploration, using dance as a starting point for these ideas.

The **Philippine binasuan** is a traditional dance that features dancers skillfully balancing glasses of wine on their palms while performing various movements. This dance, which has different versions across Southeast Asia, is known for its simplicity and artistry, making it both impressive and enjoyable to watch. A related dance, called **pandango sa ilaw**, involves dancers holding lighted candles instead of glasses, showcasing similar graceful motions with their palms always facing upward(Figure 20, (c.f., [35])).

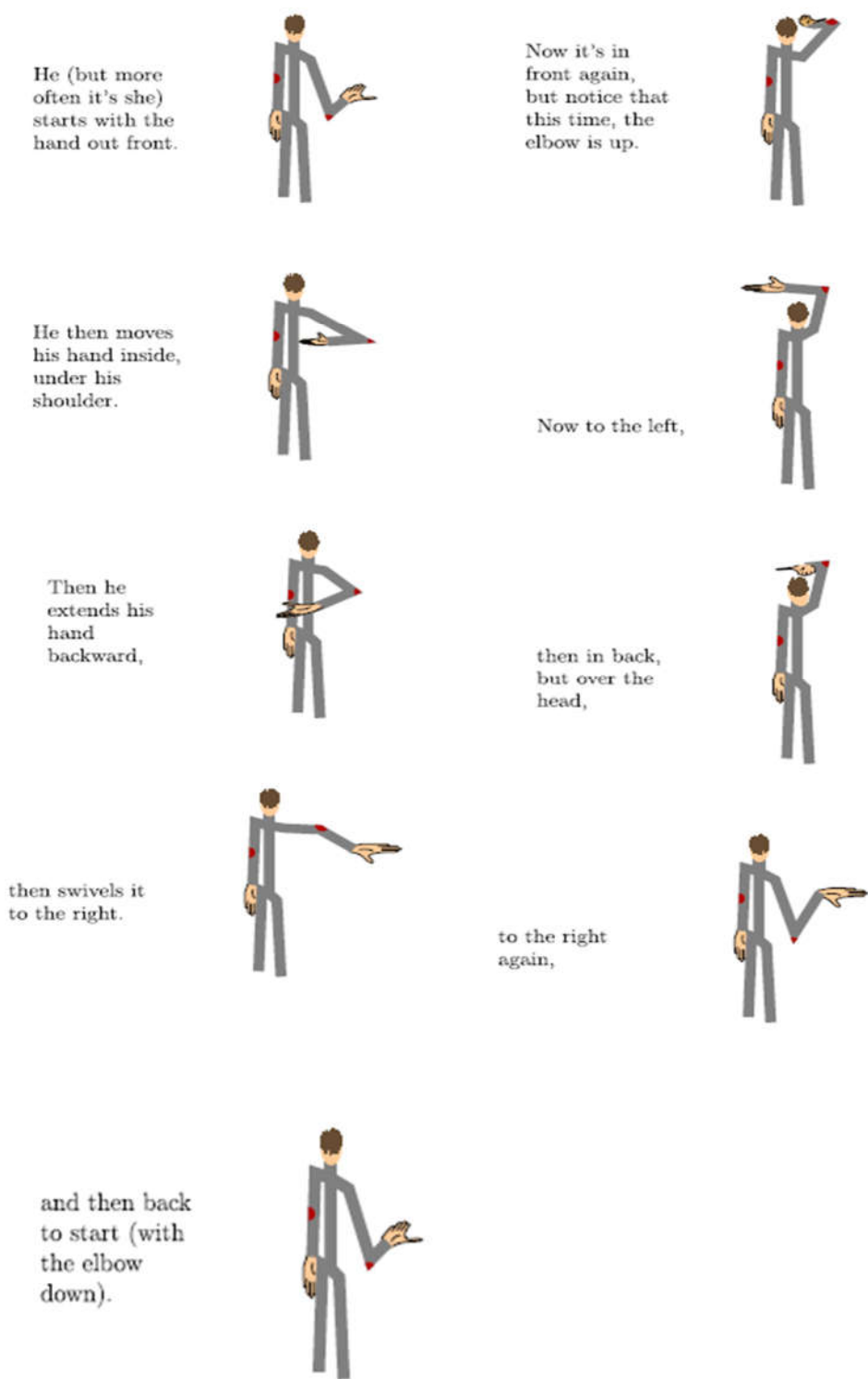


Figure 20

Experimenting where you rotate your hand two full times[35], resulting in a total rotation of 720 degrees, this movement is compared to the behavior of two connected Mo"bius strips, which have unique twisting properties. After the first rotation[35], your arm twists 180 degrees, but the second rotation cancels out that twist, allowing your arm to return to its original position without strain on your elbow.

In this context[35], the positions of dancers in a dance can be thought of as different arrangements or "permutations" of the dancers. When we focus solely on how the dancers are

positioned, each dance step can be seen as a way to rearrange them into new configurations. An example given is Zoltan Diene's dance with three dancers[36], which illustrates this concept of permutations in a simple and clear manner.

In a dance involving four dancers[35], there are 4! (4 factorial) possible arrangements, which equals 24 different positions. The question is whether all these positions can be achieved using two specific dance moves: rotating the dancers (R) and switching their places (S). Additionally[35], it asks for the maximum number of steps required to move from one position to another using these dance moves, highlighting the exploration of permutations and distances in dance choreography, as demonstrated by Figure 21 (c.f., [35]).

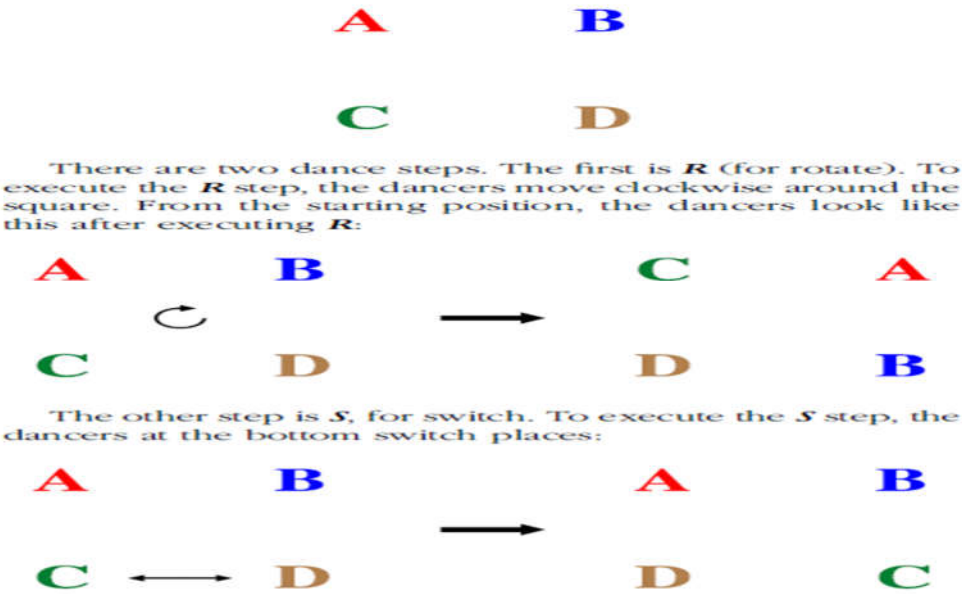


Figure 21

Square dancing is a traditional form of dance that has been around for over 300 years[35], but challenge square dancing, which is a more complex version, has only been popular for about 50 years. In challenge square dancing[35], a caller directs the dancers using a wide variety of commands, which can involve intricate mathematical concepts like geometry and combinatorics. This dance form is so rich in mathematical elements that it merits further exploration in dedicated discussions, and there are many animations available to illustrate the various calls used in the dance.

In a locomotion[35], the triangle game is a fun ice-breaker activity often used in classrooms and corporate settings to encourage teamwork. In this game[35], participants randomly choose two other players as partners and then try to move together to form an equilateral triangle. While it promotes cooperation, it can also be viewed as a mathematical dance, highlighting the interesting patterns and relationships that emerge as everyone tries to align their movements with their partners, as depicted by Figure 22 (c.f., [35]).

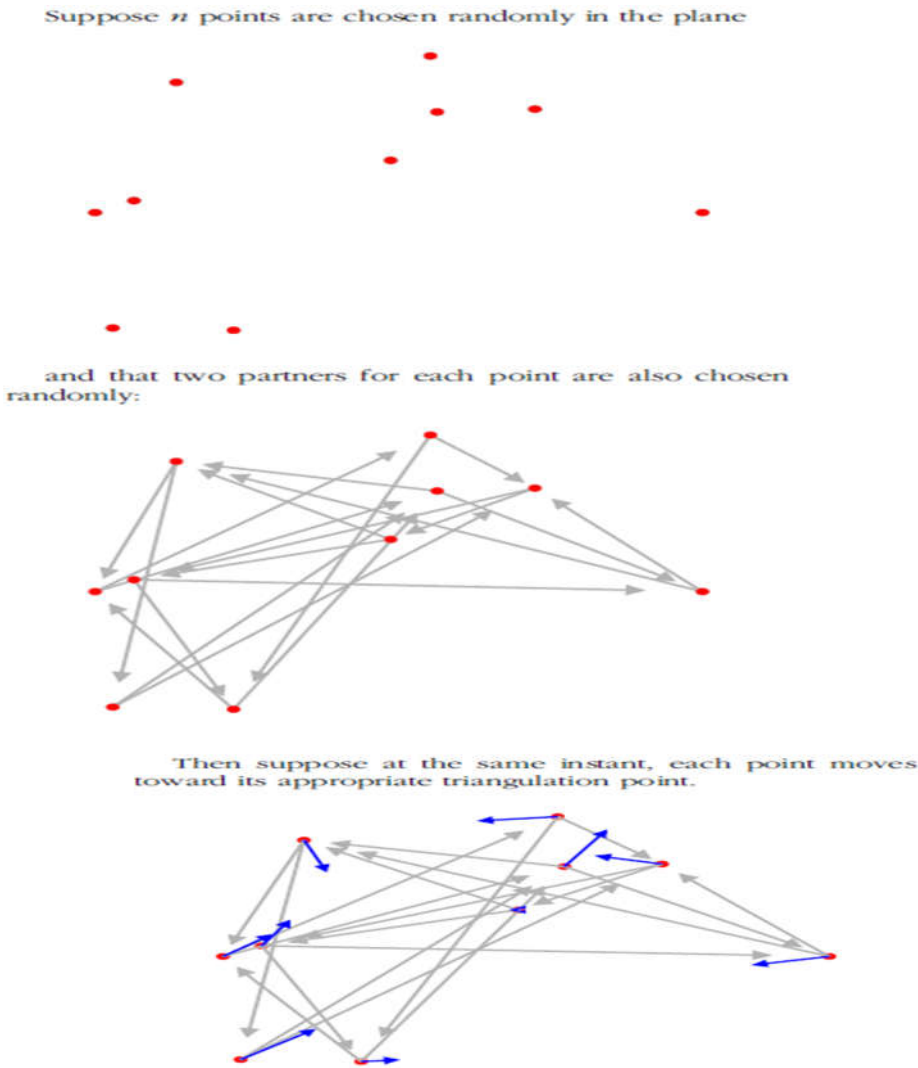


Figure 22

The Boxtrot [35] is a mathematical dance performed on a rectangular grid, where each dancer in a square move to another square based on a set of numbered instructions. For example, if a dancer is in a square labelled with a red number, they move to the square that has the corresponding blue number. This creates various loops of movement, with some dancers returning to their original positions after a certain number of steps, while others remain stationary, revealing interesting patterns in the dance's structure, as in schematics 23 and 24 (c.f., [35]), respectively.

I'll tell you about one more dance, a dance that puzzles me immensely.

Given two numbers, say 5 and 7, the 5×7 boxtrot is a dance on a 5×7 rectangle. We divide the rectangle into 35 squares. We can number the squares by rows,

0	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	32	33	34

and we can number them by columns:

0	5	10	15	20	25	30
1	6	11	16	21	26	31
2	7	12	17	22	27	32
3	8	13	18	23	28	33
4	9	14	19	24	29	34

But suppose we do both:

0	1	2	3	4	5	6
0	5	10	15	20	25	30
7	8	9	10	11	12	13
1	6	11	16	21	26	31
14	15	16	17	18	19	20
2	7	12	17	22	27	32
21	22	23	24	25	26	27
3	8	13	18	23	28	33
28	29	30	31	32	33	34
4	9	14	19	24	29	34

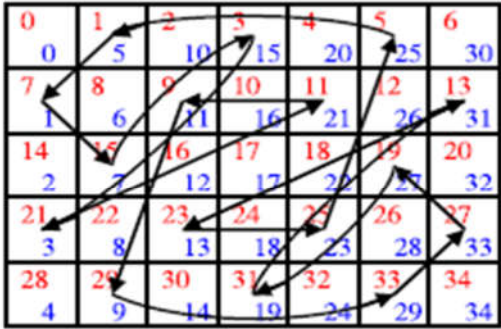
Schematic 23

Now put a dancer in each square. A step in this dance consists in each dancer looking at the red number in their square and then moving to the square that has that number as its blue number. With all the dancers moving at once, it's a bit messy.

Notice first that the dancers in the 0 square and the 34 square don't move. The dancer in the 17 square also doesn't move.

How about the dancer in the square with a red 1? He goes to the square with a blue 1, which has a red 7. In the next step he goes to the square with a blue 7, and that has a red 15. This goes on. The dancer won't be back to his

original square until the dancers have performed 16 steps. Here is his path:



For this size box (5×7), there are two loops of 16 and three dancers who never move—call those loops of 1.

What about a 4×7 box? There you have two loops of 9, two loops of 3, and four loops of 1.

What about a 5×8 box? There you have nine loops of 4, one loop of 2, and two loops of 1.

Isn't this interesting? Here are a few more:

- 4×20 : loops of 39, 39, 1, 1;
- 5×20 : loops of 30, 30, 10, 10, 6, 5, 5, 2, 1, 1;
- 15×20 : loops of 132, 132, 22, 12, 1, 1;
- 13×20 : 36, 36, 36, 36, 36, 36, 36, 2, 2, 2, 1, 1.

Mysterious!

Schematic 24

At first[37], it might seem strange to connect math and dance, as math is often seen as static and not a form of performance. Many mathematical papers focus on the results and do not show the creative process behind them, which includes the interesting ideas and visualizations that mathematicians have in their minds. The author [37] argued that sharing these narratives and analogies, like comparing a system of differential equations to a love story, can make the process of doing mathematics more engaging and relatable.

The system of differential equations

$$\frac{dx}{dt} = y \text{ and } \frac{dy}{dt} = x \tag{1}$$

has an oscillating solution

$$x = \cos(t) - \sin(t) \text{ and } y = -\cos(t) - \sin(t) \tag{2}$$

describes a relationship that oscillates over time, like how two people might interact in a love affair. In this analogy, the values of x and y represent the feelings of two individuals, where one person's love influences the other's response. By using narratives like this, mathematicians can better understand and communicate complex mathematical ideas, making them more relatable and engaging.

The "love affair" system of differential equations describes the interactions between two people, represented by the variables x and y . In this analogy, the equations show how each person's feelings change over time: as one person becomes more in love, the other becomes more disinterested, creating a dynamic relationship. This narrative helps us understand the mathematical solutions in a more relatable way, making the abstract concepts of mathematics feel more human and engaging.

In Figure 25 (c.f., [37]), the height of the output represents the level of love between two people in a mathematical model described by differential equations. Person y is portrayed as a fickle lover who becomes more excited by rejection, while person x becomes more infatuated with y as y 's affection grows. This narrative helps us understand the behavior of the equations in a more relatable way, making the math more engaging and intuitive.

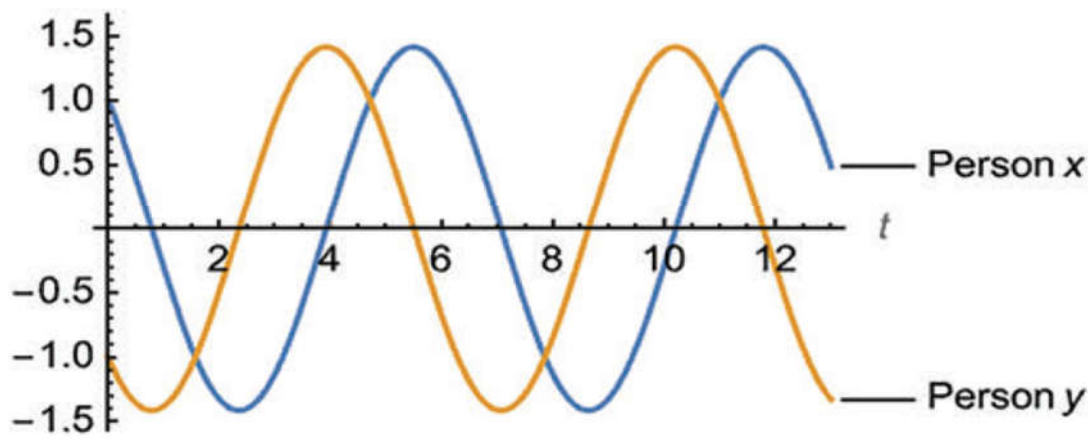


Figure 25

The author[37] suggested that while the mathematical equations are correct and complete, we can gain a deeper understanding by framing them within a narrative. Specifically, [38] illustrated how the system of differential equations can be interpreted as a love story between two people, where their emotions and interactions are represented by the mathematical relationships. This narrative approach helps to make the concepts more relatable and engaging, allowing us to visualize the dynamics of the equations in a more meaningful way.

Indeed [37], this showcases how dance can be an effective way to communicate and explain complex mathematical ideas to people who may not appreciate math. By using dance[37], educators can create a joyful and engaging experience that helps reduce math anxiety and makes the subject more relatable. The author [37]emphasized the importance of knowing the audience and using culturally relevant methods, like dance, to improve the public's perception of mathematics.

Figure 26 (c.f., [37]), "Video stills from 'Algebra, Geometry, and Topology: What's the difference?'" refers to images taken from a video that visually compares the three branches of mathematics—algebra, geometry, and topology. In the video[37], the creator uses these stills to help the audience understand how each field is distinct, like how different scientific fields like biology, chemistry, and physics are unique in their approaches. This comparison aimed to make complex

mathematical concepts more relatable and easier to grasp for those who may not have a strong background in mathematics.



Figure 26

In [39], the author created a Math-Dance video called “Algebra, Geometry, and Topology: What’s the difference?” which is available on YouTube and received an honourable mention in the NSF video competition “We Are Mathematics.” The video aimed to explain the differences between the fields of algebra, geometry, and topology by using analogies that relate these mathematical areas to familiar scientific fields like biology, chemistry, and physics. This approach [37] helped making complex mathematical concepts more accessible to the public (Figure 27, (c.f., [37])).



Figure 27

In the video stills (Figure 28(c.f., [37])), the creator uses sound and visual elements to differentiate between the fields of mathematics: geometry, topology, and algebra. Each field is represented with unique colours, props, and music styles—geometry uses a rigid hula hoop, topology features a

flexible fabric loop, and algebra is depicted with dynamic laser lights. This approach helps the audience understand the distinct characteristics of each mathematical area without needing a deep technical background.

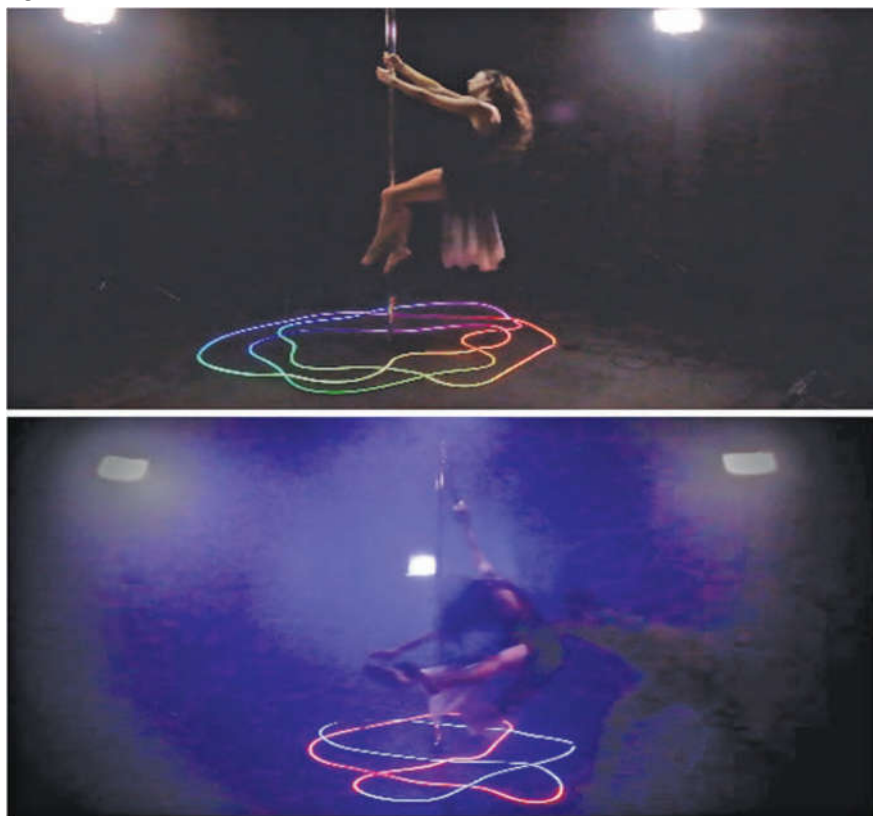


Figure 28

In her Math-Dance projects, the author[37] often gets asked how she comes up with concepts, how these projects influence her mathematical work, and how long it takes to create a video. She explained that developing a Math-Dance requires practice and the belief that it's possible to express math through dance. By simplifying complex mathematical ideas into relatable stories and movements, she enhances her communication skills, although she notes that her deep understanding of math hasn't changed significantly.

Research in embodied mathematics emphasizes the importance of the body in understanding mathematical concepts[40], yet there is still a need to find effective ways to help learners recognize and use this connection. This research examined how professional dancers utilized their physical practices, termed "physical research," to explore and express mathematical ideas during a choreographic activity related to the 2016 Rio Olympics. By comparing the dancers' approach to that of 8th grade girls[40], the study highlights how engaging the body in learning can enhance mathematical reasoning and foster creativity in a collaborative environment.

Ensemble activity emphasizes the importance of collaboration in learning[40], particularly in mathematics, where physical engagement enhances understanding. Scholars have different views on how the body contributes to learning: the conceptualist perspective sees the body as a reflection of cognitive processes[40], while the interactionist view focuses on learning through interaction with others. By using movement and teamwork[40], learners can gain deeper insights into mathematical concepts, as demonstrated by dancers finding the midpoint of a line through coordinated actions, which fosters a richer understanding of the relationships between mathematical objects.

Expanding possibilities in mathematics education [40] through ensemble learning emphasizes how individuals' movements are interconnected within a group. This approach [40] allows participants to gain insights into mathematical concepts from their own perspectives while collaborating with others, fostering new relationships and trust. By focusing on how groups learn

together[40], this method highlights the importance of empathy and trust in creating effective learning environments, particularly in mathematics, where these qualities are often lacking.

Framing movements [40] as proposals means that each movement is seen as an idea that can be changed or built upon by others. Choreographers give verbal prompts to dancers to modify their movements[40], like suggesting they perform in a diagonal line instead of straight. The dancers then respond to these prompts with their bodies, creating new movements together until they develop a shared set of movements, or "movement vocabulary," that the choreographer can further refine(Figure 29, c.f., [40]).

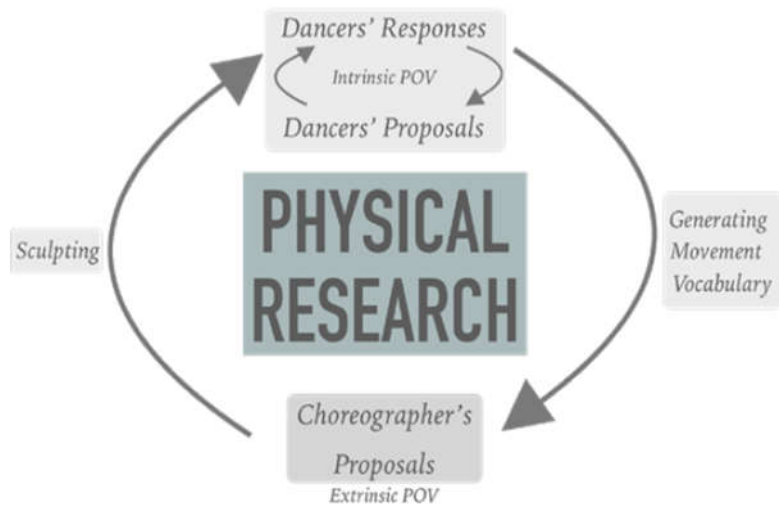


Figure 29

After the dancers created a triangle-folding pattern while standing, they felt the need to enhance their movement vocabulary. Faith proposed a new formation called the "dumpling airbag," where all four dancers would move together to puff their prop into a dumpling shape. This collaborative approach allowed them to discover a new way to find the midpoint of their prop, making it easier to manipulate and integrate their movements as they continued their physical research, as depicted by Figure 30 (c.f., [40]).

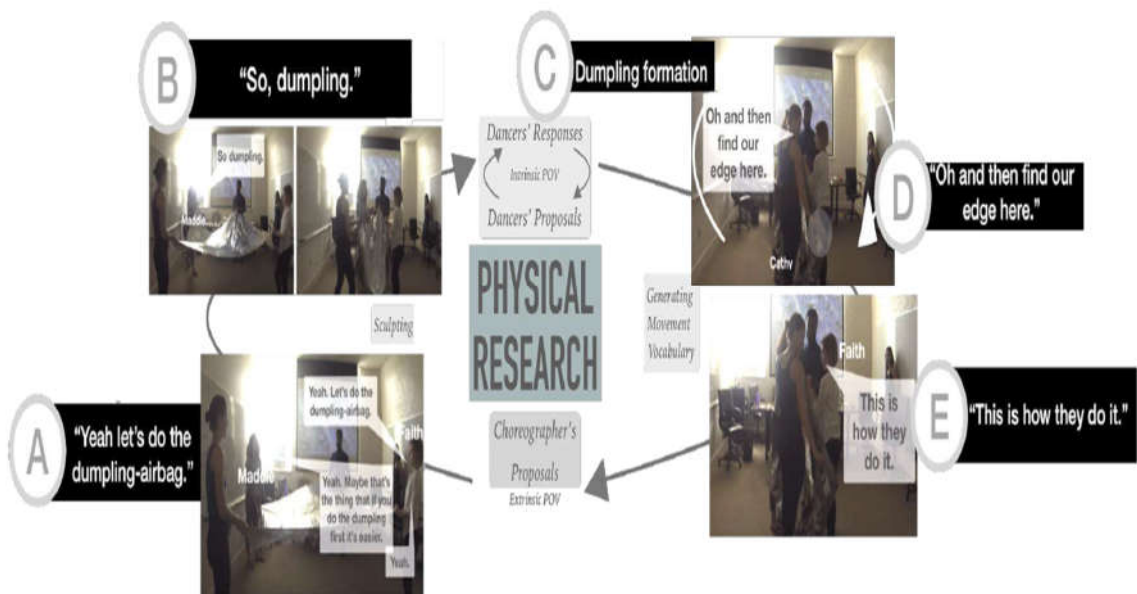


Figure 30

The importance of using physical activities[40], like folding props, to enhance mathematical understanding among students, as seen in the example of Ava and Octavia, was reasoned. The discussion about the properties of a triangle illustrates how engaging in hands-on activities can lead to deeper insights and collaborative learning. By highlighting the role of embodied sensemaking and collective inquiry[40], the paper suggests that integrating physical research into math education can help students connect their everyday experiences to mathematical

Ava and Octavia rely on traditional school-based ideas about geometry[41], which they learned from textbooks, to discuss the triangle they created by folding a large square. They refer to established mathematical rules,[40,41] which makes sense given their education, but this approach limits their ability to fully engage with the triangle's properties in a real-world context. Instead of exploring the triangle based on their physical experience [40,41], they stick to a rigid understanding of geometry that doesn't account for the dynamic nature of their activity.

An exploratory of the connection between dance and mathematics, suggesting that engaging in dance can enhance spatial abilities, which are important for developing mathematical skills, was provided by [42]. Researchers[42] worked with first-year primary school students over a month, using creative dance and movement activities to see if these would improve both their mathematical and spatial skills. While the results showed some improvement in mathematical skills, there was no significant change in spatial abilities[42], prompting the researchers to investigate why this was the case.

Zoltán Dienes [42,43] was a notable Hungarian researcher who pioneered the integration of dance and music into mathematics education, illustrating how physical movement can reveal connections between abstract mathematical and musical concepts. His work[42,43], along with that of American researchers, emphasizes that incorporating creative and interactive methods, like dance, can enhance learning in mathematics, leading to improved self-esteem and social skills among students. Programs like SHINE aim to encourage girls to pursue STEM careers by using movement-based activities to teach mathematical concepts[42,43], demonstrating significant improvements in their performance and confidence.

"An example measuring estimation – from the input set" refers to a specific task designed to assess a child's ability to estimate quantities or measurements. In the context of the study[42], this task is part of a broader set of activities aimed at evaluating mathematical abilities, focusing on how well children can make approximations without relying on exact calculations. The task is structured to encourage children to use their intuition and understanding of numbers[42], helping educators gauge their estimation skills effectively, as demonstrated by Figure 31 (c.f., [42]).

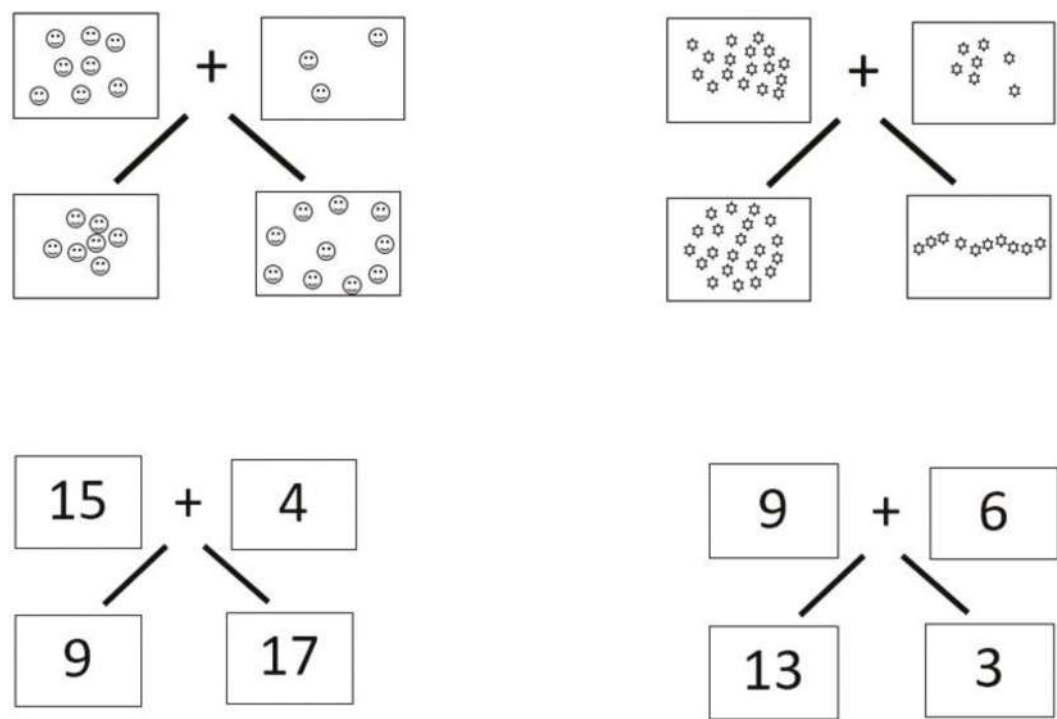


Figure 31

The input and output task sets in the study [42] were designed to assess mathematical abilities and were largely similar in structure, but they included some differences to ensure that students could not simply recall previous answers, as shown by Figure 32(c.f., [42]). This approach helps to evaluate their understanding and problem-solving skills rather than just their memory. Additionally[42], while a general time limit was set for completing each task, most students finished before the time was up, indicating their proficiency in the material.

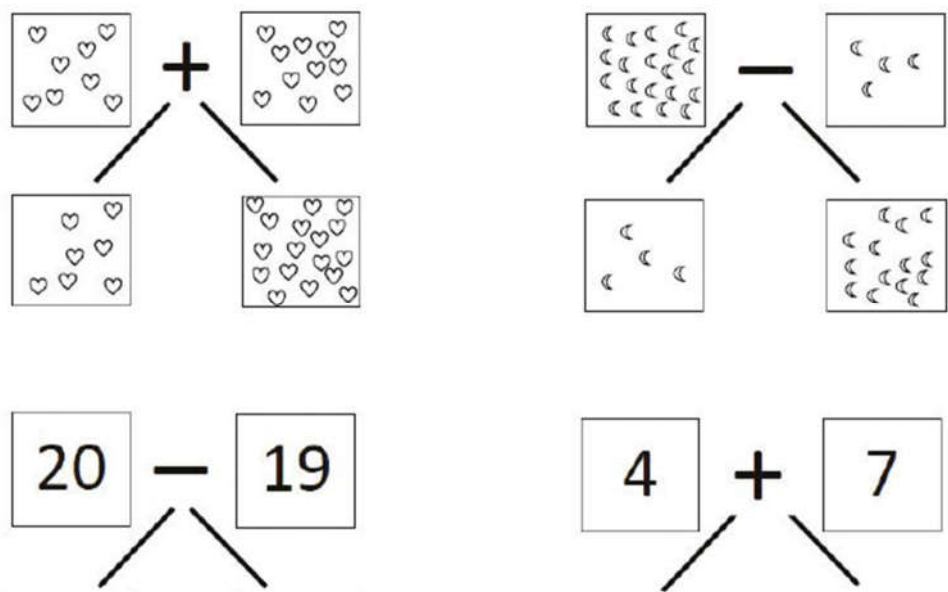


Figure 32

"An example for filling in equality and inequality" refers to a task where students are asked to determine whether two expressions are equal or to identify the correct inequality symbol (like $<$, $>$, or $=$) to compare them(Figure 33(c.f., [42])). This type of exercise helps students develop their understanding of mathematical relationships and the concept of balance in equations. In the context

of the study, such tasks are likely designed to assess students' spatial and mathematical reasoning skills, which are important for their overall cognitive development.

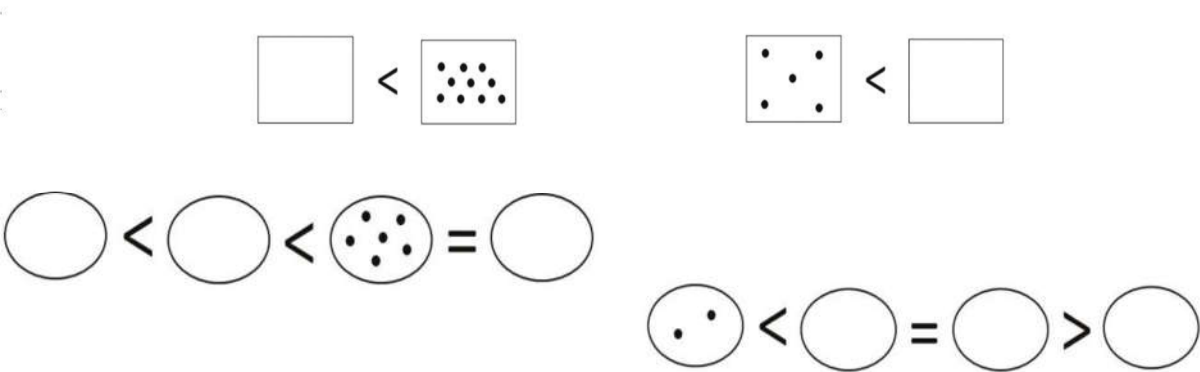


Figure 33

Figure 34 (c.f., [42]), visualizes refers to "An example for specific operations", a task or activity designed to assess mathematical or cognitive skills in students, such as first graders. In the context of the study, this could involve exercises where students perform operations like addition or subtraction using visual aids or manipulatives to enhance their understanding. These specific operations help researchers evaluate how well young learners grasp fundamental concepts in mathematics and spatial reasoning.

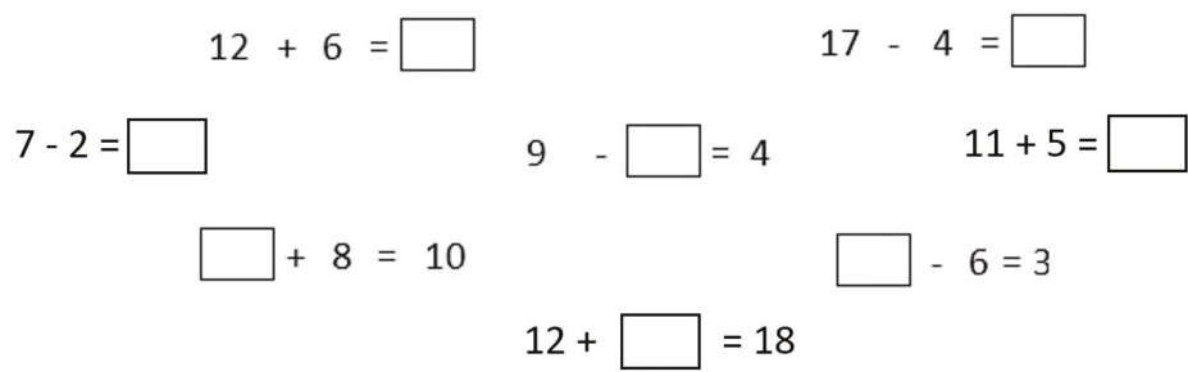


Figure 34

In the context of the study[42], in reference to research by [44] that focuses on assessing mental rotation skills, which is the ability to visualize how objects look when rotated in space. They used characters from stories and printed capital letters as stimuli for this test[42,44,45], allowing participants to engage with familiar shapes while evaluating their spatial reasoning abilities. This approach helps researchers understand how well individuals can manipulate and interpret visual information in different orientations, as in Figure 35 (c.f., [44]).

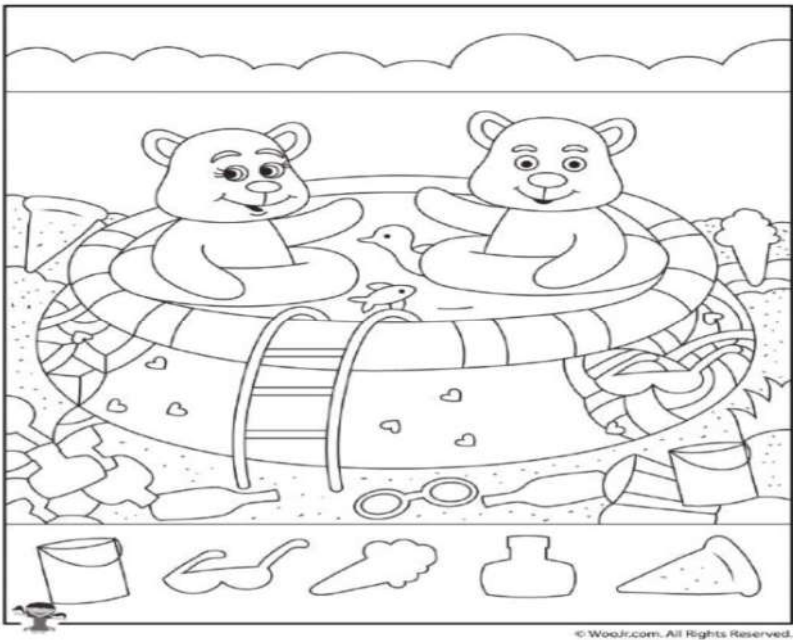


Figure 35

Mental rotation tasks [42]are cognitive exercises that assess a person's ability to visualize and manipulate objects in their mind. In these tasks[42], participants are typically shown two or more shapes and must determine if they are the same shape but rotated in space or different shapes altogether. This type of task is important for understanding spatial reasoning skills, which are relevant in fields like mathematics, engineering, and architecture, as depicted in Figure 36 (c.f., [42]).

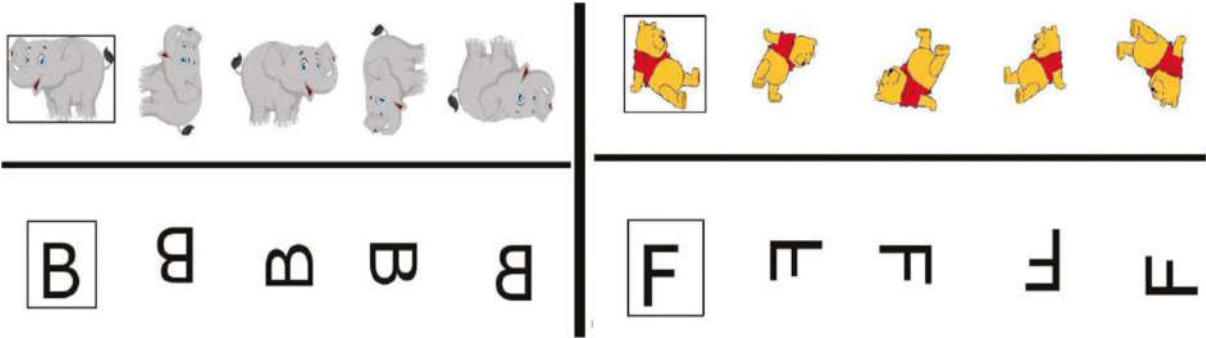


Figure 36

An example of horizontal tests[42] refers to a specific type of assessment used to evaluate children's spatial reasoning abilities. In this context[42], children are asked to "fill up" five overturned bottles by colouring water in them, which tests their understanding of perspective and how objects relate to one another in space. This task [42] helps researchers understand how children perceive and manipulate spatial information, like other tests that assess mental rotation and perspective-taking skills. The reader can visualize this through Figure 37 (c.f., [46]).



Figure 37

In brief, this consolidates the connection between spatial skills and mathematical abilities[42], suggesting that these skills overlap in the brain and can be developed through activities like creative dance. Hence, the author [42] has conducted an experimented combining dance and drama games to improve first graders' math skills, but the results were inconclusive due to factors like a small sample size and varying difficulty levels of tasks. They emphasize the importance of consistent testing conditions and tailored instructions to effectively enhance children's spatial and mathematical development.

The "Maths in Motion" (MiM) project explores how dance and body movement can be effective tools for teaching mathematics[47], focusing on concepts like rotational symmetries and the properties of triangles. The workshop includes a brief introduction, two movement activities where participants engage in creative assignments[47], and a discussion to reflect on their experiences. One specific activity, "Dancing Snowflakes," allows participants to learn about geometric symmetries while enhancing their cooperation and creativity through movement-based performance.

"Dancing Snowflakes" [47] is an interactive activity designed to teach participants about rotational symmetries and geometry through creative movement and dance. During this 30-minute session[47], participants work together to explore these mathematical concepts while enhancing their cooperation, problem-solving skills, and creativity. The activity also includes a warm-up that focuses on mirroring movements and communication through body language[47], helping participants become more comfortable with movement and collaboration.

The activity described in the workshop[47], which focuses on using dance to teach concepts like rotational symmetries, is like an exercise called "Threesies" found in [48]. This connection highlights how both activities aim to integrate movement with mathematical learning, encouraging participants to explore geometry through creative expression. Additionally[47], the reference to a workshop by [49], suggested that this approach has been previously discussed and developed in the context of math education through dance.

The warm-up activity in the "Maths in Motion" [47] workshop focuses on helping participants understand basic mirroring symmetry through movement. By working with a partner [47], participants learn to observe and imitate each other's movements, which fosters communication and creativity. This exercise also encourages comfort with physical interaction and explores various body positions[47], laying the groundwork for deeper engagement with mathematical concepts like symmetry.

In this exercise (Figure 38,(c.f., [47])) , participants work in pairs to practice mirroring each other's movements while listening to slow, melodic music. One person leads by making movements, and the other follows as if they are a mirror image[47], then they switch roles to maintain a flow of

movement. The goal is to reach a state where both participants are in sync and fully focused on their movements without speaking[47], enhancing their connection and awareness of spatial dynamics.



Figure 38

Recalling that Rotational symmetries[47], refer to arrangements where a group can rotate around a central point and still look the same from different angles. In this new activity[47], participants form a circle while holding hands and are encouraged to explore various positions that maintain this symmetry. They then practice transitioning between these positions[47], using prompts if they need help to spark their creativity(See Figure 39, (c.f., [47])).



Figure 39

In a different setup [47],participants in the program are instructed to create various body postures at three different levels: low (bottom), medium (middle), and high (top). These postures can include movements like a "crab kick," which involves using the hands and feet to lift the body off the ground, or a "gorilla hop," which mimics the movement of a gorilla. This activity (Figure 40, c.f., [47]))

helps integrate physical movement with mathematical concepts, enhancing both understanding and engagement in the learning process.

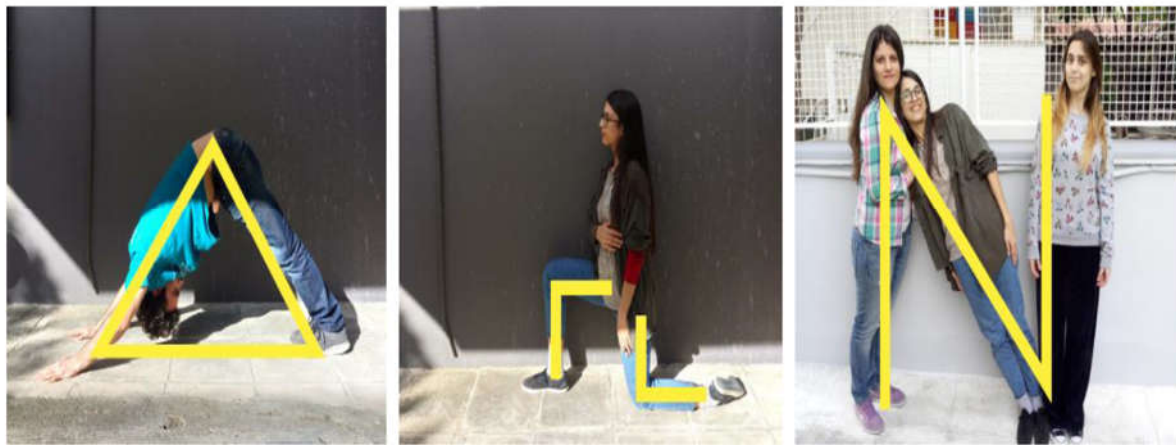


Figure 40

Participants[47] in the program are instructed to create various body postures at three different levels: low (bottom), medium (middle), and high (top). These postures can include movements like a "crab kick," which involves using the hands and feet to lift the body off the ground, or a "gorilla hop," which mimics the movement of a gorilla. This activity [47] helps integrate physical movement with mathematical concepts, enhancing both understanding and engagement in the learning process.

The project "Maths in Motion" connects experts in mathematics [47–61], dance, and education to explore how dance can enhance math learning. Research in cognitive neuropsychology shows that physical activity[47–61], like dance, positively affects cognitive skills and academic performance, suggesting that incorporating movement into math education can improve understanding and retention of mathematical concepts. Despite this[47–61], traditional math education often neglects the body, focusing solely on intellectual tasks, which highlights the need for more embodied approaches in teaching mathematics.

The curtain is drawn over this part with the following elegant piece of art :

My Dance Is Mathematics

*Down, down, down into the darkness of
the grave Gently they go, the beautiful,
the tender, the kind; Quietly they go, the
intelligent, the witty, the brave. I know.
But I do not approve. And I am not
resigned.*

From "Dirge without Music" by Edna St Vincent Millay; offered
by Hermann Weyl in a Memorial Address for Amalie
"Emmy" Noether on April 26, 1935 at Bryn Mawr
College.

They called you *der* Noether, as if
mathematics was only for men. In 1964,
nearly thirty years past your death, I saw

you in a spotlight
in a World's Fair mural, "Men of Modern Mathematics."

Colleagues praised your brilliance--but after
they had called you fat and plain, rough and
loud. Some mentioned kindness and good
humor though none, in your lifetime,
admitted it was you who led the way in
axiomatic algebra.

Direct and courageous, lacking self-
concern, elegant of mind, a poet of
logical ideas.

At a party when you were eight
years old, you spoke up to solve a
hard math puzzle. Fearless, you set
yourself apart.

I followed you and saw you choose
between mathematics and other
romance. For women only, this
exclusive standard.

I heard fathers say, "Dance with
Emmy-- just once, early in the
evening. Old Max is my friend; his
daughter likes to dance."

*If a woman's dance is mathematics,
she dances alone.*

Mothers said, "Don't tease. That strange one's
heart is kind. She helps her mother with the
house
and cannot help her curious mind."

Teachers said, "She's smart but
stubborn, contentious and loud, a
theory builder not persuaded by
our ideas."

Students said, "She's hard to follow, bores me."
A few stood firm and built new
algebras on her exacting
formulations.

In spite of Emmy's
talents, always there
were reasons not to
give her rank
or permanent
employment. She's a
pacifist, a woman. She's
a woman and a Jew.
Her abstract
thinking is
female and
abstruse.

Today, history books proclaim that
Noether is the greatest mathematician
her sex has produced. They say she was good
... for a woman.

First published in *Mathematics Magazine* in December 1995. <https://www.jstor.org/stable/2690927>

Note: Amalie "Emmy" Noether was born in Germany (1882) and educated there; she fled the Nazis to the US in 1933 and died on April 14, 1935, in Bryn Mawr, Pennsylvania. When the *NY Times* failed to publish an obituary following the death of noted algebraist Amalie "Emmy" Noether, Albert Einstein corrected the omission with a letter to the editor (noting Noether's accomplishments) that was published on May 5, 1935. In addition to his praise for one of the most accomplished mathematicians of all time, Einstein said this of mathematics: "Pure mathematics is, in its way, the poetry of logical ideas." In the 1960s, as I climbed into the male-dominated world of mathematics, Emmy Noether was one of my heroes. Many years later I wrote this poem.

PE-Based Approach to Teach Mathematics

Moderate-to-vigorous physical activity (MVPA) levels in children are currently very low[62], prompting the UK government to fund primary schools to provide 30 minutes of in-school MVPA for all students. One promising method to increase MVPA is through physically active learning (PAL), which incorporates movement into academic lessons. This study aims to evaluate the effects of a six-week PAL program called "Maths on the Move" on children's physical activity and math performance, while also exploring how children feel about participating in this program.

The findings from the study [62] indicated that the 'Maths on the Move' (MOTM) program effectively enhances primary school children's physical activity (PA) levels and academic performance in math. Additionally, the program fosters positive attitudes among students, as they reported increased willingness, enjoyment, and engagement during the learning sessions. Overall, the MOTM initiative not only promotes physical health but also supports better educational outcomes.

Research [63] suggested that physical activity (PA) during school can help improve children's academic performance, but most studies have focused on traditional forms of PA, like gym class. This study [63] aimed to explore the effects of combining juggling with math lessons on children's ability to memorize multiplication tables and their enjoyment of the lessons. While the juggling did not significantly improve mathematics performance[63], it did increase the enjoyment of the students,

indicating that integrating physical activity into learning could be beneficial for engagement in the classroom.

Another study [64] found that most primary school children do not meet the recommended 30 minutes of moderate-to-vigorous physical activity (MVPA) during school hours. Researchers tracked 122 children using accelerometers to measure their activity levels throughout the school day, discovering that they averaged only about 18 minutes of MVPA[64], with morning lessons focused on maths and English being less active compared to afternoon lessons. Breaks and lunch were identified as the most active times, highlighting the need for more engaging physical activities during academic lessons to improve overall activity levels.

Figure 41 (c.f., [64]) provides a visualization on how different schools have varying percentages of students who engage in different amounts of moderate-to-vigorous physical activity (MVPA) during school hours. Specifically[64], it categorizes students based on the time they spend in MVPA each day, with groups for those who achieve 0–9 minutes, 10–19 minutes, 20–29 minutes, and 30 or more minutes. This variability [64] highlights the differences in physical activity levels among students across different schools, suggesting that some schools may provide more opportunities for physical activity than others.

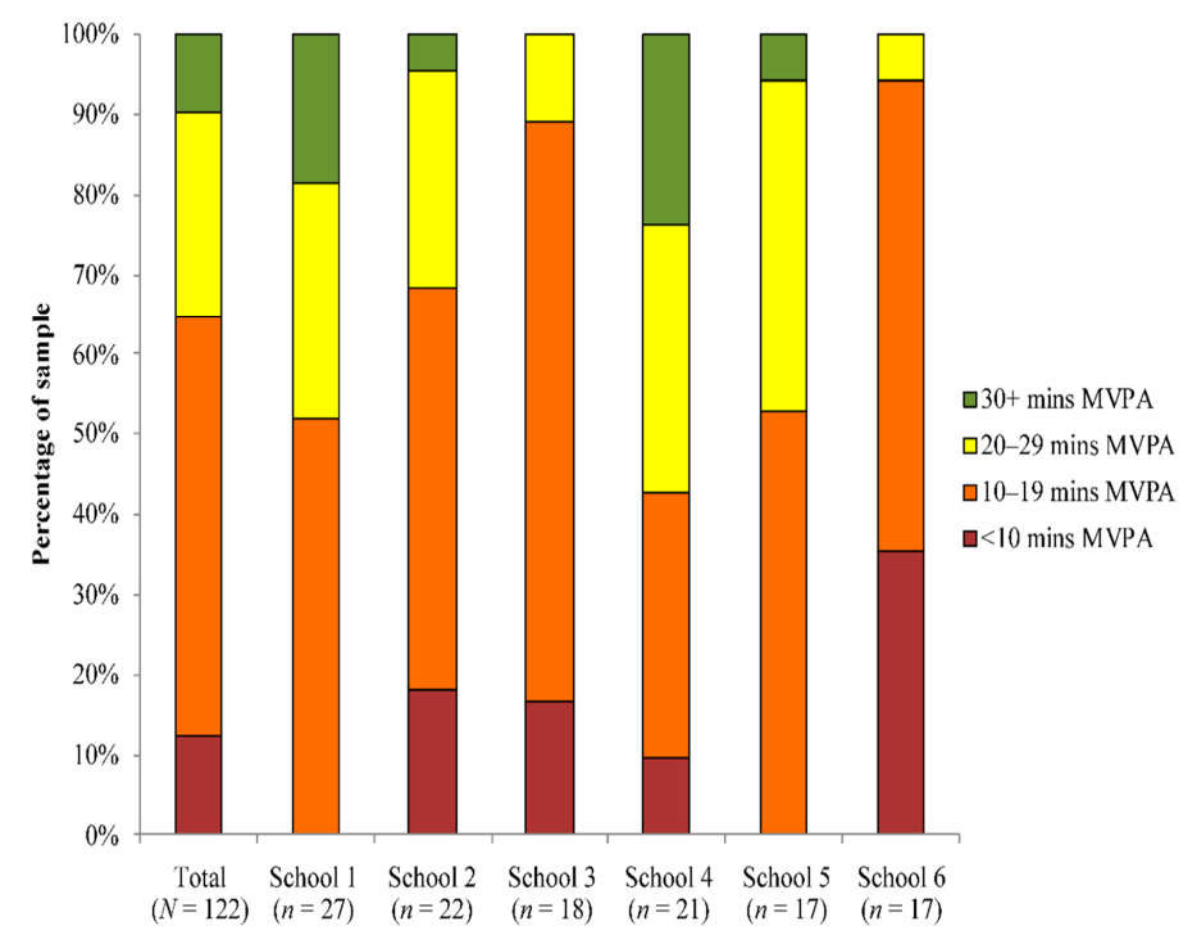


Figure 41

Figure 42 (c.f., [64]) showcases the analysis of the frequency of different subjects taught in schools during specific lesson periods. It found that Maths and English were the most commonly taught subjects, with an average of seven sessions of Maths and nine sessions of English per school each week. In the first two lessons, Maths and English accounted for the majority of classes, while a wider variety of subjects, including PE and others, were covered in the third lesson.

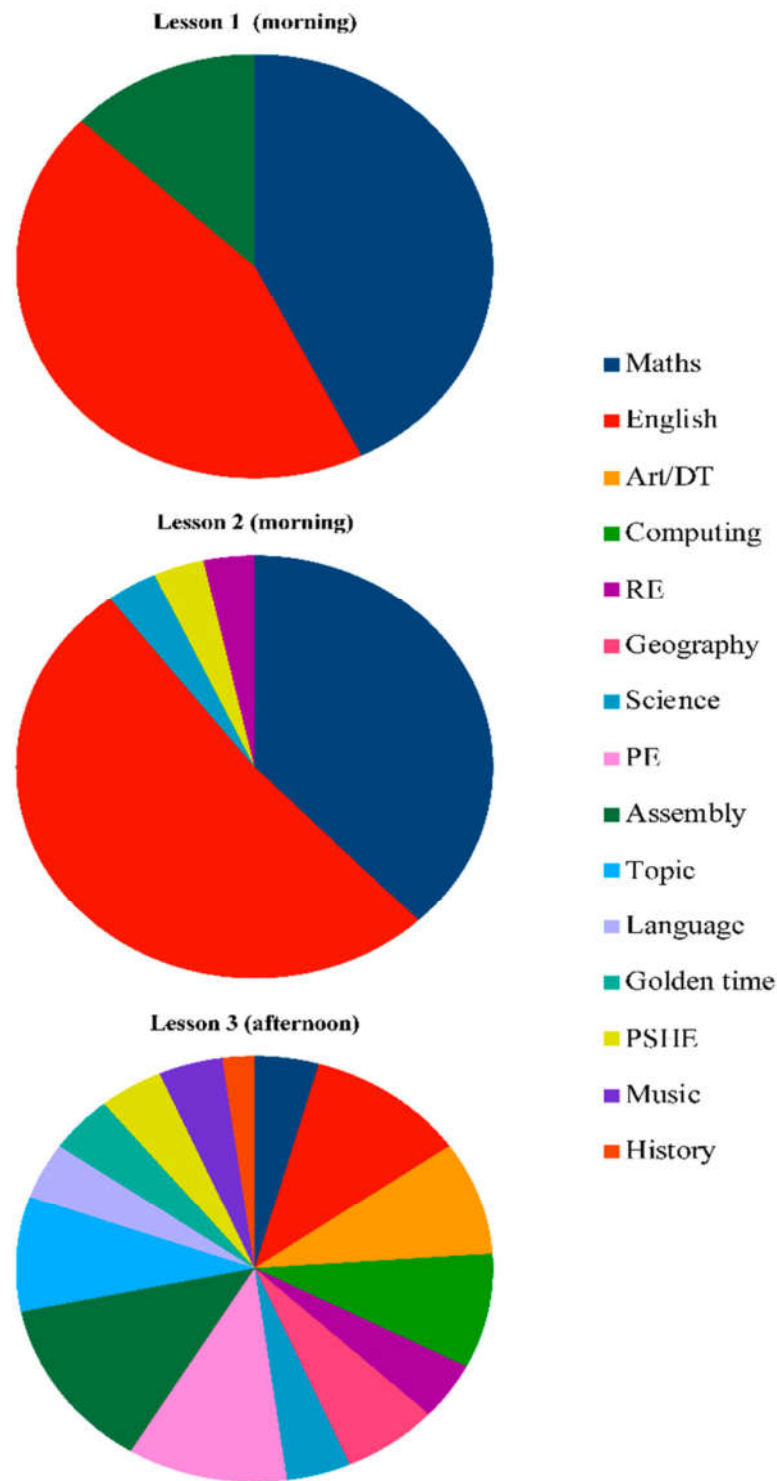


Figure 42

PSTs, or Primary School Trainees, were selected from 24 different Initial Teacher Education (ITE) programs across England to participate in the study[65]. They were invited to join through their PE subject coordinators, who acted as gatekeepers and had previously shown interest in the research through a new network focused on primary PE teacher education. All participants were enrolled in a primary education program that leads to obtaining Qualified Teacher Status (QTS), which is necessary for teaching in schools.

The "breakdown of the sample across each region in England" [65] refers to how the participants in the study are distributed geographically, as depicted by Figure 43(c.f., [65]). The data shows the

number of participants who completed school-based placements for varying durations, ranging from one week to twelve weeks, across different regions. This information helps to understand the representation of different areas in the study and highlights the focus on regions with a higher concentration of Initial Teacher Education (ITE) providers.

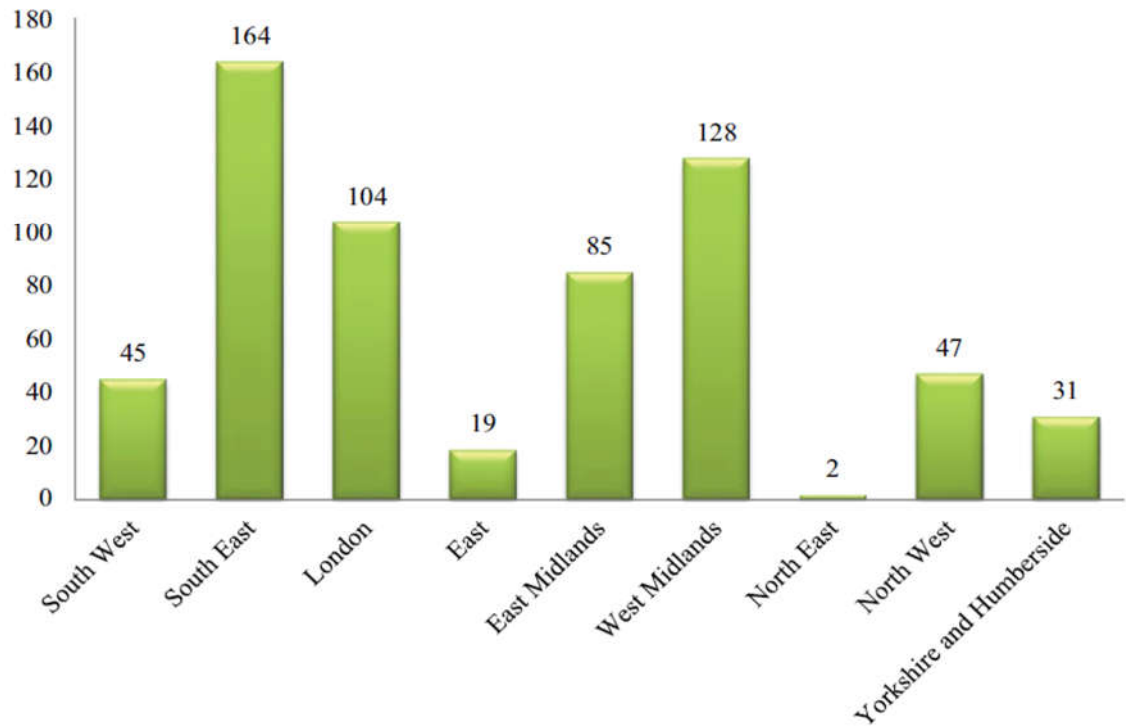


Figure 43

This qualitative case study[66] investigated how future early childhood educators view the use of play-based teaching strategies in the Foundation Phase, which includes pre-Grade R and Grade R classes. The study[66] key findings proved that these educators believe that play-based learning enhances students' thinking, problem-solving skills, and engagement in subjects like mathematics and physical education. By using interviews and narratives, the researchers aim to highlight the benefits of combining play-based methods with traditional teaching to create a more effective learning environment. This trigerrrs highlights that play fosters creativity and teamwork among students[66], helping them build confidence while learning. The findings suggest that integrating play into lessons can enhance students' motivation and understanding of mathematics, and emphasizes the importance of providing equal resources and training for teachers across different schools.

In a nutshell [66], teaching mathematics and physical education (PE) through play-based methods is shown to be effective and engaging for students, making learning more enjoyable and relevant. To ensure all schools have equal access to resources[66], it's important to centralize the procurement strategy, so that every school, regardless of its financial situation, can provide the necessary materials. Additionally[66], both pre-service (future teachers) and in-service (current teachers) training should be assessed to ensure that the teaching methods are appropriate for young learners and support this hybrid approach in the early education phase.

The Moving Maths study[67] aimed to boost physical activity and learning among third-grade students in Finland by incorporating physically active math lessons. In this study, 397 children and 22 teachers participated in two types of lessons: one that integrated physical activity directly into math learning and another that used short physical activity breaks. The findings showed[67] that

students were emotionally and socially engaged, and while they concentrated well on their work after the activities, some teachers were unsure about the overall learning outcomes, suggesting that active lessons can reduce sedentary behavior and enhance student engagement.

Figure 44 (c.f., [67]) visualizes the summary of the main results in the Integrated Physical Activity (PA) group highlights how incorporating physical activity into math lessons positively affected student engagement. Most teachers [67] observed that students became more enthusiastic about learning after they adjusted to this new approach, especially during activities that were competitive or involved movement outside the classroom. However[67], some students, particularly those needing extra support, struggled with this method and found it challenging to stay engaged, especially during less dynamic activities or when instructions were too complex.

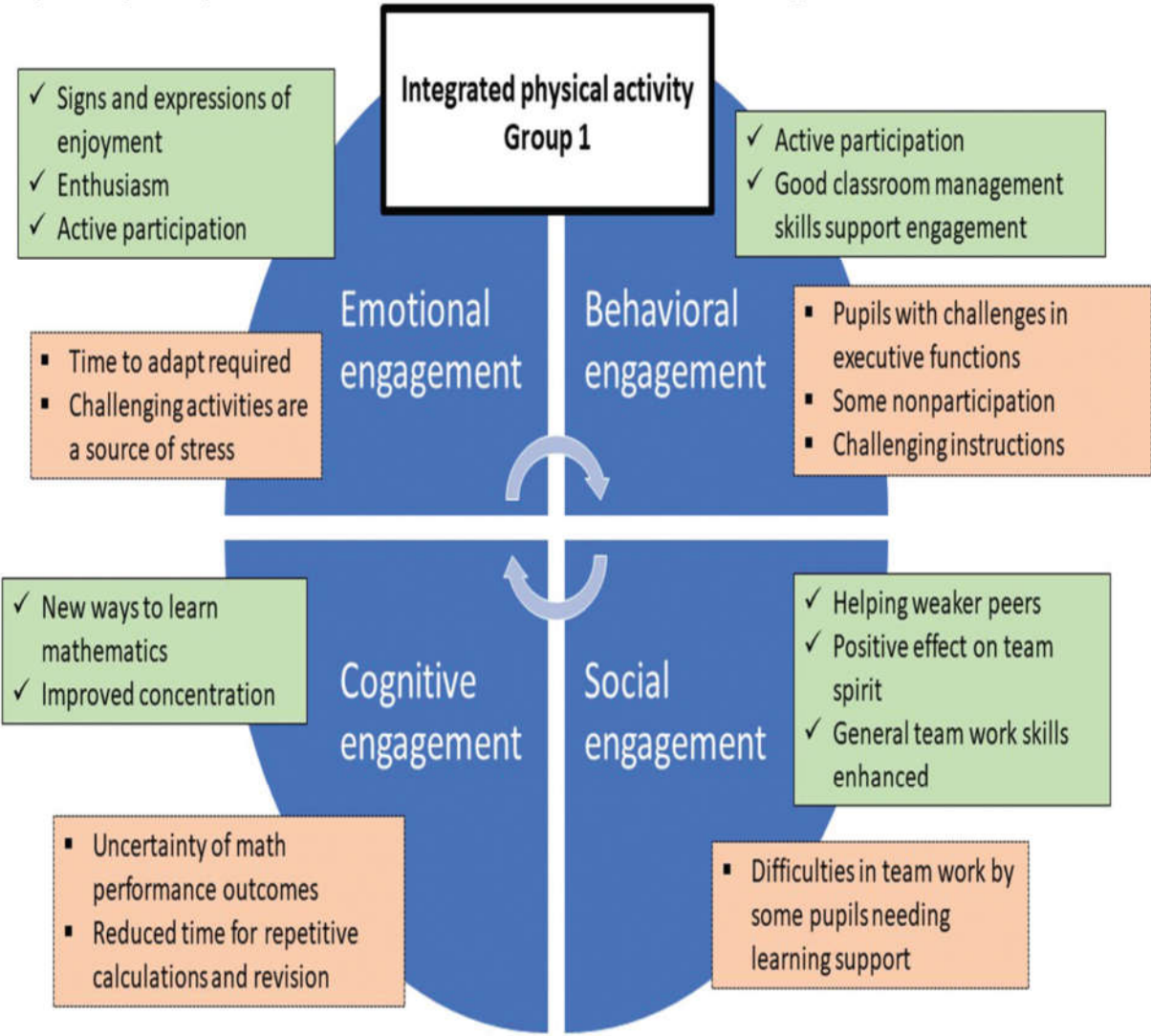


Figure 44

Figure 45 (c.f., [67]), highlights teachers' observations regarding student engagement during physical activity (PA) breaks. Some teachers noted that certain students[67], particularly those needing learning support, initially struggled with emotional engagement but showed improvement over time, while others remained consistently engaged. Overall[67], most teachers reported good behavioral engagement, with students participating enthusiastically and following instructions, although challenges arose for those who were distracted or had difficulty understanding the activities.

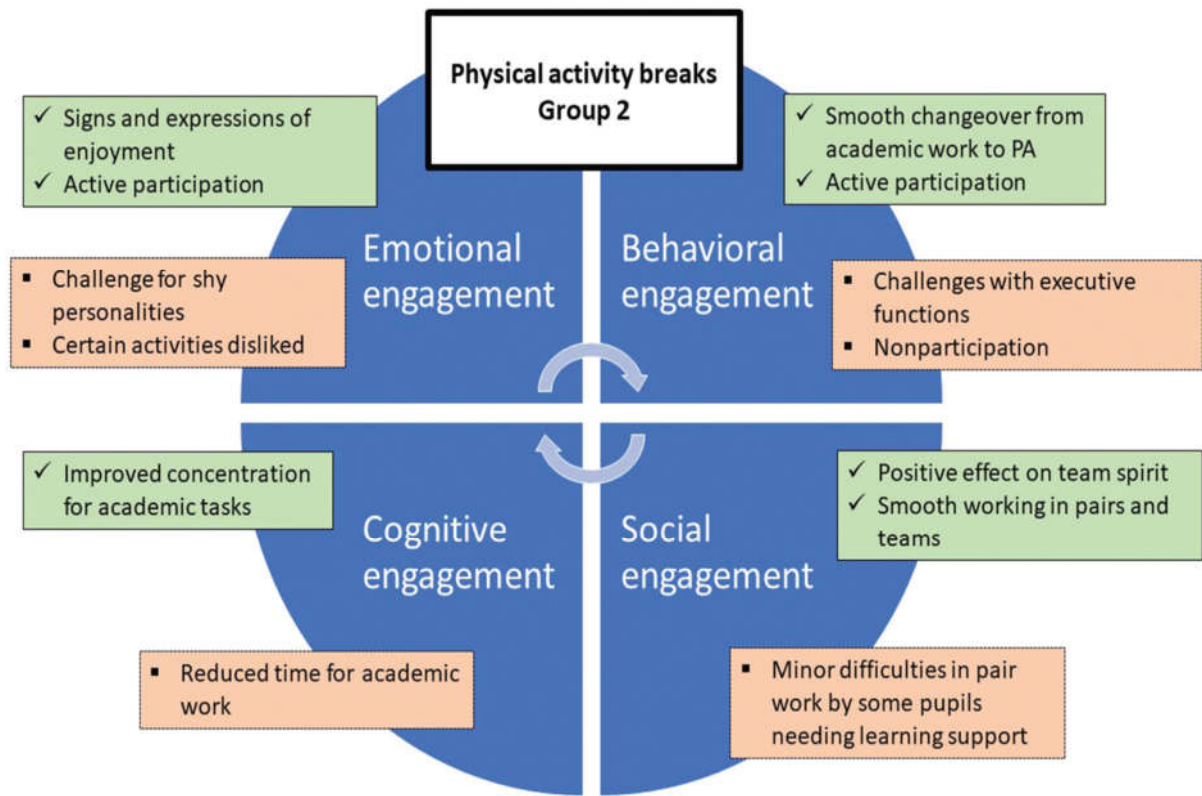


Figure 45

The study [67] has successfully managed to show how teachers evaluated their students' cognitive engagement in math by looking at whether the students met their learning goals. They used specific tests created by the study book publisher, which is a standard practice in Finnish primary schools. However[67], four out of seven teachers were uncertain or critical about the effectiveness of the activities, with some feeling that these activities took away valuable time needed for basic calculations and more complex topics, leading them to sometimes revert to traditional lesson plans.

This authors [68] have used Cultural Historical Activity Theory (CHAT) to analyze the challenges faced by a young teacher who is teaching mathematics outside of their main area of expertise in a small rural school in Australia. By examining interview transcripts[68], the authors identify key issues and changes over time related to this out-of-field teaching, as well as how it connects to regular teaching practices. They also explored the emotional and identity aspects of the teacher's experience[68], highlighting the complexities involved in teaching subjects for which one may not be fully prepared.

In the second generation of Cultural-Historical Activity Theory (CHAT)[68], the focus is on how cultural and historical factors shape the rules, roles, and community interactions that influence group activities. These elements are visually represented as points on interconnected triangles (Figure 46 (c.f., [68])) , which helps illustrate how they work together to mediate collective actions within a community. This framework emphasizes the importance of understanding the social context in which learning and engagement occur.

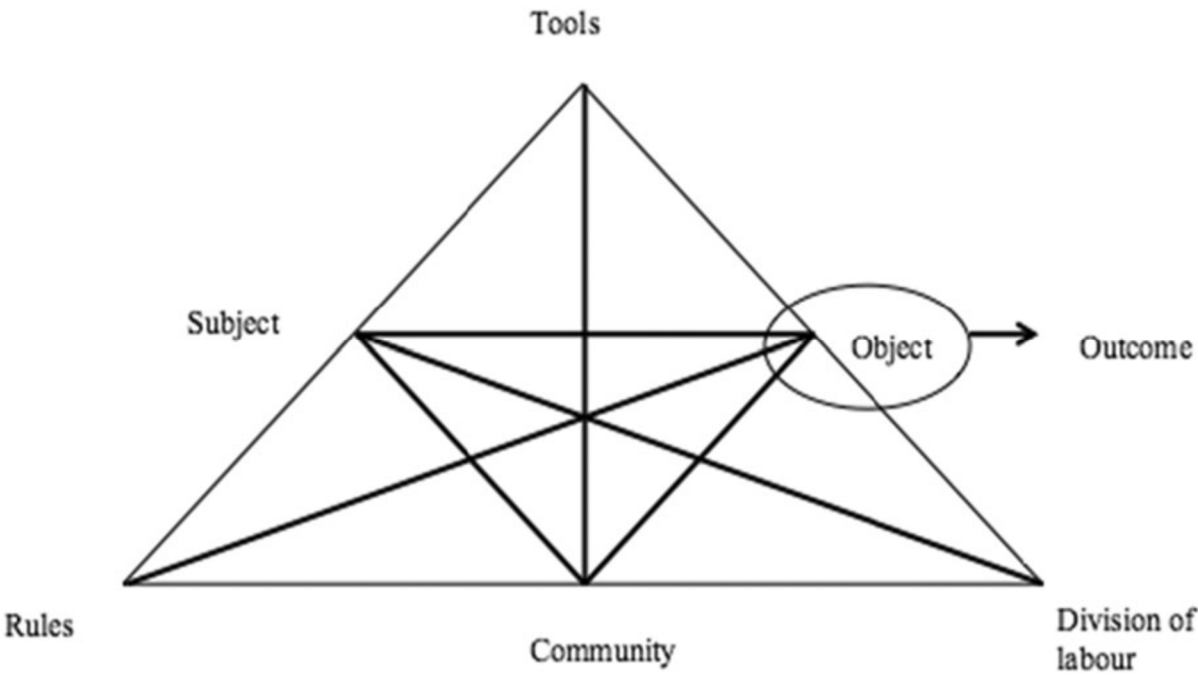


Figure 46

Figure 47 (c.f., [68]) shows two specific teaching activities of Bobby over a three-year period, particularly emphasizing his experience teaching mathematics outside of his primary area of expertise, referred to as "out-of-field" teaching. It highlights how Bobby's sociocultural background and emotional responses influenced his teaching practices and identity, as represented in a visual model called CHAT triangles. The study aims to analyze the changes and challenges Bobby faced in his mathematics teaching compared to his regular teaching subjects, providing insights into the complexities of teaching in unfamiliar areas.

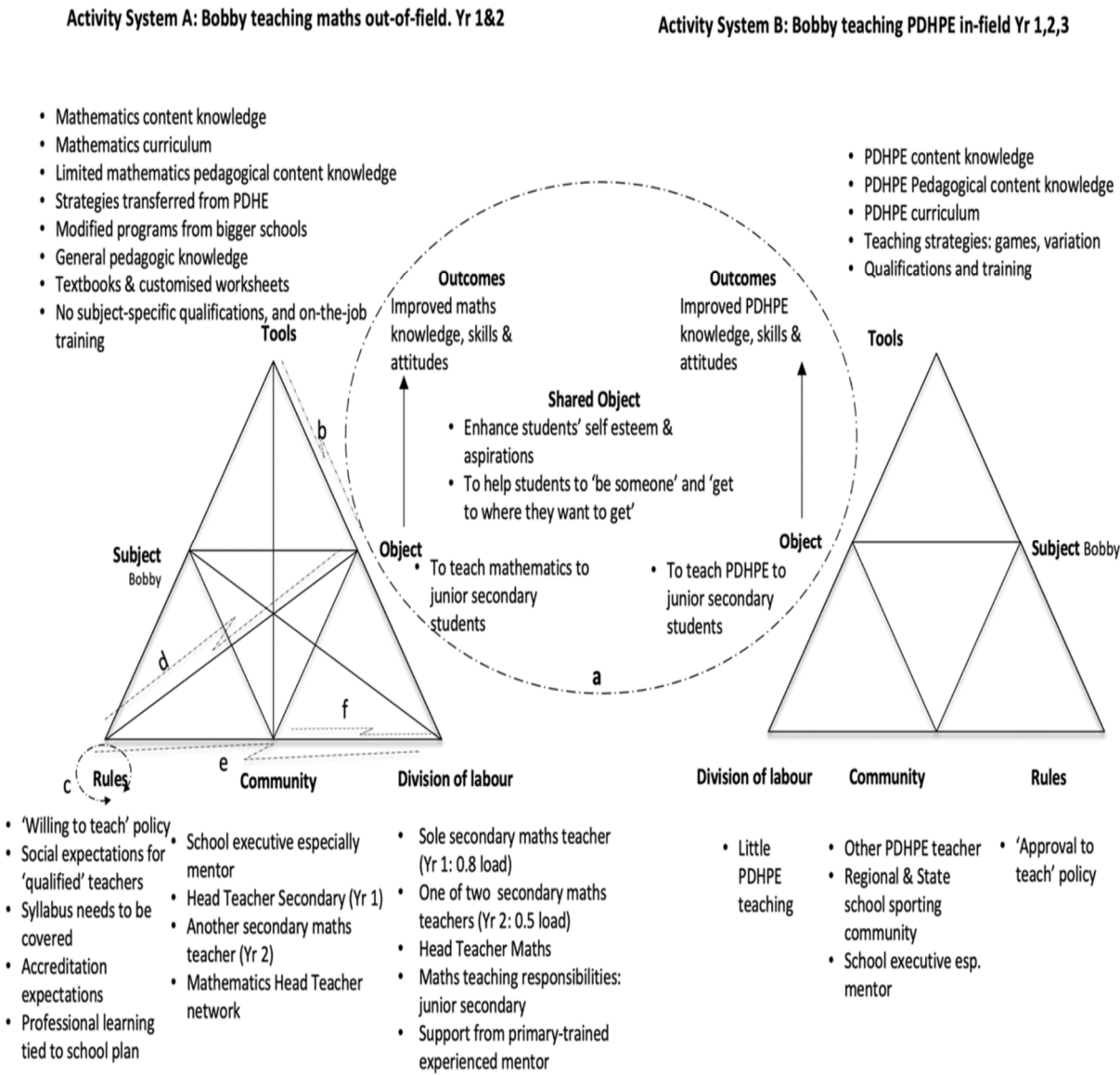


Figure 47

Unsupportive environments[68–70] can hinder teachers' ability to learn new subjects and teaching methods, which negatively affects their skills and confidence. Research [68–70]indicated that having mentors or support for teachers who are teaching outside their expertise can significantly boost their confidence and enjoyment in teaching. To maintain high teaching quality[68–70], it's essential to address teachers' emotional needs and ethical concerns about their students and subjects, ensuring they feel supported in their roles.

3. Open Problems

- The undertaken research [2] described a model that can generate high-quality videos of a specific person dancing by mimicking the movements of another dancer. However, the model has some limitations, such as difficulty accurately representing loose clothing or hair, which can lead to visual errors in the final video. This triggers the need to focus on better detecting poses, using different clothing or lighting in the videos, and reducing visual artifacts caused by textures in loose or wrinkled materials, which is till an open problem.
- This work of [6] has highlighted case studies that show how these creative therapies focus on the strengths of families to aid in their recovery. Although initial research shows positive effects

of these therapies on service members, this has emerged solving the open problem on how to expand these approaches further to support military families and enhance their resilience.

- There is a potential open problem[8] to determine the best types and amounts of participation in performing arts, like music and dance, to achieve specific health benefits. Solving this sophisticated open problem is vital to understand the importance of understanding how the physical intensity of these activities relates to the health outcomes observed
- The work in [9] posed a challenge, called relational analysis, which involves understanding how different social groups interact and connect. It is to be noted that [9] didn't observe students in various settings, like during breaks or in their leisure sports, and instead relied on focus group discussions. Therefore, it is highly suggested to gain a better understanding of how students relate to sports and physical education, future research should include observations beyond just PE classes to capture a fuller picture of their social dynamics.
- An emerging open problems arises from [21], namely, understanding what factors help mathematics teachers effectively relate math concepts to real-life situations. By exploring these factors, researchers can enhance the existing knowledge about how mathematics can be made more relevant and connected to everyday problems. This could improve teaching practices and help students see the value of mathematics in their lives.
- The study [33] found that using Realistic Mathematics Education (RME) significantly improves how well elementary school students learn. However, the research focused only on teaching fractions to fifth-grade students, triggering the provision of a solution to the open problem of exploring RME's effectiveness with different math topics and across various grade levels. This suggested that while RME shows promise, its broader application in education requires further investigation.
- The study[62] had several limitations that should be acknowledged, suggesting several open problems, which need solving. First, the researchers could not keep the outcome assessors unaware of which group the children were in, which could introduce bias. Additionally, they did not measure how much the children enjoyed math before the study, and the multiplication test was too easy, leading to many children scoring at the top (ceiling effects). Lastly, while the juggling-math lessons were longer than the regular math lessons, the researchers lacked detailed information on how well each child engaged with the program.
- The study [64] highlights the critical need to include physical activity in all school lessons, particularly in subjects like maths and English, which tend to be more sedentary. To achieve this, future programs could adopt the Creating Active Schools Framework, which considers various factors like school policies and the involvement of different groups within the school community. Additionally[64], national policies may need to shift focus to balance students' academic success with their physical and mental health.
- To improve teacher competency in primary physical education (PE)[65], it's essential for national and institutional policies to focus on the training and development needs of future teachers. This means that teacher education programs should create more opportunities for pre-service teachers (PSTs) to actively teach and engage in PE, especially if schools are outsourcing PE instruction. Additionally[65], these programs should prepare PSTs to collaborate effectively with a diverse range of professionals in the field to ensure successful teaching practices. These open problems are vital and need more investigations to find solutions.
- The study[67] has addressed some challenges related to behavioral engagement in classrooms that integrate physical activity (PA) into lessons, particularly in the Integrated PA group. Some teachers noted that their classes were restless, which could be linked to issues like conduct problems or difficulties with concentration. To improve engagement, teachers suggested creating permanent teams of students and using reward systems or differentiated activities based on students' abilities, emphasizing the importance of introducing PA from the start of school to normalize it as part of learning.

4. Concluding Remarks and Future Research Pathways

The use of dancing and physical education (PE) as potent and advantageous dynamics to guide innovative mathematics teaching is one of the important issues explored in this study. Therefore, by default, the current study bridges two interdisciplinary fields—dance and physical education—to implement a cohesive method to teaching mathematics. Fundamentally, this article offers a range of materials suitable for academics, researchers, and maths students. On a more positive side, numerous significant open problems are presented to give the research community something to think about in order to guide future developments in modern mathematics education. Future research pathways include the provision of possible solution to the proposed open problems, as well as following an exploratory approach to finding other innovative mathematics teaching to all.

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